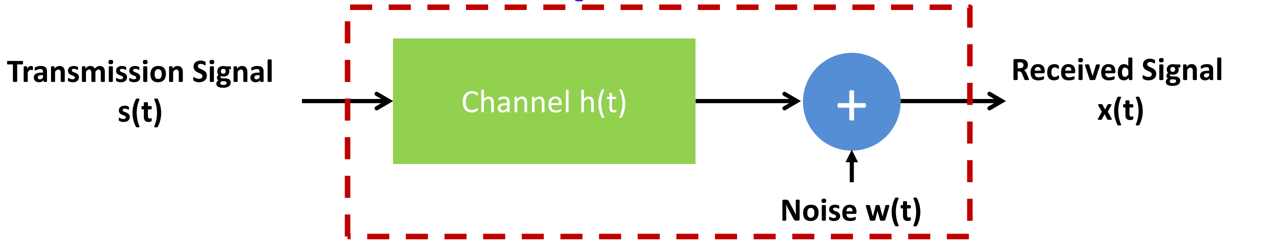
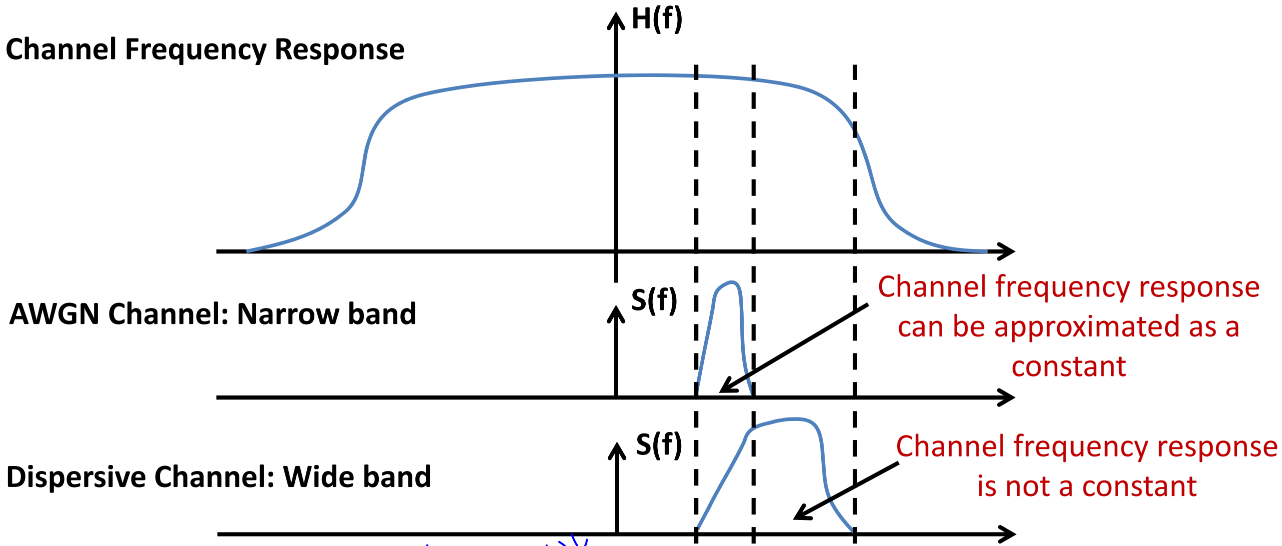
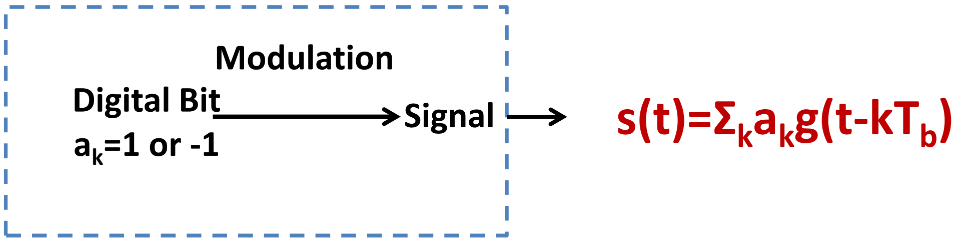
AWGN Channel vs. Dispersive Channel





信号带宽相对窄，可看作高斯信道

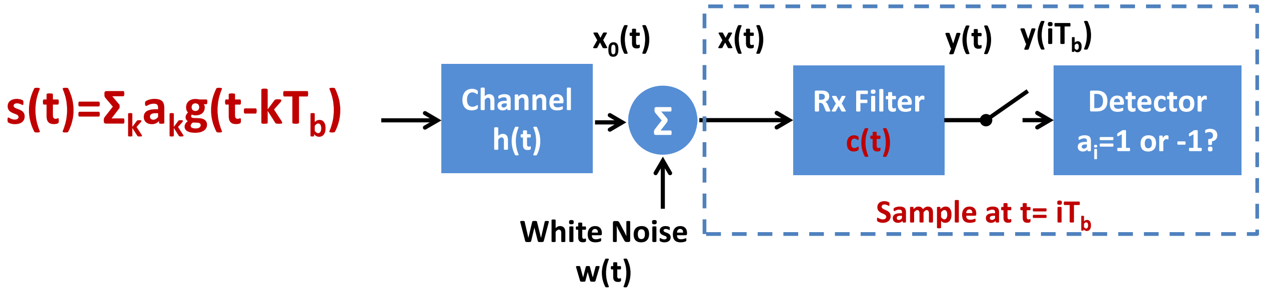
2-ary Transmitter for Dispersive Channel



- Transmitter
  - Message $\{a_k\}$: $a_k = 1$ means bit 1; $a_k = -1$ means bit 0
  - $g(t)$: modulation pulse; T_b: transmission period
  - k -th transmission signal: $a_k g(t - kT_b)$
  - Transmission signal: $s(t) = \sum_k a_k g(t - kT_b)$

