EE206 2020 Spring

通信原理习题课

Assignment No. 6

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1. An FM modulator is followed by an ideal band-pass filter with centre frequency of 500 Hz and bandwidth of 72 Hz. The gain of the filter is 1 in the pass-band. The message signal $m(t) = 10\cos(20\pi t)$ and the carrier signal is $f(t)=10\cos(1000\pi t)$. The modulation frequency sensitivity $k_f = 7$ Hz/volt.
   a. Draw the amplitude spectra of the FM signal at the input and output of the band-pass filter, respectively.
   b. Determine the signal power at the input and output of the band-pass filter.

Solution

Q1. a) $f_c = 500Hz, f_m = 10Hz, \quad A_m = 10V, \quad k_f = 7Hz/V, \quad A_c = 10V$

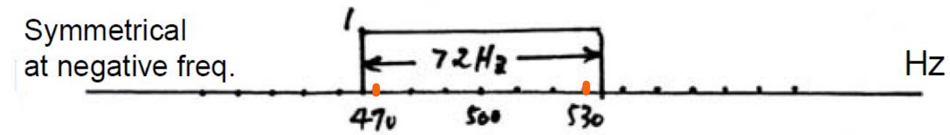
$$\beta = \frac{k_f A_m}{f_m} = \frac{7 \times 10}{10} = 7$$

For single-tone message and Wide-band FM (β>0.2):

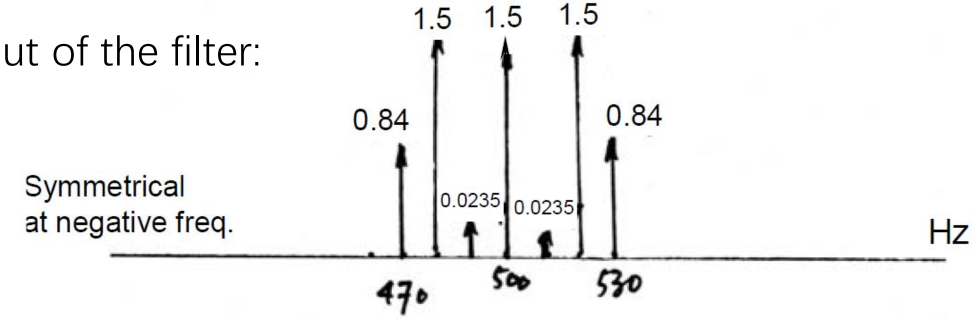
$$f_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi f_c t + 2n\pi f_m t]$$

$$= 10\sum_{n=-\infty}^{\infty} J_n(7)\cos[1000\pi t + 20n\pi t]$$

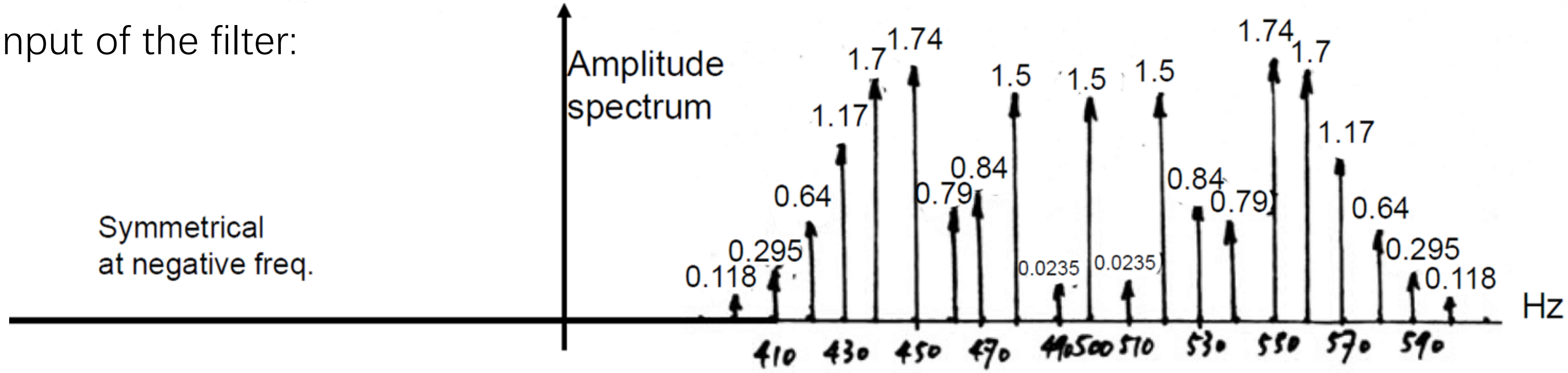
The band-pass filter:



