EE206: Communications Principles Tutorial

Assignment 2

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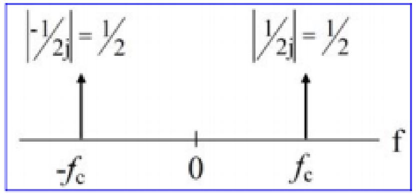
Amplitude Spectrum

- In general, the Fourier Transform $V(f)$ is a complex function of frequency f, so that it can be rewritten as

$$V(f) = |V(f)| \exp[j\,\theta(f)]$$

- Fourier Transform has 2 components:
  - Amplitude spectrum $|V(f)|$
  - Phase spectrum $\theta(f)$

❑ F.T. of $\sin(2\pi f_c t) = \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$



❑ F.T. of $\mathrm{rect}\left(\frac{t}{T}\right) = T\,\mathrm{sinc}(f\,T)$. What is the amplitude spectrum?

✓ Usually, spectrum refers to amplitude spectrum unless otherwise stated.

In general, the Fourier transform $G(f)$ is a complex function of frequency f, so that we may express it in the form

$$G(f) = |G(f)| \exp[j\theta(f)] \tag{2.6}$$

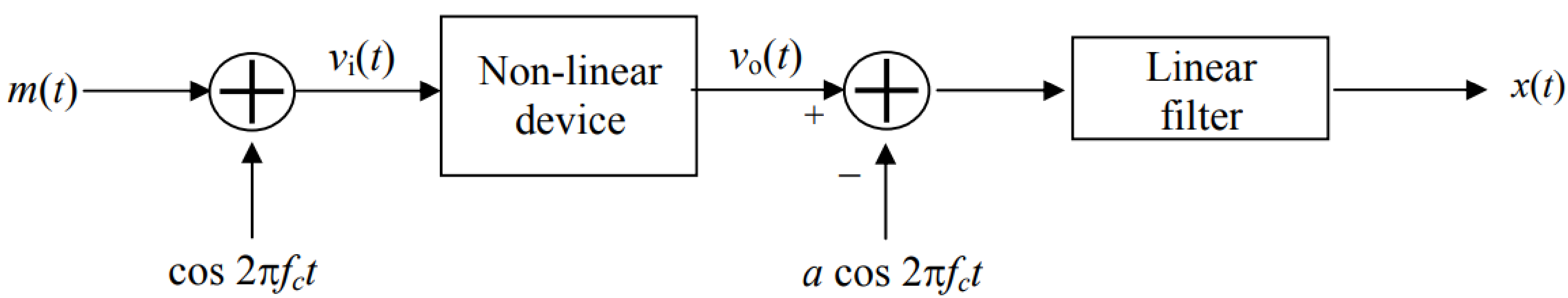
where $|G(f)|$ is called the *continuous amplitude spectrum* of $g(t)$, and $\theta(f)$ is called the *continuous phase spectrum* of $g(t)$. Here, the spectrum is referred to as a *continuous spectrum* because both the amplitude and phase of $G(f)$ are defined for all frequencies.

1. The message signal $m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$ is input to the system shown below to generate a DSBSC-AM signal $x(t)$. Assume that $v_o(t) = a\, v_i(t) + b\, v_i^2(t)$ where a and b are constants, and the carrier frequency $f_c >> 70\text{Hz}$.

   (a) Sketch the amplitude spectrum of the filter input

   (b) Determine the center frequency and bandwidth of the filter in this modulator

   (c) Determine the minimum value of f_c permitted for this modulator



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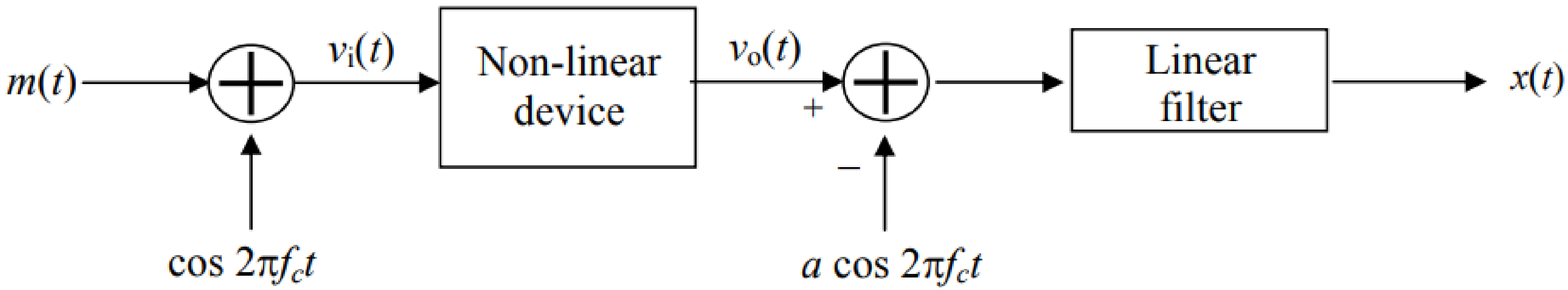


