Outline

① Introduction
② Amplitude Modulation
③ Angle Modulation

Amplitude Modulation - Topics

I. Review
   - Fourier Transform
   - Filters
   - Signal Power
   - Conventional/Full AM
II. Double-Sideband Suppressed Carrier (DSBSC) AM
III. AM Related Systems and Applications
IV. Noise Performance of AM Signals

Fourier Transform

- A mathematical operation which gives the frequency component of a time-domain signal:

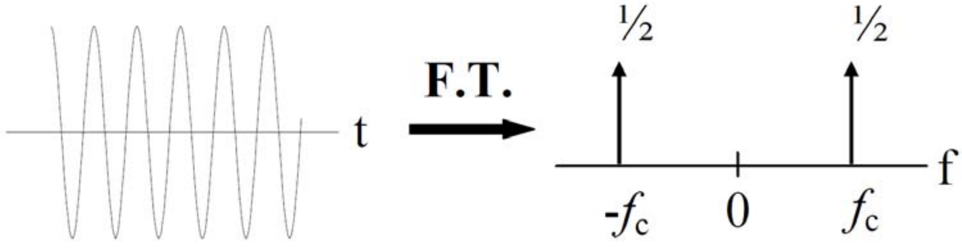
$$\text{F.T. of } v(t),\ V(f) \equiv \int_{-\infty}^{\infty} v(t)\exp(-j2\pi ft)\,dt$$

- Two math tables, *Fourier Transform Pairs* and *Fourier Transform Properties*, are placed at the end of the course notes.

Fourier Spectrum (1)

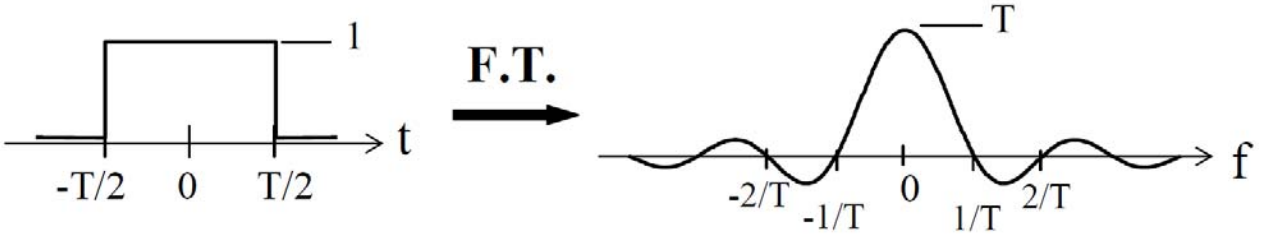
- A plot of Fourier Transform on the positive and negative frequency axes.

❑ F.T. of $\cos(2\pi f_c t) = \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$

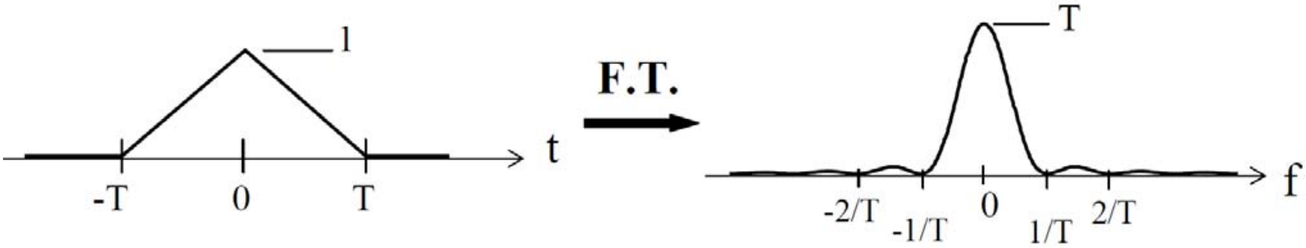


Fourier Spectrum (2)

- F.T. of $\text{rect}\left(\frac{t}{T}\right) = T\,\text{sinc}\,(f\,T)$. Note $\boxed{\text{sinc}\,(x) \equiv \frac{\sin(\pi x)}{\pi x}}$



- F.T. of $\Lambda\left(\frac{t}{T}\right) = T\,\text{sinc}^2(f\,T)$



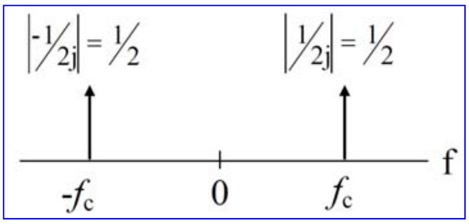
Amplitude Spectrum

- In general, the Fourier Transform $V(f)$ is a complex function of frequency f, so that it can be rewritten as

$$V(f) = |V(f)| \exp[\,j\,\theta(f)\,]$$

- Fourier Transform has 2 components:
  - Amplitude spectrum $|V(f)|$
  - Phase spectrum $\theta(f)$

- F.T. of $\sin(2\pi f_c t) = \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$



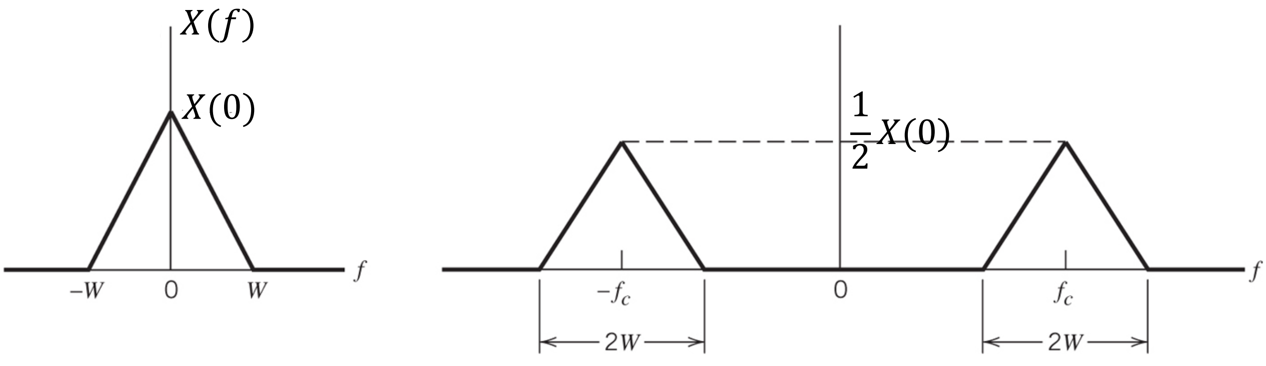
- F.T. of $\text{rect}\left(\frac{t}{T}\right) = T\,\text{sinc}\,(f\,T)$. What is the amplitude spectrum?

✓ Usually, spectrum refers to amplitude spectrum unless otherwise stated.

Modulation Theorem

- A shift in frequency domain.
- From frequency shifting property of F. T., it follows that

$$\mathcal{F}[x(t)e^{j2\pi f_c t}] = X(f - f_c)$$
$$\mathcal{F}[x(t)\cos 2\pi f_c t] = [X(f - f_c) + X(f + f_c)]/2$$

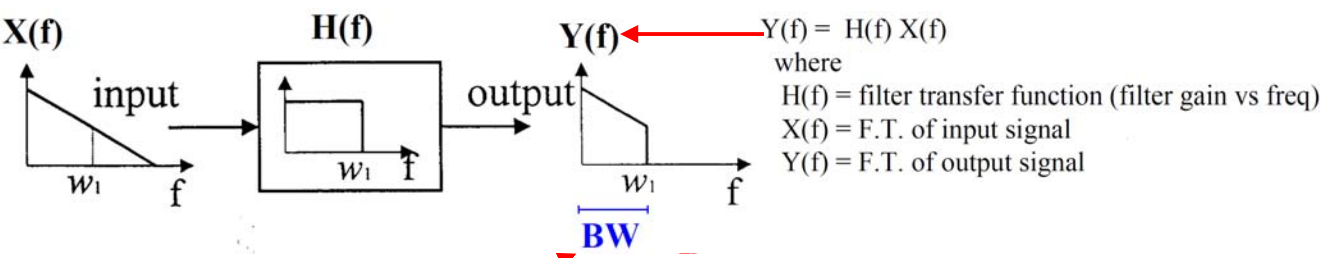


(a) (b)

Filters (1)

- A filter is a device which removes undesired frequency components of the input signal.

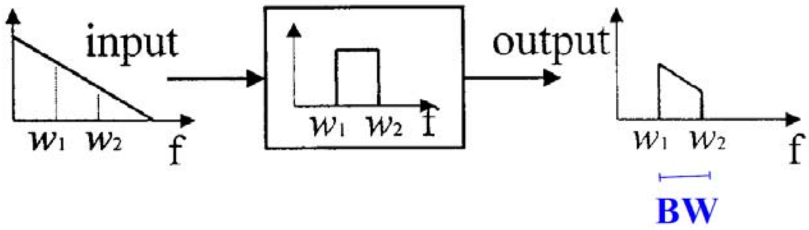
✓ *Lowpass filter* passes through all frequencies less than or equal to some w_1 and rejects all frequencies beyond w_1.



Bandwidth (BW) is defined as range of **positive** frequency occupied by a spectrum

Filters (2)

- ✓ *Bandpass filter* passes through all frequency components within an interval $w_1 \leq f \leq w_2$.



