1. Sketch the amplitude spectrum of each of the following signals:

(a) $2\cos^2(2000\pi t)$

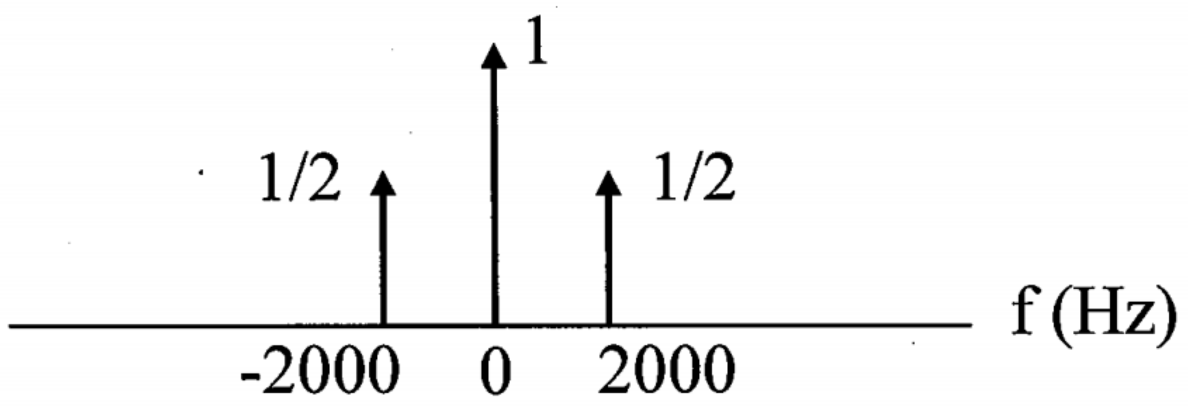
(b) rect(2000t) filtered by an ideal lowpass filter with 4KHz bandwidth

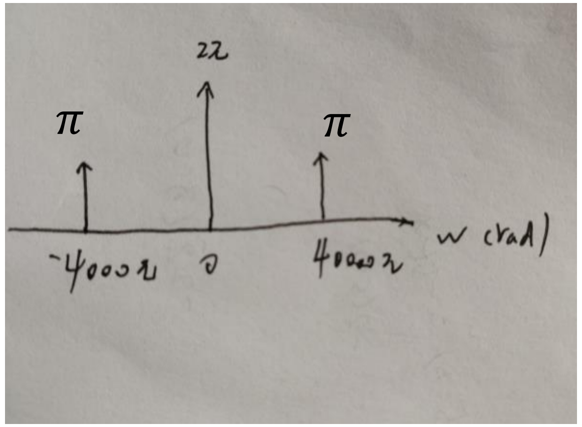
Solution:

(a) $2cos^2(2000\pi t) = 2 * \frac{1+\cos(4000\pi t)}{2} = 1 + \cos(4000\pi t)$

$$1 + \cos(4000\pi t) \xrightarrow{\text{F.T}} \delta(f) + \frac{\delta(f-2000)+\delta(f+2000)}{2}$$

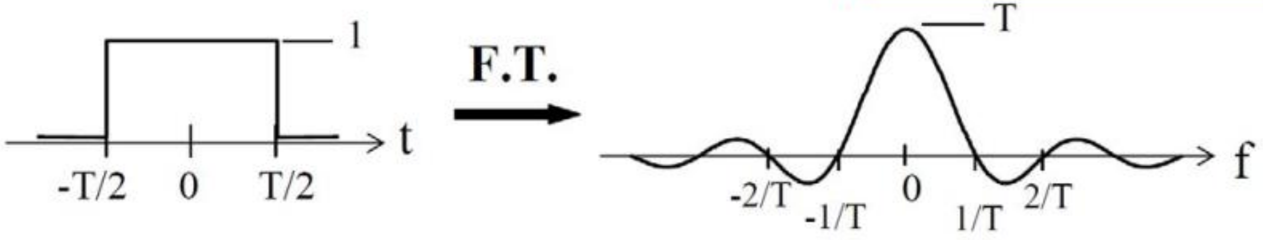
$$2\pi\delta(f) + \pi\delta(f - 4000\pi) + \pi\delta(f + 4000\pi)$$