之前发的信号经过delay之后

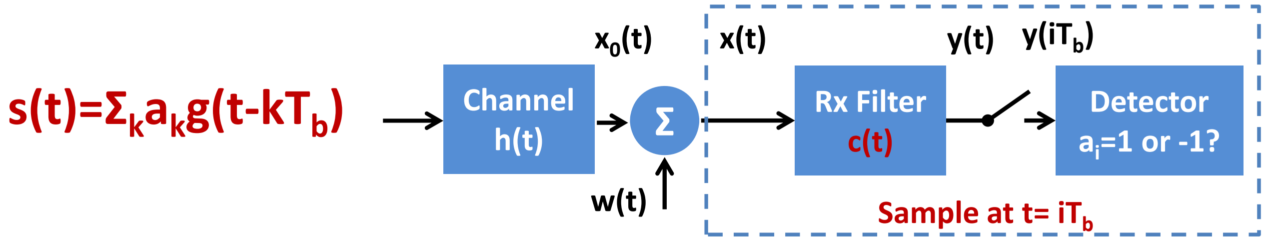
Model with Dispersive Channel



- Channel
  - After channel impulse response: $x_0(t) = s(t) * h(t)$
  - Noise: $x(t) = s(t) * h(t) + w(t)$

- Receiver
  - Structure is similar to AWGN channel receiver
  - Filtering, sampling and detection

Inter-Symbol Interference



- $x(t) = s(t) * h(t) + w(t)$
- $y(t) = s(t) * h(t) * c(t) + c(t) * w(t)$

归一化

$$= \sum_k a_k g(t - kT_b) * h(t) * c(t) + c(t) * w(t) = \sum_k a_k \mu p(t - kT_b) + n(t)$$

where $\mu p(t) = g(t) * h(t) * c(t)$, $\underline{p(0) = 1}$, $n(t) = c(t) * w(t)$

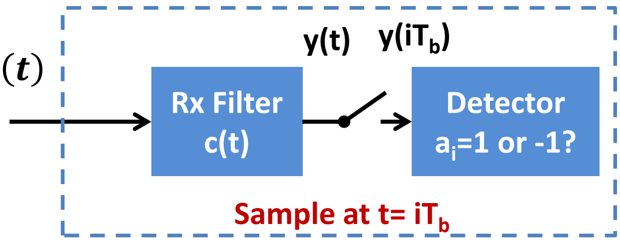
- $y(iT_b) = \sum_k a_k \mu p(iT_b - kT_b) + n(iT_b)$

$$= \underbrace{\mu a_i}_{Signal} + \underbrace{\sum_{k \neq i} a_k \mu p(iT_b - kT_b)}_{Inter-Symbol\,Interference} + \underbrace{n(iT_b)}_{Noise}$$

sampling

Receiving Filter Design

$$\boldsymbol{x(t) = s(t) * h(t) + w(t)}$$



- What's the role of Rx filter?

$$y(iT_b) = \underbrace{\mu a_i}_{Signal} + \underbrace{\sum_{k \neq i} a_k \mu p(iT_b - kT_b)}_{Inter-Symbol\,Interference} + \underbrace{n(iT_b)}_{Noise}$$

- μ ,n(t) and p(t) are all determined by c(t)
- c(t) is critical to suppress the ISI and noise

SINR

- Use Signal-to-Interference-and-Noise Ratio (SINR) to measure the quality of $y(iT_b)$
  - $y(iT_b) = \underbrace{\mu a_i}_{Signal} + \underbrace{\sum_{k \neq i} a_k \mu p(iT_b - kT_b)}_{Inter-Symbol\,Interference} + \underbrace{n(iT_b)}_{Noise}$
  - Signal Power: $P_s = \mu^2$
  - Interference Power

交叉项的期望为0。

$$P_I = \mathbb{E}\left\{\left[\sum_{k \neq i} a_k \mu p(iT_b - kT_b)\right]^2\right\} = \sum_{k \neq i} \mu^2 p^2(iT_b - kT_b)$$

  - Noise Power

$$P_N = \mathbb{E}[n^2(iT_b)] = \int_{-\infty}^{\infty} \frac{|C(f)|^2\, N_0}{2} df$$

$$\boldsymbol{SINR = \frac{\mu^2}{\mu^2 \sum_{k \neq i} p^2(iT_b - kT_b) + \int_{-\infty}^{\infty} \frac{|C(f)|^2\, N_0}{2} df}}$$