- ✓ High-pass filter
- ✓ Band-stop filter

Signal Power

- The power of a signal $v(t)$ can be calculated in 2 domains.
- ✓ Time domain:

$$\overline{v^2(t)} = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |v(t)|^2 \, dt$$

where the “overbar” means “taking average of”, T_0 is the repetition period of $v(t)$, and | | means absolute value.

- ✓ Frequency domain:

$$\overline{v^2(t)} = \int_{-\infty}^{\infty} S_v(f)\, df$$

where $S_v(f)$ is the power spectral density (PSD) of $v(t)$.

$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 \rightarrow \frac{1+\cos(2x)}{2} = \cos^2 x$

Signal Power - Exercise

- Given $v(t) = A\cos(2\pi f_c t + \phi)$, find its power.

$\overline{v^2(t)} = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |v(t)|^2 dt$

❑ Time-domain approach

❑ Frequency-domain approach

✓ The answer is $A^2/2$, which does not depend on frequency f_c and phase ϕ, hence the result applies to the sine function too.

Types of AM

- Conventional/Full AM
  - Analog broadcast radio
- DSB Suppressed-Carrier AM
  - Satellite Communications
- Single Sideband AM
  - Long distance telephone links
- Vestigial Sideband AM
  - Analog broadcast TV
- Quadrature AM
  - PC modem, wireless LAN, digital TV

See Section 3.5 on pages 88-91

Conventional AM

- Consider a sinusoidal carrier wave given by $A_c \cos(2\pi f_c t)$. Let $m(t)$ denote the modulating signal (i.e., message). The full AM is given by

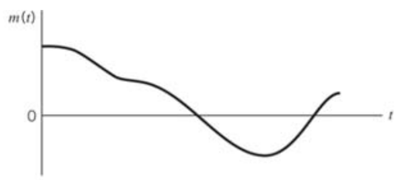
Amplitude sensitivity, a constant (1/volt)

$$x(t) = A_c[1 + \boxed{k_a} m(t)] \cos 2\pi f_c t$$
$$= \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} + \underbrace{A_c k_a m(t) \cos 2\pi f_c t}_{\text{sidebands}}$$

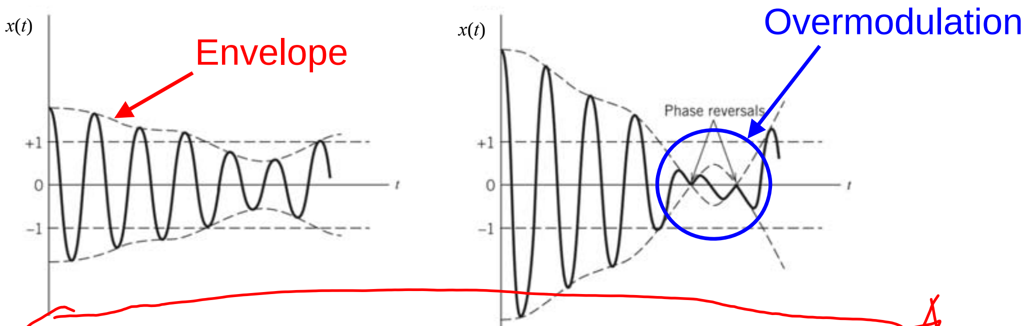
- Fourier transform of $x(t)$:

$$X(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$
$$+ \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

Waveform of Conventional AM



Message signal



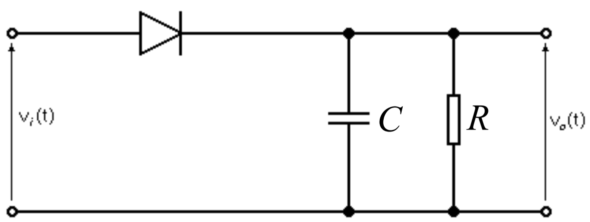
AM wave with $|k_a m(t)|$<1 AM wave with $|k_a m(t)|$>1

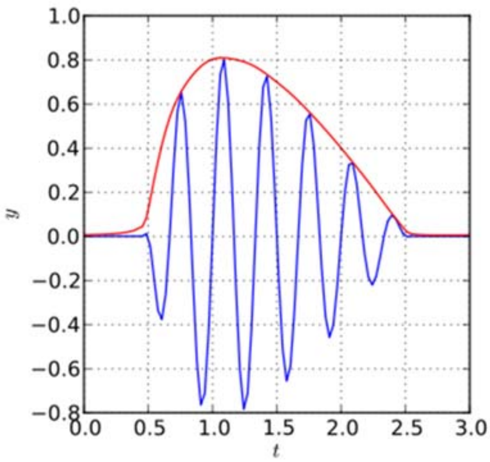
✓ The envelope of $x(t)$ has the same shape as the message, provided that:

1. $|k_a m(t)|$<1 for all t;
2. $f_c \gg W$, where W is the message bandwidth.

Envelope Detection (包络检测)

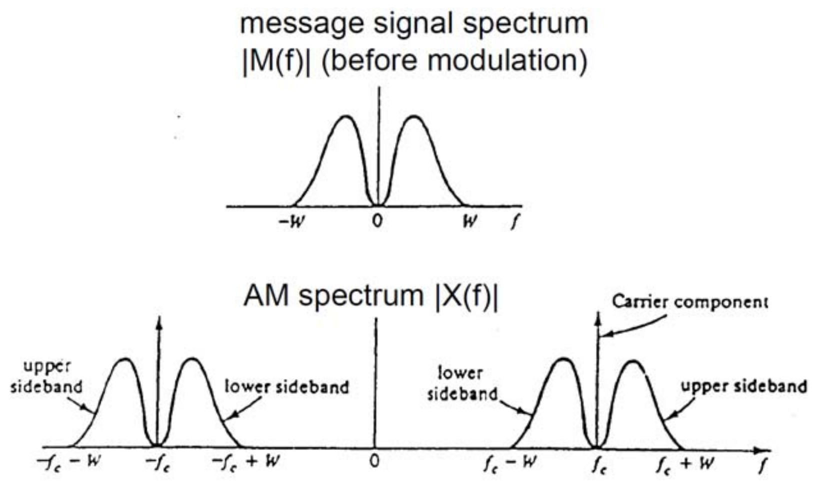
- The simplest form of **envelope detector** is the **diode detector:**





- In order for this diode detector to work, it is required that $\frac{1}{f_c} \ll RC \ll \frac{1}{W}$.

Spectrum of Conventional AM



- Upper sideband and lower sideband are symmetrical.
- $f_c > W$ to ensure the sidebands do not overlap.
- Transmission bandwidth of a conventional AM is exactly twice the message bandwidth W, that is, $2W$.

Example: Single-Tone AM

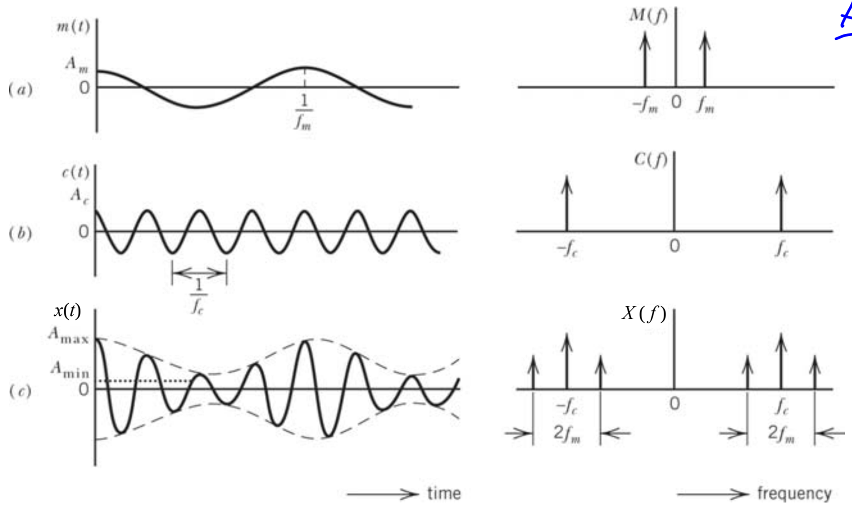
$= A_c \cos 2\pi f_c t + \mu \cos 2\pi f_m t \cos 2\pi f_c t$

- The message contains a single tone/frequency component, i.e., $m(t) = A_m \cos 2\pi f_m t$.
- The single-tone AM is given by $x(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$, where $\mu = k_a A_m$.

Modulation index of AM

- To avoid overmodulation, μ must be kept below 1.

$\frac{A_c}{2}[\delta(f+f_c) + \delta(f-f_c)]$



(a) Modulating wave

(b) Carrier wave

(c) AM wave

Amplitude Modulation - Topics

I. Review
  - Fourier Transform
  - Filters
  - Signal Power
  - Conventional/Full AM

II. Double-Sideband Suppressed Carrier (双边带抑制载波) (DSBSC) AM

III. AM Related Systems and Applications

IV. Noise Performance of AM Signals

Double-Sideband Suppressed-Carrier AM

What we will discuss:

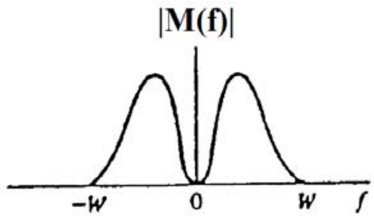
1. Why need to suppress the carrier?
2. Time-domain expression
3. Frequency-domain expression
4. How to demodulate?

- The transmission of the carrier in conventional AM is a waste of power.
- In DSBSC-AM, we suppress the transmission of the carrier component.