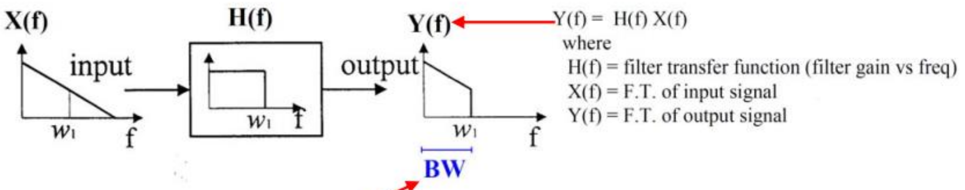




❑ F.T. of $\text{rect}\left(\frac{t}{T}\right) = T\,\text{sinc}(f\,T)$. Note $\boxed{\text{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}}$

$$A\,\text{sinc}(2Wt) \rightleftharpoons \frac{A}{2W}\,\text{rect}\left(\frac{f}{2W}\right)$$



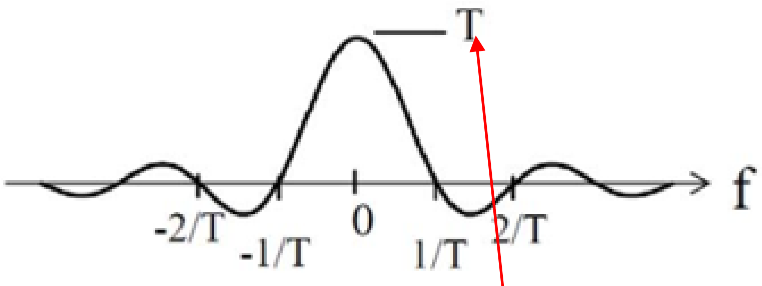


Bandwidth (BW) is defined as range of **positive** frequency occupied by a spectrum

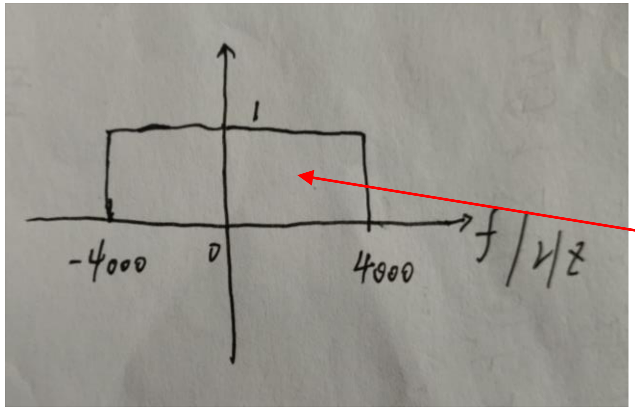
Solution:

(b)

$$rect(2000t) \xrightarrow{\text{F.T}} \frac{1}{2000}\,sinc\left(f * \frac{1}{2000}\right)$$

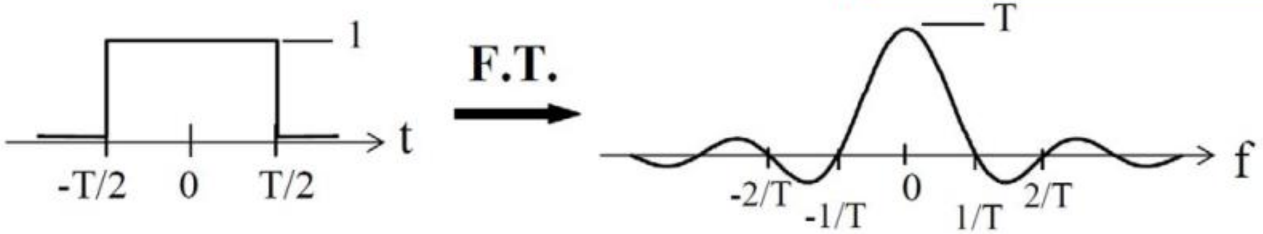


$\frac{1}{2000}$

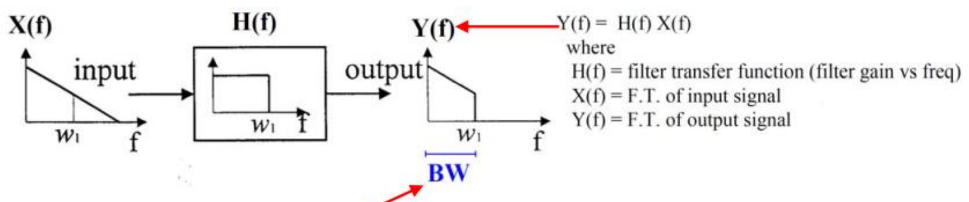


H_{Lp}

- F.T. of $\mathrm{rect}\left(\frac{t}{T}\right) = T\,\mathrm{sinc}(f\,T)$. Note $\boxed{\mathrm{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}}$



$$A\,\mathrm{sinc}(2Wt) \rightleftharpoons \frac{A}{2W}\,\mathrm{rect}\left(\frac{f}{2W}\right)$$



