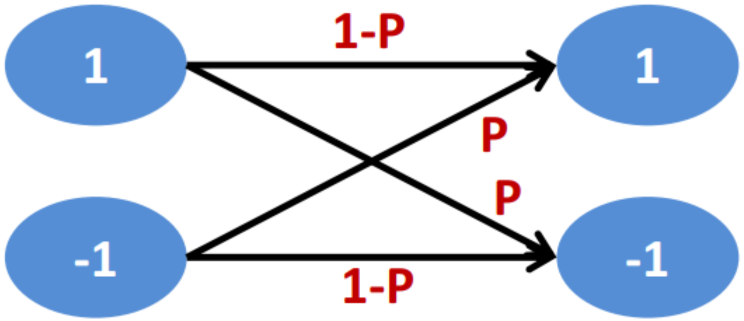
- D1.1

Given the following communication channel, if each information bit is repeated 4 times (code rate = 1/5) at the transmitter and P=0.9, how to achieve a good bit error rate at the receiver? What will the bit error rate be?

Binary Symmetric Channel:



Solution:

可以在接收端进行相应处理-1 -> 1 , 1 -> -1
经过相应处理后的bit error rate：

$$\mathrm{P}_r = \sum_{i=3}^{5}\binom{5}{i}(1-P)^i P^{5-i}=0.00856$$

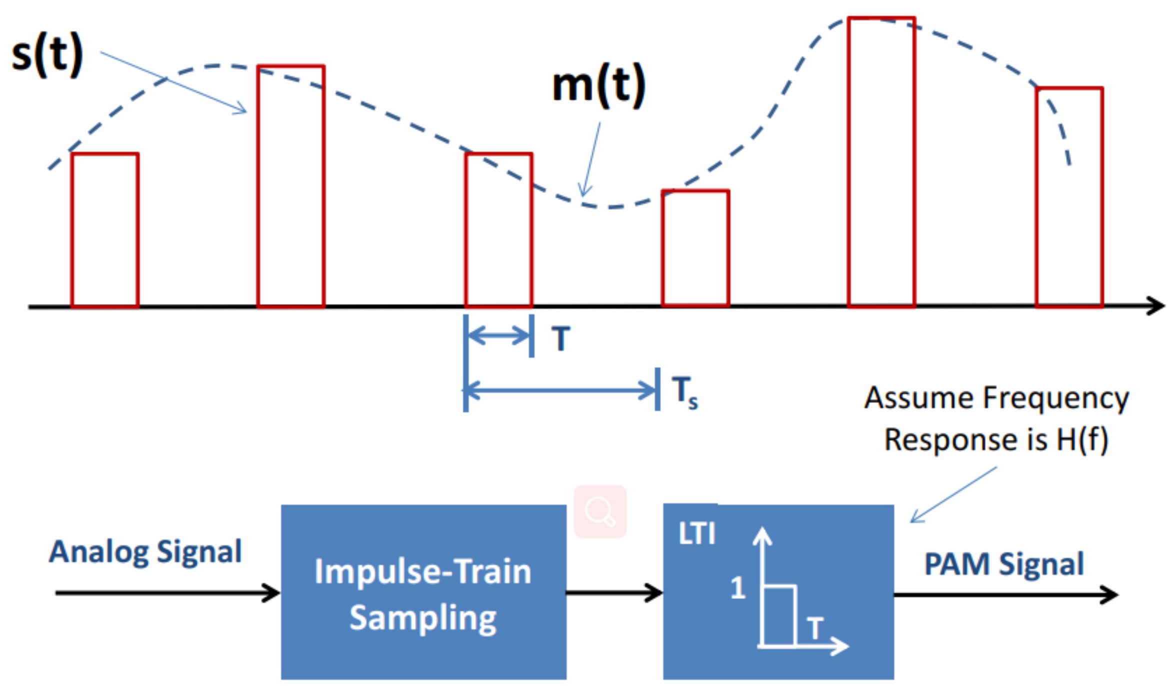
- D2.1

Plot the spectrum of a PAM wave produced from the following modulating signal

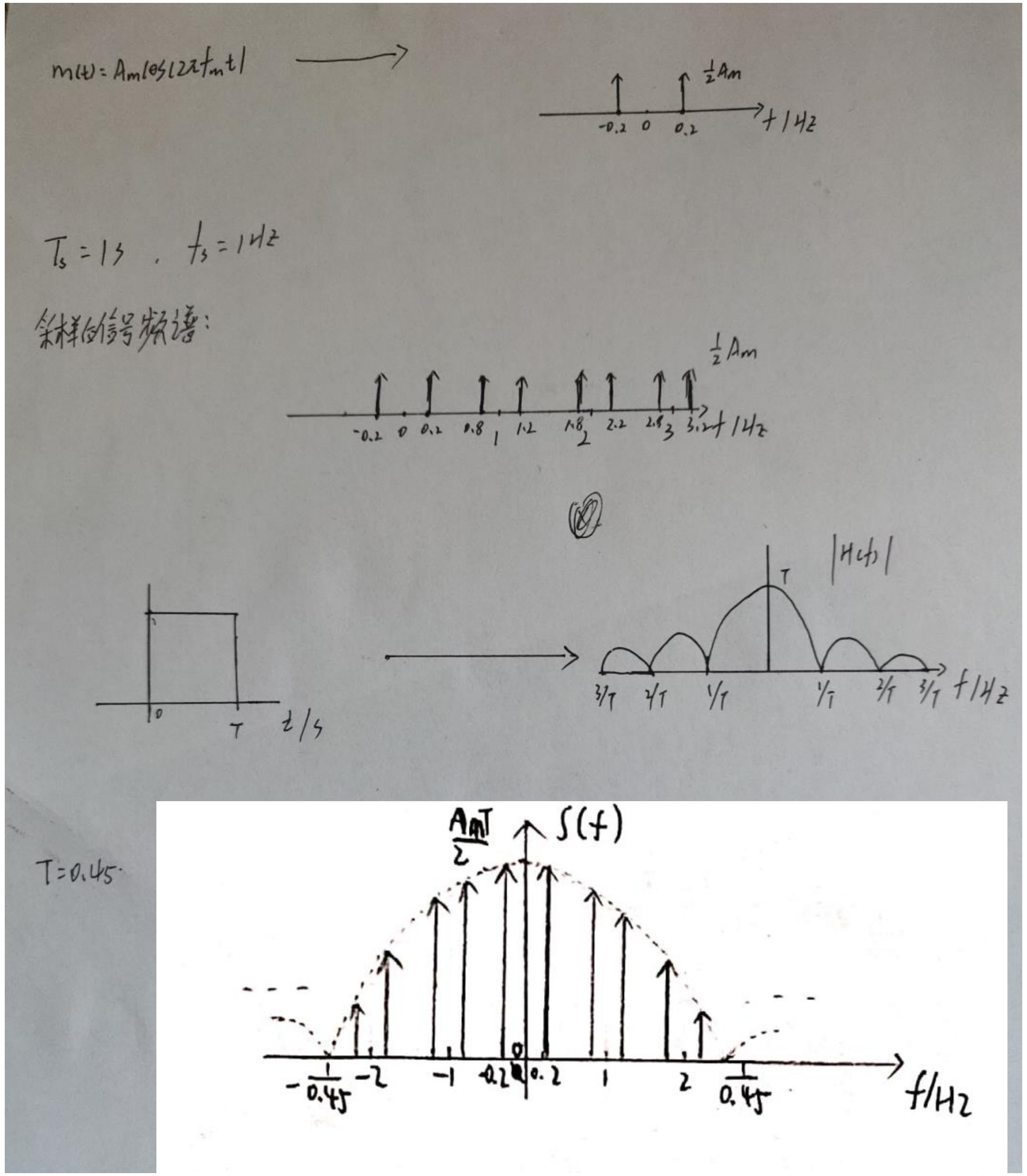
$$\boldsymbol{m(t) = A_m \cos(2\pi f_m t)}$$

assuming $f_m = 0.2Hz$, PAM sampling period $T_s = 1s$, and pulse duration $T = 0.45s$.

Solution:



s(t)
m(t)
T
T_s
Assume Frequency Response is H(f)
Analog Signal
Impulse-Train Sampling
LTI
1
T
PAM Signal



$m(t) = A_m \cos(2\pi f_m t)$
$\frac{1}{2}A_m$
f / Hz
$T_s = 1s$, $f_s = 1Hz$
采样的信号频谱:
$\frac{1}{2}A_m$
f / Hz
|H(f)|
T
t / s
f / Hz
T=0.4s
$\frac{A_m T}{2}$
S(f)
f/Hz

- D3.1

A compact disc (CD) records audio signals digitally by using PCM. Assume the audio signal bandwidth to be 15kHz.

(a) What is the Nyquist rate?

(b) If the Nyquist samples are quantized into *L*=65,536 levels and then binary coded, determine the number of binary digits per second (bit/s) required to encode the audio signal.