Optimization

- Optimal: given g(t) and h(t), choose c(t) to maximize SINR

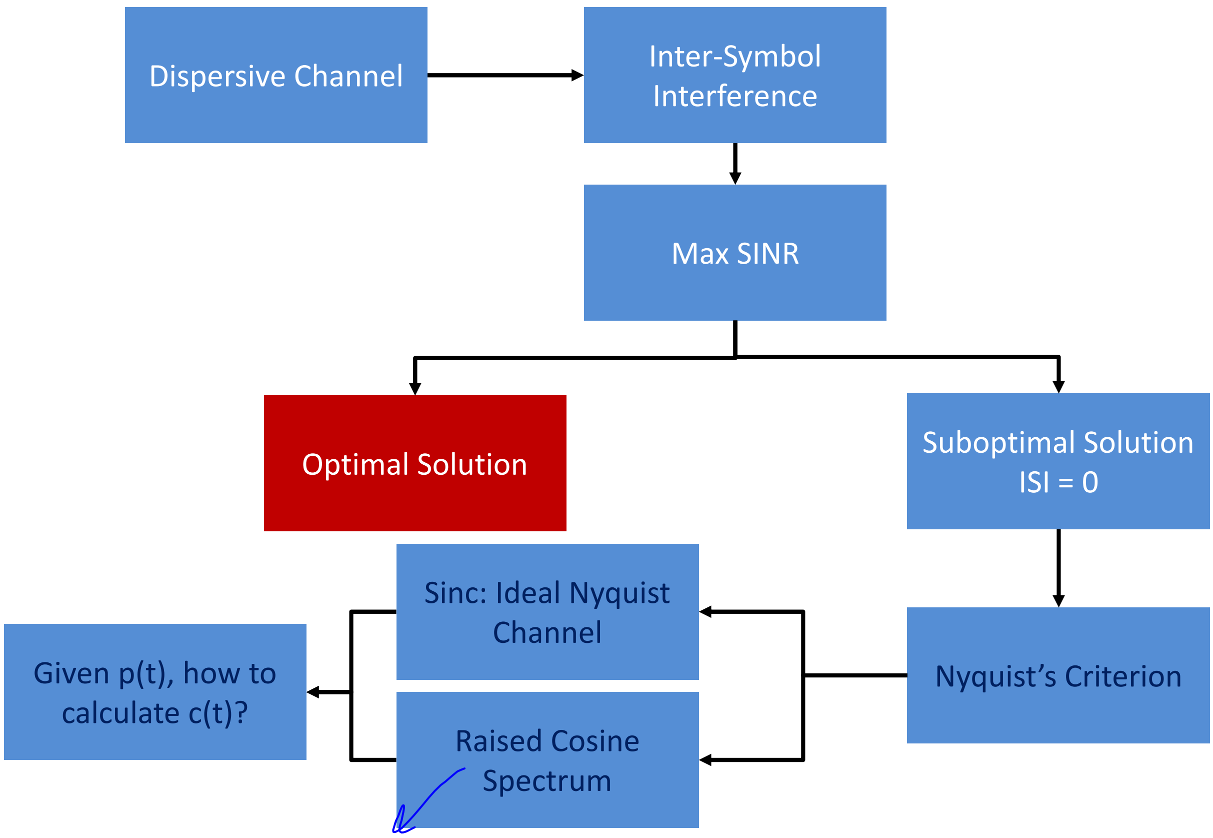
很难求出

$$max_{c(t)} SINR = max_{c(t)} \frac{\mu^2}{\mu^2 \sum_{k \neq i} p^2(iT_b - kT_b) + \int_{-\infty}^{\infty} \frac{|C(f)|^2 N_0}{2} df}$$

- Suboptimal: given g(t) and h(t), choose c(t) to force ISI to zero (ISI is usually dominant)

Find c(t), such that $\sum_{k \neq i} p^2(iT_b - kT_b)$=0

1. **Find a p(t) that can cancel ISI.**
2. **Derive the corresponding c(t) as g(t) and h(t) are known.**



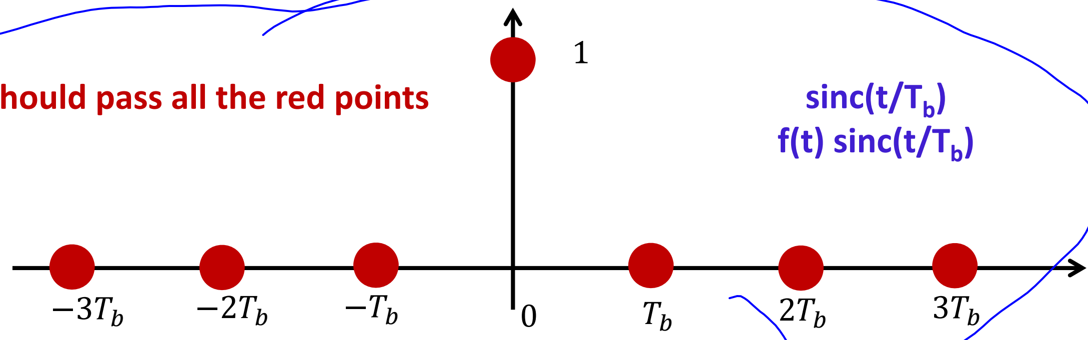
升余弦

Suboptimal: ISI Free Receiving

- $\sum_{k \neq i} p^2(iT_b - kT_b)$=0
- $\forall k \neq i, p(iT_b - kT_b) = p((i-k)T_b) = 0$
- $p(t)$ should satisfy

$$p(kT_b) = \begin{cases} 1, & k = 0 \\ 0, & k \text{ is an integer and } k \neq 0 \end{cases}$$

p(t) should pass all the red points



sinc(t/T_b)
f(t) sinc(t/T_b)

Nyquist's Criterion

- Let $p(t) \leftrightarrow P(f)$
- After impulse chain sampling $p(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_b)$, we have

$$p(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_b) \leftrightarrow \frac{1}{T_b}\sum_{n=-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right)$$

采样

- Since

$$\boldsymbol{p(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_b) = \delta(t)},$$

we have

$$\frac{1}{T_b}\sum_{n=-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right)=1$$

or

$$\sum_{n=-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right) = \mathrm{T_b}$$

What does this mean?

