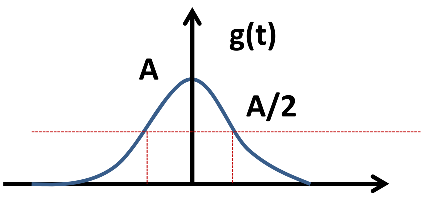
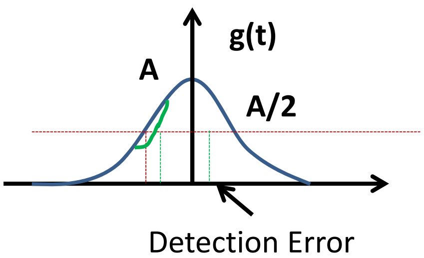
Outline

- Why do we need digital communications?
- Semi-digital representation of analog signals
  - Sampling: digitalization in time domain
  - Analog pulse modulation schemes: PAM, PDM, PPM
- Generation, detection and analysis of PPM
- Digital representation of analog signals
  - Quantization: digitalization in signal scale
  - Quantization noise
  - Digital modulation schemes: PCM, DM

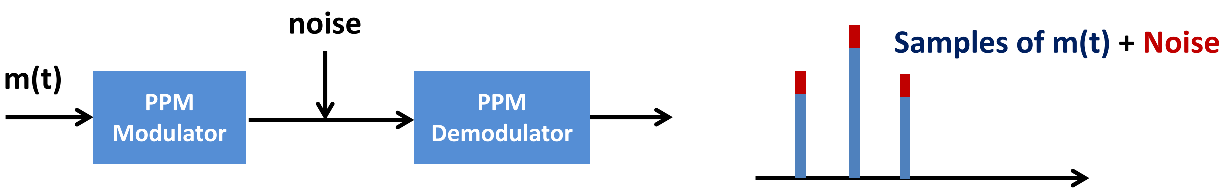
Noise Effect in PPM



Ideally, slicer + delay is able to detect the exact position of pulse peak.



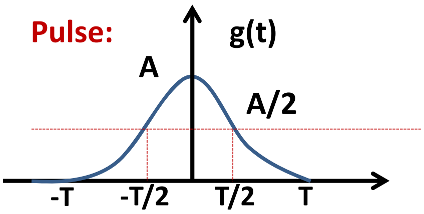
Due to the effect of noise, there might be detection error on the peak position.



How to measure the effect of noise (SNR) in PPM detection?

SNR Before Receiver

Pulse:

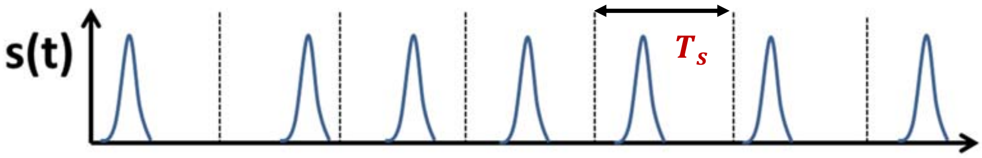


Pulse Shape: $g(t) = \frac{A}{2}[1 + \cos(\pi B_T t)] \quad -T \leq t \leq T, B_T = 1/T$

Modulating Signal: $m(t) = \frac{A_m}{2} sin 2\pi f_m t$

Slicing level = A/2

PPM:



Before receiver: Average Signal Power $= \frac{1}{T_s}\int_{-T}^{T} g^2(t)dt$

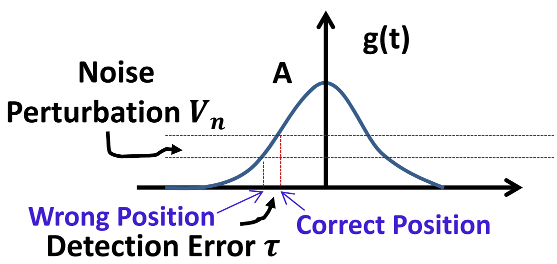
$$= \frac{1}{T_s}\int_{-T}^{T} \frac{A^2}{4}[1+\cos(\pi B_T t)]^2 dt$$

$$= \frac{3A^2 T}{4T_s} = \frac{3A^2}{4T_s B_T}$$

Nosie Power $= N_0 W$ **(W is the bandwidth of receiving filter)**

Channel SNR $= \frac{3A^2}{4T_s B_T W N_0}$

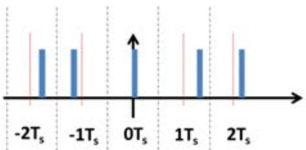
Noise Perturbation



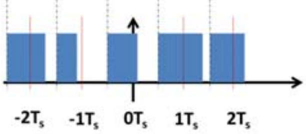
Assume noise level is much smaller than signal level

