EE206: Communications Principles Tutorial

Assignment 4

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Power Ratios and Decibels

In communications systems, the power levels may vary from megawatts for the case of a television transmitter to 10^{-12} watts at a satellite receiver. To manage this wide range of powers, it is customary practice to use a unit called the *decibel*. The decibel, commonly abbreviated as dB, is one tenth of a larger unit, the *bel*. The unit bel is named in honor of Alexander Graham Bell. In addition to inventing the telephone, Bell was the first to use logarithmic power measurements in sound and hearing research. In practice, however, we find that for most applications the bel is too large a unit, hence, the wide use of decibel as the unit for expressing power ratios.

Let P denote the power at some point of interest in a system. Let P_0 denote the reference power level with respect to which power P is compared. The number of decibels in the power ratio P/P_0 is defined as

$$\left(\frac{P}{P_0}\right)_{dB} = 10\log_{10}\left(\frac{P}{P_0}\right)$$

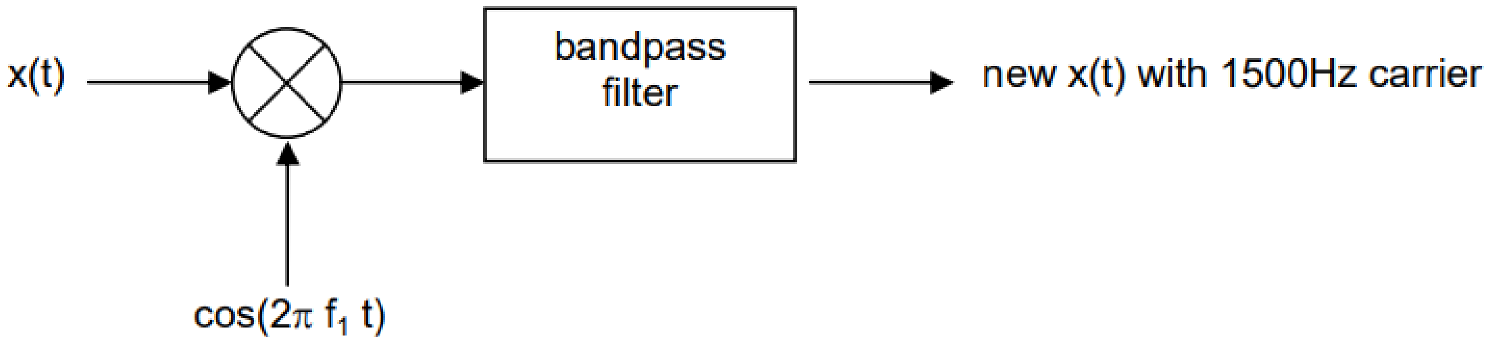
For example, a power ratio of 2 approximately corresponds to 3 dB, and a power ratio of 10 exactly corresponds to 10 dB.

We may also express the signal power relative to one watt or one milliwatt. In the first case, we express the signal power P in dBW as $10\log_{10}(P/1\text{W})$ where W is the abbreviation for watt. In the second case, we express the signal power P in dBm as $10\log_{10}(P/1\text{ mW})$, where mW is the abbreviation for milliwatt.

1. The signal $s(t) = \sin(200\pi t + \pi/3) + \cos(200\pi t + \pi/3)$ is modulated using a cosine carrier signal with carrier frequency 500Hz and zero phase to generate a Suppressed-Carrier AM signal $x(t)$.

   (a) Write $s(t)$ as a single cosine term. Then find $x(t)$.

   (b) Use the mixer below to shift the carrier frequency of $x(t)$ to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.



1. The signal s(t) = sin(200πt + π/3) + cos(200πt + π/3) is modulated using a cosine carrier signal with carrier frequency 500Hz and zero phase to generate a Suppressed-Carrier AM signal x(t).