Nyquist's criterion for distortionless transmission

没变形

Condition for Distortionless Transmission

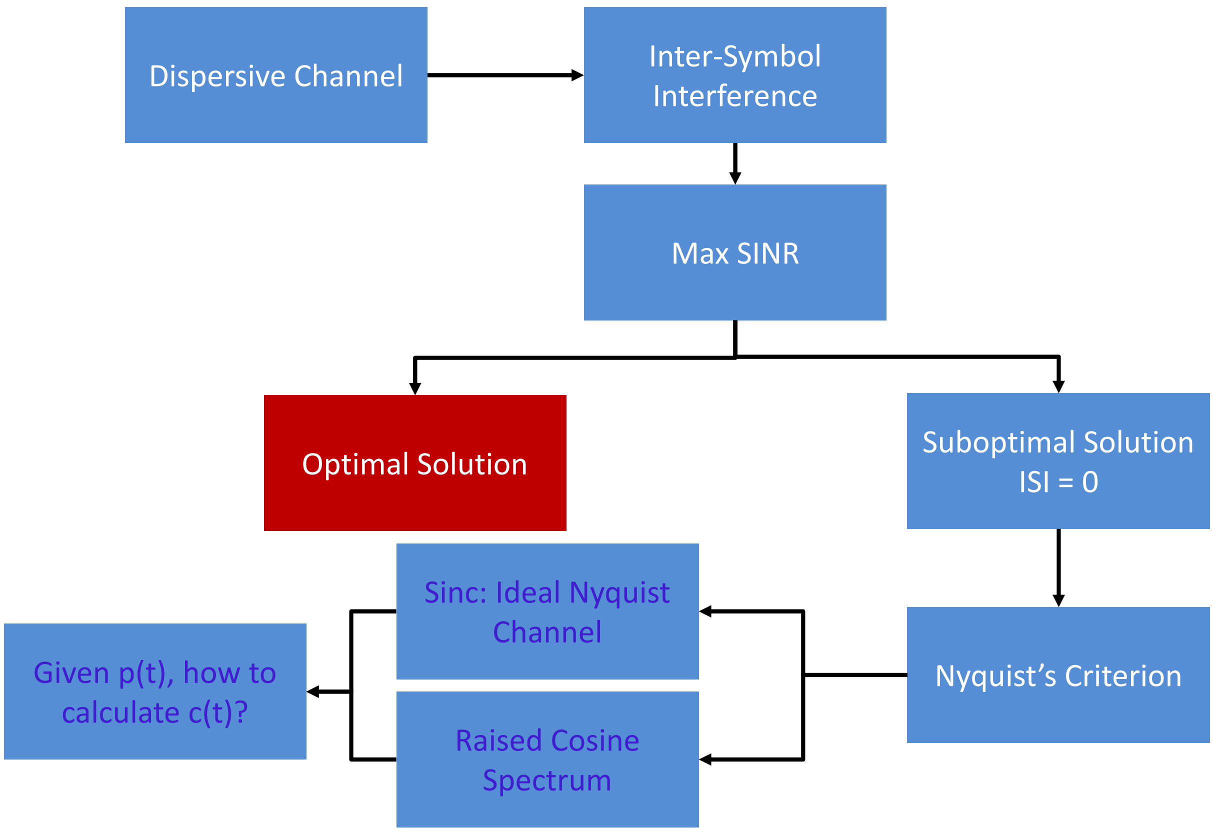
- Time domain:

$$p(kT_b) = \begin{cases} 1, & k = 0 \\ 0, & k \text{ is an integer and } k \neq 0 \end{cases}$$

- Frequency domain (Nyquist's Criterion):

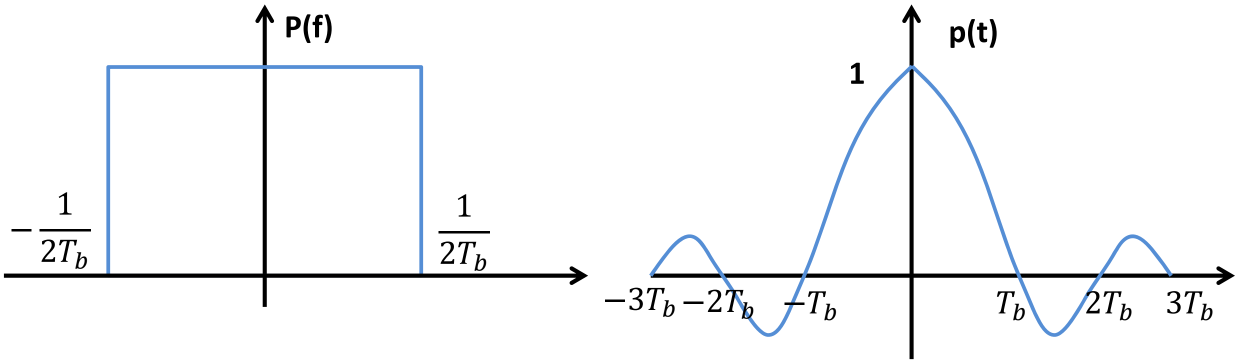
$$\sum_{n=-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right) = \mathrm{T_b}$$

- The two conditions are equivalent.
- Without noise, they can guarantee distortionless communications.



Sinc Wave: Ideal Nyquist Channel

- Sinc wave can satisfy the Nyquist's criterion



- Choose c(t), such that

$$p(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} = sinc(t/T_b)$$

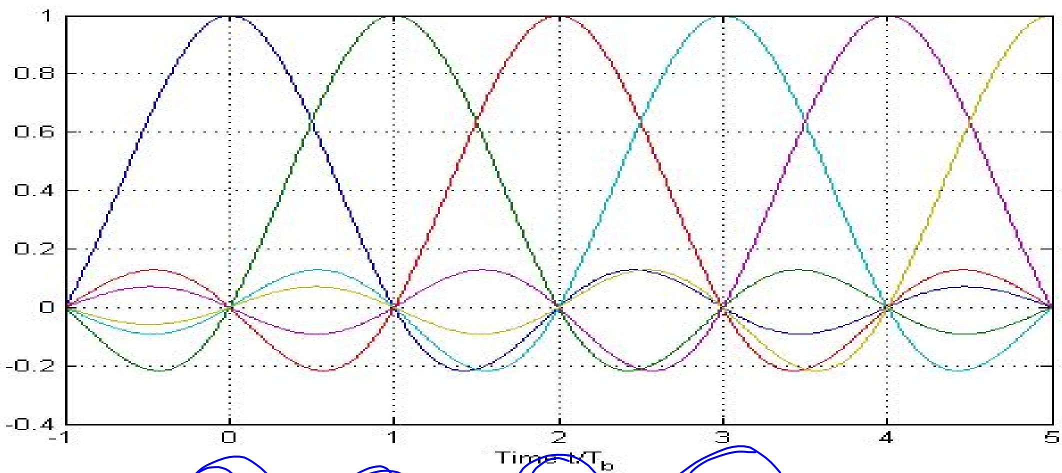
Ideal Nyquist channel

Example

$$\boldsymbol{y(t) = \sum_k a_k \mu p(t - kT_b) + n(t)} \qquad p(\mathrm{t}) = \mathrm{sinc}(\frac{\mathrm{t}}{\mathrm{T_b}})$$