=>Noise perturbation happens around slicing level

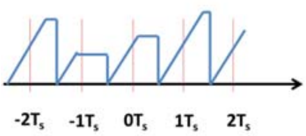
$$V_n = \tau \frac{dg(t)}{dt}\bigg|_{t=-\frac{T}{2}} = \tau\frac{\pi B_T A}{2} \Rightarrow \tau = \frac{2V_n}{\pi B_T A}$$



$k_p m(nT_s) + \tau$



$Area\text{:}\ h(\frac{T_s}{2} + k_p m(nT_s) + \tau)$



$\frac{Area - \frac{hT_s}{2}}{hk_p}$ = m(nT$_s$) + τ/k_p

Noise Power = $E[\frac{\tau^2}{{k_p}^2}]$

$$= E[\frac{4V_n^2}{\pi^2 B_T^2 A^2 k_p^2}]$$

$$= \frac{4N_0 W}{\pi^2 B_T^2 A^2 k_p^2}$$

wherein V_n^2 is the noise power within receiver's bandwidth, hence

$$E[V_n^2] = \frac{N_0}{2}2W = N_0 W$$

Figure of Merit

After receiver: Signal Power= $\dfrac{\int_0^{1/f_m} m^2(t)dt}{1/f_m} = \dfrac{A_m^2}{8}$,

Nosie Power= $E\left[\dfrac{\tau^2}{k_p^2}\right] = \dfrac{4N_0W}{\pi^2 B_T^2 A^2 k_p^2}$,

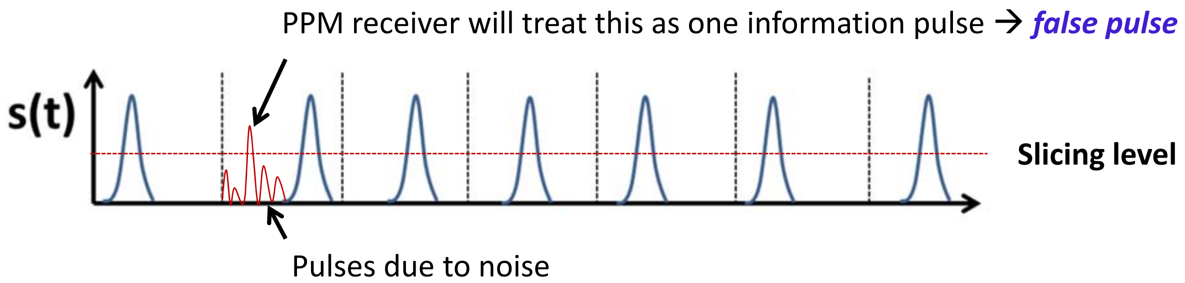
SNR= $\dfrac{\pi^2 B_T^2 A_m^2 A^2 k_p^2}{32 N_0 W}$,

Figure of Merit = $\dfrac{SNR\ after\ Rx}{SNR\ before\ Rx} = \dfrac{\dfrac{\pi^2 B_T^2 A_m^2 A^2 k_p^2}{32N_0W}}{\dfrac{3A^2}{4T_sB_TWN_0}} = \dfrac{\pi^2}{24} B_T^3 T_s A_m^2 k_p^2$

Figure of merit shows the gain of receiver for PPM signal, where we can observe that 稳固的

- Larger B_T leads to better receiving gain. This is because the narrower pulse is more robust against noise perturbation. 扰乱
- Given A_m, Larger k_p leads to better receiving gain. This is due to larger dynamic range of pulse position.

False Pulses



- Due to the randomness of noise, it is possible that the instantaneous noise level is larger than the slicing level, leading to false pulse
- Noise power depends on the bandwidth of s(t), denoted as W
- ***Threshold effect:*** when W is large, noise power is large, the probability of false pulse is also large
  - We can increase peak pulse power, and choose larger slicing level

