EE206 2020 Spring

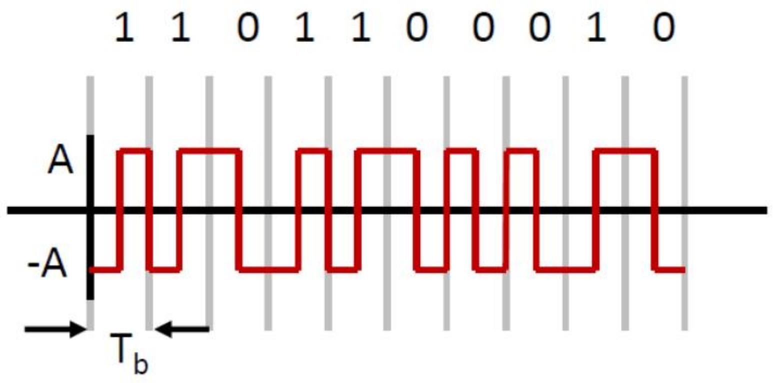
通信原理
习题课

Digital Assignment 6&7

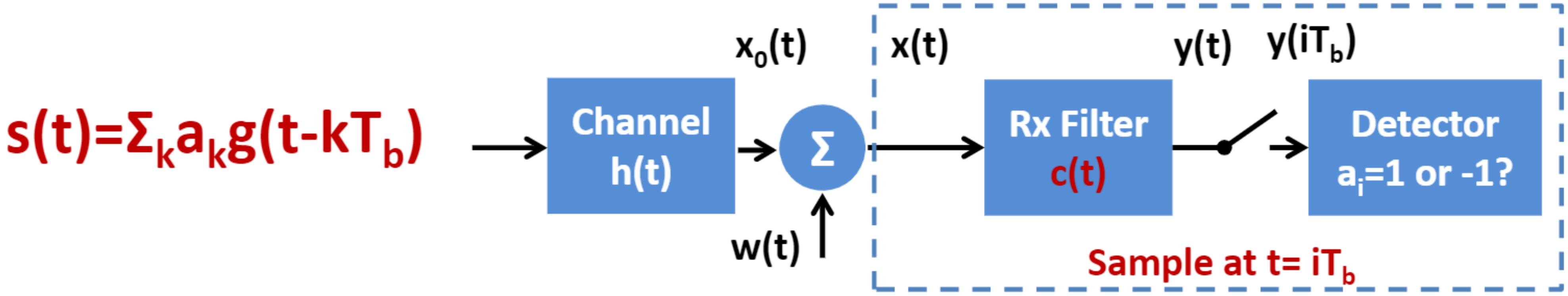
Homework #D6

- D6.1

Please design the receiving filter c(t) for the following *Split-Phase Manchester Code* by both ideal Nyquist channel and raised cosine spectrum.



Split-Phase Manchester Code



- $x(t) = s(t) * h(t) + w(t)$
- $y(t) = s(t) * h(t) * c(t) + c(t) * w(t)$

$$= \sum_k a_k g(t - kT_b) * h(t) * c(t) + c(t) * w(t) = \sum_k a_k \mu p(t - kT_b) + n(t)$$

where $\mu p(t) = g(t) * h(t) * c(t),\ p(0) = 1,\ n(t) = c(t) * w(t)$

- $y(iT_b) = \sum_k a_k \mu p(iT_b - kT_b) + n(iT_b)$

$$= \underbrace{\mu a_i}_{Signal} + \underbrace{\sum_{k \neq i} a_k \mu p(iT_b - kT_b)}_{Inter-Symbol\ Interference} + \underbrace{n(iT_b)}_{Noise}$$

Condition for Distortionless Transmission

- Time domain:

$$p(kT_b) = \begin{cases} 1, & k = 0 \\ 0, & k \text{ is an integer and } k \neq 0 \end{cases}$$

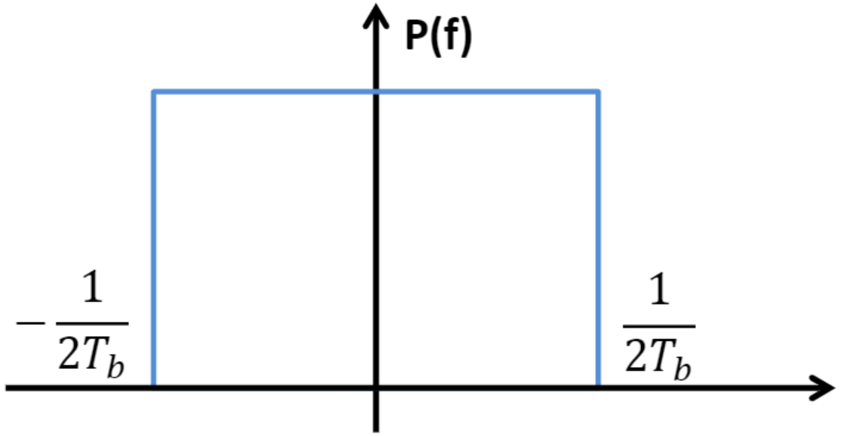
- Frequency domain (Nyquist's Criterion):