$\boldsymbol{a_1 \mu p(t - T_b)}$ $\boldsymbol{a_3 \mu p(t - 3T_b)}$

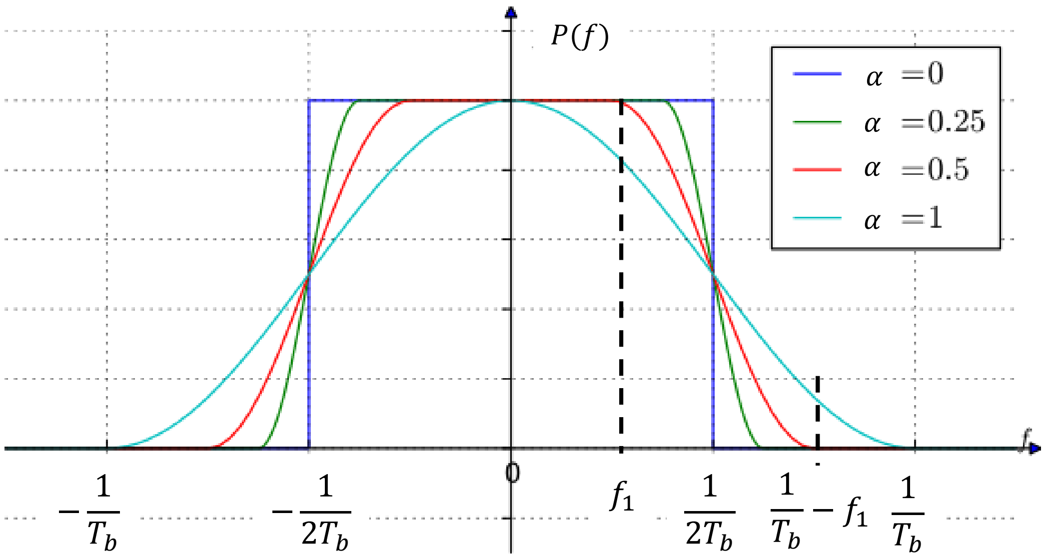
$\boldsymbol{a_0 \mu p(t)}$ $\boldsymbol{a_2 \mu p(t - 2T_b)}$ $\boldsymbol{a_4 \mu p(t - 4T_b)}$



$\boldsymbol{y(0T_b)}$ $\boldsymbol{y(T_b)}$ $\boldsymbol{y(2T_b)}$ $\boldsymbol{y(3T_b)}$