尽量让它大于(noise)max

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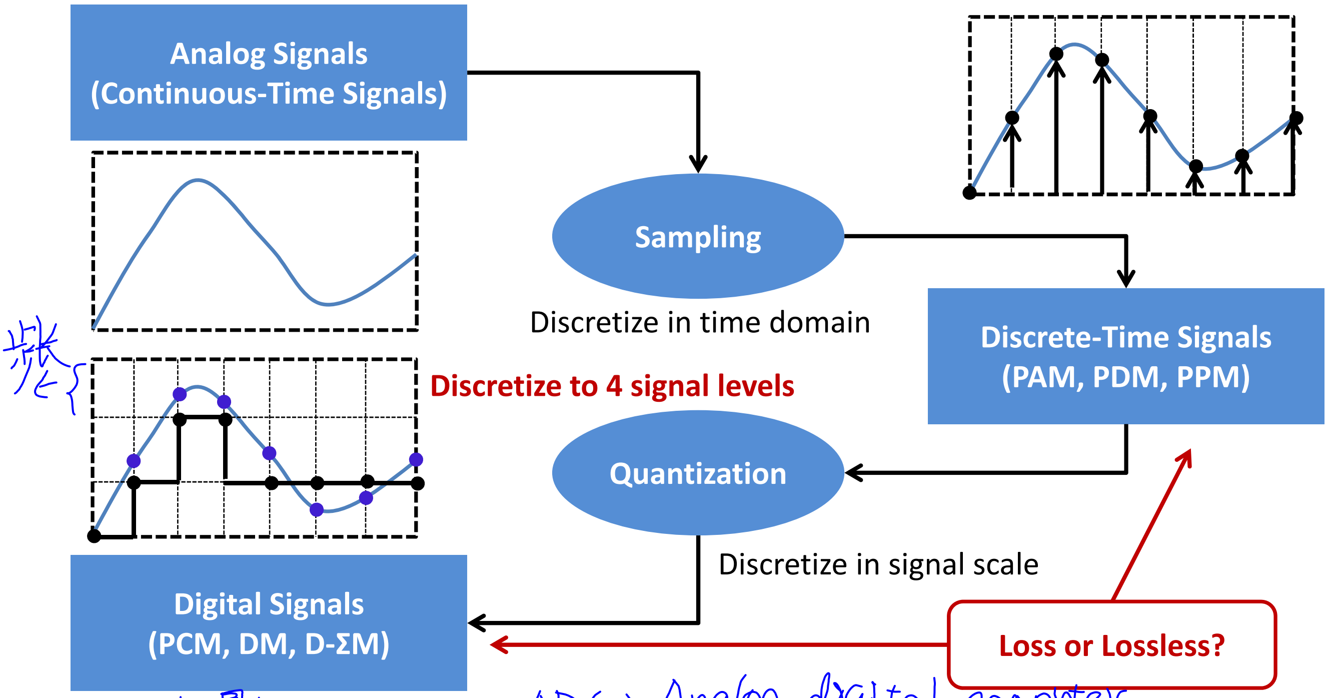
Quantization Overview

- PAM, PDM or PPM can never transmit a real-valued sample precisely due to noise (why?)
  – Information of a real number is infinite
- What's the capability of a communication system?
  – Finite: transmit one element from a set with finite cardinality (size)
- Quantization is a procedure to convert a real-valued signal to discrete-valued (and usually finite) signal
  – Discard some information to fit the communication systems
- Plenty of quantization example
  – digital camera, MP3 and etc.