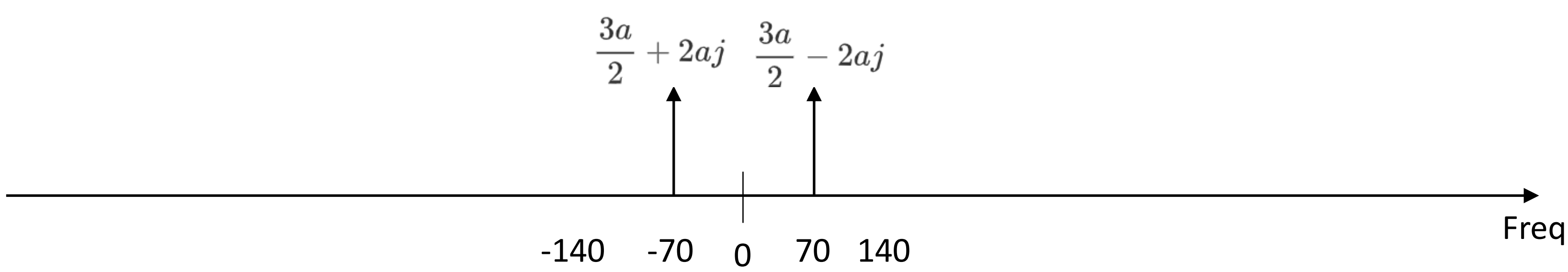
(a) Sketch the amplitude spectrum of the filter input

$$
\begin{aligned}
m(t) &= 3\cos(2\pi 70t) + 4\sin(2\pi 70t) \\
v_i(t) &= m(t) + \cos(2\pi f_c t) \\
v_o(t) &= a v_i(t) + b v_i^2(t) \\
&= a[m(t) + \cos(2\pi f_c t)] + b[m(t) + \cos(2\pi f_c t)]^2 \\
&= am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + a\cos(2\pi f_c t) + b\cos^2(2\pi f_c t) \\
&= am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + a\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t) \\
\text{filter input} &= v_o(t) - a\cos(2\pi f_c t) \\
&= am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)
\end{aligned}
$$

$$m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$$

$$\text{filter input} = am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)$$

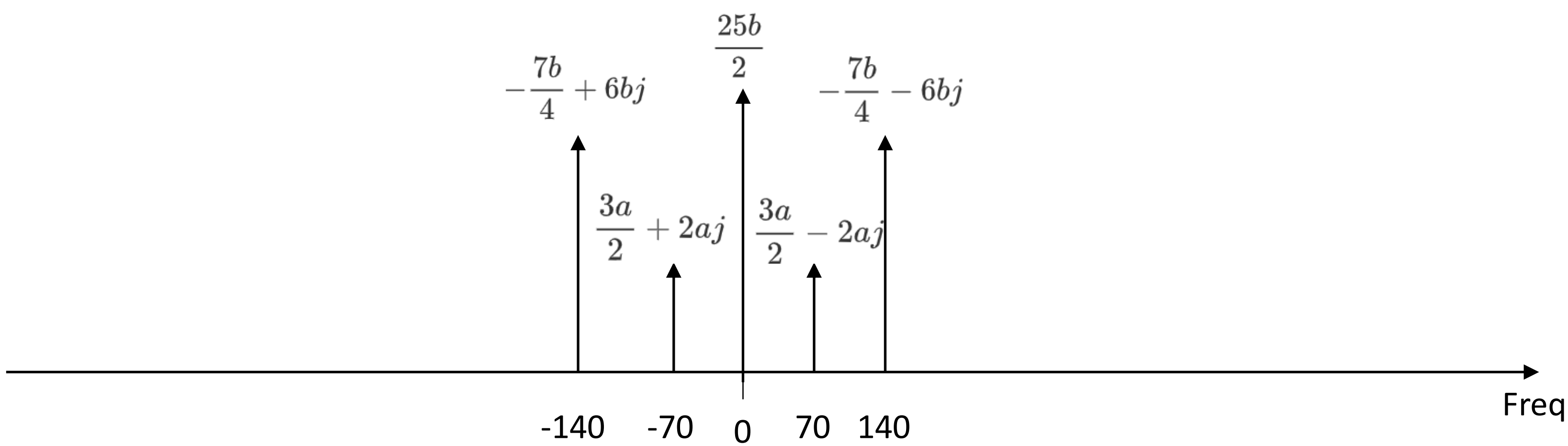
$$am(t) = 3a\cos(2\pi 70t) + 4a\sin(2\pi 70t)$$
$$= (\frac{3a}{2} - 2aj)e^{j2\pi 70t} + (\frac{3a}{2} + 2aj)e^{-j2\pi 70t}$$



