# 通信原理
# 习题课

Assignment No. 2

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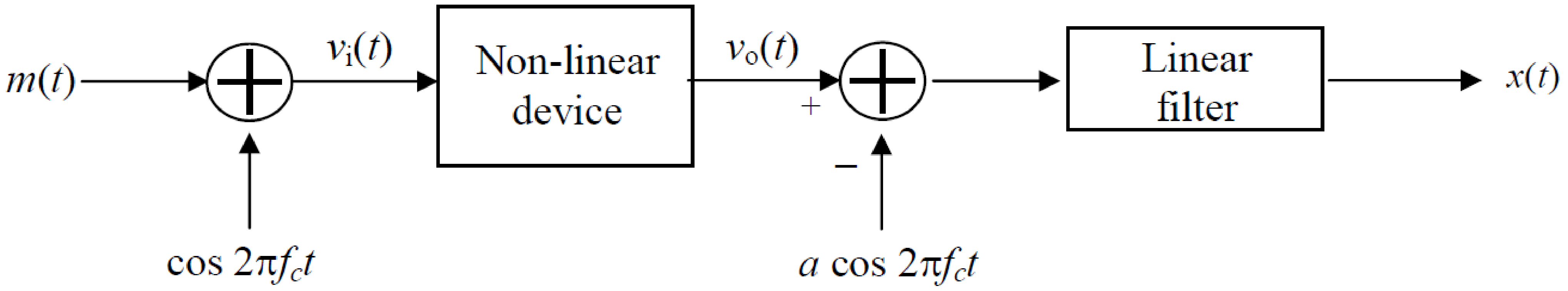
1. The message signal $m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$ is input to the system shown below to generate a DSBSC-AM signal $x(t)$. Assume that $v_o(t) = a\, v_i(t) + b\, v_i^2(t)$ where $a$ and $b$ are constants, and the carrier frequency $f_c >> 70$Hz.

   (a) Sketch the amplitude spectrum of the filter input

   (b) Determine the center frequency and bandwidth of the filter in this modulator

   (c) Determine the minimum value of $f_c$ permitted for this modulator



## Solution of (a)

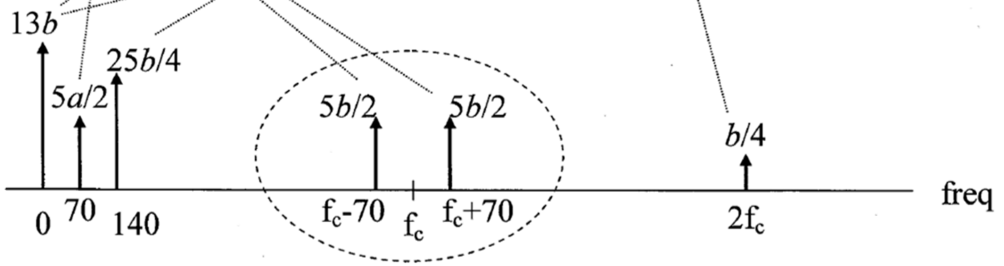
$$v_o(t) = av_i(t) + bv_i^2(t)$$
$$= a\left[m(t) + \cos(2\pi f_c t)\right] + b\left[m(t) + \cos(2\pi f_c t)\right]^2$$
$$= a\,m(t) + b\,m^2(t) + 2b\,m(t)\cos(2\pi f_c t) + a\cos(2\pi f_c t) + \overbrace{b\cos^2(2\pi f_c t)}^{\frac{b}{2}+\frac{b}{2}\cos(4\pi f_c t)}$$

$m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t) = 5\cos(2\pi 70t - \theta)$ ← from Trigo Table

