Fig. 1

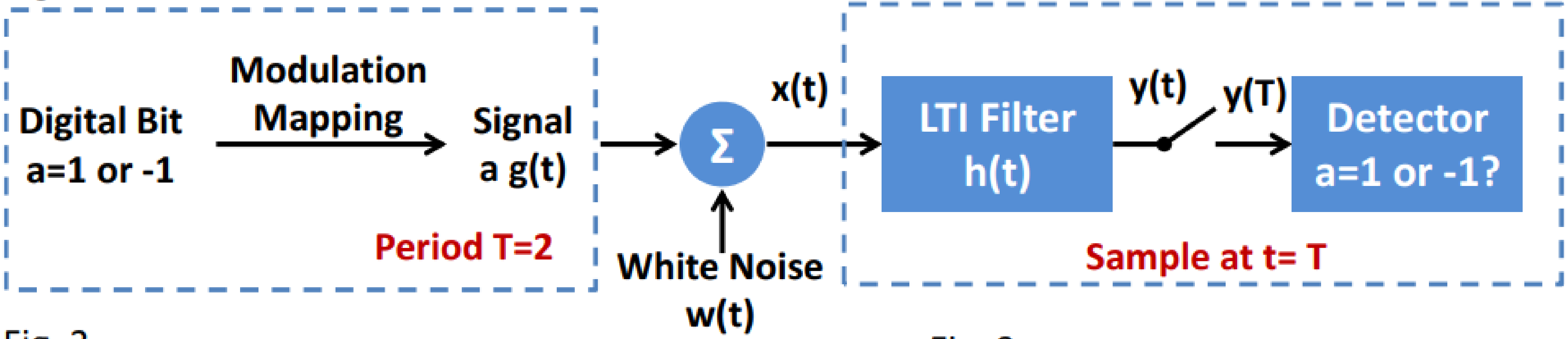


Fig. 2

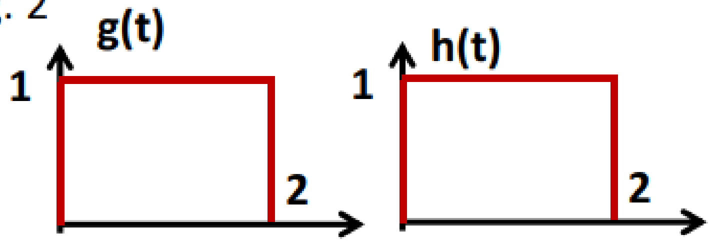
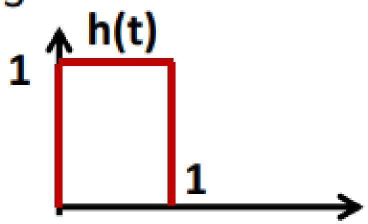


Fig. 3



- D4.1

Consider the baseband transceiver in Fig. 1, where g(t) and h(t) are given by Fig. 2,

(a) Please sketch the PSD of noise in y(t).

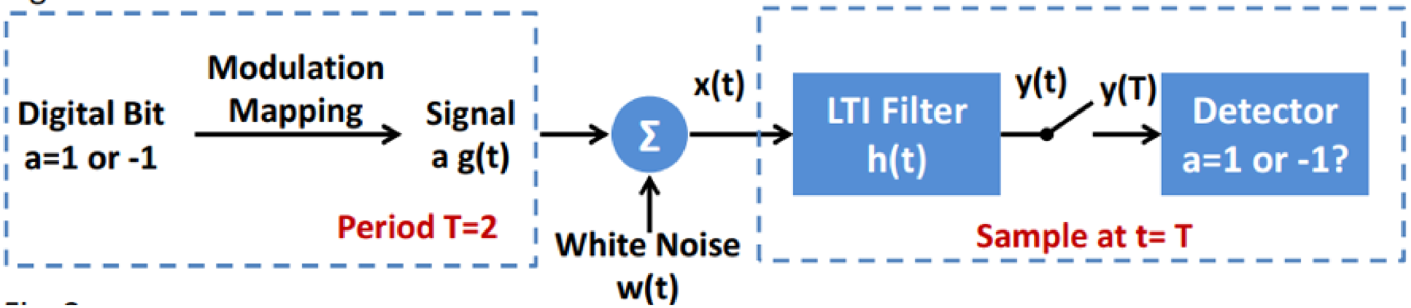
(b) What is the signal power in y(T)? What is the noise power in y(T)? What is the SNR of y(T)?