$$m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$$

$$\text{filter input} = am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)$$

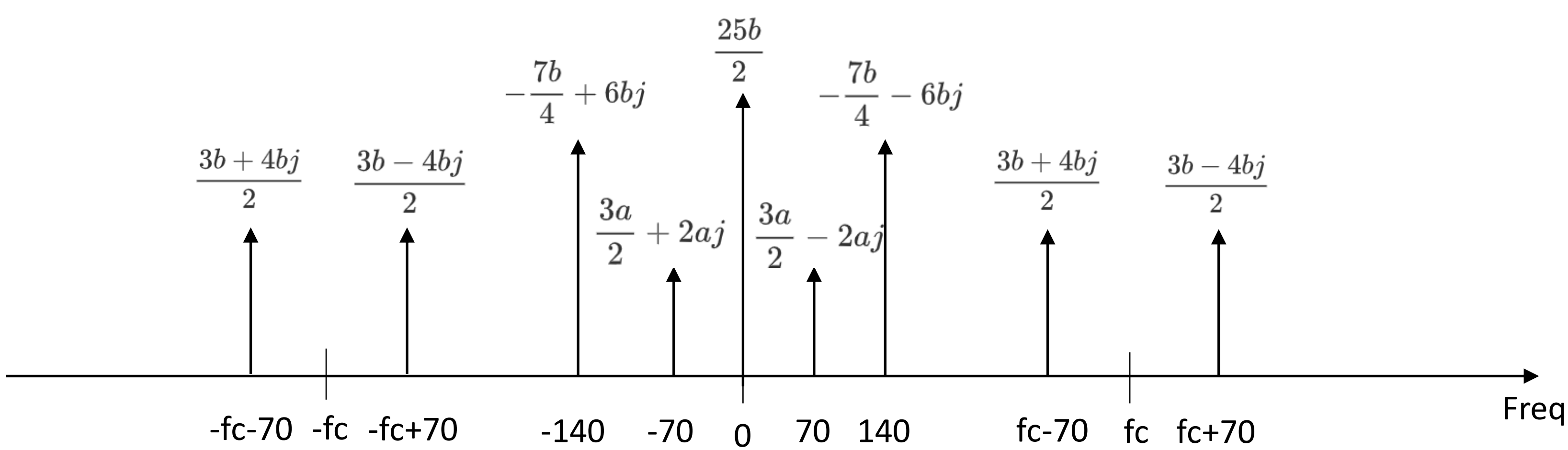
$$\begin{aligned}
bm^2(t) &= 9b\cos^2(2\pi 70t) + 24b\cos(2\pi 70t)\sin(2\pi 70t) + 16b\sin^2(2\pi 70t) \\
&= \frac{9b}{2} + \frac{9b}{2}\cos(2\pi 140t) + 12b\sin(2\pi 140t) + 8b - 8b\cos(2\pi 140t) \\
&= \frac{25b}{2} - \frac{7b}{2}\cos(2\pi 140t) + 12b\sin(2\pi 140t) \\
&= \frac{25b}{2} - (\frac{7b}{4} + 6bj)e^{j2\pi 140t} - (\frac{7b}{4} - 6bj)e^{-j2\pi 140t}
\end{aligned}$$



$$m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$$

$$\text{filter input} = am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)$$

