EE206: Communications Principles Tutorial

Assignment 12

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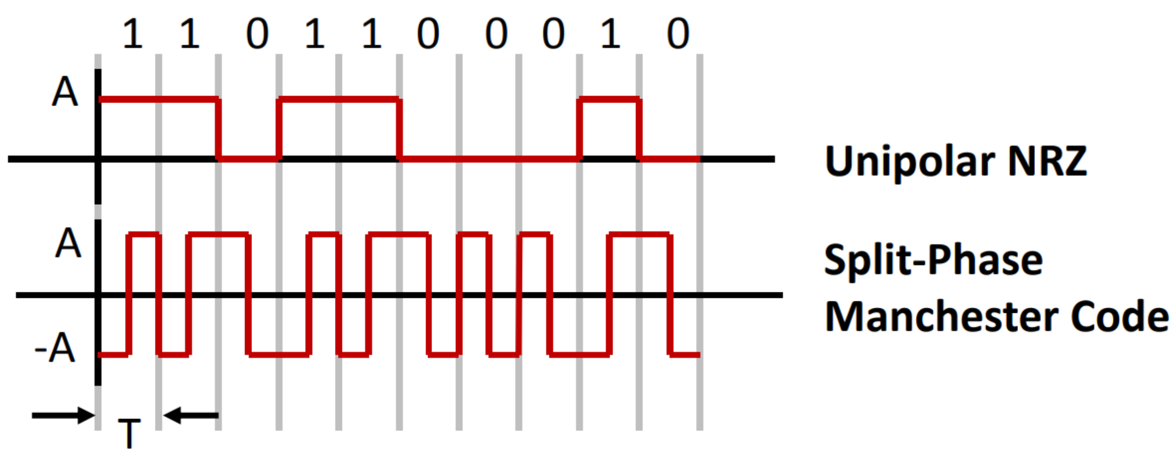
- D5.1

(a) If the Unipolar NRZ code as follows is used at the transmitter, how to design the receiver? What is the BER?

(b) If the Split-Phase Manchester Code as follows is used at the transmitter, how to design the receiver? What is the BER?



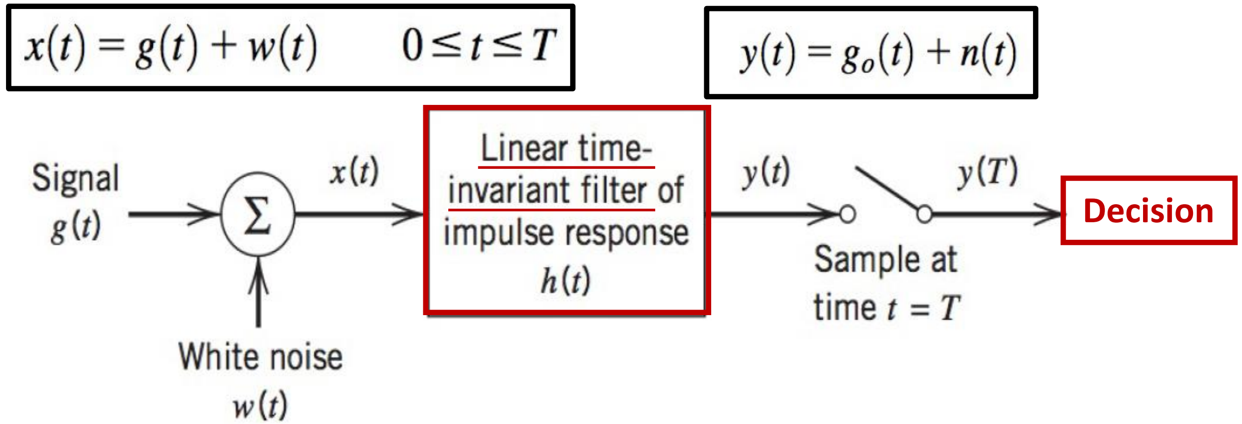


FIGURE 8.2 Linear receiver.

1. **It is assumed that channel is relatively ideal. The transmitted signal spectra g(t) is unchanged at the receiver. Transmission at low data rate over a short cable is an example for this.**
2. **It is assumed that the receiver has knowledge of the waveform of the pulse signal g(t). ← Start & end time of pulse, pulse shape.**
3. **The function of the receiver is to detect the pulse signal g(t) in an optimum manner. ← LTI match filter h(t), decision threshold λ.**

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Match Filtering: equivalent to maximizing the *peak pulse signal-to-noise ratio*, defined as

$$\boxed{\eta = \frac{|g_o(T)|^2}{\mathbf{E}[n^2(t)]}} \qquad (8.3)$$

$\frac{E}{N_0}$ is the signal energy-to-noise spectral density ratio.

$$\eta = \frac{\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi T)\,df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

Schwarz's Inequality →

$$\eta \le \frac{2}{N_0}\int_{-\infty}^{\infty}|G(f)|^2\,df \qquad \eta_{\max} = \frac{(kE)^2}{(k^2N_0E/2)} = \frac{2E}{N_0}$$

$$\boxed{\begin{aligned} h_{\text{opt}}(t) &= k\int_{-\infty}^{\infty} G(-f)\exp[-j2\pi f(T-t)]\,df \\ &= kg(T-t) \end{aligned}}$$