该时刻码间干扰为零

Raised Cosine Spectrum



$$\sum_{n=-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right) = T_b$$

Rolloff factor $\alpha = 1 - 2T_b f_1$

Raised Cosine Spectrum

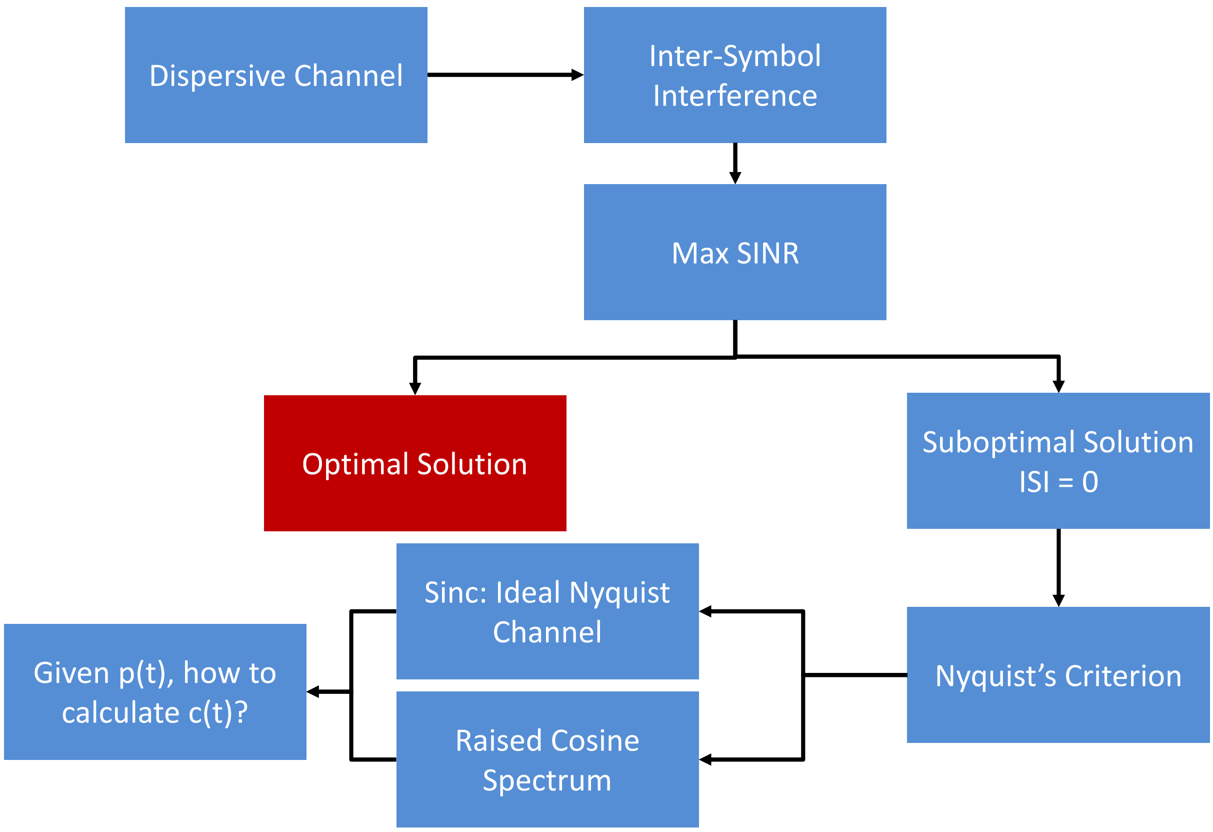
- Raised cosine spectrum is given by

$$P(f) = \begin{cases} T_b, & 0 \le |f| < f_1 \\ \frac{T_b}{2}\left\{1 - \sin\left[\frac{\pi(|f|-W)}{2W-2f_1}\right]\right\}, & f_1 \le |f| < \frac{1}{T_b} - f_1 \\ 0, & |f| \ge \frac{1}{T_b} - f_1 \end{cases}$$

$$p(t) = sinc\left(\frac{t}{T_b}\right)\left(\frac{\cos(2\pi\alpha W t)}{1 - 16\alpha^2 w^2 t^2}\right)$$

Where $W = \frac{1}{2T_b}$

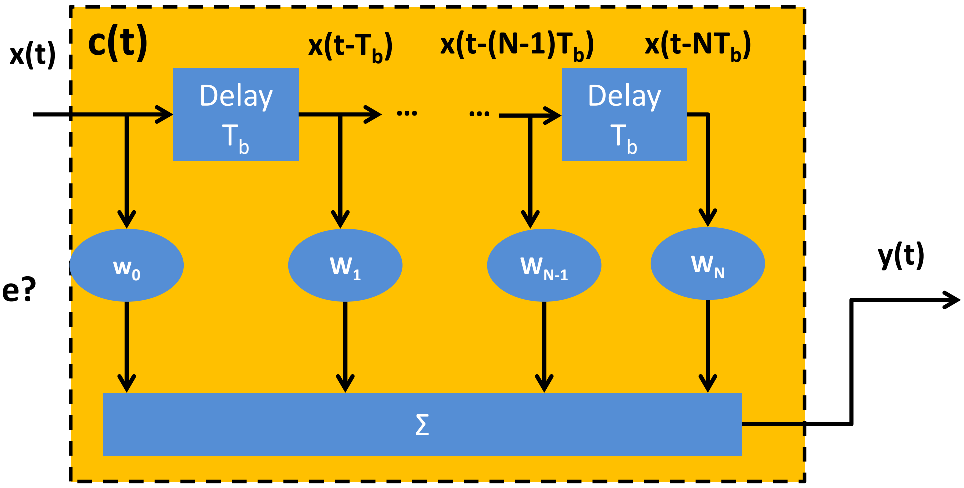
- Rolloff factor $\alpha = 1 - 2T_b f_1$ ($\alpha = 0?$)



However …

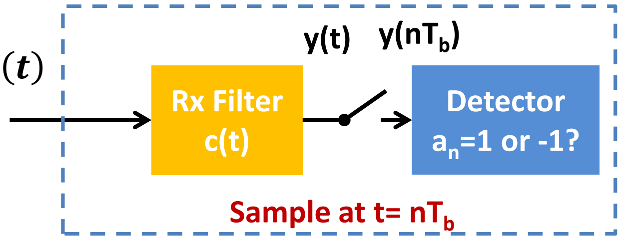
- In order to achieve raised cosine spectrum P(f), the cost of implementing c(t) may still be high.
- Engineers' choice
  - Low-cost structure of receive filter c(t): tapped-delay-line filter
  - May not perfectly cancel the ISI

Tapped-Delay-Line Filter



Impulse Response?

$$\boldsymbol{x(t) = s(t) * h(t) + w(t)}$$



Tapped-Delay-Line Equalization

- Impulse response: $c(t) = \sum_{k=0}^{N} w_k \delta(t - kT_b)$

- $y(t) = s(t) * h(t) * c(t) = [\sum_i a_i g(t - iT_b)] * h(t) * \left[\sum_{k=0}^{N} w_k \delta(t - kT)\right] = \sum_i \sum_{k=0}^{N} a_i w_k g(t - (i + k)T_b) * h(t) = \sum_i \sum_{k=0}^{N} a_i w_k \beta(t - (i + k)T_b)$
  - Where $\beta(t) = g(t) * h(t)$
  - Assume no noise

- $y(nT_b) = \sum_i a_i \sum_{k=0}^{N} w_k \beta\big((n - i - k)T_b\big) = \sum_i a_i \sum_{k=0}^{N} w_k \beta_{n-i-k} = a_n \sum_{k=0}^{N} w_k \beta_{-k} + \sum_{i \neq n} a_i \sum_{k=0}^{N} w_k \beta_{n-i-k}$
  - Where$\beta_{n-i-k} = \beta\big((n - i - k)T_b\big)$

- If we wants to detect a_n from $y(nT_b)$, we need
  - Maintain the desired message: $\sum_{k=0}^{N} w_k \beta_{-k}$=1
  - Cancel the interference as much as possible

$\sum_{k=0}^{N} w_k \beta_{n-k-i} = 0$ (i=n-1,n-2,…,n- N **why?**)

Matrix Expression

$$\underbrace{\begin{bmatrix} \beta_0 & \beta_{-1} & \beta_{-2} & \cdots & \beta_{-N} \\ \beta_1 & \beta_0 & \beta_{-1} & \cdots & \beta_{-N+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta_N & \beta_{N-1} & \beta_{N-2} & \cdots & \beta_0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \cdots \\ w_N \end{bmatrix}}_{W} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix}}_{I}$$

$$\boldsymbol{B\,W} = \boldsymbol{I}$$
$$\boldsymbol{W} = \boldsymbol{B^{-1}I}$$

See also: Rake Receiver of CDMA systems
http://www.wirelesscommunication.nl/reference/chaptr05/cdma/rake.htm