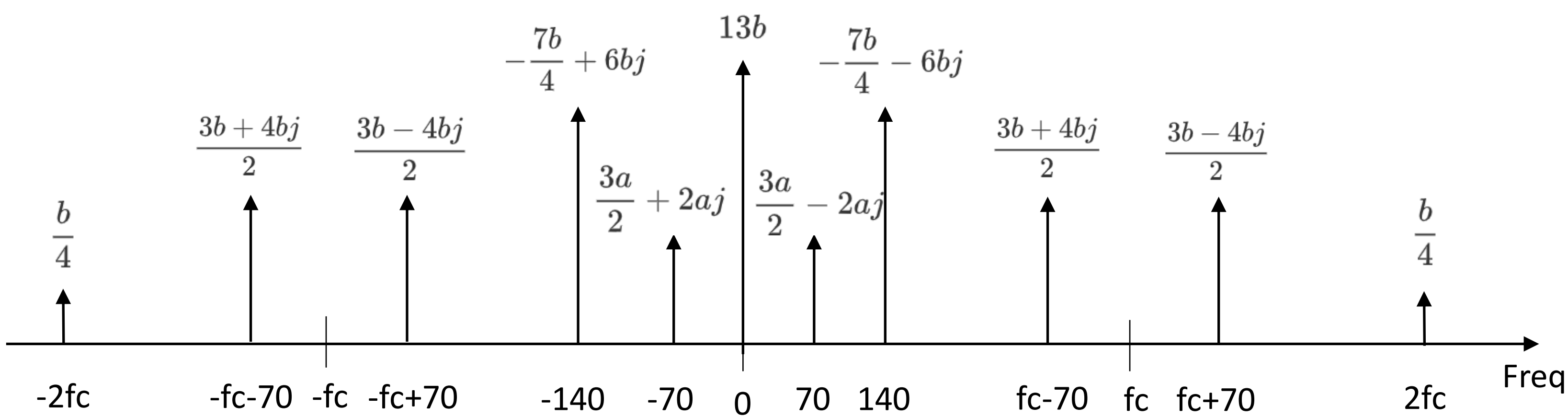
$$\begin{aligned}
2bm(t)\cos(2\pi f_c t) &= 2b(3\cos(2\pi 70t) + 4\sin(2\pi 70t))\cos(2\pi f_c t) \\
&= ((3b - 4bj)e^{j2\pi 70t} + (3b + 4bj)e^{-j2\pi 70t})(\frac{1}{2}e^{j2\pi f_c t} + \frac{1}{2}e^{-j2\pi f_c t}) \\
&= \frac{3b - 4bj}{2}e^{j2\pi(f_c+70)t} + \frac{3b - 4bj}{2}e^{j2\pi(-f_c+70)t} + \frac{3b + 4bj}{2}e^{j2\pi(f_c-70)t} + \frac{3b + 4bj}{2}e^{j2\pi(-f_c-70)t}
\end{aligned}$$