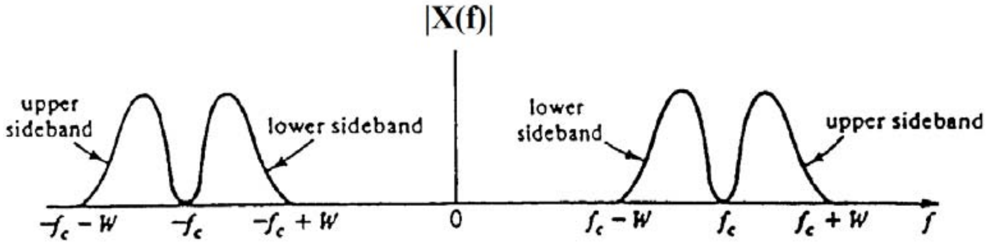
Expression & Spectrum of DSBSC-AM

- The DSBSC AM modulated signal $x(t)$ is
  $x(t) = m(t) \cdot A_c \cos 2\pi f_c t$
- The signal spectrum is obtained as

$$X(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$



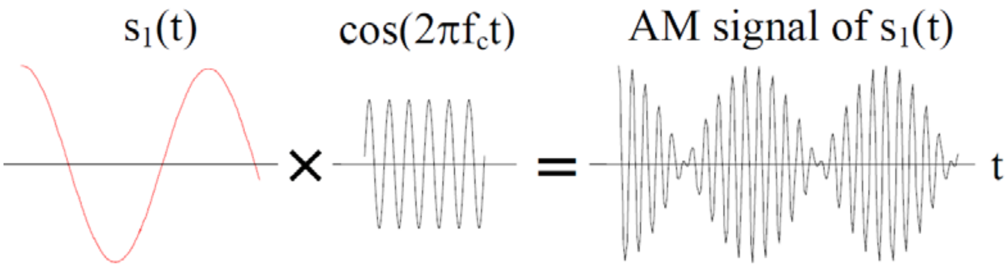
Note: DSBSC-AM shifts the center freq. from 0 to $\pm f_c$, and doubles the BW, while maintaining the spectral shape



Examples of DSBSC-AM (1)

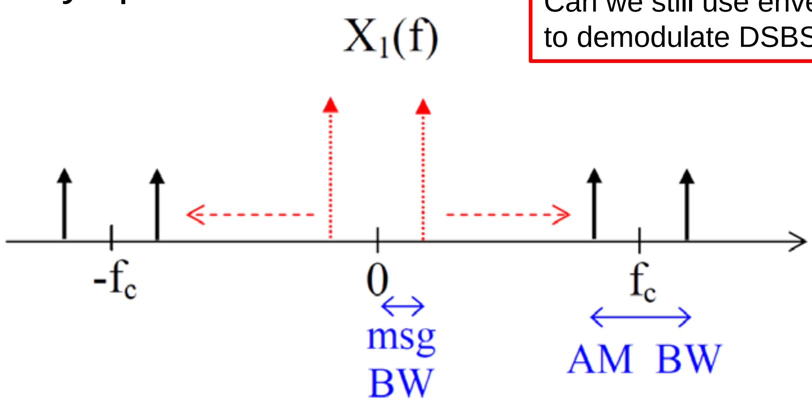
- Time waveform:

The AM signal has an envelope that follows the message waveform



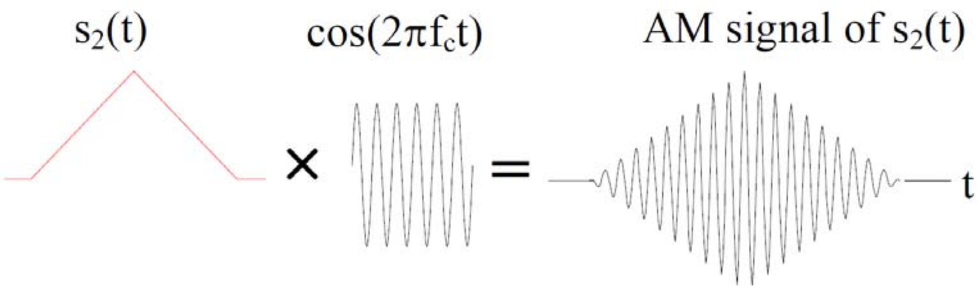
- Frequency spectrum

Can we still use envelope detector to demodulate DSBSC-AM signals?

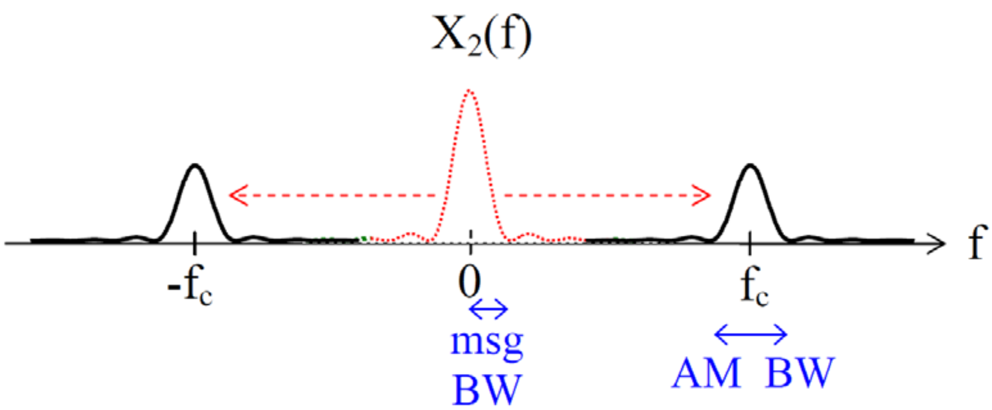


Examples of DSBSC-AM (2)

- Time waveform:

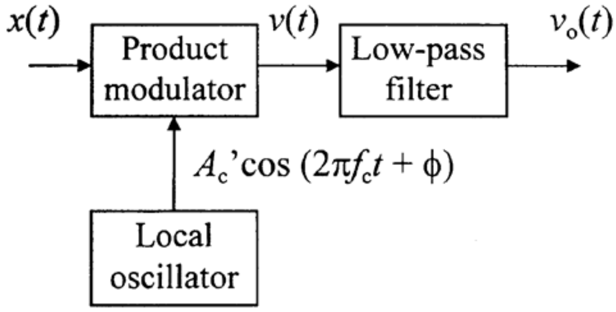


- Frequency spectrum



(相干解调)　(同步解调)

Coherent/Synchronous Demodulation



The job of demodulation is to recover the message $m(t)$ from the DSBSC-AM signal $x(t)$.

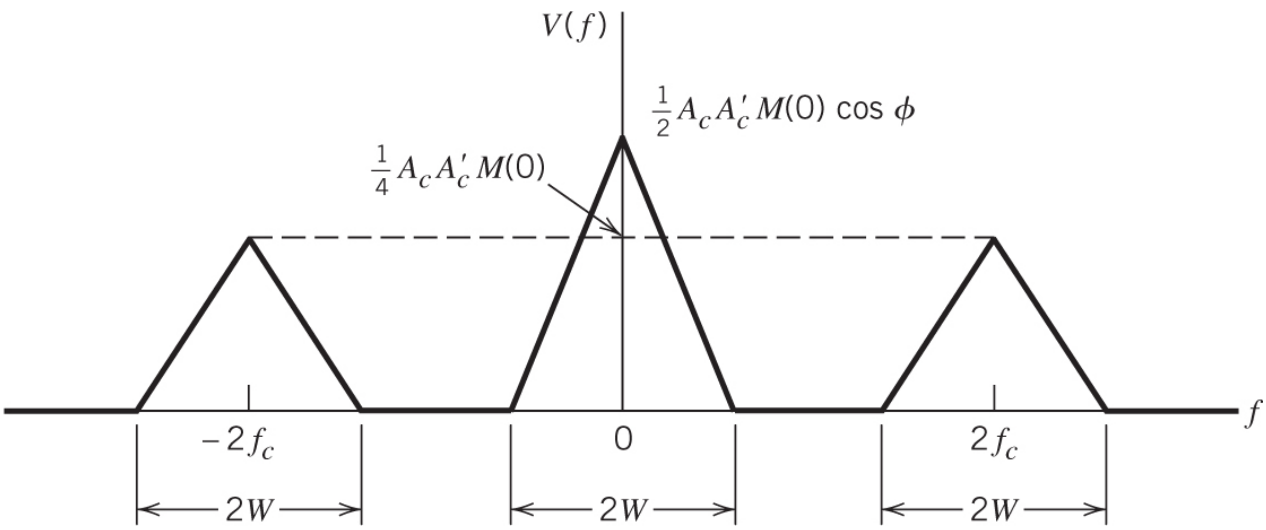
1. Input (DSBSC): $x(t) = m(t) \cdot A_c \cos 2\pi f_c t$
2. $v(t) = A_c{}' \cos(2\pi f_c t + \phi) \cdot m(t) \cdot A_c \cos 2\pi f_c t$

$$= \frac{1}{2} A_c{}' A_c m(t) \cos\phi + \frac{1}{2} A_c{}' A_c m(t) \cos(4\pi f_c t + \phi)$$

3. $v_0(t) = \frac{1}{2} A_c{}' A_c m(t) \cos\phi = \text{constant} \times m(t)$

✓ The output contains a phase error, ϕ. Output = 0 if $\phi = \pm\pi/2$. Output is max if $\phi = 0$, i.e., the local oscillator has the same frequency & phase (synchronized, coherent) as the carrier component in the AM signal.

