EE206: Communications Principles Tutorial

Assignment 8&9

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1. An FM signal has an amplitude of 5 volts, carrier frequency of 120 MHz and peak frequency deviation of 75 KHz. The modulating message signal is a single tone of 4 KHz with amplitude of 2 volts. The FM signal is transmitted through a transmission line using a bandwidth based on Carson's rule. The gain of the transmission line is 1 in the pass-band. At the output of the transmission line, the FM signal is demodulated using a demodulator including a limiter, differentiator, envelop detector, D.C. blocking and an ideal low-pass filter which has a bandwidth of 4 KHz and unity gain in the pass-band. The signal amplitude at the output of the limiter is assumed to be 2 volt. The white noise at the input of the transmission line has a double-sided PSD of $\eta/2 = 10^{-6}$ W/Hz.

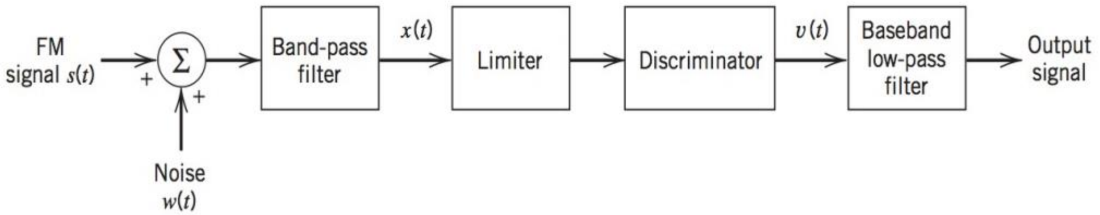
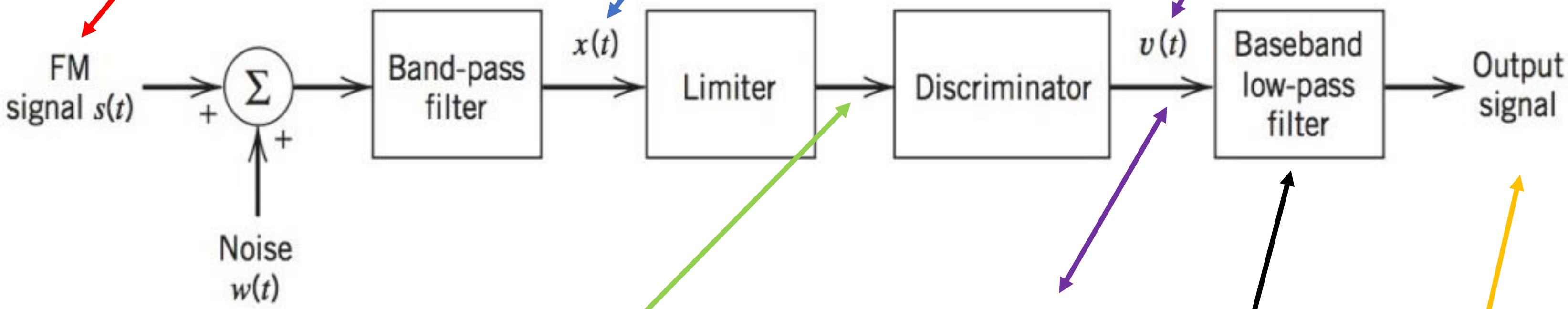


FIGURE 6.7 Noisy model of an FM receiver.