Quantization Formulation

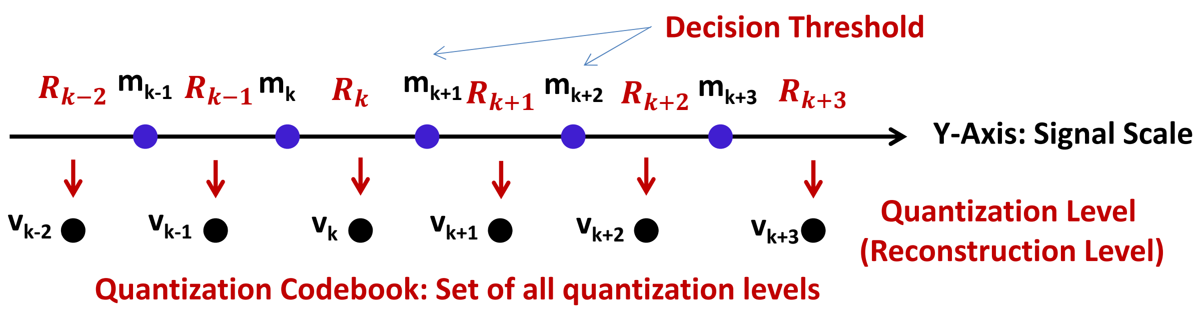


均匀量化

非均匀量化

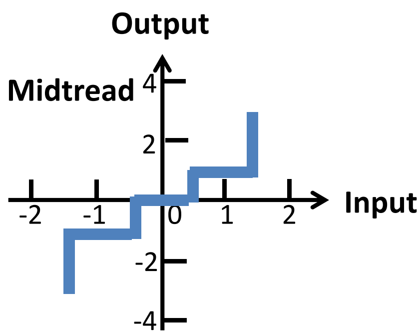
ADC: Analog digital converter

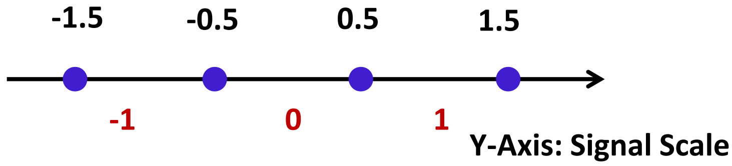
DAC: Digital analog converter



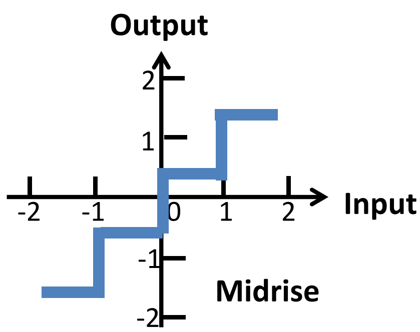
- The dynamic range of signal is divided into finite number of regions
  - E.g., $R_k\!: \{m_k < m \le m_{k+1}\}$
- Let m and v be the signal scale before and after quantization, then the quantization procedure can be written as $v = g(m)$
  - $v_k = g(m),\ \forall m \in R_k$
  - g: Quantization Characteristic

Midtread & Midrise

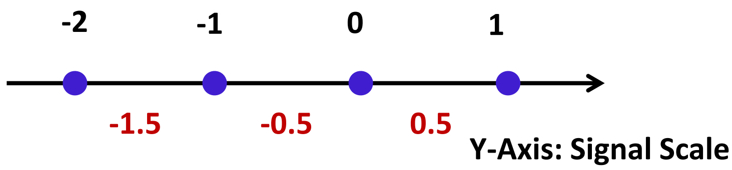




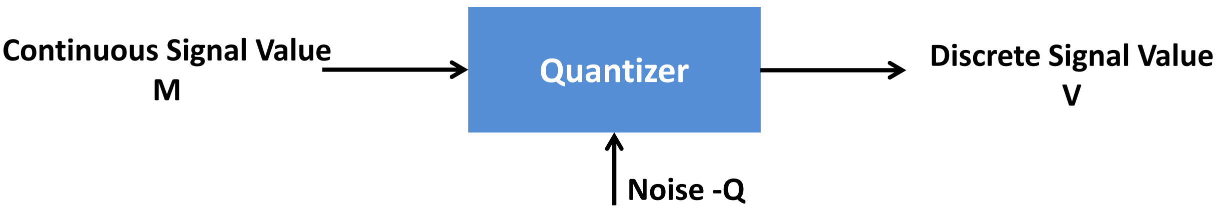
The origin is between two decision thresholds



The origin is one of decision thresholds



Quantization Noise



$Quantization\ \boldsymbol{Noise}\ \boldsymbol{Q} = \boldsymbol{M} - \boldsymbol{V}$,
Random variables **M** and **V** are the quantizer input and output respectively

Power of signal is measured by $E[M^2]$

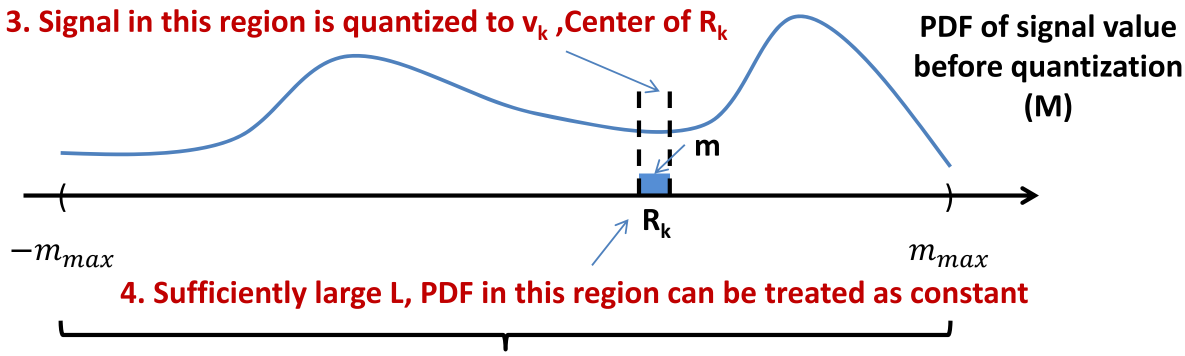
Power of noise is measured by $E[Q^2]$

Depends on signal itself, assumed to be P

The quality of quantization is measured by $SNR = \frac{E[M^2]}{E[Q^2]}$

How to measure?