Illustrating the spectrum of $v(t)$



维持同步

✓ The local oscillator at the receiver must maintain perfect synchronization, in both frequency and phase, with the carrier wave used at the transmitter.

Example of Coherent Demodulation

Let message signal = sinc(t/50) carrier signal = cos(2π 70 t)

DSBSC-AM signal: x(t) = sinc(t/50) cos(2π 70 t)

The coherent demodulator multiplies x(t) by local carrier cos(2π 70 t) to get:

$x(t)\cos(2\pi\, 70\, t)$

$= \mathrm{sinc}(t/50)\ \underline{\cos^2(2\pi\, 70\, t)}$

$= \mathrm{sinc}(t/50) \times \underline{\tfrac{1}{2}\,[1+\cos 2(2\pi\, 70\, t)]}$

$= \tfrac{1}{2}\,\mathrm{sinc}(t/50) + \tfrac{1}{2}\,\mathrm{sinc}(t/50)\cos(2\pi\, 140t)$

= recovered message signal + unwanted high-freq component

After LPF with BW = 1/100 Hz (message signal BW)

output signal = ½ sinc(t/50)

Can you show the effect of demodulation on the AM signal spectrum?

Remarks:

- In Full AM systems, the sidebands are transmitted in full together with the carrier. The demodulation of such AM signal can be accomplished rather simply by using an envelope detector or a square-law detector.

- In DSBSC-AM systems, the receiver is more complex because additional circuitry is needed to recover the carrier (maintain frequency and phase synchronization). However, it requires much less power to transmit the same amount of information, which makes the transmitter less expensive.

$\cos 2x = 2\cos^2 x - 1 \rightarrow \frac{1+\cos 2x}{2} = \cos^2 x$

$1+k_a^2 m^2(t)+2k_a m(t)$ $\quad$ $a v_i(t) = aA_c[1+k_a m(t)]\cos 2\pi f_c t$

Exercise:

$b v_i^2(t) = bA_c^2[1+k_a m(t)]^2 \cos^2 2\pi f_c t$

Consider a square-law detector, using a nonlinear device defined by

$= [bA_c^2 + bA_c^2 k_a^2 m^2(t) + 2k_a m(t)][\frac{1}{2}+\frac{1}{2}\cos(4\pi f_c t)]$

$$v_o(t) = av_i(t) + b{v_i}^2(t)$$

where a and b are constants, $v_i(t)$ is the input, and $v_o(t)$ is the output. The input consists of the AM wave

$$v_i(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$$

a) Evaluate the output $v_o(t)$.

b) Find the conditions for which the message signal $m(t)$ can be recovered from $v_o(t)$.

预习到这

Amplitude Modulation - Topics

I. Review
- Fourier Transform
- Filters
- Signal Power
- Conventional/Full AM

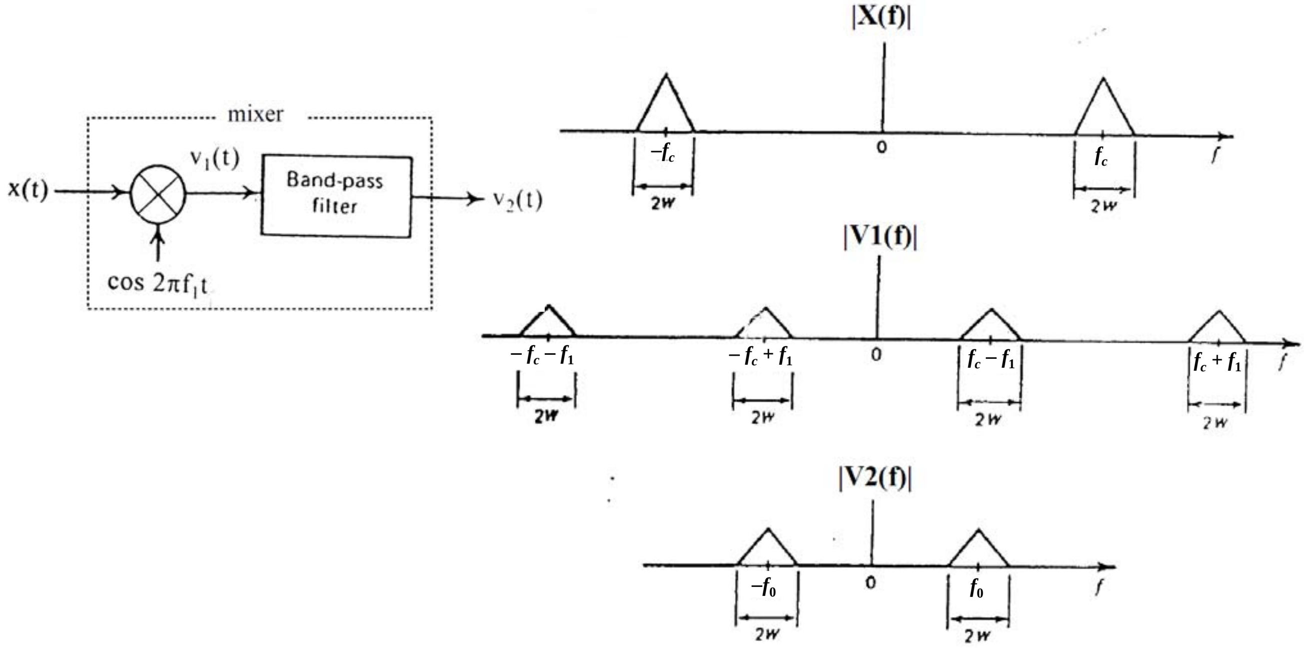
II. Double-Sideband Suppressed Carrier (DSBSC) AM

III. AM Related Systems and Applications

IV. Noise Performance of AM Signals

Frequency Translation (1)

- During the processing of signals in a communication system, it is often necessary to translate a modulated signal upward or downward in the frequency domain.



Frequency Translation (2)

- For a modulated signal $x(t) = m(t)\cos 2\pi f_c t$,

$$\begin{aligned} v_1(t) &= x(t) \cdot \cos 2\pi f_1 t \\ &= m(t)\cos 2\pi f_c t \cos 2\pi f_1 t \\ &= \frac{m(t)}{2}[\cos 2\pi(f_c - f_1)t + \cos 2\pi(f_c + f_1)t] \end{aligned}$$

- Assuming $f_c > f_1$, if $v_1(t)$ is passed through a bandpass filter with a centre frequency $f_0 = f_c - f_1$, then $x(t)$ will occupy a new frequency band. That is

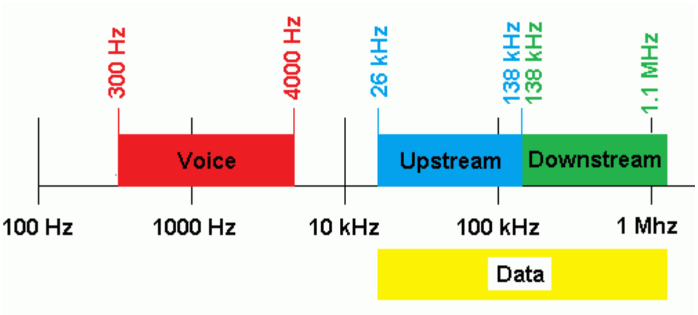
$$\begin{aligned} v_2(t) &= [v_1(t)]_{BP} \\ &= \frac{1}{2}m(t)\cos 2\pi(f_c - f_1)t = \frac{1}{2}m(t)\cos 2\pi f_0 t \end{aligned}$$

- The device that carries out the frequency translation of a modulated signal is called mixer, and the operation itself is called mixing.

Frequency-Division Multiplexing

- Multiplexing is a technique that combines multiple signals for transmission over a common channel, e.g., one submarine cable for many IDD calls, one ADSL phone line for simultaneous tel+internet calls.
- The multiplexed/combined signals must be separable at the receiver without interfering with each other. This is accomplished by separating the different signals in the frequency or time domain.
- The multiplexing technique that separates signals in the frequency domain is called Frequency Division Multiplexing (FDM).
- Basically, FDM ensures that the spectra of different signals are sufficiently separated and do not overlap.

FDM in ADSL:



FDM Example

- Consider 3 message signals:

  $s_1(t)$ = triangular signal $\Lambda(t)$, $s_2(t) = \text{sinc}^2(t)$, $s_3(t) = \text{sinc}(t)$

- ❑ They cannot be transmitted at the same time by simple summation because they overlap both in time and frequency.