(c) If h(t) is given by Fig. 3, what is your answer of question (b)?

(d) Compare the SNR of question (b) and (c), which impulse response h(t) is better for receiver?

Solution：

Fig. 1

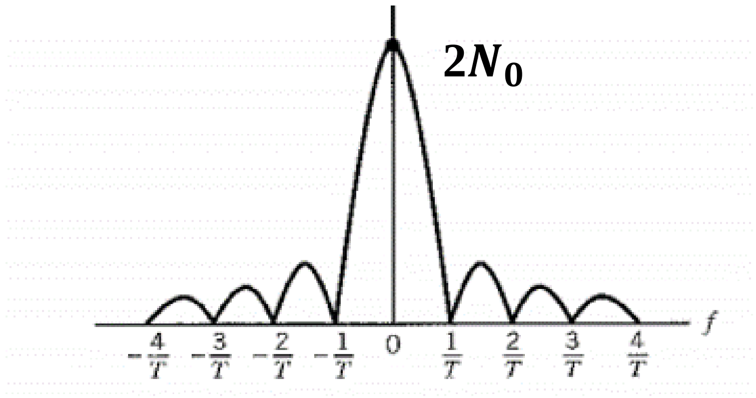


$x(t) = ag(t) + w(t)$

signal noise

$y(t) = x(t)\otimes h(t) = (ag(t) + w(t))\otimes h(t) = \boxed{ag(t)\otimes h(t)} + \boxed{w(t)\otimes h(t)}$

a) PSD of noise in y(t) = $\frac{N_0|H(f)|^2}{2}$

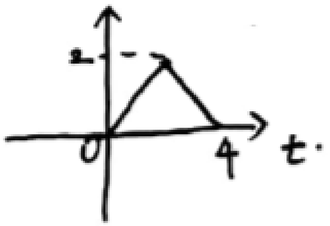


T = 2

b) $y(T) = ag(t)\otimes h(t)|_{t=T} + w(t)\otimes h(t)\,|_{t=T} = g_0(T) + n(T)$

Signal power $= |ag(t)\otimes h(t)|_{t=T}|^2 = |g_0(T)|^2$

$g(t)\otimes h(t)$ $\longrightarrow$



Signal power = $2^2 = 4$

Noise power = $E\left(n^2(T)\right) = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df = \frac{N_0}{2}\int_0^2 |h(t)|^2 dt = N_0$

SNR = $\frac{4}{N_0}$

Solution:

c) $y(T) = \ ag(t)\otimes h(t)|_{t=T} + w(t)\otimes h(t)\ |_{t=T} = g_0(T) + n(T)$

Signal power $= |ag(t)\otimes h(t)|_{t=T}|^2 = |g_0(T)|^2$

Signal power $= 1^2 = 1$

Noise power $= E\left(n^2(T)\right) = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df = \frac{N_0}{2}\int_0^1 |h(t)|^2 dt = \frac{N_0}{2}$

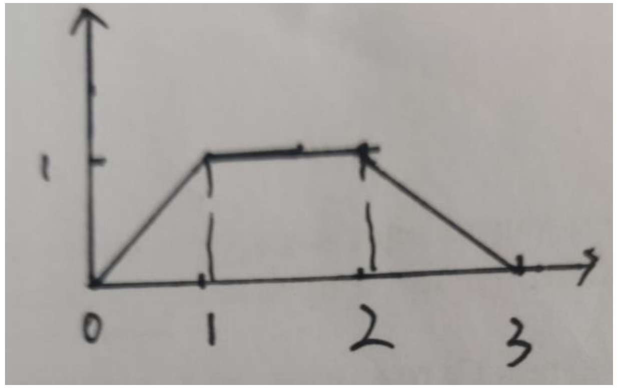
SNR $= \frac{1}{\frac{N_0}{2}} = \frac{2}{N_0}$

d)