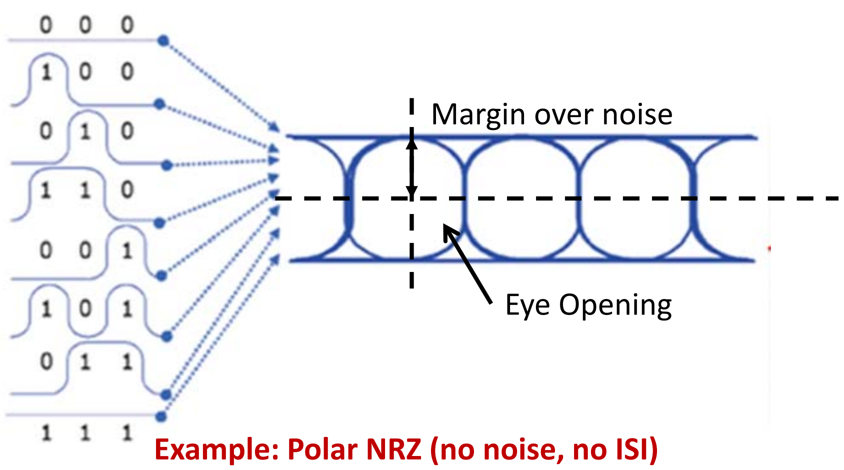
Summary

- Baseband transmission of digital signal
- Transmitter
  - Information: a binary bit sequence
  - M-ary modulation: each pulse can represent $\log_2 M$ bits
- Receiver of AWGN channel
  - Match filter
  - Detector: minimize the error probability (BER)
- Receiver of dispersive channel
  - Inter-symbol interference
  - Optimal receive filter: maximize the SINR
  - Suboptimal receive filter (Nyquist's criterion): ideal Nyquist channel, raised cosine spectrum
  - Tapped-delay-line filter

Eye Pattern

Eye Pattern

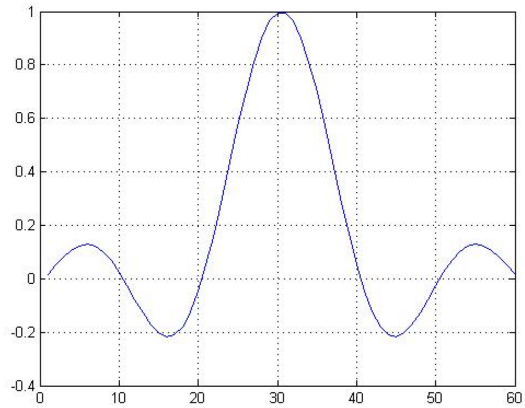
- Eye Pattern (Eye Diagram): useful tool to evaluate the quality of received signal
- How to generate eye pattern?



Example: Polar NRZ (no noise, no ISI)
Repetitively draw all possible curves of received signal within a certain period

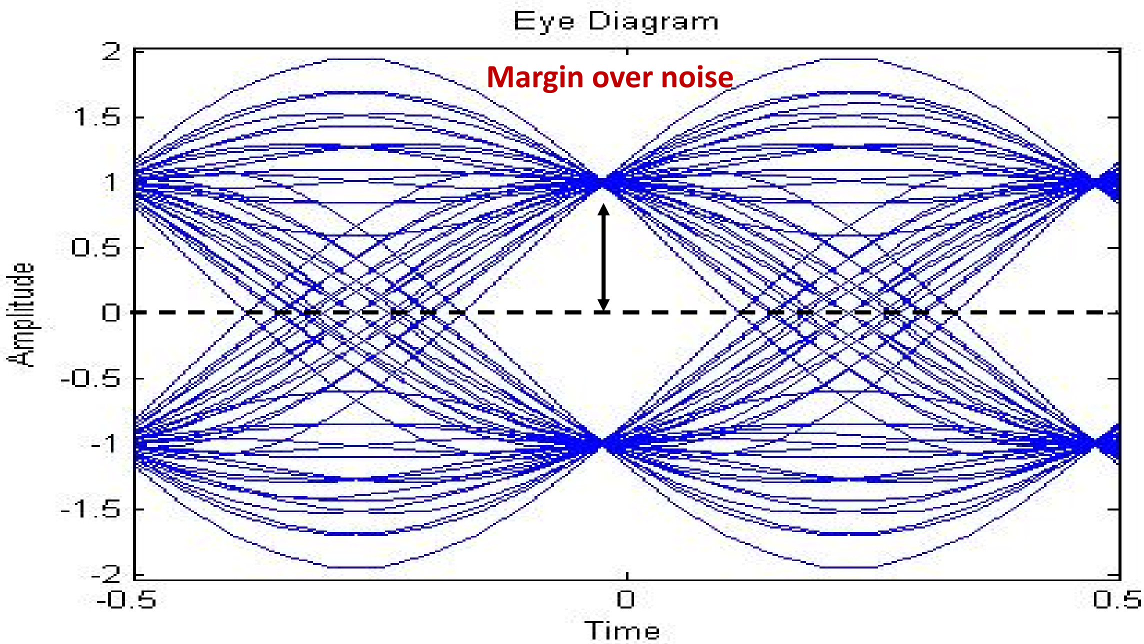
Figure from: digital.ni.com/public.nsf/allkb/0B20F0575F5F3CFF86257B04003F841C