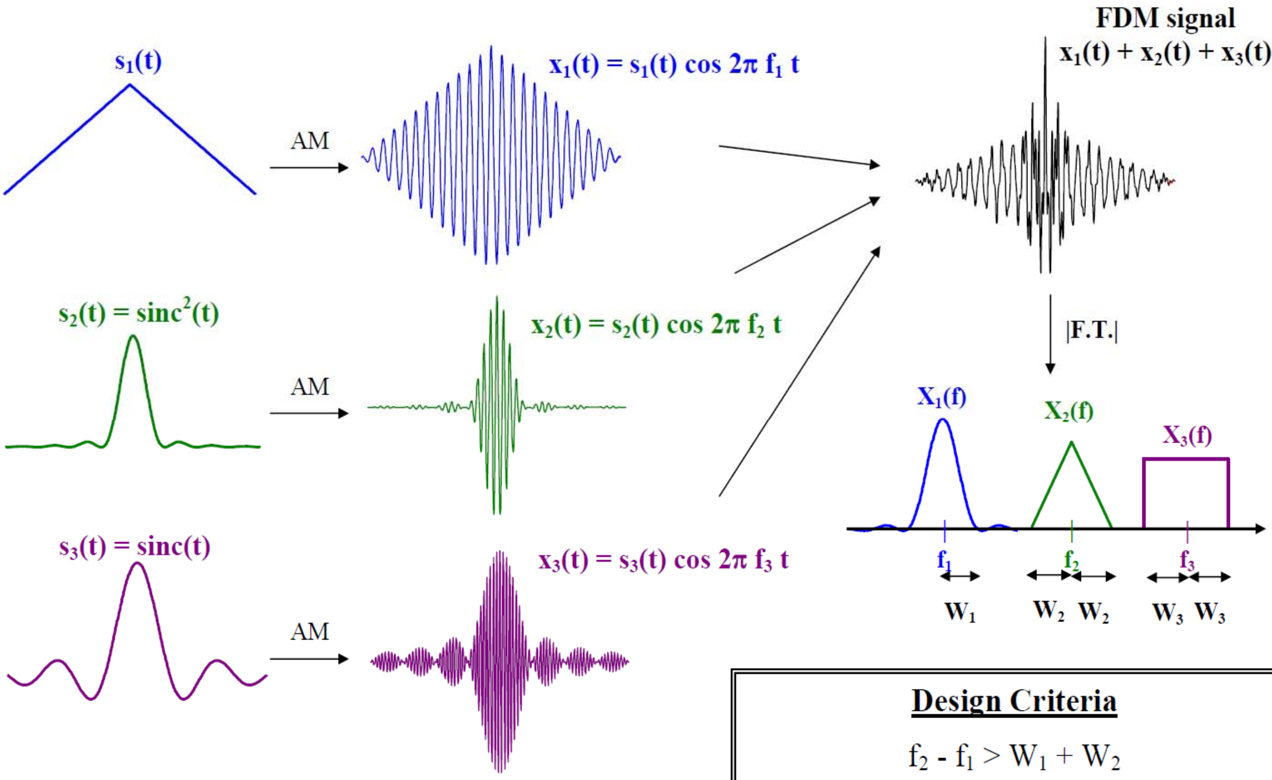
- Now, modulate $s_1(t)$, $s_2(t)$ and $s_3(t)$ using different carrier freq:

  $x_1(t) = s_1(t)\cos(2\pi f_1 t)$, $x_2(t) = s_2(t)\cos(2\pi f_2 t)$, $x_3(t) = s_3(t)\cos(2\pi f_3 t)$

- ❑ The summed signal $x_1(t)+ x_2(t)+ x_3(t)$ overlaps in time, but can be made to be non-overlapping in frequency with careful design of the carrier frequencies f_1, f_2 and f_3.

- At the receiver, $x_1(t)+ x_2(t)+ x_3(t)$ can be separated into individual $x_1(t)$, $x_2(t)$ and $x_3(t)$ by bandpass filtering, followed by respective AM demodulation to recover $s_1(t)$, $s_2(t)$ and $s_3(t)$.
- ❑ Hence, AM modulation provides a means to multiplex signals in the frequency domain → FDM.

Multiplexing



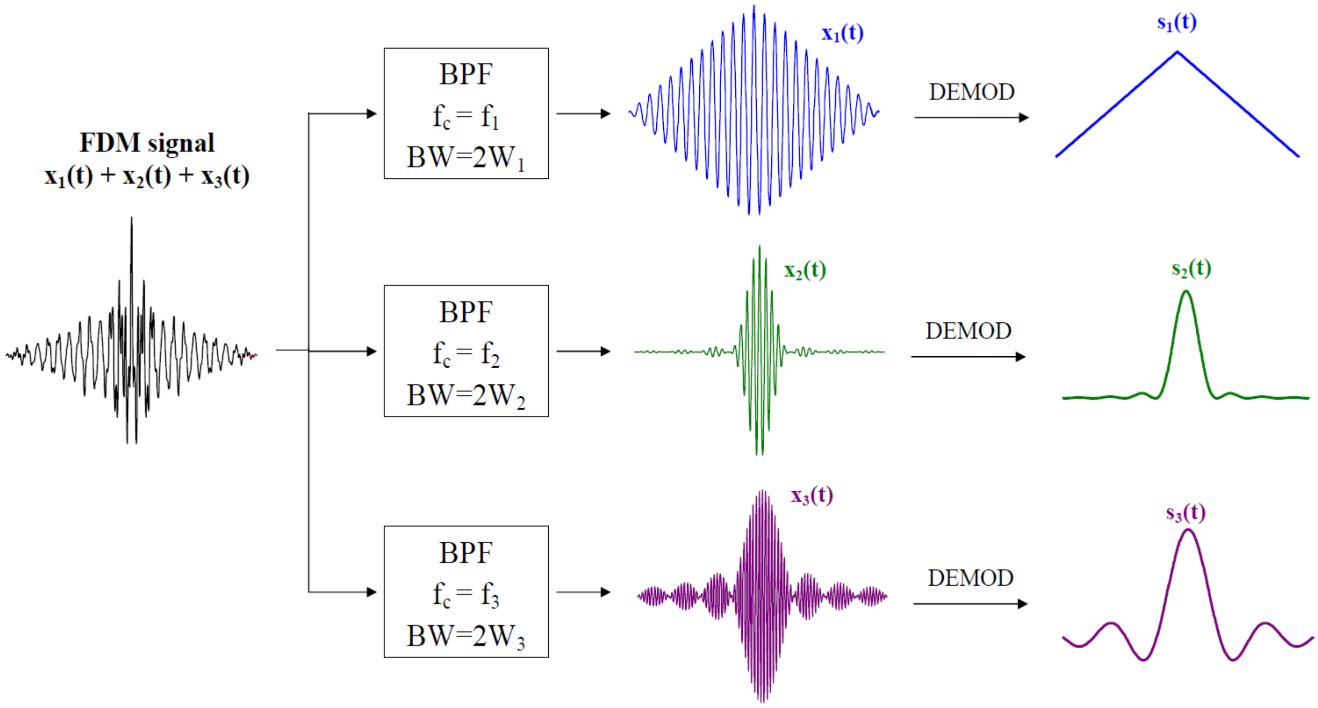
Design Criteria

$f_2 - f_1 > W_1 + W_2$

$f_3 - f_2 > W_2 + W_3$

f_i = carrier freq of $x_i(t)$, W_i = BW of $s_i(t)$.

De-multiplexing

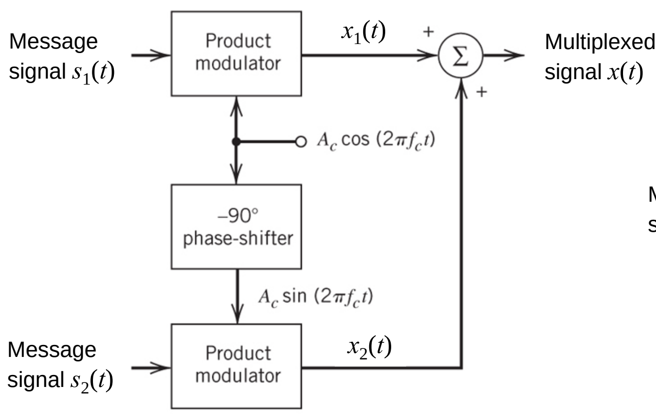


Quadrature Amplitude Modulation (QAM)

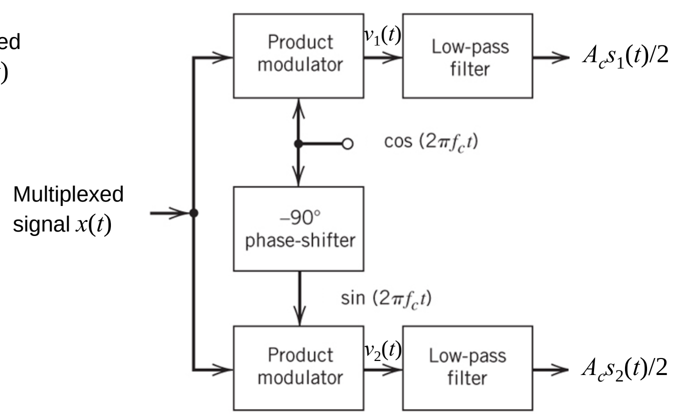
- Also called Quadrature Carrier Multiplexing.
- QAM enables two DSBSC AM signals to occupy the same transmission bandwidth by using the same carrier frequency.
- QAM is used in almost all broadband systems:
  - High-speed telephone modem
  - Cable modem
  - Digital TV
  - 54 Mbps wireless LAN
  - 4G mobile cellular
  - Power line communication

QAM Generation and Detection

QAM Generation:



QAM Detection:



QAM Generation:

$x_1(t) = A_c\, s_1(t) \cos 2\pi f_c t$ = in-phase (I) signal

$x_2(t) = A_c\, s_2(t) \sin 2\pi f_c t$ = quadrature (Q) signal

$\therefore\ x(t) = x_1(t) + x_2(t) = A_c\, s_1(t) \cos 2\pi f_c t + A_c\, s_2(t) \sin 2\pi f_c t$

where $s_1(t)$ and $s_2(t)$ are two different message signals.