Optimal Detection: **optimizing λ to make smallest P_e (first kind error + second kind error).**

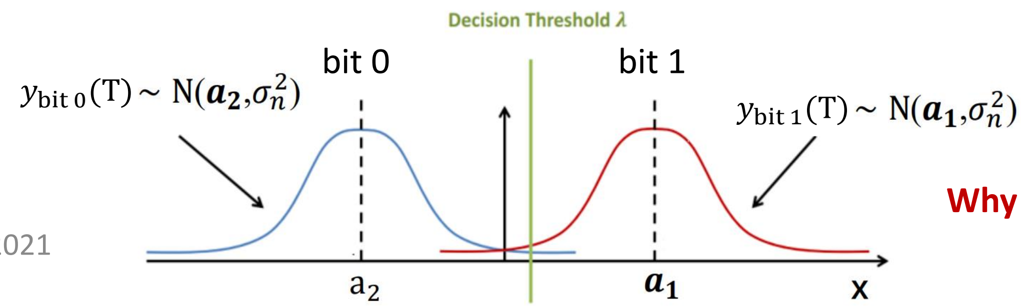
$$P_e = P(a=1)P(error|a=1) + P(a=0)P(error|a=0)$$

$$= P(a=1)\int_{-\infty}^{\lambda}\frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{(x-a_1)^2}{2\sigma_n{}^2}}dx + P(a=0)\int_{\lambda}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{(x-a_2)^2}{2\sigma_n{}^2}}\text{dx}$$

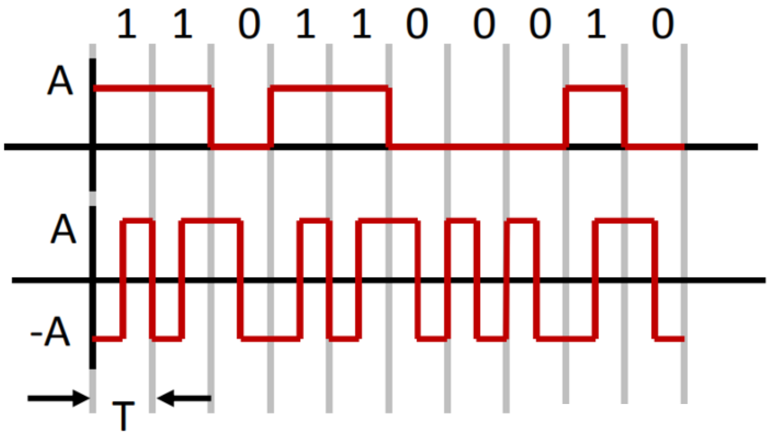
, where a is transmitted bit.

$$\boxed{\lambda = \frac{\text{a}_1+\text{a}_2}{2} + \frac{\sigma_n^2}{\text{a}_1-\text{a}_2}\ln\left(\frac{P(a=0)}{P(a=1)}\right)}$$

→ **If $P(a=0) = P(a=1) = \frac{1}{2}$** $\quad \lambda = \frac{a_1+a_2}{2}$



Why Gaussian?



Unipolar NRZ

Split-Phase Manchester Code

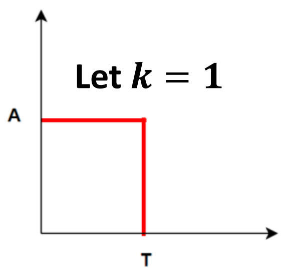
$$n(T) = \int_0^T A w(t)dt$$

$$\begin{aligned}
\sigma_N^2 &= \mathbf{E}[N^2] \\
&= A^2\mathbf{E}[\int_0^T\int_0^T w(t)w(u)dtdu] \\
&= A^2\int_0^T\int_0^T \mathbf{E}[w(t)u(t)]dtdu \\
&= A^2\int_0^T\int_0^T \boxed{R_W(t,u)}dtdu \\
&= A^2\int_0^T\int_0^T \boxed{\frac{N_0}{2}\delta(t-u)}dtdu \\
&= \frac{A^2TN_0}{2}
\end{aligned}$$

U-NRZ

h(t):

Let $k = 1$



$$BER = \int_{\frac{A^2T}{2}}^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{\frac{A^2TN_0}{2}}} e^{-\frac{x^2}{A^2TN_0}}\,dx$$

$$= Q(\sqrt{\frac{E_b}{N_0}})$$

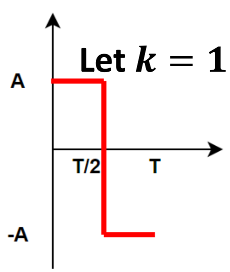
$$E_b = (A^2T+0)/2 = A^2T/2$$

λ: $\frac{a_1+a_2}{2} = \frac{A^2T+0}{2} = \frac{A^2T}{2}$

SPMC

h(t):

Let $k = 1$



$$BER = \int_0^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{\frac{A^2TN_0}{2}}} e^{-\frac{(x+A^2T)^2}{A^2TN_0}}\,dx$$

$$= Q(\sqrt{\frac{2E_b}{N_0}})$$

$$E_b = (A^2T+A^2T)/2 = A^2T$$

λ: $\frac{a_1+a_2}{2} = \frac{A^2T-A^2T}{2} = 0$