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= A_c \cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \psi(t)] \end{aligned}$$

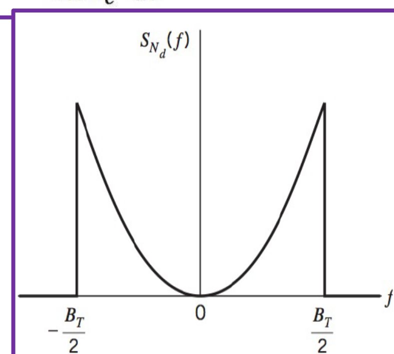
$$\begin{aligned} v(t) &= \frac{1}{2\pi}\frac{d\theta(t)}{dt} \\ &\simeq k_f m(t) + n_d(t) \end{aligned}$$



$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin[\psi(t) - \phi(t)]}{A_c + r(t)\cos[\psi(t) - \phi(t)]}\right\}$$

$$\theta(t) \simeq 2\pi k_f \int_0^t m(t)\, dt + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]$$

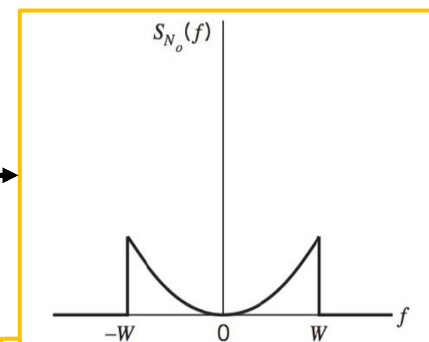
$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\{r(t)\sin[\psi(t) - \phi(t)]\}$$



$$S_{N_d}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \le \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$

LPF

Output noise



$$S_{N_o}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \le W \\ 0, & \text{otherwise} \end{cases}$$

Signal amplitude of Limiter output is omitted in this slide.

Reference: Textbook P215-P219

a. Determine the SNR at the input and the output of the demodulator, respectively.

$$A_c = 5\text{V}, \quad f_c = 120\text{MHz}$$

$$\triangle f = k_f A_m = 75\text{kHz}, \quad A_m = 2\text{V}, \quad f_m = 4\text{kHz}$$

$$\frac{\eta}{2} = 10^{-6}\text{W/Hz}$$

By Carson's Rule,

$$\text{BW} = 2(\beta + 1) f_m = 2(\frac{\triangle f}{f_m} + 1) f_m = 2(\frac{75\text{k}}{4\text{k}} + 1)4k = 158\text{kHz}$$

Input signal power:

$$S_i = \frac{1}{2} A_c^2 = 12.5\text{W}$$

Input noise power:

$$N_i = \frac{\eta}{2} \times 2 \times \text{BW} = 2 \times 10^{-6} \times 158\text{k} = 0.316\text{W}$$

Input SNR:

$$\text{SNR}_\text{i} = \frac{S_i}{N_i} = 15.97\text{dB}$$

a. Determine the SNR at the input and the output of the demodulator, respectively.

Output signal:

$$y(t) = A_l 2\pi k_f m(t)$$

where $V_l = 2\text{V}$, denotes the signal amplitude at the output of the limiter.

Output signal power:

$$S_o = \frac{1}{2} A_l^2 (2\pi k_f)^2 A_m^2 = \frac{1}{2} A_l^2 (2\pi \triangle f)^2$$

Output noise power:

$$N_o = \frac{A_l^2 8\pi^2 \eta f_{LPF}^3}{3A_c^2}$$

Output SNR:

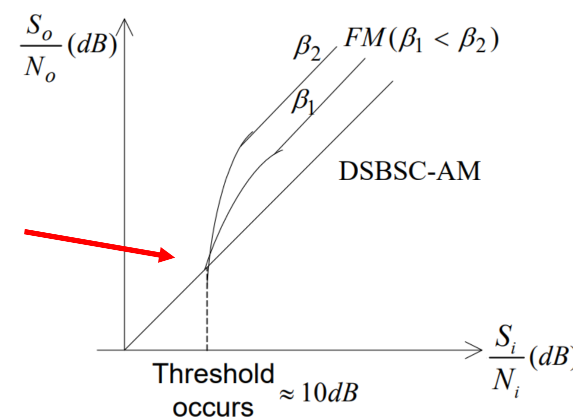
$$SNR_o = \frac{S_o}{N_o} = \frac{3\triangle f^2 A_c^2}{4\eta f_{LPF}^3} = 59.16\text{dB}$$

b. Determine if the system is under the threshold.