$$m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$$

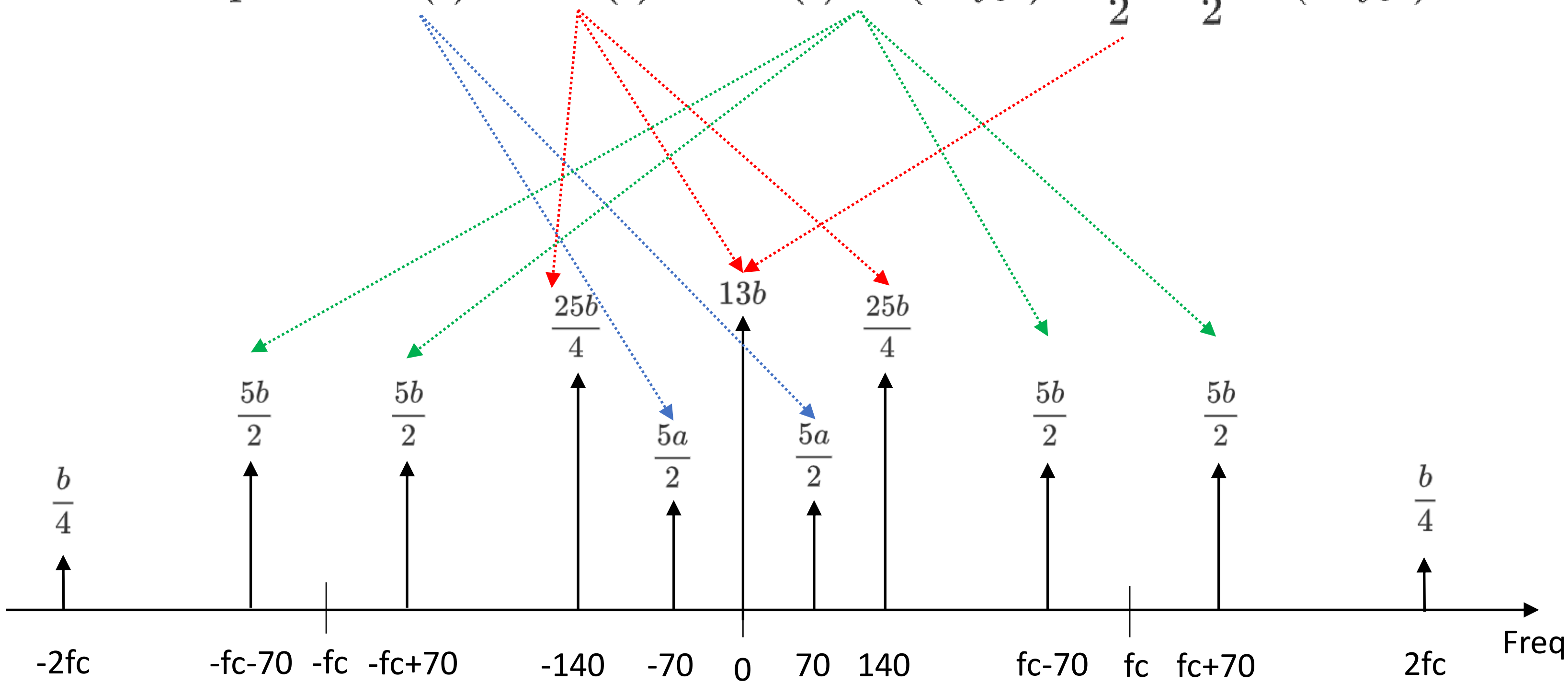
$$\text{filter input} = am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)$$

$$\frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t) = \frac{b}{2} + \frac{b}{4}e^{j2\pi 2 f_c t} + \frac{b}{4}e^{-j2\pi 2 f_c t}$$

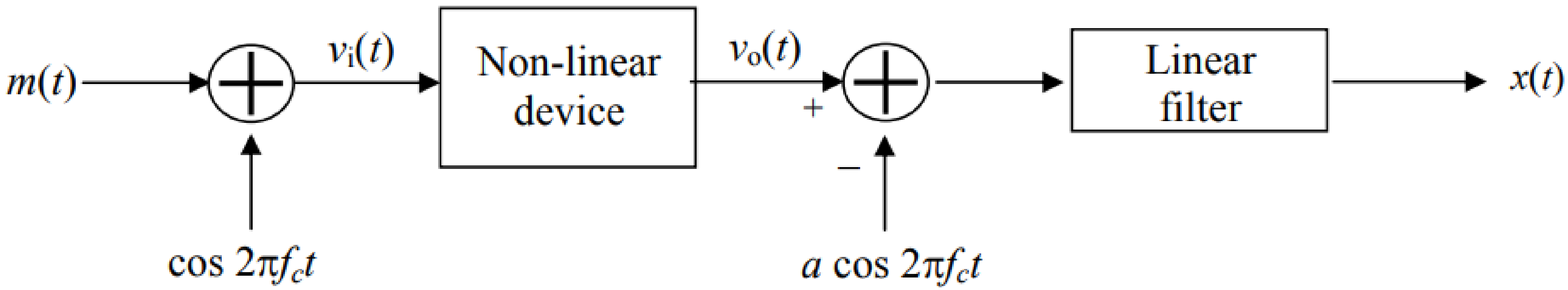


$$m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$$
$$= 5\cos(2\pi 70t - \theta)$$
$$\theta = \tan^{-1}\frac{4}{3}$$

$$\text{filter input} = am(t) + bm^2(t) + 2bm(t)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)$$



1. The message signal $m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$ is input to the system shown below to generate a DSBSC-AM signal $x(t)$. Assume that $v_o(t) = a\,v_i(t) + b\,v_i^2(t)$ where a and b are constants, and the carrier frequency $f_c >> 70$Hz.