1. Assume input signal range is $(-m_{max}, m_{max})$



2. Uniform Quantizer: Uniformly divide into L quantization levels

$$Setp-size\ \boldsymbol{\Delta} = \frac{\boldsymbol{2m_{max}}}{\boldsymbol{L}}$$

Given $\boldsymbol{V = v_k}$, $\boldsymbol{M \sim unif}\left(\mathrm{v_k} - \frac{\Delta}{2}, \mathrm{v_k} + \frac{\Delta}{2}\right)$, therefore $\boldsymbol{Q = M - v_k \sim unif}\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$

The PDF of quantization error is $\boldsymbol{f_Q(x)} = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < \mathbf{q} \leq \frac{\Delta}{2} \\ \mathbf{0}, & \boldsymbol{Otherwise} \end{cases}$

- The power of quantization noise power is given by

$$\boldsymbol{Noise\ Power\ \ E[Q^2]} = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \boldsymbol{x^2 f_Q(x) dx} = \frac{\boldsymbol{\Delta^2}}{\boldsymbol{12}}$$

- Let P be the average power of input continuous sample, the SNR of uniform quantizer is

$$\boldsymbol{SNR} = \frac{\boldsymbol{E[M^2]}}{\boldsymbol{E[Q^2]}} = \frac{\boldsymbol{12P}}{\boldsymbol{\Delta^2}}$$

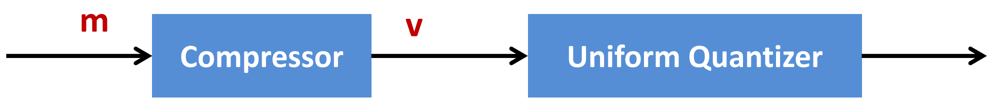
- L quantization levels can be represented by $R = \log_2 L$ bits, hence,

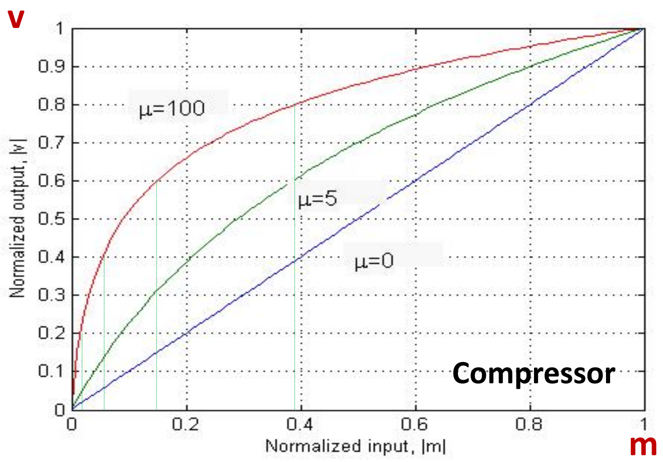
$$\boldsymbol{\Delta} = \frac{\boldsymbol{2m_{max}}}{\boldsymbol{L}} = \frac{\boldsymbol{2m_{max}}}{\boldsymbol{2^R}} \quad \boldsymbol{and}\ \ \boldsymbol{SNR} = \frac{\boldsymbol{3P}}{\boldsymbol{m_{max}}^2}\boldsymbol{2^{2R}}$$

SNR increases exponentially with the number of information bits.

Nonuniform Quantization

- Why nonuniform quantization?
  - Low power signal is more sensitive to the noise
- How to use uniform quantizer to achieve non-uniform quantization?





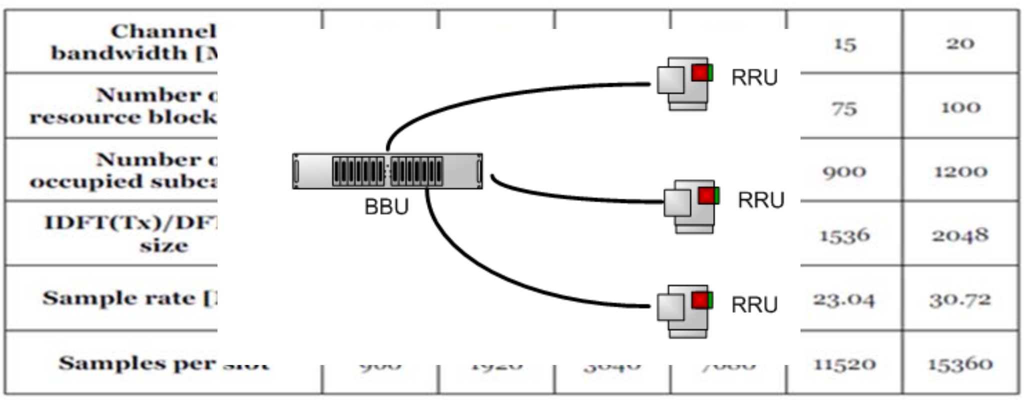
μ-law: $|v| = \frac{log(1+\mu|m|)}{log(1+\mu)}$

$$\frac{d|m|}{d|v|} = \frac{log(1+\mu)}{\mu}(1+\mu|m|)$$

Larger signal value, larger step-size

Quantization in LTE

- Sampling frequency: 1.92, 3.84, 7.68, 15.36, 23.04, 30.72MHz
- Sampled bits per I or Q: 8~20
- Number of antennas: 4(LTE), 8(LTE-A)
- Calculation: 15 * 2 * 7.68M * 8 = 1.8432Gbps