(a) Write s(t) as a single cosine term. Then find x(t).

$$
\begin{aligned}
s(t) &= \sin(2\pi 100t + \frac{\pi}{3}) + \cos(2\pi 100t + \frac{\pi}{3}) \\
&= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{3} - \tan^{-1}\frac{1}{1}) \\
&= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{3} - \frac{\pi}{4}) \\
&= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12}) \\
x(t) &= s(t)\cos(2\pi 500t) \\
&= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12})\cos(2\pi 500t)
\end{aligned}
$$

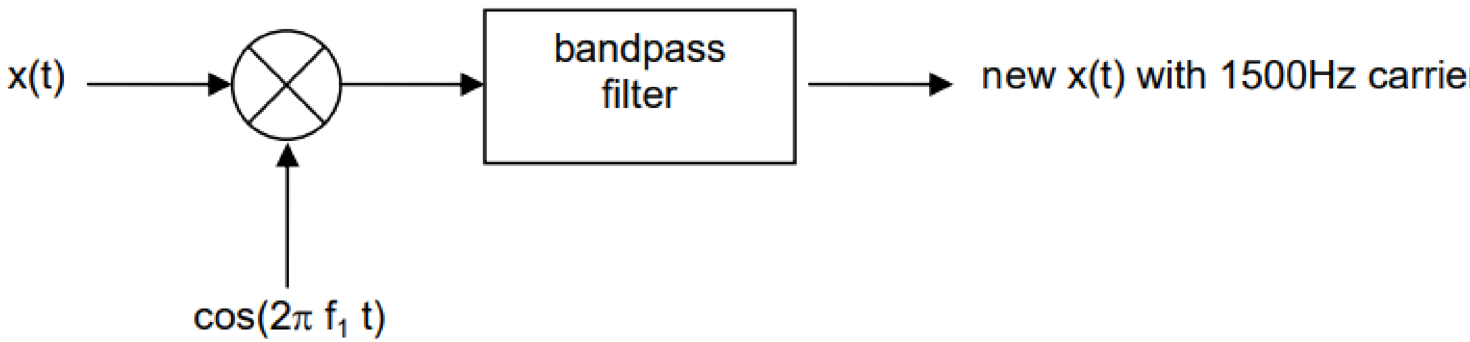
$a\cos x + b\sin x$ can be written as $R\cos(x-\alpha)$

where

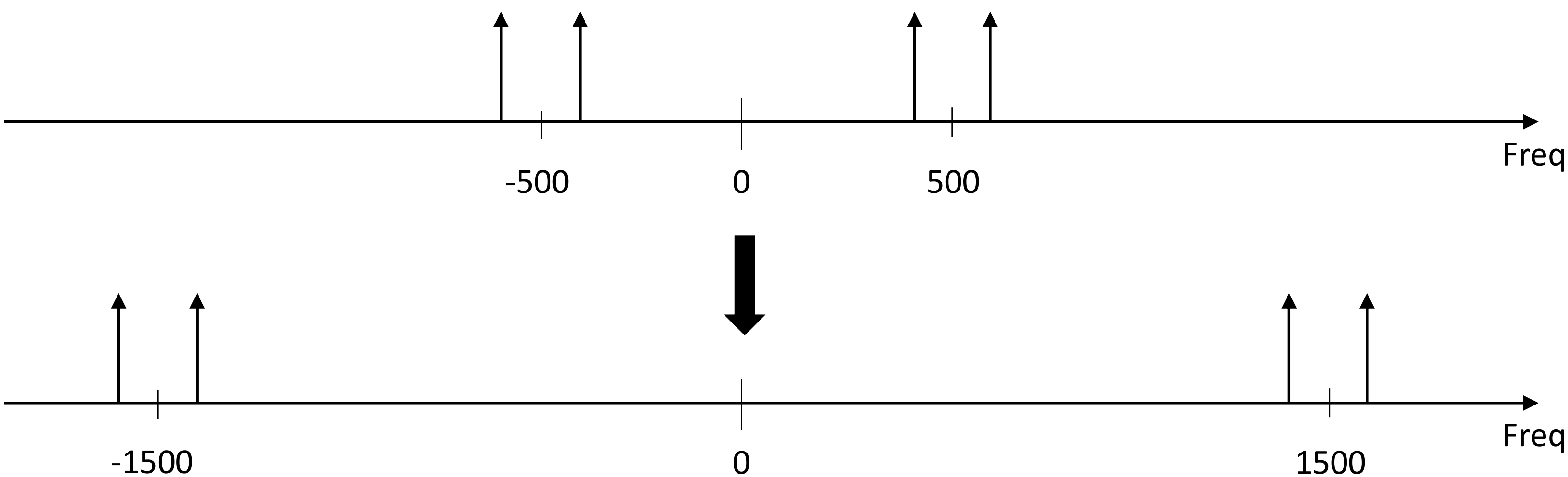
$$R = \sqrt{a^2+b^2}, \qquad \tan\alpha = \frac{b}{a}$$

https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-rcostheta-alpha-2009-1.pdf

(b) Use the mixer below to shift the carrier frequency of x(t) to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.



$$x(t) = \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12})\cos(2\pi 500t)$$



(b) Use the mixer below to shift the carrier frequency of x(t) to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.