Eye Pattern Example

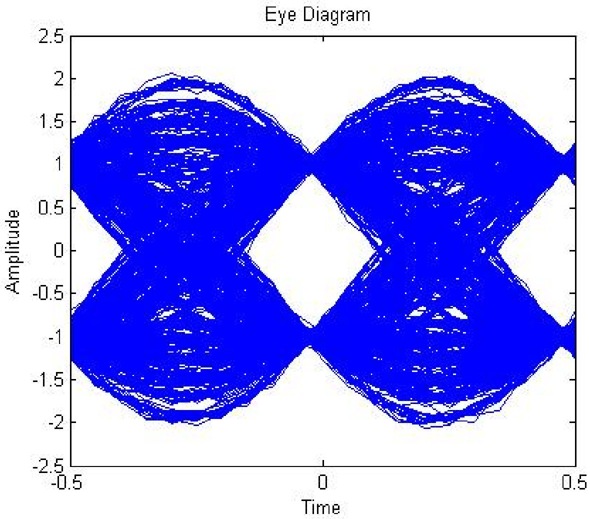


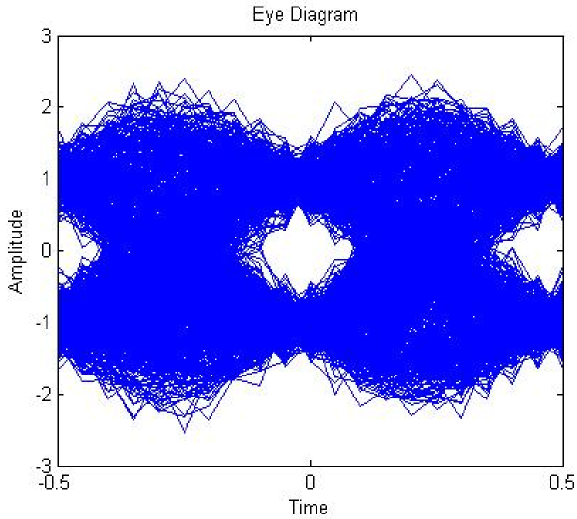
Pulse shape $g(t)$ is a truncated sinc function

Eye Pattern without Noise or ISI



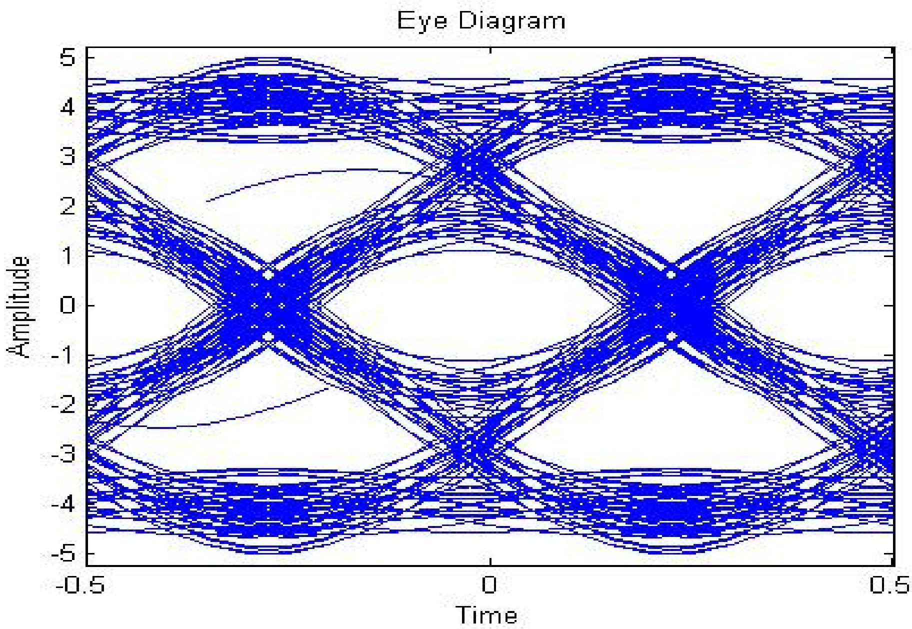
Eye Pattern with Noise





- Increasing noise power leads to closing eye

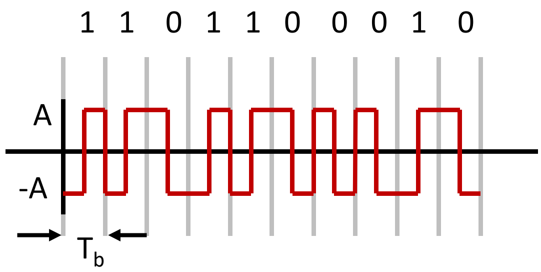
Eye Pattern with ISI



Homework #D6

- D6.1

Please design the receiving filter c(t) for the following *Split-Phase Manchester Code* by both ideal Nyquist channel and raised cosine spectrum.



Split-Phase Manchester Code

Appendix --- Eye Pattern Generation

```
clear;
clc;
% Generate the messaage
MsgLength = 2000;
Msg = 2*(randi([0 1],1,MsgLength)-0.5);
% Generate the waveform of pulse
p = sinc(-2.95:0.1:2.95);
% Generate the modulated signal
Y = zeros(1,10*MsgLength+50);
Y(1:length(p)) = Msg(1) * p;
for i=2:MsgLength
   Signal = Msg(i) * p;
   Y((i-1)*10+1:(i-1)*10+length(p)) = Y((i-1)*10+1:(i-1)*10+length(p)) + Signal;
end;
% Plot the pulse waveform
plot(p);
grid;
% Plot a clip of modulated signal
figure;
plot(Y(31:500));
grid;
% Plot eye diagram without noise and ISI
eyediagram(Y(31:end-30),20);
% Add noise
Y1 = Y + randn(size(Y))*0.05;
eyediagram(Y1(31:end-30),20);
Y2 = Y + randn(size(Y))*0.2;
eyediagram(Y2(31:end-30),20);

% Add ISI
H = [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1];
H = H / sqrt(H*H');
Y3 = conv(Y,H);
eyediagram(Y3(31:end-30),20,1,7);
```