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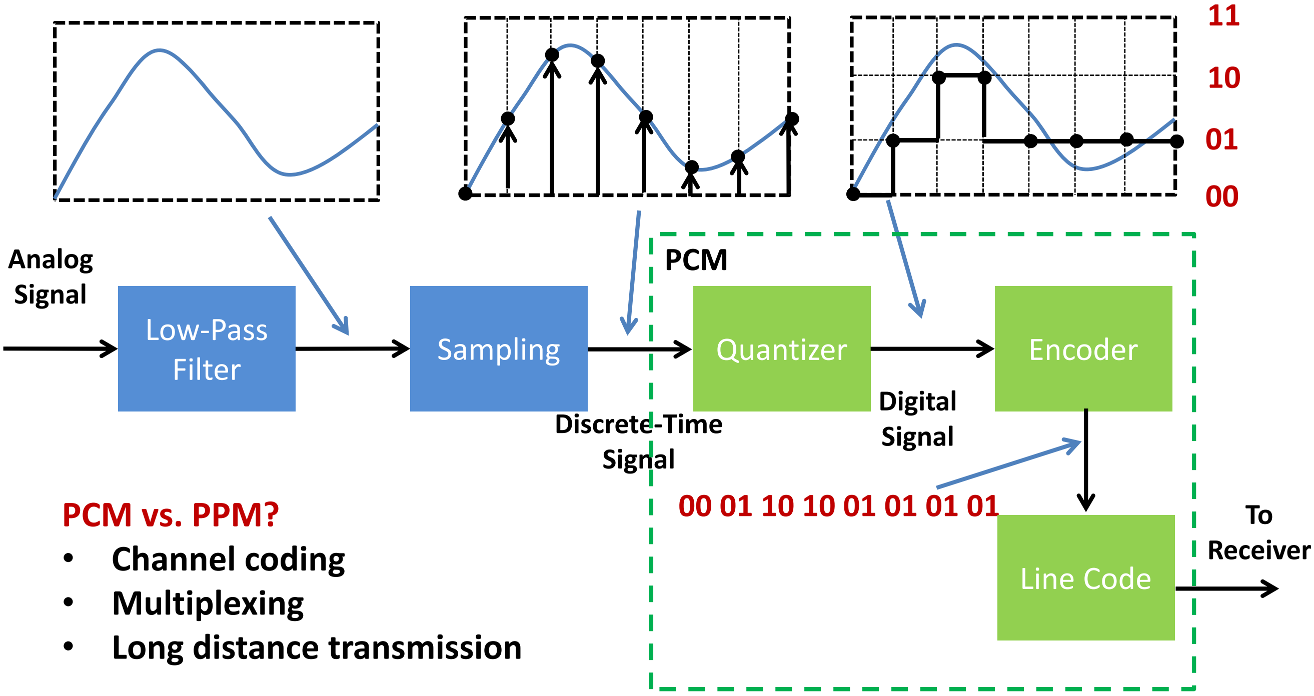
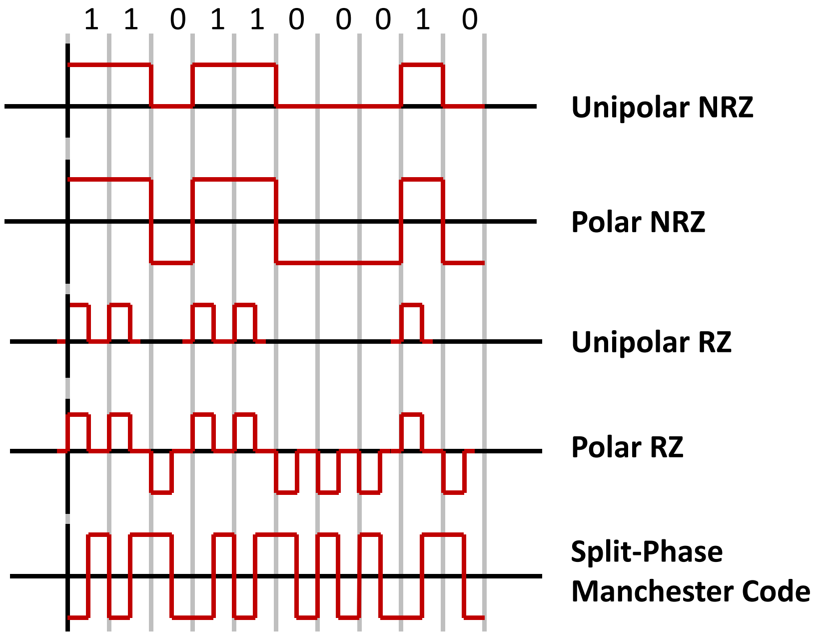
Pulse Code Modulation