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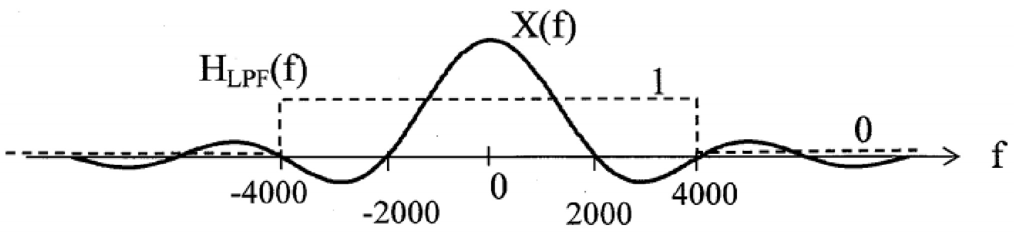
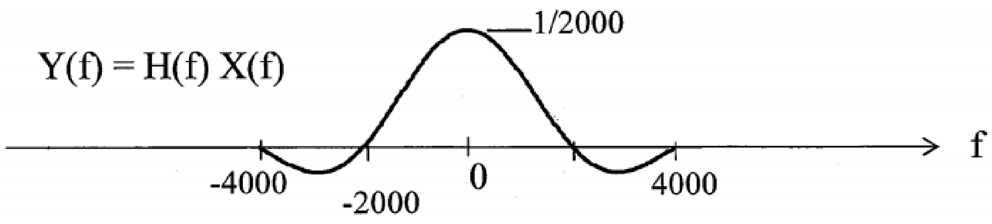
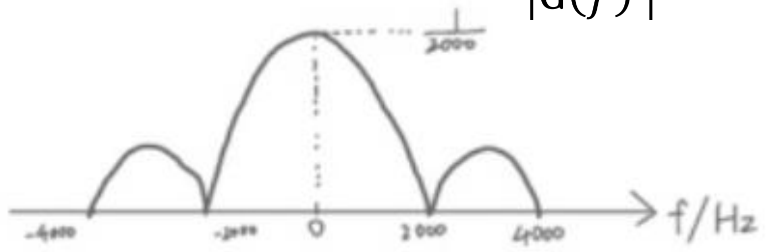
$$rect(2000t) \xrightarrow{\text{F.T}} \frac{1}{2000}\,sinc\left(f * \frac{1}{2000}\right)$$





$|G(f)|$



2. Determine the power of the signal s(t) = 6cos(200πt) + 8 sin(200πt) using the time-domain and frequency-domain formulas. Do not use superposition of power.

<u>Note</u>: PSD of $A\cos(2\pi f_0 t + \theta)$ is $\frac{A^2}{4}\left[\delta(f - f_0) + \delta(f + f_0)\right]$, where θ is any phase angle

and $\delta(f)$ is a delta dirac function in f.

✓ Time domain:

$$\overline{v^2(t)} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |v(t)|^2 \, dt$$

✓ Frequency domain:

$$\overline{v^2(t)} = \int_{-\infty}^{\infty} S_v(f)\, df$$

Solution:

(1) Time domain

$$s(t) = 6\cos(200\pi t) + 8\sin(200\pi t)$$

$$= 10\cos(\boxed{200\pi t - \phi}) \quad (\phi = \tan^{-1}(\tfrac{4}{3}))$$

$$\overline{s^2(t)} = \frac{100}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} cos^2(x)\, dt = \frac{100}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{1+\cos(2x)}{2}\, dt = \frac{100}{T_0} * \frac{T_0}{2} = 50$$