Textbook P217:
We assume that the carrier-to-noise ratio measured at the discriminator input is large compared with unity.

Threshold Phenomenon

- The output SNR formula was derived based on the small noise assumption. For large noise, "threshold phenomenon" occurs in the FM receiver.
- As discussed in the capture effect, when noise is small, the desired signal captures the noise.
- When noise is strong, it can effectively capture the signal.
- The "threshold" is defined as the minimum S_i / N_i that validates the output SNR formula and makes the FM receiver work properly.
- This threshold occurs when $\frac{S_i}{N_i} \approx 10$ dB.

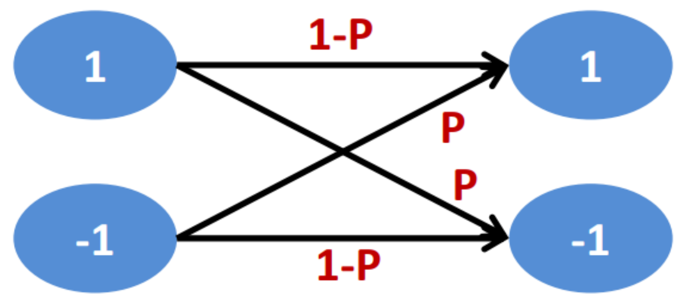


Answer: **Because $SNR_i > 10dB$, the system is not under the threshold.**

- D1.1

Given the following communication channel, if each information bit is repeated 4 times (code rate = 1/5) at the transmitter and P=0.9, how to achieve a good bit error rate at the receiver? What will the bit error rate be?

Binary Symmetric Channel:



Invert -1, 1 in receiver.

$$\Pr(\boldsymbol{Error\ Occurs})$$
$$= \Pr\left(\boldsymbol{More\ than\ \frac{n}{2}\ bits\ are\ wrong}\right)$$
$$= \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \boldsymbol{Pr(i\ bits\ are\ wrong)}$$
$$= \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \binom{n}{i} P^i (1-P)^{n-i}$$

- D2.1

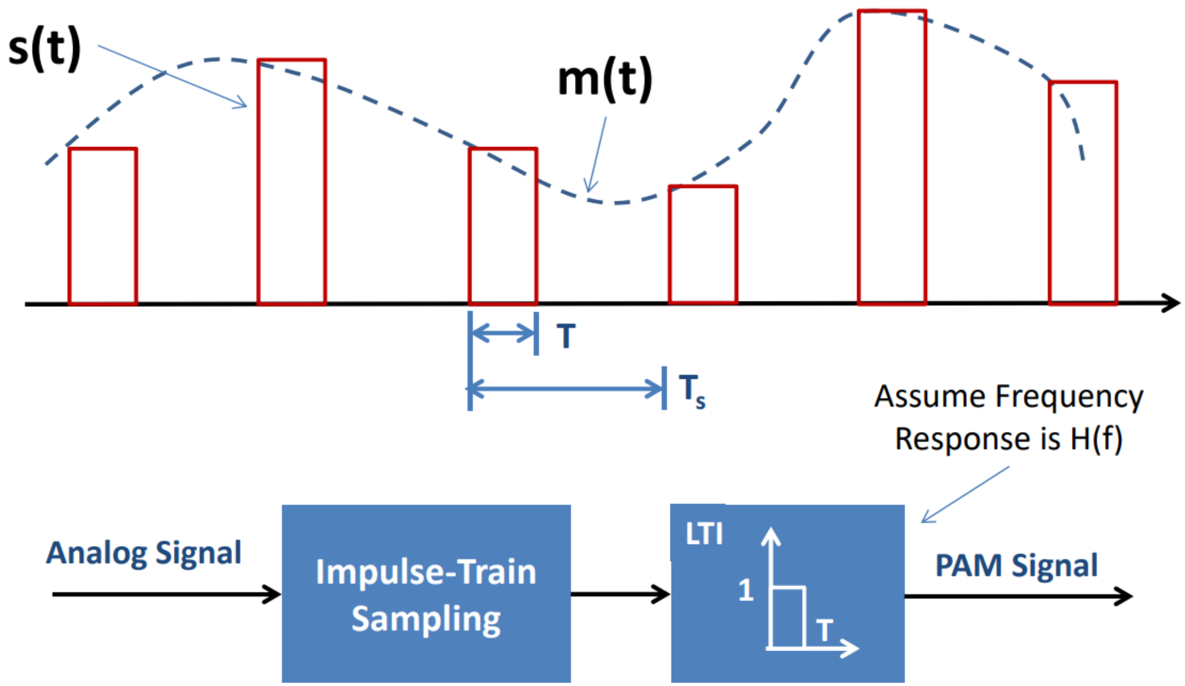
Plot the spectrum of a PAM wave produced from the following modulating signal