PCM vs. PPM?
- **Channel coding**
- **Multiplexing**
- **Long distance transmission**

Line Code

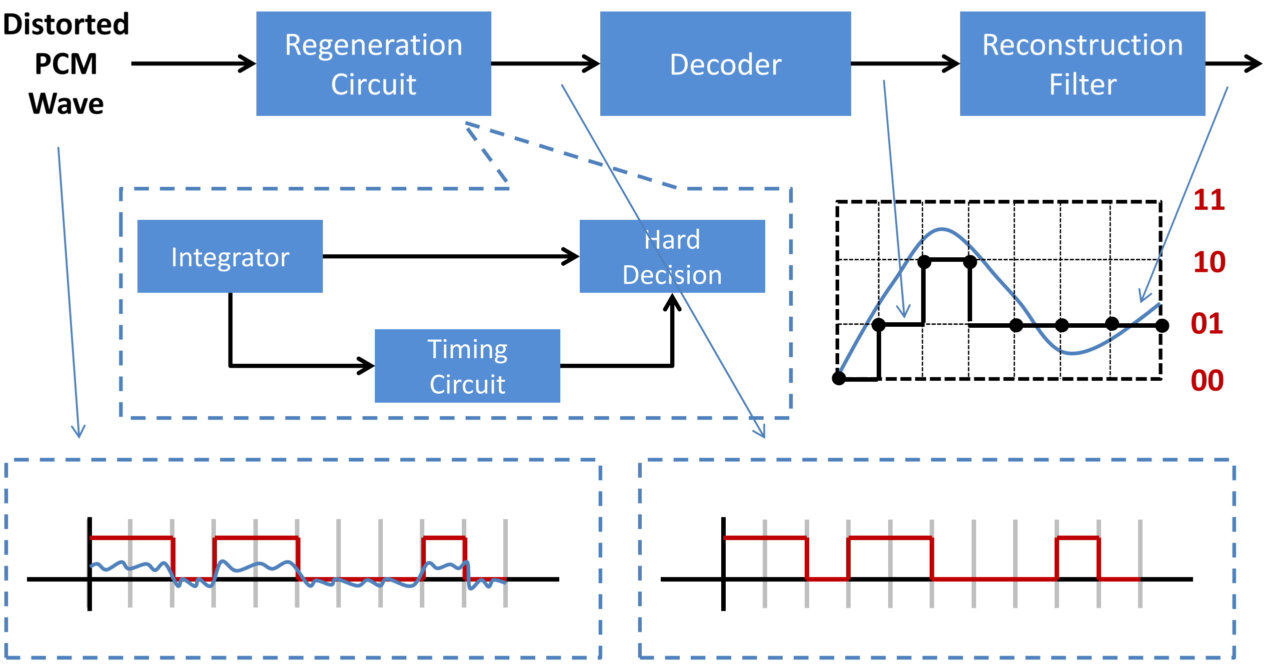
- Line codes: baseband modulation of binary (digital) signals