(2) Frequency domain

PSD of s(t) $\frac{100}{4}\delta(f - 100) + \delta(f + 100)$

$$\overline{s^2(t)} = \frac{100}{4} * 2 = 50$$

3. Determine and compare the bandwidths of the following signals:

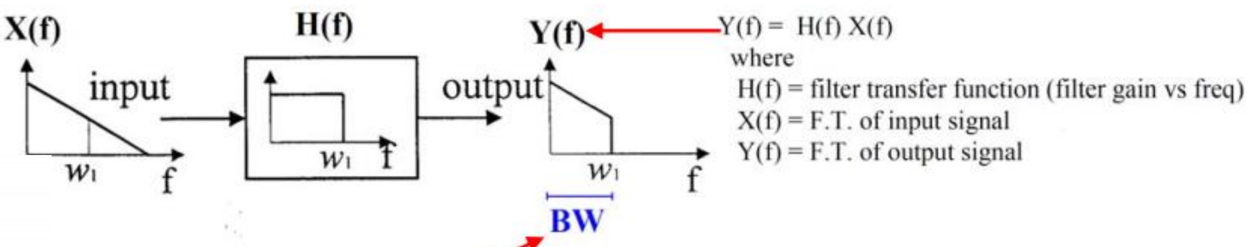
(a) $\text{sinc}\left(\dfrac{t}{50}\right)$

(b) $\text{sinc}\left(\dfrac{t-4}{50}\right)$

(c) $\text{sinc}\left(\dfrac{t}{50}\right) - 4$

(d) $\text{sinc}\left(\dfrac{t}{50}\right)\sin(5000\pi t)$

$$A\,\text{sinc}(2Wt) \rightleftharpoons \frac{A}{2W}\,\text{rect}\left(\frac{f}{2W}\right)$$



Bandwidth (BW) is defined as range of **positive** frequency occupied by a spectrum

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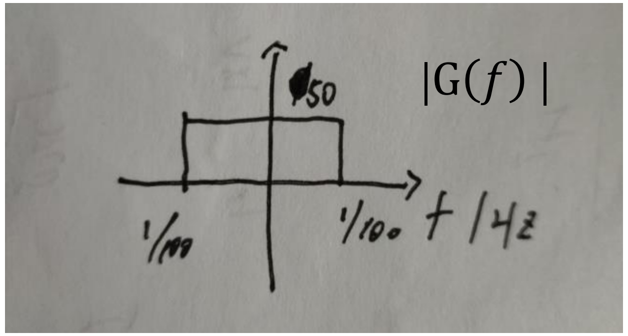
Solution:

$$sinc\left(\frac{t}{50}\right) \xrightarrow{\text{F.T}} 50rect\left(\frac{f}{\frac{1}{50}}\right)$$

$$\text{rect}(t) = \begin{cases} 1, & -\dfrac{1}{2} < t < \dfrac{1}{2} \\ 0, & |t| \geq \dfrac{1}{2} \end{cases} \quad (2.7)$$

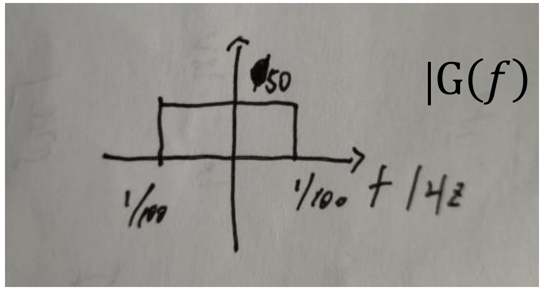
a)



BW = 0.01 HZ.

Solution:

b) $sinc(\frac{t-4}{50}) \xrightarrow{\text{F.T}} 50rect(\frac{f}{\frac{1}{50}}) * e^{-j2\pi f*4}$



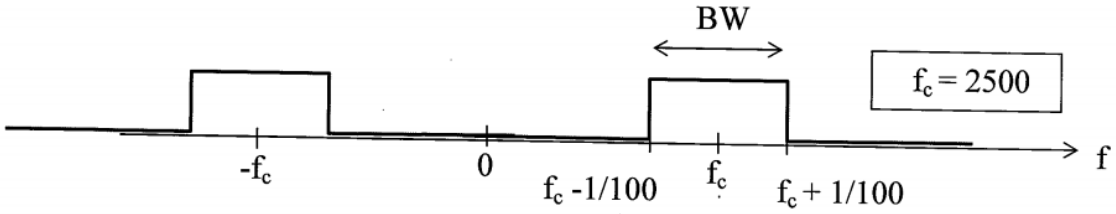
$|G(f)|$

BW = 0.01 HZ.

c)

$sinc(\frac{t}{50})$ -4 $\xrightarrow{\text{F.T}}$ $50rect(\frac{f}{\frac{1}{50}})$-$4\delta(f)$ BW = 0.01HZ.

d) $sinc(\frac{t}{50}) * \sin(5000\pi t) \xrightarrow{\text{F.T}} X(f) \otimes Y(f)$



BW = 0.02 HZ.