Frequency Translation (2)

- For a modulated signal $x(t) = m(t)\cos 2\pi f_c t$,

$$v_1(t) = x(t) \cdot \cos 2\pi f_1 t$$
$$= m(t)\cos 2\pi f_c t \cos 2\pi f_1 t$$
$$= \frac{m(t)}{2}[\cos 2\pi(f_c - f_1)t + \cos 2\pi(f_c + f_1)t]$$

- Assuming $f_c > f_1$, if $v_1(t)$ is passed through a bandpass filter with a centre frequency $f_0 = f_c - f_1$, then $x(t)$ will occupy a new frequency band. That is

$$v_2(t) = [v_1(t)]_{BP}$$
$$= \frac{1}{2}m(t)\cos 2\pi(f_c - f_1)t = \frac{1}{2}m(t)\cos 2\pi f_0 t$$

- The device that carries out the frequency translation of a modulated signal is called mixer, and the operation itself is called mixing.

47

(b) Use the mixer below to shift the carrier frequency of x(t) to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.

$$x(t)\cos(2\pi f_1 t) = \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12})\cos(2\pi 500t)\cos(2\pi f_1 t)$$

$$f_1 - 500 = 1500 \text{ or } f_1 + 500 = 1500$$

$$f_1 = 1000Hz \text{ or } 2000Hz$$

filter center frequency: $1500Hz$

bandwidth: $200Hz$

2. a) A received signal a(t) has SNR 13dB and **noise** power 64 μW (μW = 10^{-6} Watt). Another received signal b(t) also has SNR 13dB but **total** (signal+noise) power of 64 μW. Determine the useful signal power in mW in each of these signals.

$$\begin{aligned}
\mathrm{SNR}_a &= 10\log_{10}\frac{P_{m_a}}{64\times 10^{-6}} = 13\mathrm{dB} \\
P_{m_a} &= 0.001277W \\
&= 1.277mW \\
\mathrm{SNR}_b &= 10\log_{10}\frac{P_{m_b}}{64\times 10^{-6} - P_{m_b}} = 13\mathrm{dB} \\
P_{m_b} &= 0.000061W \\
&= 0.061mW
\end{aligned}$$

b) An AM signal x(t) is received with 6mW signal power, 20KHz bandwidth and carrier freq 100MHz. Another AM signal y(t) is received with 100mW signal power, 3MHz bandwidth and carrier freq 500MHz. The channel contains white noise. Which signal has better quality?

$$
\begin{aligned}
\mathrm{SNR_x} &= 10\log_{10}\frac{6\times10^{-3}}{20\times10^{3}\times2\times\frac{\eta}{2}} \\
&= 10\log_{10}(\frac{3}{\eta}\times10^{-7}) \\
\mathrm{SNR_y} &= 10\log_{10}\frac{100\times10^{-3}}{3\times10^{6}\times2\times\frac{\eta}{2}} \\
&= 10\log_{10}(\frac{1}{3\eta}\times10^{-7}) \\
\mathrm{SNR_x} &> \mathrm{SNR_Y}
\end{aligned}
$$

signal x has better quality.