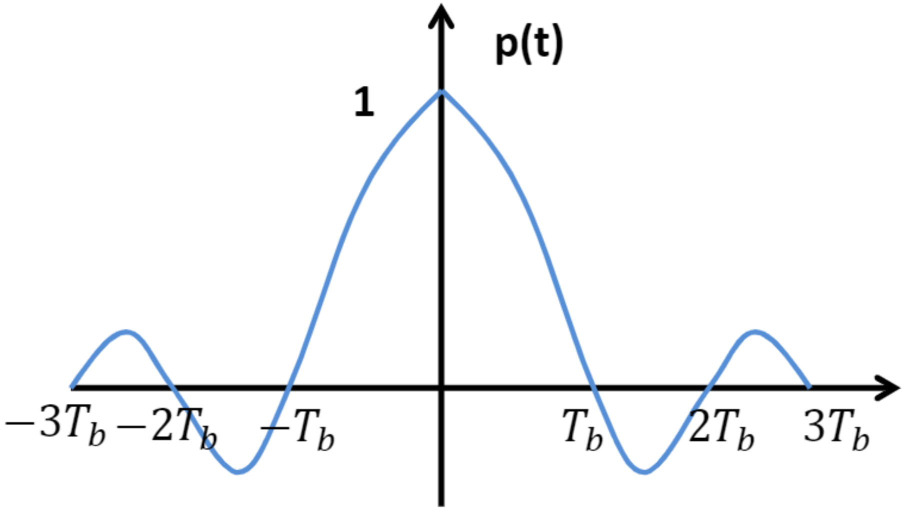
$$\sum_{n=-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right) = \mathrm{T_b}$$

- The two conditions are equivalent.
- Without noise, they can guarantee distortionless communications.

Sinc Wave: Ideal Nyquist Channel

- Sinc wave can satisfy the Nyquist's criterion





- Choose c(t), such that

$$p(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} = sinc(t/T_b)$$

Ideal Nyquist channel

Raised Cosine Spectrum

- Raised cosine spectrum is given by

$$P(f) = \begin{cases} T_b, & 0 \le |f| < f_1 \\ \frac{T_b}{2}\left\{1 - \sin\left[\frac{\pi(|f|-W)}{2W-2f_1}\right]\right\}, & f_1 \le |f| < \frac{1}{T_b} - f_1 \\ 0, & |f| \ge \frac{1}{T_b} - f_1 \end{cases}$$

$$p(t) = sinc\left(\frac{t}{T_b}\right)\left(\frac{\cos(2\pi\alpha Wt)}{1-16\alpha^2 w^2 t^2}\right)$$

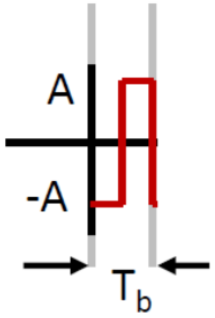
Where $W = \frac{1}{2T_b}$

- Rolloff factor $\alpha = 1 - 2T_b f_1$ $\quad$ ($\alpha = 0?$)

Solution

$$\mu p(t) = g(t) * h(t) * c(t)$$

$g(t)$:



$$G(f) = \int_{0}^{T_b/2} -Ae^{-i2\pi ft}dt + \int_{T_b/2}^{T_b} Ae^{-i2\pi ft}dt$$

$$= \frac{iA}{2\pi f}\left(2e^{-i\pi Tbf} - e^{-i2\pi Tbf} - 1\right)$$

$$h(t) \overset{FT}{\leftrightarrow} H(f)$$

For ideal Nyquist channel,

$$p(t) = sinc\left(\frac{t}{T_b}\right)$$

$$P(f) = T_b rect(tTb)$$

For Raised Cosine Spectrum,

$$P(f) = \begin{cases} T_b, & 0 \le |f| < f_1 \\ \frac{T_b}{2}\left\{1 - \sin\left[\frac{\pi(|f|-W)}{2W-2f_1}\right]\right\}, & f_1 \le |f| < \frac{1}{T_b} - f_1 \\ 0, & |f| \ge \frac{1}{T_b} - f_1 \end{cases}$$

$$p(t) = sinc(\frac{t}{T_b})\left(\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 w^2 t^2}\right) \quad \text{Where } W = \frac{1}{2T_b}$$

Rolloff factor $\alpha = 1 - 2T_b f_1$

$$C(f) = \frac{\mu P(f)}{G(f)H(f)}$$

- D7.1

Please design a receiver of the following band-pass modulation for AWGN channel. What is the BER?