QAM Detection:

$v_1(t) = x(t) \cos 2\pi f_c t = A_c\, s_1(t) \cos^2 2\pi f_c t + A_c\, s_2(t) \sin 2\pi f_c t \cos 2\pi f_c t$

$\quad = \frac{1}{2} A_c\, s_1(t) + \frac{1}{2} A_c\, s_1(t) \cos 4\pi f_c t + \frac{1}{2} A_c\, s_2(t) \sin 4\pi f_c t$

Similarly, we get

$v_2(t) = x(t) \sin 2\pi f_c t$

$\quad = \frac{1}{2} A_c\, s_2(t) - \frac{1}{2} A_c\, s_2(t) \cos 4\pi f_c t + \frac{1}{2} A_c\, s_1(t) \sin 4\pi f_c t$

Thus, $\frac{1}{2} A_c\, s_1(t)$ and $\frac{1}{2} A_c\, s_2(t)$ can be obtained after the lowpass filters.

Hence, the two message signals $s_1(t)$ and $s_2(t)$ can be recovered.

Amplitude Modulation - Topics

I. Review
   - Fourier Transform
   - Filters
   - Signal Power
   - Conventional/Full AM

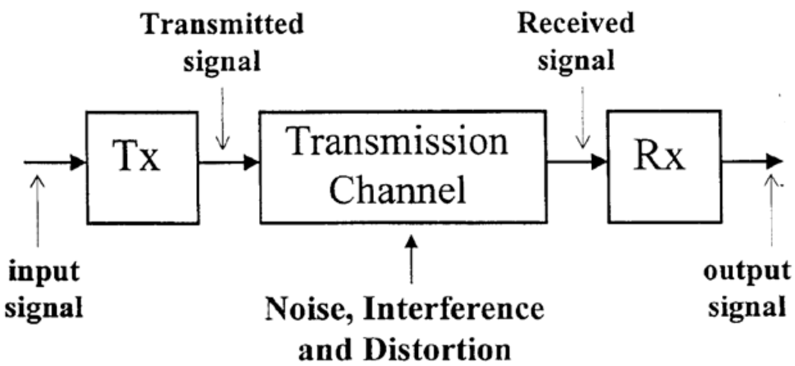
II. Double-Sideband Suppressed Carrier (DSBSC) AM

III. AM Related Systems and Applications

IV. Noise Performance of AM Signals

Channel Impairments in Communications

- In a communication system, there are various undesirable effects in the course of signal transmission.
- Attenuation is undesirable as it reduces the signal strength at the receiver. However, it does not alter the signal waveform. More serious are distortion, interference, and noise.



扰动

- **Distortion** is waveform perturbation caused by imperfect response of the system to the desired signal itself, e.g., non-linear devices, filter, etc.

污染

- **Interference** is contamination by signals from other sources such as other transmitters, power lines, switching circuits, etc.

前提: mean=0

- **Noise** refers to random and unpredictable electrical signals produced by natural processes both internal and external to the system.

在数学上 power = variance

✓ Hence, when designing a communication system, one primary concern besides power, bandwidth, and cost, is to reduce **noise**, **interference** and **distortion**. → $r(t) = \sqrt{c}\, s(t)$

干扰

✓ The time values of noise are random → not very useful. Instead, the **statistical/average parameters** are more deterministic → more useful.

$r(t) = s(t) + n(t)$ noise

$r(t) = s(t) + v(t)$ interference

Parameters of Noise

a) Power $\overline{n^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |n(t)|^2\, dt$

b) Power Spectral Density (PSD)

Let $S_N(f)$ be the PSD of $n(t)$. The noise power has another formula: $\overline{n^2(t)} = \int_{-\infty}^{\infty} S_N(f)\, df$

c) Signal-to-Noise Ratio (SNR)

SNR is defined as the ratio of signal power (S) to noise power (N):

$$\frac{\text{S}}{\text{N}} = \frac{\overline{s^2(t)}}{\overline{n^2(t)}} = 10\log_{10}\frac{\overline{s^2(t)}}{\overline{n^2(t)}}\ \text{dB}$$

Higher SNR → Better signal quality

很 unit 跟 scale

d) PSD of filtered noise

If a signal $x(t)$ with PSD $S_X(f)$ is filtered by a filter with transfer function $H(f)$, the filtered signal $y(t)$ will have PSD $S_Y(f)$ given by

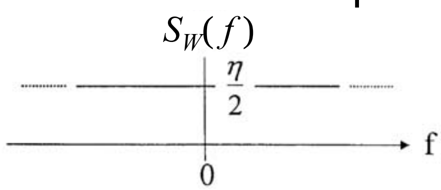
$$S_Y(f) = |H(f)|^2\, S_X(f)$$

White Noise

理想化的

- It is an idealized form of noise which is usually used for the noise analysis of a communication system.
- White noise has a flat power spectral density over all frequencies. The PSD of white noise is expressed as

$$S_W(f) = \frac{\eta}{2} \quad \text{W/Hz}$$



where η is a constant.

- The total power of white noise is

$$\overline{n(t)^2_{\text{white}}} = \int_{-\infty}^{\infty} \frac{\eta}{2}\, df \to \infty$$

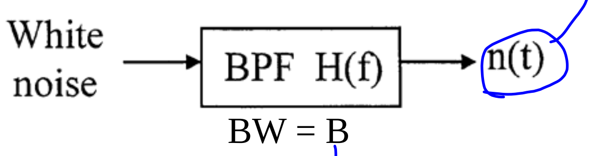
插入的

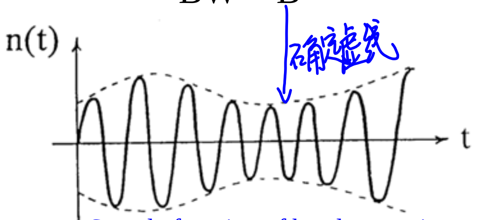
- A bandpass filter (BPF) is usually inserted before the demodulator to remove as much 'white noise' as possible. Such filtered noise is called 'bandpass (white) noise'. (带通白噪声)

通过带通滤波器后的

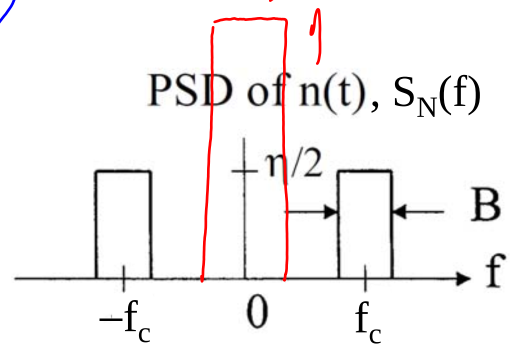
Bandpass Noise

$S_{Nc}(f) = S_{Ns}(f)$





Sample function of bandpass noise



Note:

带宽变小,通过的功率变小,频率越固定

a) $\boxed{B \ll f_c}$

b) **Bandpass noise** $n(t)$ has an appearance of a sinusoid. As B becomes smaller, the average amplitude of the noise becomes smaller and the waveform is more sinusoidal.

c) **Bandpass noise** $n(t)$ has slowly varying random amplitude and phase.

Time Representation of Bandpass Noise

$S_N(f)$

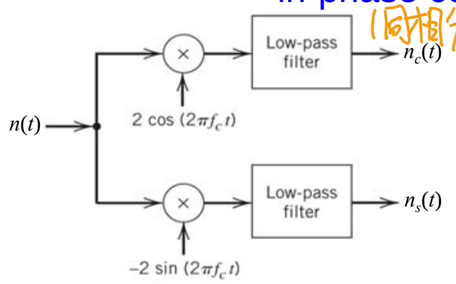
In light of the theory of bandpass signals and systems, we may represent bandpass noise $n(t)$ as:

参考QAM

$$n(t) = \underbrace{n_c(t)}_{\text{in-phase component}} \cos 2\pi f_c t - \underbrace{n_s(t)}_{\text{quadrature component}} \sin 2\pi f_c t$$

B is BW

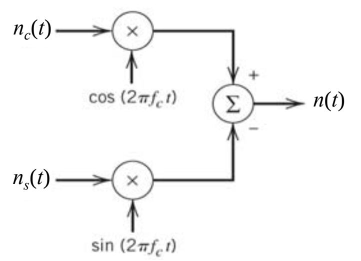
(同相分量) (正交部分)



- Both $n_c(t)$ and $n_s(t)$ are lowpass signals
- Both $n_c(t)$ and $n_s(t)$ have the same PSD, given by

$$S_{Nc}(f) = S_{Ns}(f) = \begin{cases} S_N(f-f_c) + S_N(f+f_c), & -B/2 \le f \le B/2 \\ 0, & \text{otherwise} \end{cases}$$

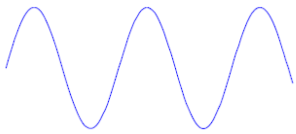
- Both $n_c(t)$ and $n_s(t)$ have PSD centered at frequency = 0, and bandwidth half of $n(t)$



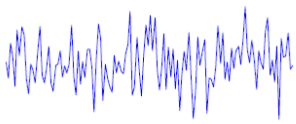
- Power of $n(t)$ = Power of $n_c(t)$ = Power of $n_s(t)$
  i.e., $\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$

Examples of noise-corrupted signal

Signal:

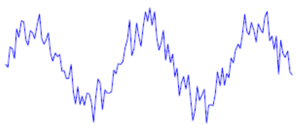


Noise:

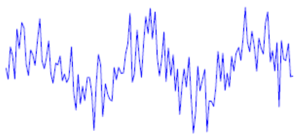


Signal + Noise:

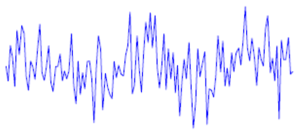
SNR = 10dB



SNR = 0dB



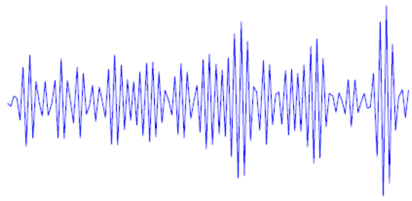
SNR = -10dB



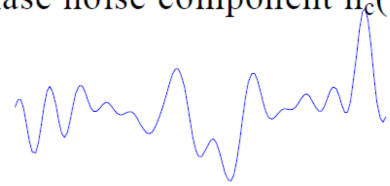
Example of bandpass noise

Bandpass noise:

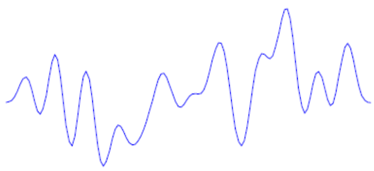
$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$



I-phase noise component $n_c(t)$:



Q-phase noise component $n_s(t)$:



Power of Bandpass Noise

Demodulator input noise (bandpass noise):

$$n_i(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t = r(t) \cos [2\pi f_c t + \varphi(t)]$$

where envelope $r(t) = [n_c^2(t) + n_s^2(t)]^{1/2}$, phase $\varphi(t) = \tan^{-1}[n_s(t)/n_c(t)]$.

- We need to know the power of $n_c(t)$, $n_s(t)$ as the demodulator output noise usually contains $n_c(t)$ or $n_s(t)$, not $n_i(t)$.
- Power of $n_i(t)$ = Power of $n_c(t)$ = Power of $n_s(t)$

  i.e., $\overline{n_i^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$
- Power of $n_i(t)$ can be calculated form its PSD as

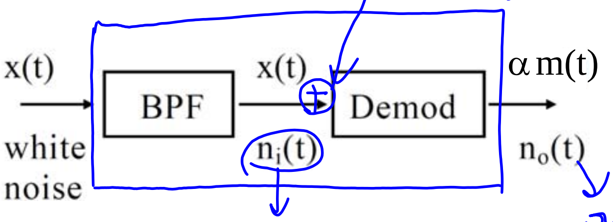
$$\overline{n_i^2(t)} = \int_{-\infty}^{\infty} S_N(f)df$$

$$= \frac{\eta}{2} \times B \times 2$$

$$= \eta B$$

Receiver Model

receiver input (针对解调器)



band pass noise n(t)

output(针对调解器)

$\alpha m(t)+n_0(t)$

BPF

- Should filter away as much white noise as possible
- Should not filter/distort the AM signal $x(t)$ at all

⇒ $\boxed{\text{BPF bandwidth = AM bandwidth}}$

x(t)

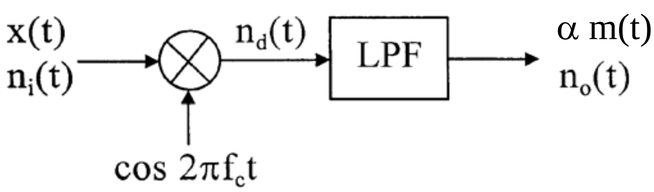
Demodulator input:

- $x(t)$ = received AM signal
- Input noise $n_i(t)$
  = bandpass noise
  = $n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$

Demodulator output:

- Demodulated signal = $\alpha\, m(t)$
- Output noise = $n_0(t)$
- α and $n_0(t)$ depend on modulation as well as demodulation type

DSBSC-AM with Coherent Detection (1)



Input:

- Signal $x(t) = m(t)\cos 2\pi f_c t$ (assuming A_c=1)

$\therefore\ S_i = \overline{x^2(t)} = \frac{1}{2}\overline{m^2(t)}$

- Noise $n_i(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$

$\therefore\ N_i = \overline{n_i^2(t)} = \eta B$

- SNR of demodulator input: $(\frac{S}{N})_i = \frac{S_i}{N_i} = \frac{\overline{m^2(t)}}{2\overline{n_i^2(t)}}$

DSBSC-AM with Coherent Detection (2)

Output:

- Signal

$$\alpha m(t) = [x(t)\cos 2\pi f_c t]_{LP}$$
$$= [\frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 4\pi f_c t]_{LP}$$
$$= \frac{1}{2}m(t)$$

$$\Rightarrow \quad \therefore\ S_0 = \overline{[\frac{1}{2}m(t)]^2} = \frac{1}{4}\overline{m^2(t)}$$

- Noise

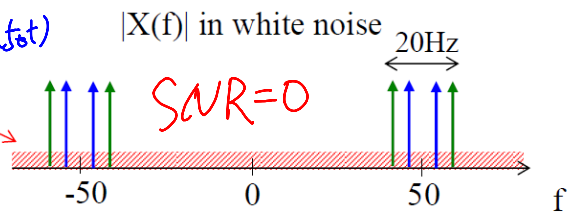
$$n_0(t) = [n_i(t)\cos 2\pi f_c t]_{LP}$$
$$= [n_c(t)\cos^2 2\pi f_c t - n_s(t)\sin 2\pi f_c t \cos 2\pi f_c t]_{LP}$$
$$= [\frac{1}{2}n_c(t) + \frac{1}{2}n_c(t)\cos 4\pi f_c t - \frac{1}{2}n_s(t)\sin 4\pi f_c t]_{LP} = \frac{1}{2}n_c(t)$$

$$\therefore\ N_0 = \overline{[\frac{1}{2}n_c(t)]^2} = \frac{1}{4}\overline{n_c^2(t)}$$

- SNR of demodulator output: $(\frac{S}{N})_0 = \frac{S_0}{N_0} = \frac{\overline{m^2(t)}}{\overline{n_c^2(t)}} = 2(\frac{S}{N})_i$

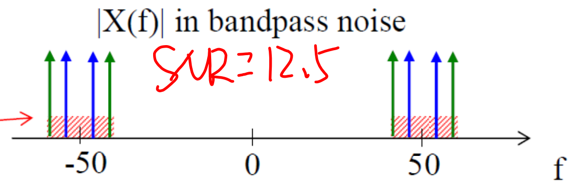
Example of SNR Calculation

- x(t) = [sin(10πt)+sin(20πt)] cos(100πt) = m(t) cos(2πf₀t)
- PSD of white noise = 10^{-3} Watt/Hz = $\frac{1}{2}$



- AM bandwidth = 20Hz

→ BW of BPF before demodulator
= 20Hz
= BW of bandpass noise $n_i(t)$

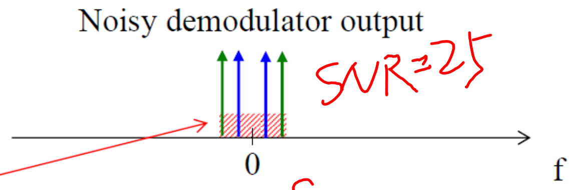


- message m(t) = sin(10πt)+sin(20πt)

$$\overline{m^2}(t) = \overline{\sin^2(x)} + \overline{\sin^2(y)} + 2\overline{\sin(x)\sin(y)}$$
$$= \frac{1}{2} + \frac{1}{2} + 0 = 1$$

$$\overline{n_i^2}(t) = (10^{-3}\times 20)\times 2 = 0.04$$

$$S_O/N_O = \overline{m^2}(t)\Big/\overline{n_c^2}(t) = \overline{m^2}(t)\Big/\overline{n_i^2}(t) = 1/0.04 = 25 = 13.98dB$$



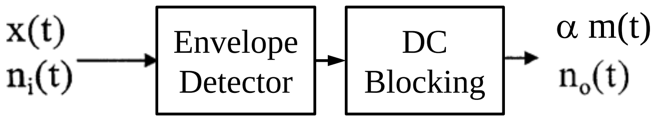
$$\frac{S_i}{N_i} = 12.5$$

Conventional AM with Envelope Detection

- Consider a conventional AM signal (assuming A_c=1)
  $x(t) = [1 + k_a m(t)] \cos 2\pi f_c t$
- Demodulator input:

$$S_i = [1 + k_a^2 \overline{m^2(t)}]/2$$

$$N_i = \overline{n_i^2(t)}$$



- Demodulator output:

$$\begin{aligned} y(t) &= \text{envelope of } [x(t) + n_i(t)] \\ &= \{[1 + k_a m(t) + n_c(t)]^2 + n_s^2(t)\}^{1/2} \\ &\approx 1 + k_a m(t) + n_c(t) \end{aligned}$$

- After DC blocking, the SNR at the demodulator output is:

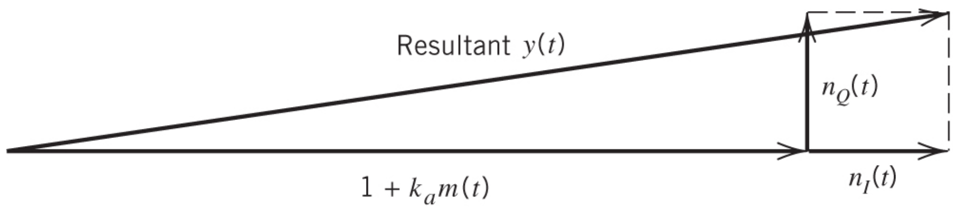
$$(\frac{S}{N})_0 = \frac{S_0}{N_0} = \frac{k_a^2 \overline{m^2(t)}}{\overline{n_c^2(t)}} \Longrightarrow \frac{SNR_0}{SNR_i} = \frac{2k_a^2 \overline{m^2(t)}}{1 + k_a^2 \overline{m^2(t)}}$$

Phasor Diagram for Conventional AM

$$x(t) + n_i(t) = [1 + k_a m(t) + n_c(t)]\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$$

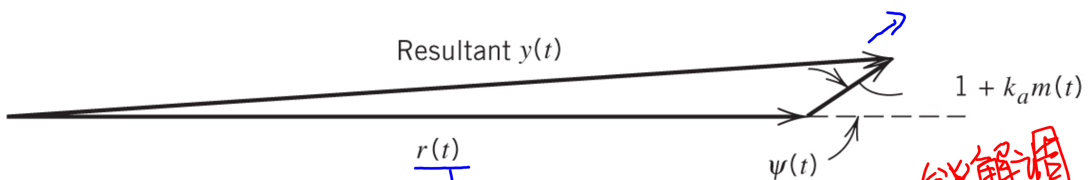
$= + n_s(t)\cos(2\pi f_c t + \frac{\pi}{2})$

Phasor diagram: represent each component by means of phasor.



(a) Phasor diagram for the case of high SNR_i

噪声很强, $y(t) \approx r(t)$



(b) Phasor diagram for the case of low SNR_i → 不能解调

带通噪声本身包络, 见page 63

Exercise: Conventional Single-Tone AM

Consider a single-tone AM signal given by $A_c = 1$

$x(t) = \cancel{A_c}[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$. This AM signal is demodulated using an envelope detector. Find $\mathrm{SNR}_0/\mathrm{SNR}_i$.

$$\frac{SNR_o}{SNR_i} = \frac{2K_a^2\overline{m^2(t)}}{1+K_a^2\overline{m^2(t)}} = \frac{2\cdot\frac{1}{2}\mu^2}{1+\frac{1}{2}\mu^2} = \frac{2\mu^2}{2+\mu^2} = \frac{2}{\frac{2}{\mu^2}+1}$$

✓ When $\mu = \mu_{max} = 1$, the above figure of merit achieves its max value equal to $2/3$.

✓ The noise performance of conventional AM is always inferior to that of DSBSC-AM, due to the wastage of carrier power.

Threshold Effect of Envelope Detection

- When the input SNR is small compared with unity, the noise term dominates.
- From the phasor diagram, the envelope detector output is

$$y(t) = \text{envelope of } [x(t) + n_i(t)] \approx r(t) + \cos\varphi(t) + k_a m(t) \cos\varphi(t)$$

which has no component proportional to $m(t)$. The last term contains $m(t)$ multiplied by noise in the form of $\cos\varphi(t)$.

- As $\varphi(t)$ is random, the detector output contains no $m(t)$ at all.

✓ The loss of message in a demodulator that operates at a low input SNR is referred to as the threshold effect.

- There is an input SNR threshold, below which the noise performance of a demodulator deteriorates much more quickly than proportionately to the input SNR.
- All nonlinear detectors exhibit a threshold effect. 门限效应

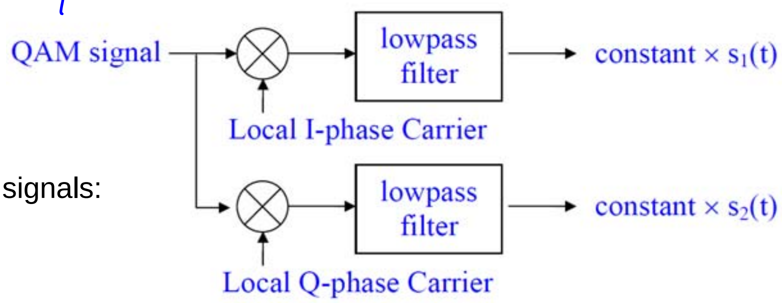
Summary of Amplitude Modulation (1)

Conventional AM
- Modulation and demodulation procedures
- Time-domain expression and amplitude spectrum
- Overmodulation, Envelope detection

DSBSC-AM
- Time-domain expression and amplitude spectrum
- Coherent demodulator

Single Sideband AM
Vestigial Sideband AM 仅了解

QAM
- Time-domain expression
- Coherent demodulator for QAM signals:



Frequency translation
- Mixer, mixing, frequency up/down conversion

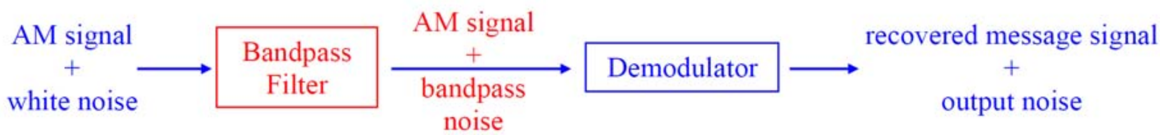
Summary of Amplitude Modulation (2)

FDM

- Minimum separation of FDM carrier frequencies: adjacent FDM channel spectrum should not overlap

Noise analysis

- Power of a signal v(t) = $\overline{v^2(t)} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}[\text{square of v}(t)]\,dt$ or $\int_{-\infty}^{\infty}[\text{PSD of v}(t)]\,df$
- Signal-to-noise ratio (SNR) = signal power ÷ noise power = 10 Log_{10}(SNR) dB
- Time-domain expression of bandpass noise n(t) = $n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$
- Power of n(t) = power of $n_c(t)$ = power of $n_s(t)$ → $\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$
- Demodulator structure in the presence of white noise =



Appendix A: Trigonometric Identities

1	$\exp(\pm jx) = \cos x \pm j\sin x$
2	$\cos x = \frac{1}{2}\left[\exp(jx) + \exp(-jx)\right]$
3	$\sin x = \frac{1}{2j}\left[\exp(jx) - \exp(-jx)\right]$
4	$\sin^2 x + \cos^2 x = 1$
5	$\cos^2 x - \sin^2 x = \cos 2x$
6	$\cos^2 x = \frac{1}{2}\left(1 + \cos 2x\right)$
7	$\sin^2 x = \frac{1}{2}\left(1 - \cos 2x\right)$
8	$\cos^3 x = \frac{1}{4}\left(3\cos x + \cos 3x\right)$
9	$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
10	$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
11	$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
12	$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$
13	$\cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$
14	$\sin x \cos y = \frac{1}{2}\left[\sin(x-y) + \sin(x+y)\right]$
15	$A\cos x + B\sin x = C\cos(x-\theta)$ where $C = \sqrt{A^2 + B^2}$ and $\theta = \tan^{-1}\dfrac{B}{A}$

Appendix B: Fourier Transform Pairs

Fourier Transform: $G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt$

Inverse Fourier Transform: $g(t) = \int_{-\infty}^{\infty} G(f)e^{j2\pi ft}df$

	Time Function	**Fourier Transform**
1	$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & \lvert t\rvert \le T/2, \\ 0, & \text{otherwise.} \end{cases}$	$T\ \mathrm{sinc}(fT)$
2	$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\ \mathrm{rect}\left(\frac{f}{2W}\right)$
3	$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
4	$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
5	$\exp\left[-\pi\left(\frac{t}{T}\right)^2\right]$	$T\exp\left[-\pi(fT)^2\right]$
6	$\Lambda\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T, \\ 0, & \text{otherwise.} \end{cases}$	$T\ \mathrm{sinc}^2(fT)$
7	$T\ \mathrm{sinc}^2(tT)$	$\Lambda\left(\frac{f}{T}\right)$
8	$\delta(t)$	1
9	1	$\delta(f)$
10	$\delta(t - t_0)$	$\exp(-j2\pi ft_0)$
11	$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
12	$\cos(2\pi f_c t + \theta)$	$\frac{1}{2}\left[\delta(f - f_c)e^{j\theta} + \delta(f + f_c)e^{-j\theta}\right]$
13	$\sin(2\pi f_c t)$	$\frac{1}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]$
14	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
15	$\frac{1}{\pi t}$	$-j\ \mathrm{sgn}(f)$
16	$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
17	$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Appendix C: Fourier Transform Properties

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants.
2. Time Scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a \rvert} G\left(\frac{f}{a}\right)$ where a is a constant.
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t) g(t) \rightleftharpoons G(f - f_c)$
6. Modulation Theorem	$g(t)\cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\left[G(f - f_c) + G(f + f_c)\right]$ $g(t)\sin(2\pi f_c t) \rightleftharpoons \frac{1}{2j}\left[G(f - f_c) - G(f + f_c)\right]$
7. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\, dt = G(0)$
8. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\, df$
9. Differentiation in the time domain	$\frac{d}{dt} g(t) \rightleftharpoons j2\pi f G(f)$
10. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2}\delta(f)$
11. Complex conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau)\, d\tau \rightleftharpoons G_1(f) G_2(f)$
13. Multiplication in the time domain	$g_1(t) g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda)\, d\lambda$
14. Multiplication by t^n	$t^n g(t) \Leftrightarrow (-j2\pi)^{-1} \frac{d^n G(f)}{df^n}$