3. Two message signals, $s_1(t) = 2$ and $s_2(t) = 10\sin(20\pi t)$, are modulated to form a QAM signal $x(t)$ with carrier frequency 500Hz. $s_1(t)$ is modulated onto the I-phase, $s_2(t)$ onto the Q-phase. During transmission, $x(t)$ is corrupted by white noise with 2-sided PSD of 10^{-5} Watt/Hz. At the receiver, it is demodulated using a coherent demodulator. Determine the SNR of the Q-branch output signal in dB.

$$
\begin{aligned}
x(t) &= s_1(t)\cos(2\pi 500t) + s_2(t)\sin(2\pi 500t) \\
&= 2\cos(2\pi 500t) + 10\sin(2\pi 10t)\sin(2\pi 500t) \\
n(t) &= n_c(t)\cos(2\pi 500t) - n_s(t)\sin(2\pi 500t) \\
x(t)\sin(2\pi 500t) &= 2\cos(2\pi 500t)\sin(2\pi 500t) + 10\sin(2\pi 10t)\sin(2\pi 500t)\sin(2\pi 500t) \\
&= \sin(2\pi 1000t) + 5\sin(2\pi 10t) - 5\sin(2\pi 10t)\cos(2\pi 1000t) \\
n(t)\sin(2\pi 500t) &= n_c(t)\cos(2\pi 500t)\sin(2\pi 500t) - n_s(t)\sin(2\pi 500t)\sin(2\pi 500t) \\
&= \frac{1}{2}n_c(t)\sin(2\pi 1000t) - \frac{1}{2}n_s(t) + \frac{1}{2}n_s(t)\cos(2\pi 1000t)
\end{aligned}
$$

3. Two message signals, $s_1(t) = 2$ and $s_2(t) = 10\sin(20\pi t)$, are modulated to form a QAM signal x(t) with carrier frequency 500Hz. $s_1(t)$ is modulated onto the I-phase, $s_2(t)$ onto the Q-phase. During transmission, x(t) is corrupted by white noise with 2-sided PSD of 10^{-5} Watt/Hz. At the receiver, it is demodulated using a coherent demodulator. Determine the SNR of the Q-branch output signal in dB.

After LPF, the demodulated Q-branch signal is,

$$5\sin(2\pi 10t).$$

The Q-branch output noise is,

$$-\frac{1}{2}n_s(t).$$

Signal power is $\frac{5^2}{2} = 12.5W$, and noise power is $\frac{1}{4}\overline{n_s^2(t)} = \frac{1}{4}\overline{n_i^2(t)} = \frac{1}{4} \times 2 \times B \times \mathrm{PSD}_{2-sided} = \frac{1}{4} \times 2 \times 20 \times 10^{-5} = 10^{-4}W$.

$\mathrm{SNR}_o = 10\log_{10}\frac{12.5}{10^{-4}} = 50.97\mathrm{dB}$.

Things related to PSD:

PROPERTY 2

The mean-square value of a wide-sense stationary random process equals the total area under the graph of the power spectral density; that is,

$$\mathbf{E}[X^2(t)] = \int_{-\infty}^{\infty} S_X(f)\, df \tag{5.94}$$

This property follows directly from Eq. (5.92) by putting $\tau = 0$ and noting that $R_X(0) = \mathbf{E}[X^2(t)]$.

PROPERTY 3

The power spectral density of a wide-sense stationary random process is always nonnegative; that is,

$$S_X(f) \geq 0 \quad \text{for all } f \tag{5.95}$$

This property arises from the fact that the power spectral density $S_X(f)$ is closely related to the expected value of the magnitude squared of the amplitude spectrum of the random process $X(t)$, as shown by

$$S_X(f) \approx \mathbf{E}[|P(f)|^2]$$

The random process $P(f)$ with parameter f is the Fourier transform of $X(t)$. That is, every sample function of $P(f)$ is the Fourier transform of a sample function of $X(t)$.

PROPERTY 4

The power spectral density of a real-valued random process is an even function of frequency; that is,

$$S_X(-f) = S_X(f) \tag{5.96}$$

Textbook P169-P171

Double-Sided PSD & Single-Sided PSD

Both descriptions give the same result. We often use the one-sided noise power spectral density (PSD) because for real-valued processes the negative frequencies are redundant, so defining the PSD for positive frequencies is sufficient. You just have to scale the noise power spectrum such that integrating the one-sided power spectral density (PSD) over the positive frequencies gives the same result as integrating the two-sided PSD over positive and negative frequencies. I.e., for white noise with a constant PSD, defining the one-sided PSD as N_0 means that we would need to define the two-sided PSD as $N_0/2$ such that the noise power remains the same: $N_0W = N_0/2 \cdot 2W$

https://dsp.stackexchange.com/questions/71161/difference-between-double-sided-and-single-sided-awgn-noise-after-bandpass-filte