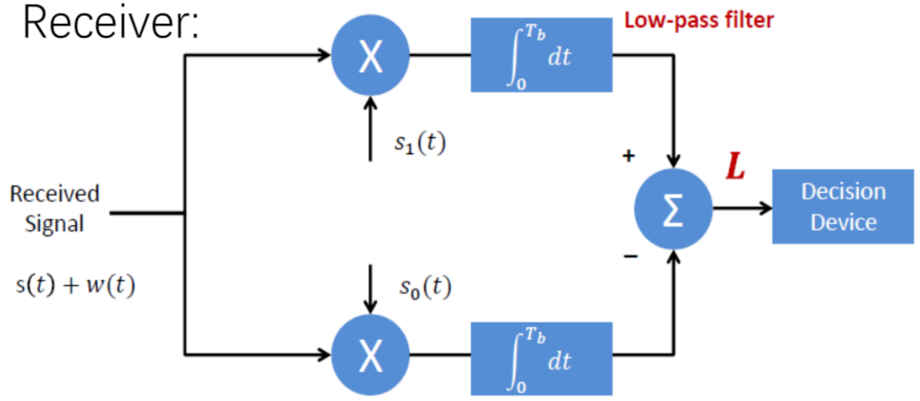
$$
\begin{aligned}
Bit\ 1:&\ \ s(t) = s_1(t) = A_c \cos(2\pi f_c t) & 0 \le t \le T_b \\
Bit\ 0:&\ \ s(t) = s_0(t) = A_c \cos(2\pi f_c t + \phi) & 0 \le t \le T_b
\end{aligned}
$$

$$Bit\ 1:\ s(t) = s_1(t) = A_c\cos(2\pi f_c t) \quad 0 \le t \le T_b$$
$$Bit\ 0:\ s(t) = s_0(t) = A_c\cos(2\pi f_c t + \phi) \quad 0 \le t \le T_b$$

Solution

Assume AWGN channel with white noise of *zero-mean* and *spectral density* $N_o/2$

Receiver:



The receiver output L is given by

$$L = \int_0^{T_b} x(t)[s_1(t) - s_0(t)]dt$$

BER is given by

$$P_e = Q\left(\sqrt{\frac{\boxed{E_b}(1-\boxed{\rho})}{N_0}}\right)$$

Signal Energy

$$E_b = \int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_0^2(t)dt = \frac{A_c{}^2T_b}{2}$$

Correlation coefficient

$$\rho = \frac{\int_0^{T_b} s_0(t)s_1(t)dt}{E_b} \in [-1,1]$$
$$= cos\phi$$

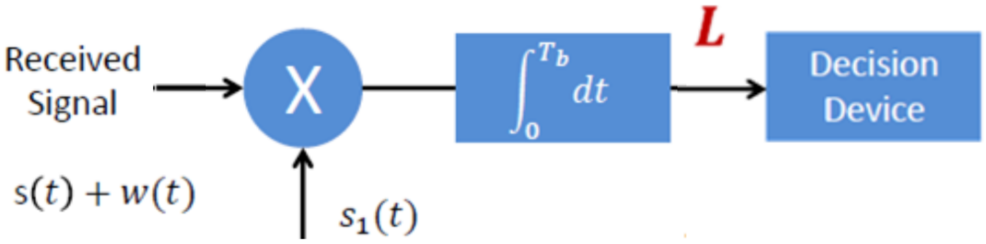
$$\text{Bit 1: } s(t) = s_1(t) = A_c \cos(2\pi f_c t) \quad 0 \le t \le T_b$$
$$\text{Bit 0: } s(t) = s_0(t) = A_c \cos(2\pi f_c t + \phi) \quad 0 \le t \le T_b$$

Discuss

Assume $\phi = \pi$, *BPSK*

Due to s0(t) is the negative of s1(t), the receiver reduces to a single path

Receiver:



The receiver output L is given by

$$L = \int_0^{T_b} x(t)\boxed{[s_1(t) - s_0(t)]}dt = \int_0^{T_b} x(t)\boxed{2s_1(t)}dt$$

Signal Energy

$$E_b = \int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_0^2(t)dt = \frac{{A_c}^2 T_b}{2}$$