$SNR_b > \ SNR_c$

The impulse response h(t) in b is better

$g(t)\otimes h(t)$

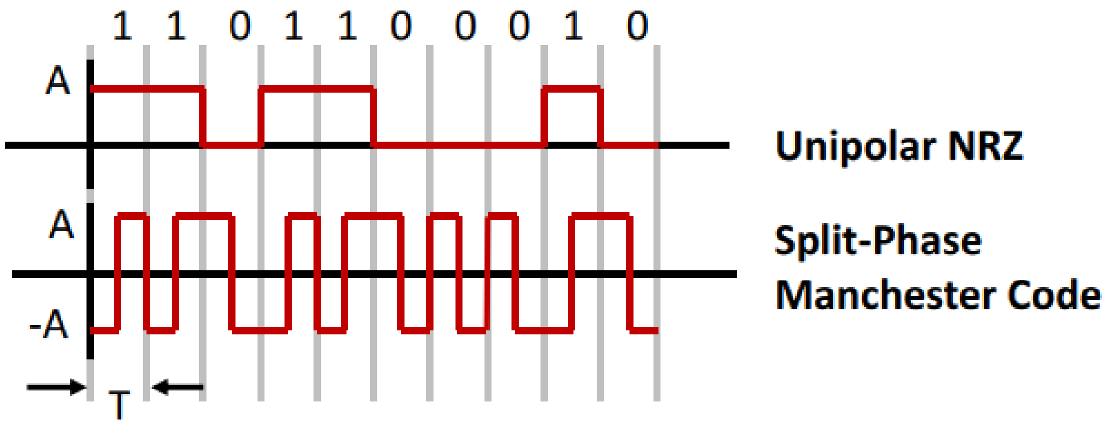


Homework #D5

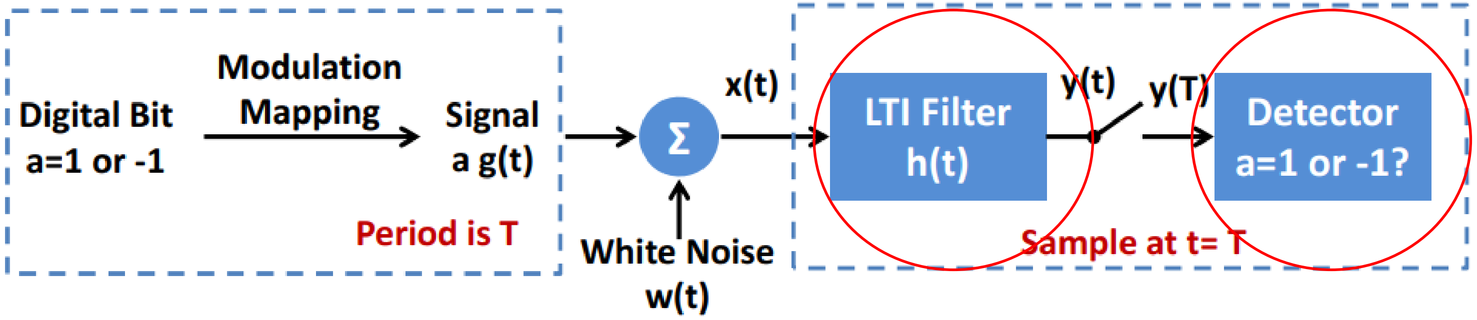
- D5.1

(a) If the Unipolar NRZ code as follows is used at the transmitter, how to design the receiver? What is the BER?

(b) If the Split-Phase Manchester Code as follows is used at the transmitter, how to design the receiver? What is the BER?