$$\boldsymbol{m(t) = A_m \cos(2\pi f_m t)}$$

assuming $f_m = 0.2Hz$, PAM sampling period $T_s = 1s$, and pulse duration $T = 0.45s$.

Pulse Amplitude Modulation

- PAM: sampled signal value is represented by the amplitude of pulses



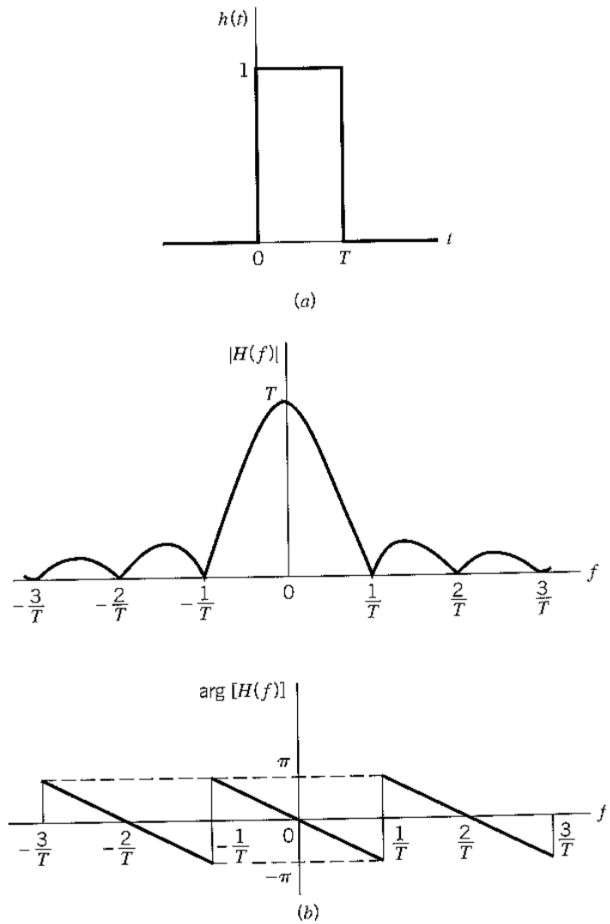


FIGURE 7.6 (a) Rectangular pulse $h(t)$. (b) Spectrum $H(f)$.

$$h(t) = \begin{cases} 1, & 0 < t < T \\ \frac{1}{2}, & t = 0,\ t = T \\ 0, & \text{otherwise} \end{cases}$$

By definition, the instantaneously sampled version of $m(t)$ is given by

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) \qquad (7.12)$$

$$\begin{aligned} m_\delta(t) \star h(t) &= \int_{-\infty}^{\infty} m_\delta(\tau)h(t-\tau)\,d\tau \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s)h(t-\tau)\,d\tau \qquad (7.13) \\ &= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t-\tau)\,d\tau \end{aligned}$$

Using the sifting property of the delta function, we thus obtain

$$m_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s) \qquad (7.14)$$

TextBook
245-246

$$S(f) = M_\delta(f)H(f) \qquad (7.16)$$

$$M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \qquad (7.17)$$