Solution:

a) $F_s \geq 2B = 30khz$

b) $L = 65536 = 2^{16}$ so 16 binary digits are required to encode every sample

$30 \times 10^3\ sample/s \times 16 bits/sample = 480\ kbits/s$

- D3.2

Show that, with a non-uniform quantizer, the average power (mean-square value) of the quantization error is approximately equal to $(1/12)\sum_i \Delta_i^2 p_i$ where Δ_i is the i-th step size and p_i is the probability that the input signal amplitude lies within the i-th interval R_i. Assume that the step-size Δ_i is small compared with the range of input signal, such that the signal can be treated as uniformly distribution within each step size.

Hints:

(1) Let Q be the quantization error, the expectation of Q^2 is given by

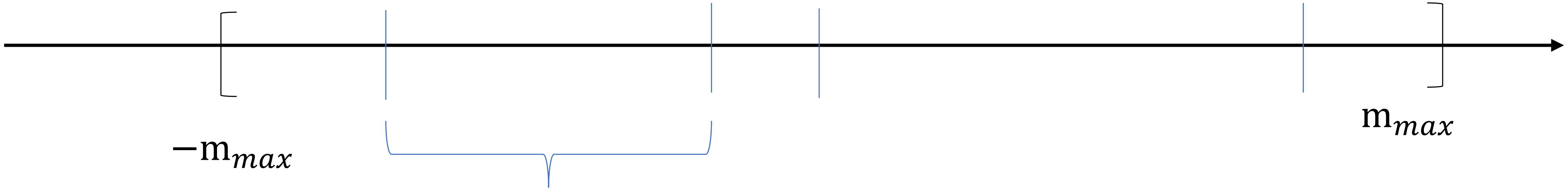
$$E[Q^2] = \sum_i \boldsymbol{E}[Q^2|\ signal\ is\ in\ the\ i-th\ step\ size]\ \boldsymbol{Pr}\ [signal\ is\ in\ the\ i-th\ step\ size]$$

(2) The mean and variance of a uniform distributed random variable within [a,b] are given by $\frac{1}{2}(a+b)$ and $\frac{1}{12}(b-a)^2$, respectively.

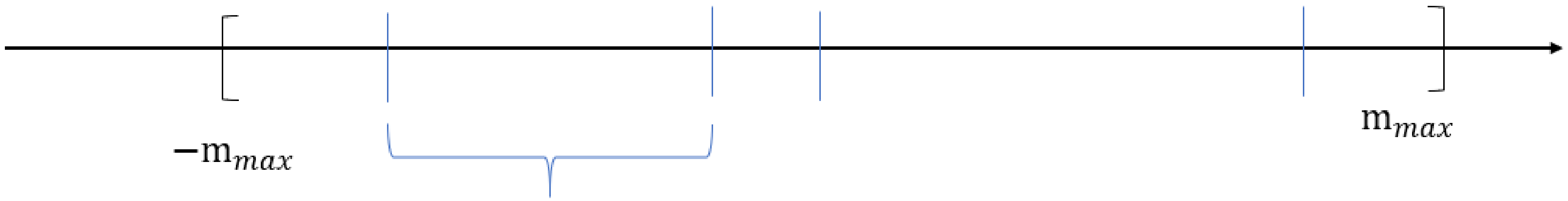
Solution:

1. 假设需要输入信号得范围是（$-\mathrm{m}_{max}$, m_{max} ）

2. 非均匀量化器：将区间分成若干个小区间



假设此区间信号量化为v_k， 区间长度为Δ_k



假设此区间信号量化为v_k， 区间长度为Δ_k

对于落在该区间得输入信号 $M_k \sim unif(v_k - \frac{\Delta_k}{2}, v_k + \frac{\Delta_k}{2})$

$$Q_k = M_k - v_k \sim unif(-\frac{\Delta_k}{2}, \frac{\Delta_k}{2})$$

$$E({Q_k}^2) = \int_{-\frac{\Delta_k}{2}}^{\frac{\Delta_k}{2}} x^2 \frac{1}{\Delta_k} dx = \frac{{\Delta_k}^2}{12}$$

$$E[Q^2] = \sum_i \boldsymbol{E}[Q^2 |\ signal\ is\ in\ the\ i-th\ step\ size]\ \boldsymbol{Pr}\ [signal\ is\ in\ the\ i-th\ step\ size]$$

$$E(Q^2) = \sum_i E(Q^2 | signal\ in\ the\ i-th\ step\ size)\, p_i = E({Q_i}^2) p_i = \sum_i \frac{{\Delta_i}^2}{12} p_i$$