Solution:



为了实现最佳接收,接收端需要确定:

1. 滤波器 $h(t)$。
2. 抽样判决,判决界限。

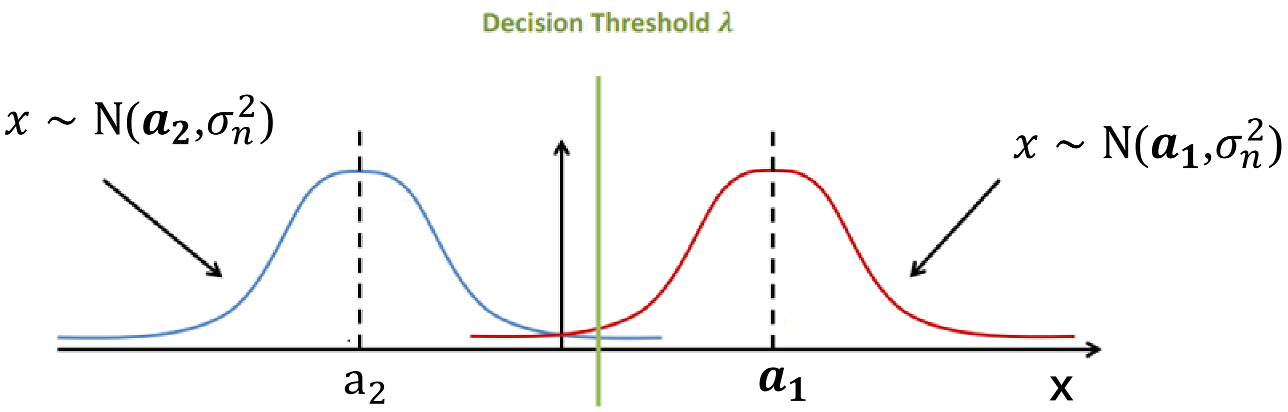
1. 匹配滤波器

$h(t) = kg(T - t)$ 推导过程见Digital-5 page22

2. 抽样判决,判决界限。

$$y(t) = x(t) \otimes h(t) = (ag(t) + w(t)) \otimes h(t) = ag(t) \otimes h(t) + w(t) \otimes h(t) = y_s(t) + n(t)$$

抽样处理后信号 $\dfrac{y(T)}{kE} = a + \dfrac{n(T)}{kE}$



$$P_e = P(a=1)P(error|a=1) + P(a=-1)P(error|a=-1)$$

最佳门限 λ，即令$\frac{\partial P_e}{\partial \lambda}$ =0

$$P_e = P(a=1)P(error|a=1) + P(a=0)P(error|a=0)$$

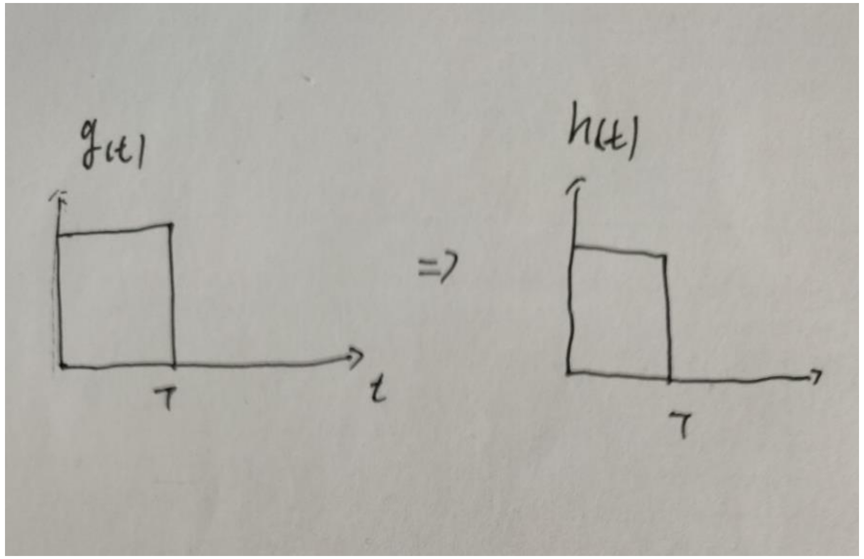
$$= P(a=1)\int_{-\infty}^{\lambda} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(x-a_1)^2}{2\sigma_n{}^2}}\, dx + P(a=0)\int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(x-a_2)^2}{2\sigma_n{}^2}}\, \mathrm{d}x$$

$$\lambda = \frac{\mathrm{a}_1+\mathrm{a}_2}{2} + \frac{\sigma_n^2}{\mathrm{a}_1-\mathrm{a}_2}\ln\left(\frac{P(a=0)}{P(a=1)}\right)$$