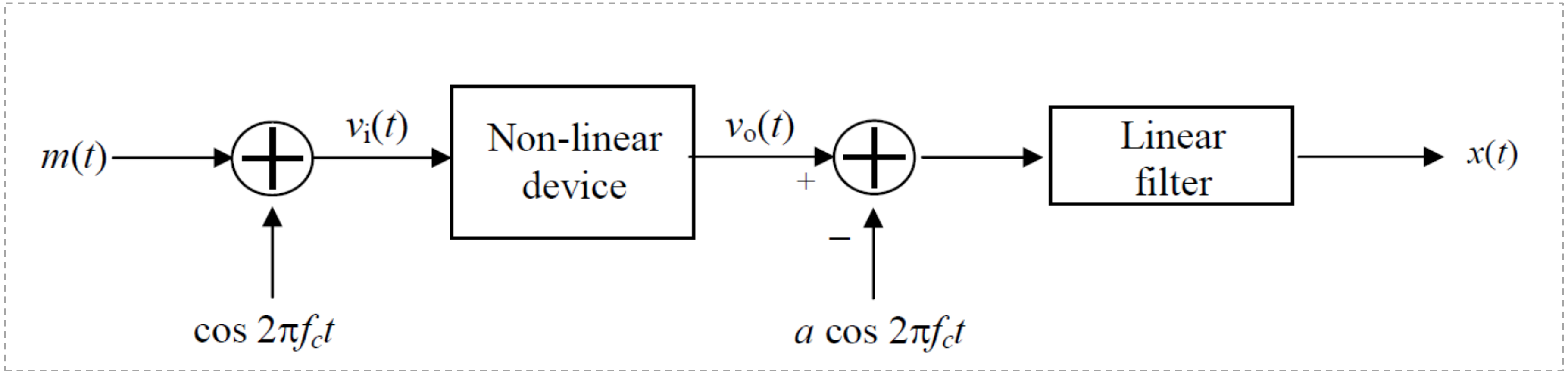
$$m^2(t) = \frac{25}{2} + \frac{25}{2}\cos(2\pi 140t - 2\theta) \text{ where } \theta = \tan^{-1}\frac{4}{3}$$

So,

$$\text{filter input} = v_o(t) - a\cos(2\pi f_c t)$$
$$= 5a\cos(2\pi 70t - \theta) + \frac{25b}{2} + \frac{25b}{2}\cos(2\pi 140t - 2\theta)$$
$$+ 10b\cos(2\pi 70t - \theta)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)$$




*desired DSBSC-AM signal*



## Solution of (b)

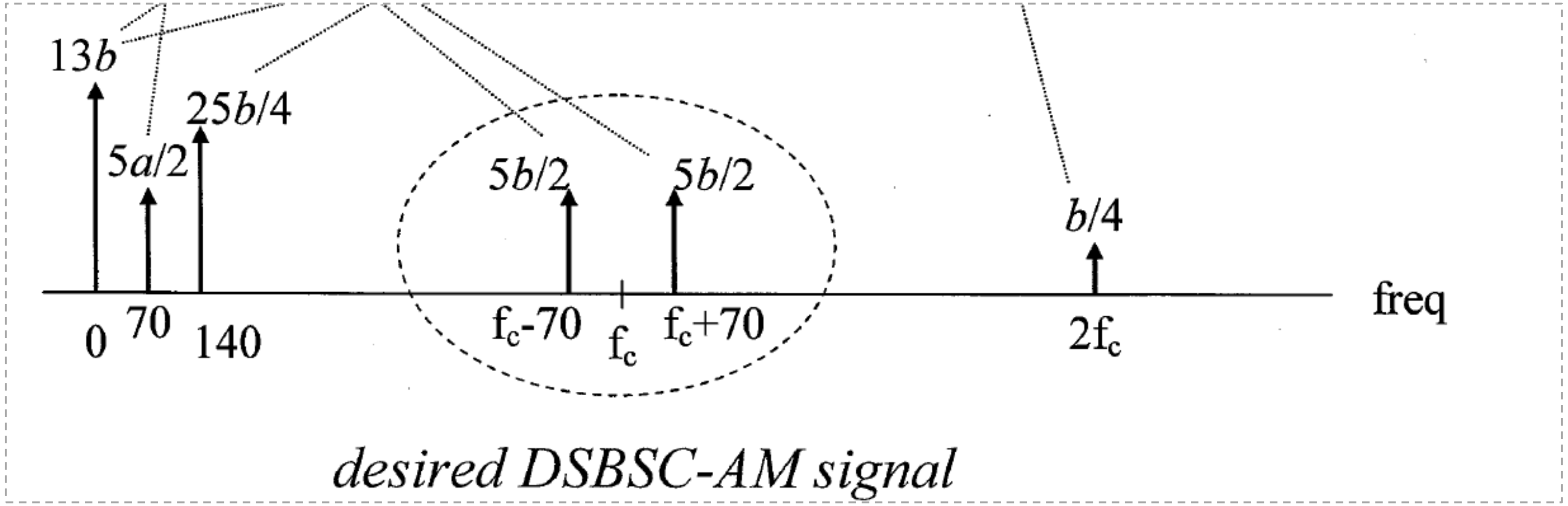
Since the message signal $m(t)$ is a $\cos(2\pi 70t - \theta)$ signal, its DSBSC-AM signal should be the $\cos(2\pi 70t-\theta)\cos(2\pi f_c t)$ term. Hence the linear filter should have center freq $= f_c$ and bandwidth = 140Hz in order to separate it from the other unwanted signal components.