Use a number of signal periods to represent one sample of modulating signal

PCM Receiver

- Receiver



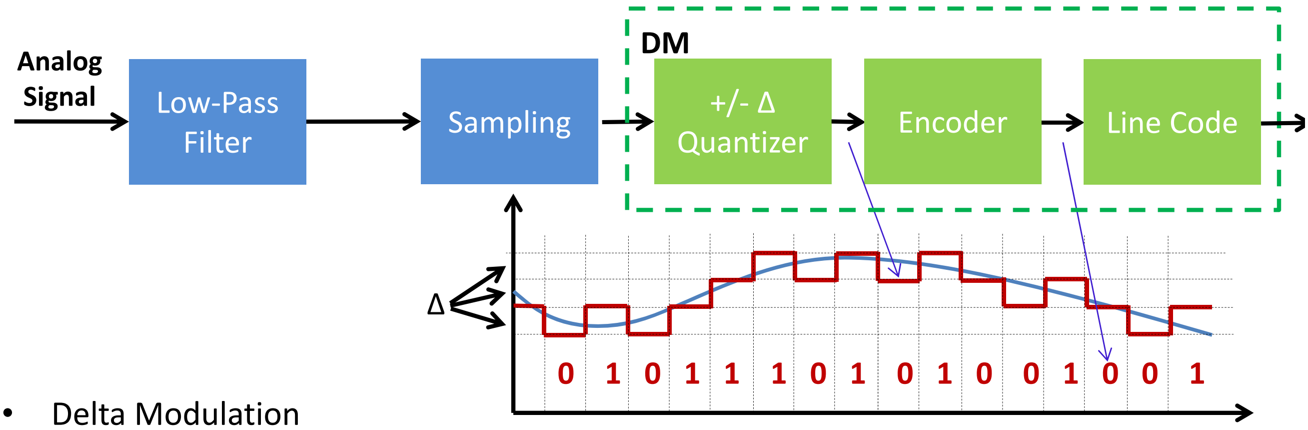
Detection of Line Code

- Integrator + threshold
- How about Manchester Code?
- Optimal detector design will be introduced in the next chapter

T1 System

- T1 system: digital communication of voice signal pioneered by Bell System (AT&T)
- Technology
  - Frequency of voice signal 300~3.1kHz
  - Sampling frequency: 8kHz
  - PCM: 255 quantization level (8 bits/sample), approximated μ-law compressor
  - TDM: multiplex 24 voice channels; additional one bit for synchronization
  - Frame size = 24*8+1 = 193bits
  - Data rate = 193 * 8k = 1.544 Mb/s
  - Further TDM: T2 = 4*T1, T3=7*T2
- T1 system was mainly adopted in US, Canada and Japan
- E1 system is the European counterpart of T1
  - 32 voice signals

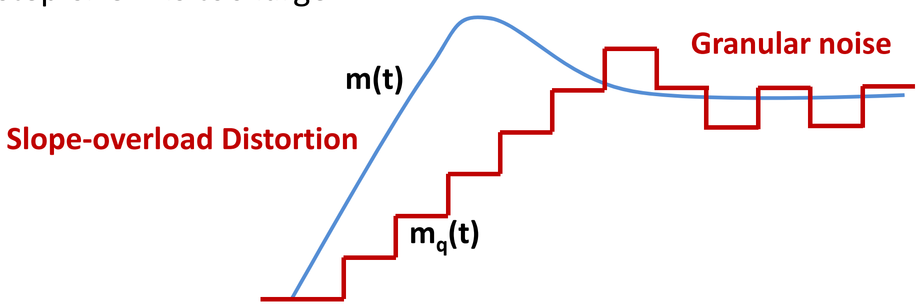
Delta Modulation



- Delta Modulation
  - Exploit the correlation between samples
  - Sampling frequency should be sufficiently large
- Comparison with PCM
  - Less bits in each sampling
  - Larger sampling frequency
- Voice
  - PCM: 8bits/sample, 8k samples/sec => 64k bits/sec
  - DM: 1bit/sample, 16k~32k samples/sec => 16k~32k bits/sec

Quantization Noise

- Two types of quantization error (noise)
- Slope over-load distortion → 斜率过载失真
  - Occurs when the step-size Δ is too small
  - Maximum slope of staircase curve is $\frac{\Delta}{T_s}$
  - Therefore, it is required that $\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$
- Granular noise
  - Occur when the step-size Δ is too large



Summary

Sampling Freq. > Nyquist Rate

Quantization Noise Incurred

Analog Signal → **Sampling** → **Sampled Analog Signal** → **Quantizer** → **Digital Signal**

Use pulses to deliver sampled signal
Schemes: PAM, PPM, PDM
Error is unavoidable due to noise
Info(Comm. Sys.) < Info(Real No.)

Schemes: PCM, DM
Different approaches are used in quantization

Homework #D3

- D3.1

A compact disc (CD) records audio signals digitally by using PCM. Assume the audio signal bandwidth to be 15kHz.

(a) What is the Nyquist rate?

(b) If the Nyquist samples are quantized into *L*=65,536 levels and then binary coded, determine the number of binary digits per second (bit/s) required to encode the audio signal.

- D3.2

Show that, with a non-uniform quantizer, the average power (mean-square value) of the quantization error is approximately equal to $(1/12)\sum_i \Delta_i^2 p_i$ where Δ_i is the i-th step size and p_i is the probability that the input signal amplitude lies within the i-th interval R_i. Assume that the step-size Δ_i is small compared with the range of input signal, such that the signal can be treated as uniformly distribution within each step size.

Hints:

(1) Let Q be the quantization error, the expectation of Q^2 is given by

$$E[Q^2] = \sum_i \boldsymbol{E}[Q^2 |\ signal\ is\ in\ the\ i-th\ step\ size]\ \boldsymbol{Pr}\ [signal\ is\ in\ the\ i-th\ step\ size]$$

(2) The mean and variance of a uniform distributed random variable within [a,b] are given by $\frac{1}{2}(a+b)$ and $\frac{1}{12}(b-a)^2$, respectively.