EE206: Communications Principles Tutorial

Assignment 10

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- D3.1

A compact disc (CD) records audio signals digitally by using PCM. Assume the audio signal bandwidth to be 15kHz.

(a) What is the Nyquist rate?

(b) If the Nyquist samples are quantized into L=65,536 levels and then binary coded, determine the number of binary digits per second (bit/s) required to encode the audio signal.

(a)
According to Nyquist Sampling Theorem, $f_s > 2f_{max} = 30\text{kHz}$.
Nyquist rate: 30kHz.

(b)
$L = 65536 = 2^{16}$, so 16 bits are required to encode every sample.
Since sample rate should be larger than Nyquist rate,

$$30 \times 10^3 \frac{\text{sample}}{\text{s}} \times 16 \frac{\text{bit}}{\text{sample}} = 480\text{kbit/s}$$

So 480k bit/s required to encode the audio signal.

- D3.2

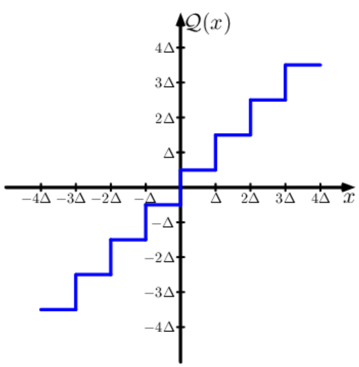
Show that, with a non-uniform quantizer, the average power (mean-square value) of the quantization error is approximately equal to $(1/12)\sum_i \Delta_i^2 p_i$ where Δ_i is the i-th step size and p_i is the probability that the input signal amplitude lies within the i-th interval R_i. Assume that the step-size Δ_i is small compared with the range of input signal, such that the signal can be treated as uniformly distribution within each step size.

Hints:

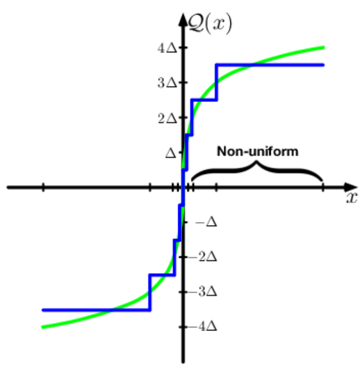
(1) Let Q be the quantization error, the expectation of Q^2 is given by

$$E[Q^2] = \sum_i \boldsymbol{E}[Q^2|\ signal\ is\ in\ the\ i-th\ step\ size]\ \boldsymbol{Pr}\ [signal\ is\ in\ the\ i-th\ step\ size]$$

(2) The mean and variance of a uniform distributed random variable within [a,b] are given by $\frac{1}{2}(a+b)$ and $\frac{1}{12}(b-a)^2$, respectively.

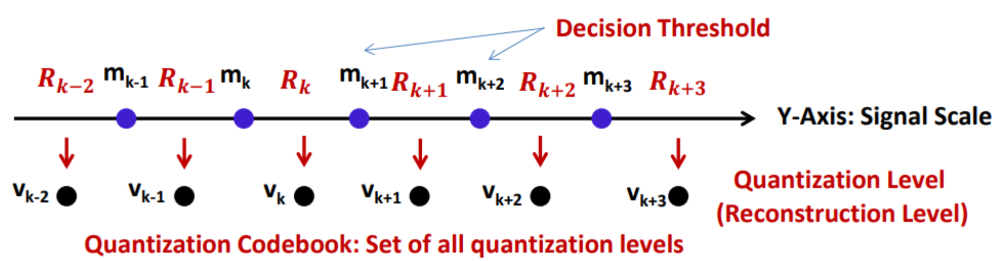


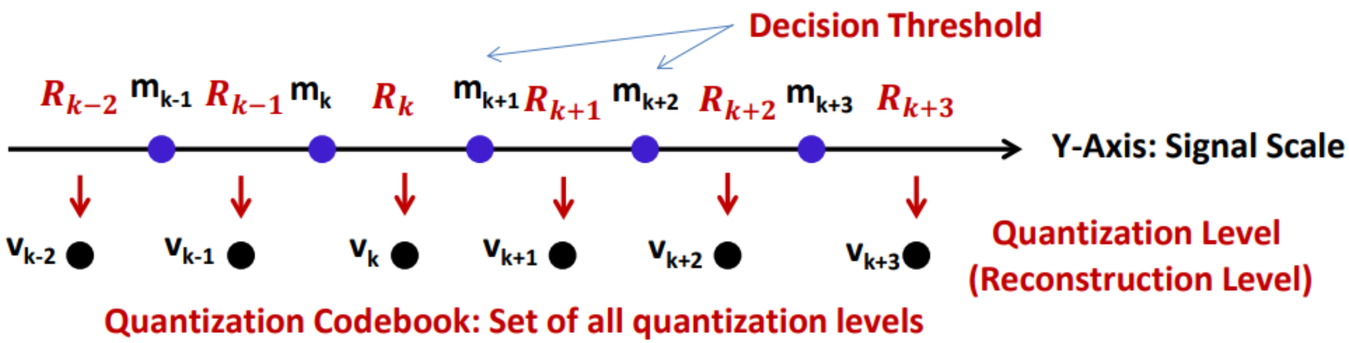
(a)



(b)

Non-uniform quantizer is that R_i is different from each other.





The sampled signal amplitude is a discrete random variable M.

For i-th interval R_i, probability that sampled signal amplitude M lies within it is p_i, and step size of the interval is Δ_i.

Assuming that random variable M_i is the sampled signal amplitude within interval R_i. We know that $M_i \sim U\left[v_i - \frac{\Delta_i}{2}, v_i + \frac{\Delta_i}{2}\right]$, where v_i is the middle value in interval R_i.

Quantization error $Q_\mathrm{i} = \mathrm{M_i} - \mathrm{v_i}$. It is also a random variable, and $Q_i \sim U(-\frac{\Delta_i}{2}, \frac{\Delta_i}{2})$.

$$\boldsymbol{E}[Q_i^2] = \int_{\frac{\Delta_i}{2}}^{-\frac{\Delta_i}{2}} x^2 \frac{1}{\Delta_i} dx = \frac{\Delta_i^2}{12}$$

$$\boldsymbol{E}[Q^2] = \sum_i \boldsymbol{E}[Q^2 | \text{signal is in the } \mathrm{i_{th}} \text{step size}]\, \boldsymbol{Pr}[\text{signal is in the i\_th step size}]$$

$$= \sum_i \boldsymbol{E}[Q_i^2] p_i = \frac{1}{12} \sum_i \Delta_i^2 p_i$$