## Solution of (c)

From the spectrum of part (a),

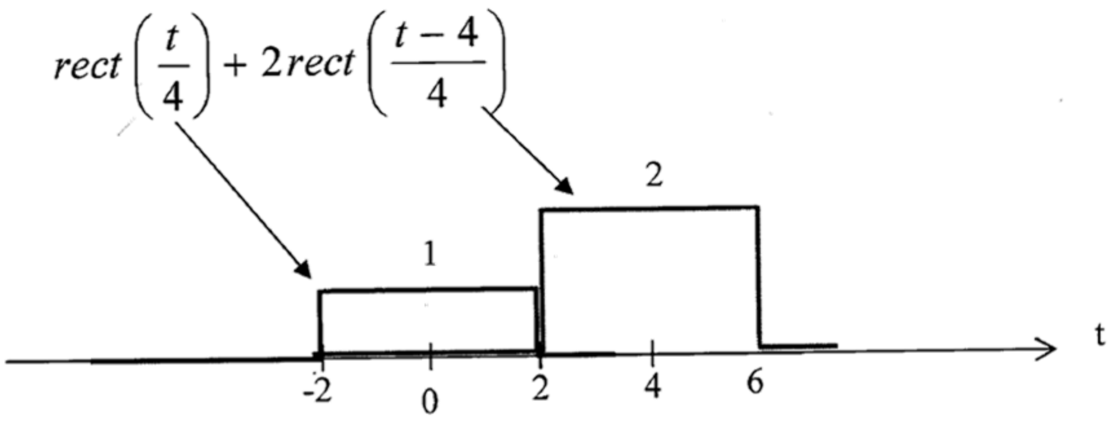
$f_c - 70 > 140$

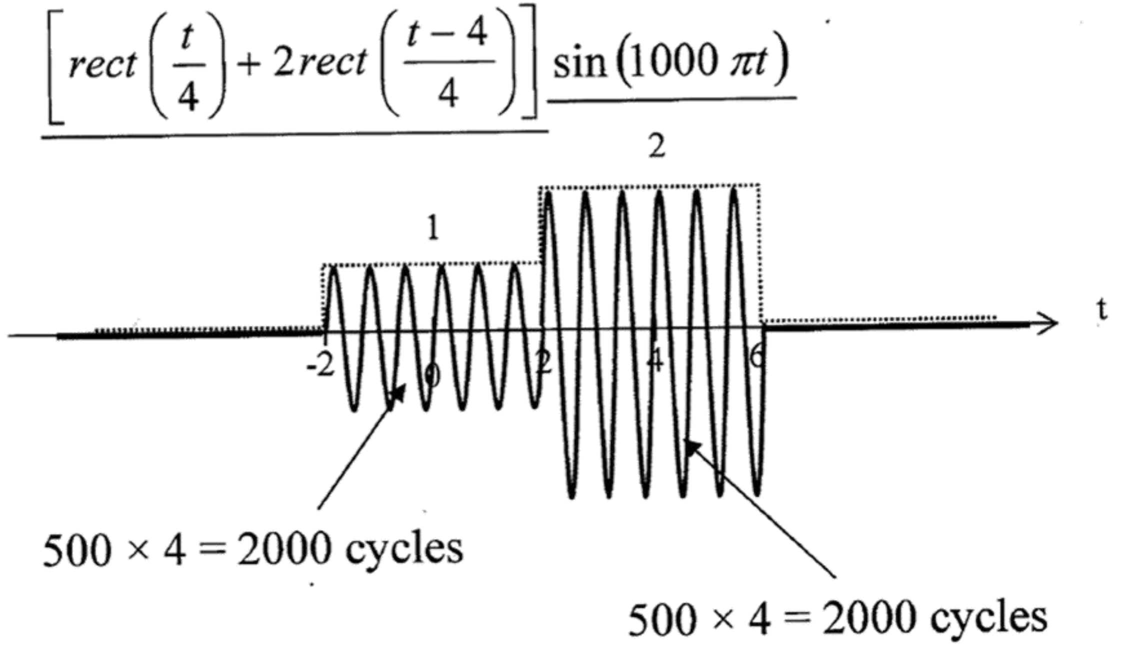
$\Rightarrow f_c > 210$, or $f_c > 3 \times$ message bandwidth



2. A suppressed-carrier AM signal $x_1(t)$ is generated by modulating $s_1(t) = rect\left(\frac{t}{4}\right) + 2rect\left(\frac{t-4}{4}\right)$ with $\sin(1000\pi t)$. Sketch the time waveform of $x_1(t)$.

**Solution**



3. A DSBSC-AM signal is

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

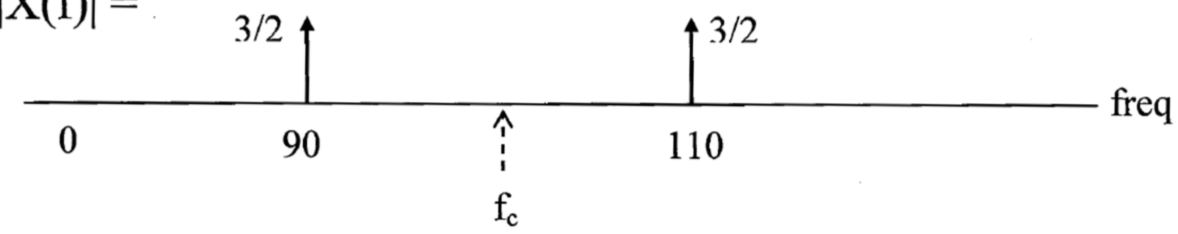
(a) Sketch the amplitude spectrum of x(t) to deduce the carrier frequency in x(t)

(b) Given that x(t) was generated using a sine carrier signal with phase 0, demodulate x(t).

## Solution

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

$|X(f)| =$




Carrier freq = centre freq of AM spectrum $\Rightarrow f_c = 100$Hz

To demod x(t), multiply it by $\sin(2\pi\, 100t)$ followed by LPF.

From X(f), AM signal BW = 110-90 = 20Hz.

LPF bandwidth = message signal BW = ½ × 20Hz = 10Hz.

Demodulator output

$$= [\,(3 \sin 180\pi t + 3 \sin 220\pi t) \sin 200\pi t\,]_{LPF}$$

$$= 3\,[\,\sin180\pi t \sin200\pi t + \sin220\pi t \sin200\pi t\,]_{LPF}$$

$$= 3/2\,[\,\cos(180\text{-}200)\pi t - \cos(180\text{+}200)\pi t$$

$$+ \cos(220\text{-}200)\pi t - \cos(220\text{+}200)\pi t]_{LPF}$$

$$= 3/2\,[\,2 \cos 20\pi t - \cos 380\pi t - \cos 420\pi t]_{LPF}$$

$$= 3 \cos 20\pi t$$

Since the modulation process would halve the amplitude of message signal, a filter with a gain of 2 should be added into demodulator. Then we would have de demodulator output as

$$6cos20\pi t$$