(b) Determine the center frequency and bandwidth of the filter in this modulator

$$
\begin{aligned}
m(t) &= 3\cos(2\pi 70t) + 4\sin(2\pi 70t) \\
&= 5\cos(2\pi 70t - \theta)
\end{aligned}
$$

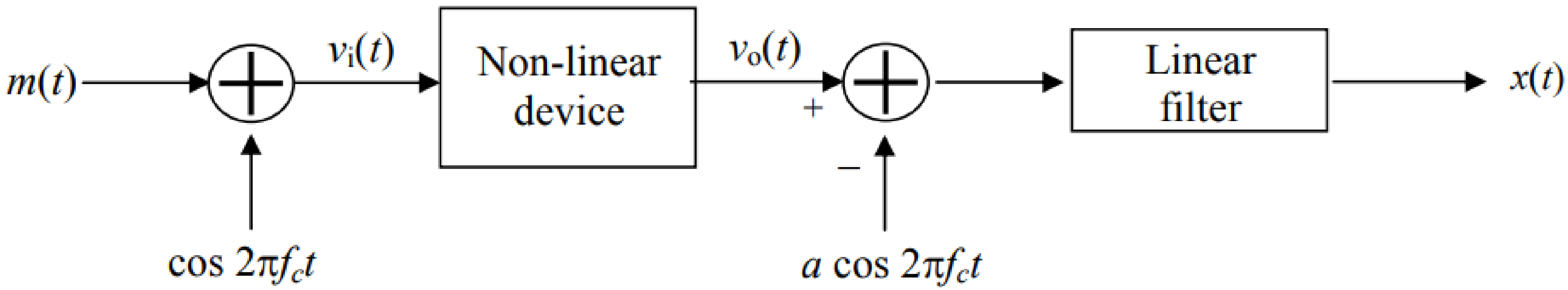
$$\theta = \tan^{-1}\frac{4}{3}$$

Bandwidth (BW) is defined as range of **positive** frequency occupied by a spectrum

The DSBSC-AM signal should be $m(t)\cos(2\pi f_c t)$.

In order to separate it from the other unwanted components, the center freq $= f_c$, and bandwidth $= 140Hz$.

1. The message signal $m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$ is input to the system shown below to generate a DSBSC-AM signal $x(t)$. Assume that $v_o(t) = a\, v_i(t) + b\, v_i^2(t)$ where a and b are constants, and the carrier frequency $f_c >> 70\text{Hz}$.