Correlation coefficient

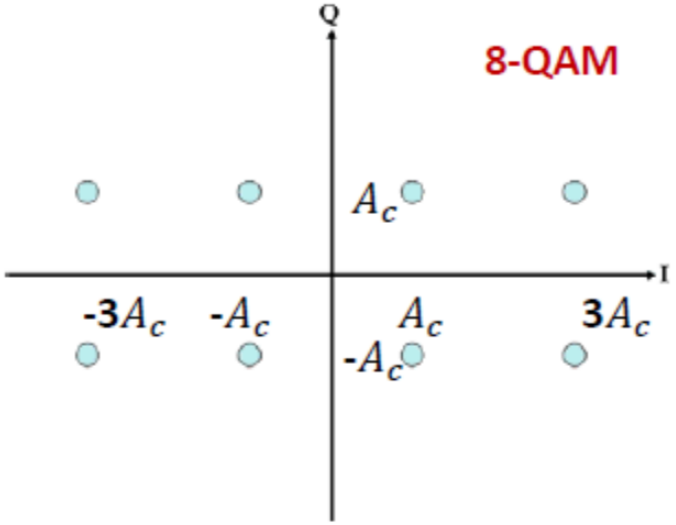
$$\rho = \frac{\int_0^{T_b} s_0(t)s_1(t)dt}{E_b} \in [-1,1]$$
$$= -1$$

BER is given by

$$P_e = Q\left(\sqrt{\frac{E_b(1-\rho)}{N_0}}\right) = Q\left(\sqrt{\frac{\boxed{2E_b}}{N_0}}\right)$$

- D7.2

What is the average transmission power of the following 8-QAM modulation scheme? Suppose each symbol is transmitted with equal probability, and the symbol duration is T.

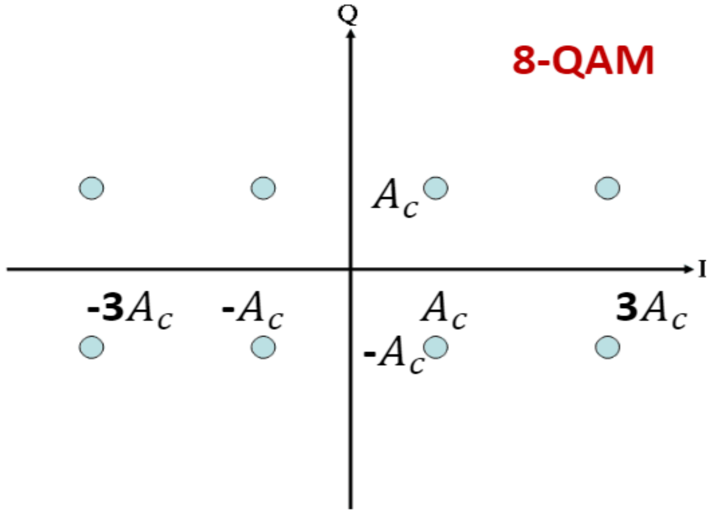


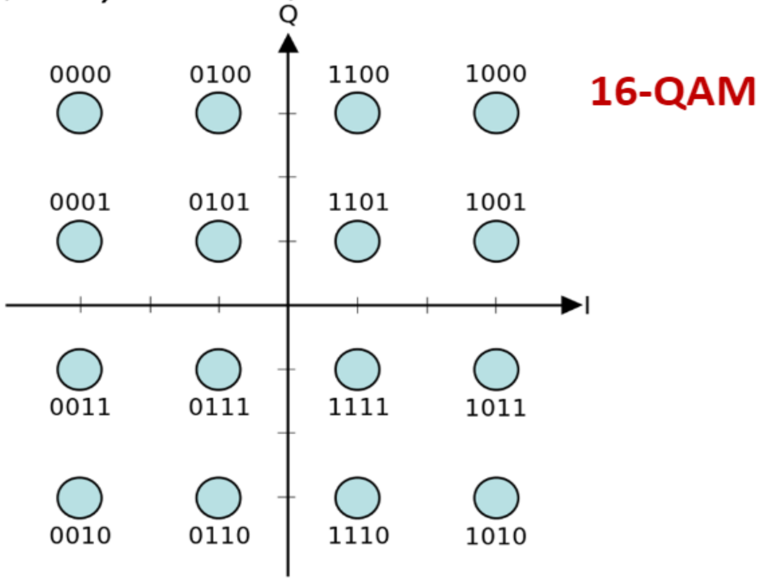
M-QAM

- M-Array Quadrature Amplitude Modulation

$$s(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$$
$$g_I = \pm A_c, \pm 3A_c, \dots, \pm(2N_I+1)A_c$$
$$g_Q = \pm A_c, \pm 3A_c, \dots, \pm(2N_Q+1)A_c$$

- Complex envelope $\tilde{s} = g_I + jg_Q = \pm(2n_I+1)A_c \pm (2n_Q+1)A_c j$
- QPSK = 4-QAM, 8-QAM, 16-QAM, 64-QAM, 256-QAM…





Solution

8 symbols total power

$$P_{total} = 4 \times \left(\frac{1}{2}\left(\sqrt{2}A_c\right)^2\right) + 4 \times \left(\frac{1}{2}\left(\sqrt{10}A_c\right)^2\right) = 24{A_c}^2$$

average transmission power with equal probability

$$P_{symbol} = \frac{E_{total}}{8} = 3{A_c}^2$$