Unipolar NRZ code

λ = 0.5

$$P_e = P(a=1)P(error|a=1) + P(a=-1)P(error|a=-1)$$



抽样值 r = $\frac{y(T)}{kE} = a + \frac{n(T)}{KE} \sim$ N(a, $\frac{N_0}{2E}$)

$P(a=1) = P(a=-1) = \frac{1}{2}$

$P(error|a=1) = P(error|a=-1)$

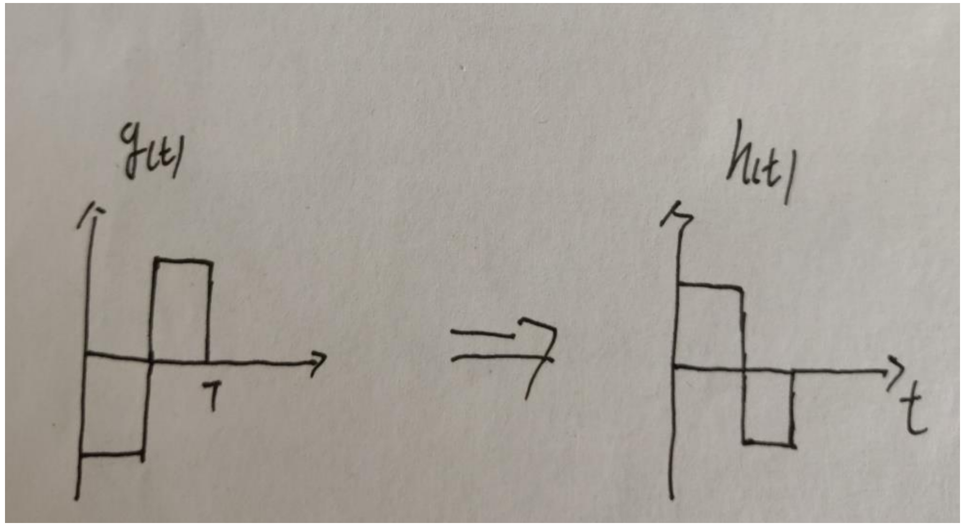
$$P_e = P(error|a=0) = \int_{\frac{1}{2}}^{\infty} \frac{1}{\sqrt{\pi N_0/E}} e^{-\frac{x^2}{N_0/E}} dx = \int_{\frac{1}{2}\sqrt{2E/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \quad (z = x\sqrt{\frac{2E}{N_0}})$$

$$= \int_{\sqrt{E/2N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = Q(\sqrt{\frac{E}{2N_0}}) = Q(\sqrt{\frac{E_b}{N_0}})$$

E is the energy for match filter E = 2 E_b (k is constant can be ignored)

E_b is the transmitter signal energy per bit = $\frac{A^2}{2}T$

Split phase Manchester code



$\lambda = 0$

$$P_e = P(a=1)P(error|a=1) + P(a=-1)P(error|a=-1)$$

抽样值 r = $\frac{y(T)}{kE} = a + \frac{n(T)}{KE} \sim$ N(a, $\frac{N_0}{2E}$)

$P(a=1) = P(a=-1) = \frac{1}{2}$

$P(error|a=1) = P(error|a=-1)$

$$P_e = P(error|a=0) = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0/E}} e^{-\frac{(x+1)^2}{N_0/E}} dx = \int_{0\sqrt{2E/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \quad (z = (x+1)\sqrt{\frac{2E}{N_0}})$$

$$= \int_{\sqrt{\frac{2E}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = Q(\sqrt{\frac{2E}{N_0}}) = Q(\sqrt{\frac{2E_b}{N_0}})$$

E_b is the transmitter signal energy per bit = A^2T

E is the energy for match filter, so E = E_b (k is constant can be ignored)