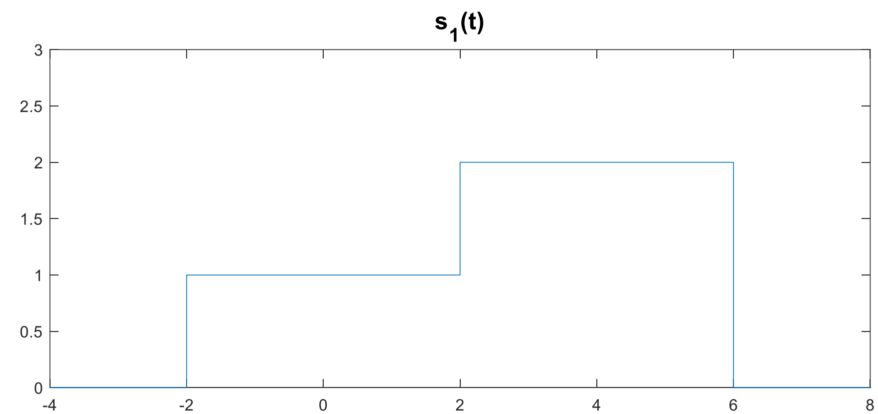


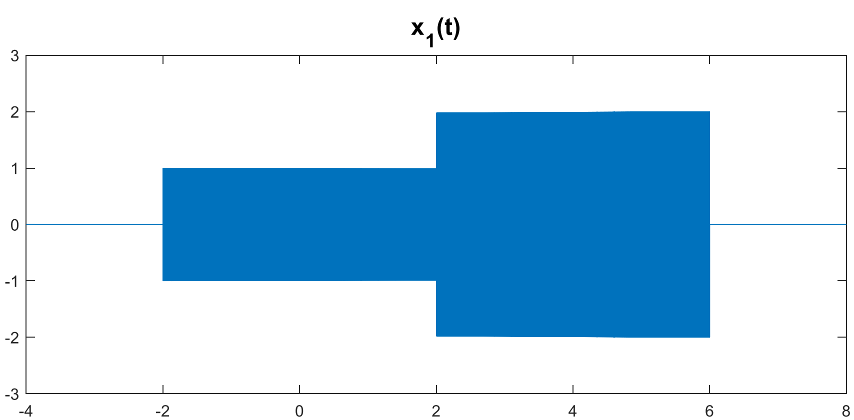
(c) Determine the minimum value of f_c permitted for this modulator

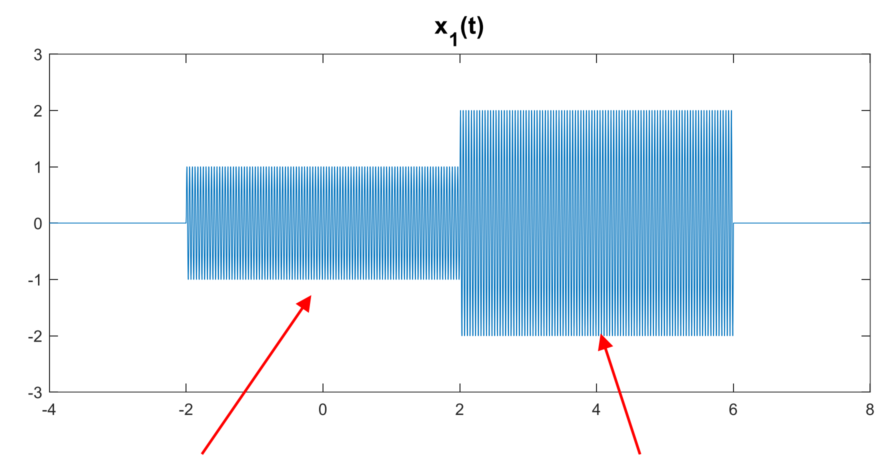
$$f_c - 70 > 140$$
$$\rightarrow f_c > 210Hz$$

2. A suppressed-carrier AM signal $x_1(t)$ is generated by modulating $s_1(t) = rect\left(\frac{t}{4}\right) + 2rect\left(\frac{t-4}{4}\right)$ with $\sin(1000\pi t)$. Sketch the time waveform of $x_1(t)$.

$$x_1(t) = s_1(t)\sin(1000\pi t)$$







$500 \times 4 = 2000$ cycles $\quad$ $500 \times 4 = 2000$ cycles

3. A DSBSC-AM signal is

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

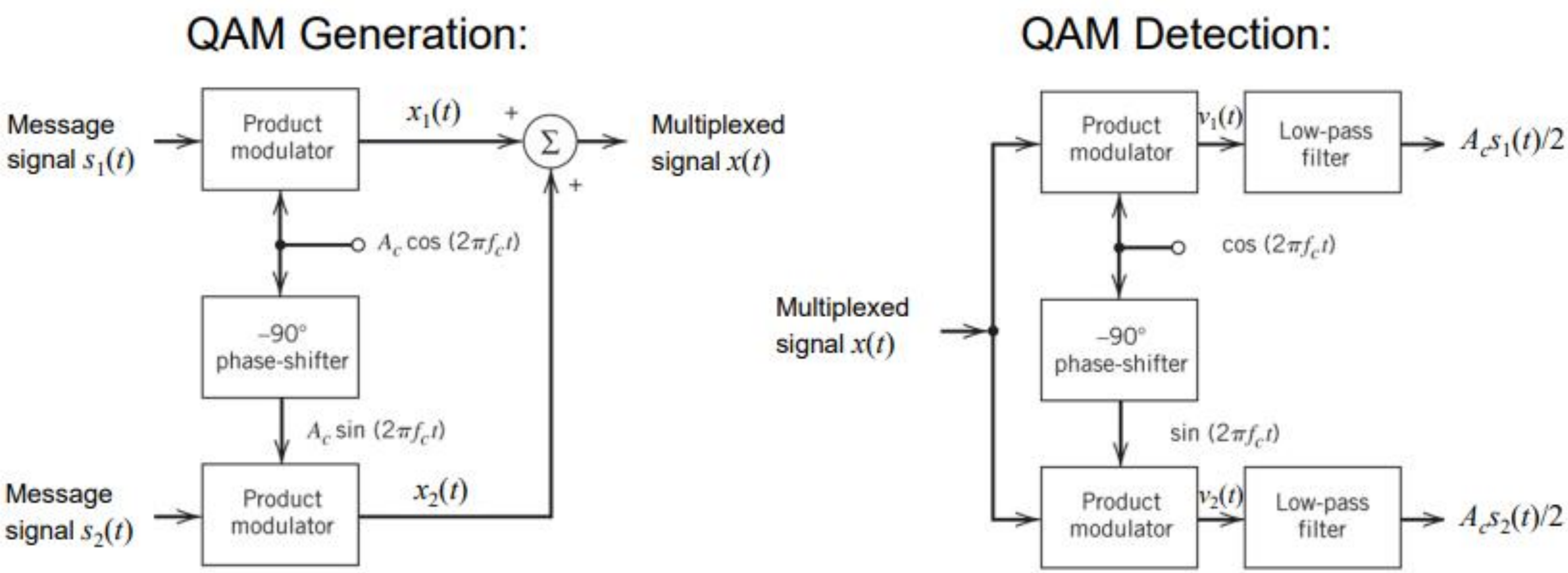
(a) Sketch the amplitude spectrum of x(t) to deduce the carrier frequency in x(t)

carrier frequency $f_c = 100Hz$.

3. A DSBSC-AM signal is

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

(b) Given that x(t) was generated using a sine carrier signal with phase 0, demodulate x(t).



3. A DSBSC-AM signal is

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

(b) Given that x(t) was generated using a sine carrier signal with phase 0, demodulate x(t).

QAM Generation:

$x_1(t) = A_c\, s_1(t) \cos 2\pi f_c t$ = in-phase (I) signal

$x_2(t) = A_c\, s_2(t) \sin 2\pi f_c t$ = quadrature (Q) signal

$\therefore\ x(t) = x_1(t) + x_2(t) = A_c\, s_1(t) \cos 2\pi f_c t + A_c\, s_2(t) \sin 2\pi f_c t$

where $s_1(t)$ and $s_2(t)$ are two different message signals.

QAM Detection:

$$v_1(t) = x(t) \cos 2\pi f_c t = A_c\, s_1(t) \cos^2 2\pi f_c t + A_c\, s_2(t) \sin 2\pi f_c t \cos 2\pi f_c t$$
$$= \frac{1}{2} A_c\, s_1(t) + \frac{1}{2} A_c\, s_1(t) \cos 4\pi f_c t + \frac{1}{2} A_c\, s_2(t) \sin 4\pi f_c t$$

Similarly, we get

$$v_2(t) = x(t) \sin 2\pi f_c t$$
$$= \frac{1}{2} A_c\, s_2(t) - \frac{1}{2} A_c\, s_2(t) \cos 4\pi f_c t + \frac{1}{2} A_c\, s_1(t) \sin 4\pi f_c t$$

Thus, $\frac{1}{2} A_c\, s_1(t)$ and $\frac{1}{2} A_c\, s_2(t)$ can be obtained after the lowpass filters.

Hence, the two message signals $s_1(t)$ and $s_2(t)$ can be recovered.

3. A DSBSC-AM signal is

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

(b) Given that x(t) was generated using a sine carrier signal with phase 0, demodulate x(t).

$$\begin{aligned}(3\sin 180\pi t + 3\sin 220\pi t)\sin 200\pi t &= 3(\sin 180\pi t \sin 200\pi t + \sin 220\pi t \sin 200\pi t) \\ &= \frac{3}{2}(\cos(180-200)\pi t - \cos(180+200)\pi t + \cos(220-200)\pi t - \cos(220+200)\pi t) \\ &= \frac{3}{2}(2\cos 20\pi t - \cos 380\pi t - \cos 420\pi t)\end{aligned}$$

after lowpass filter ($10Hz$ bandwidth)

$$\text{output} = 3\cos 20\pi t$$