Output of the filter:



Input of the filter:



$|J_n(7)|>0.01$

n	0	1	2	3	4	5	6	7	8	9	10
$J_n(7)$	0.3001	-0.0047	-0.3014	-0.1676	0.1578	0.3479	0.3392	0.2336	0.1280	0.0589	0.0235

b) The signal power at the input of band-pass filter is given by

$$P_{in} = 2\times\left(\frac{A_c}{2}\right)^2 \sum_{n=-\infty}^{\infty} J_n^{\ 2}(\beta) = \frac{A_c^2}{2}\sum_{n=-\infty}^{\infty} J_n^{\ 2}(\beta) = 50 \qquad (W)$$

The band-pass filter passes frequencies components at

$$f_{c,} \quad f_c \pm f_m, \quad f_c \pm 2f_m, \quad f_c \pm 3f_m .$$

The signal power at the output of band-pass filter is given by

$$P_{out} = \frac{A_c^2}{2}\sum_{n=-3}^{3} J_n^{\ 2}(\beta)$$

$$= \frac{A_c^2}{2}\left[J_0^{\ 2}(\beta) + 2[J_1^{\ 2}(\beta) + J_2^{\ 2}(\beta) + J_3^{\ 2}(\beta)]\right]$$

$$= 50\times\left[(0.3)^2 + 2[(0.0047)^2 + (0.3014)^2 + (0.1676)^2]\right]$$

$$= 50\times 0.328 \quad = 16.4 \ \ (W)$$

2. Show that unlike AM, the mean power of an FM signal in the form of $A_c \cos[\omega_c t + \beta \sin \omega_m t]$ is independent of modulation index, β (Hint: make use of the property $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$).

Solution

$$\overline{f_{FM}^{\ 2}(t)} = \overline{\{A_c \cos[\omega_c t + \beta \sin(\omega_m t)]\}^2}$$

$$= \overline{[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c t + n\omega_m t)]^2}$$

$$= \frac{A_c^{\ 2}}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta) \quad (\because \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1)$$

$$= \frac{A_c^{\ 2}}{2}$$

3. For an FM signal $f_{FM}(t) = 6\cos(2\pi 10^9 t + 4\sin 2\pi 10^3 t)$, calculate the total mean power of the significant sideband components and the carrier component within the bandwidth, where

a. the bandwidth is determined using 1% rule, and

b. the bandwidth is determined using Carson's rule.

Solution

Q3. **(a)** $f_{FM}(t) = 6\cos(2\pi 10^9 t + 4\sin 4\pi 10^3 t) \Rightarrow \beta = 4.$ $|J_n(4)|>0.01$

From the Bessel function table, no. of significant side-band pairs n'=7.

$$P = \frac{A_c^2}{2}\sum_{n=-7}^{7} J_n^2(\beta) = \frac{6^2}{2}\{(0.3971)^2 + 2[(0.06604)^2 + (0.3641)^2 + (0.4302)^2$$

$$+(0.2811)^2 + (0.1321)^2 + (0.04909)^2 + (0.01518)^2]\} = \frac{6^2}{2}\times\boxed{0.99991} = 17.99838$$

$\boxed{99.991\%}$ of the total signal power is included in the bandwidth.

n	0	1	2	3	4	5	6	7
$J_n(4)$	0.3971	0.06604	0.3641	0.4302	0.2811	0.1321	0.04909	0.01518

(b) Using Carson's rule

$$BW = 2(\beta+1)f_m = 2(4+1)f_m = 10f_m$$

$\Rightarrow$ The first 5 side-band pairs are included in the bandwidth.

$$P = \frac{A_c^2}{2}\sum_{n=-5}^{5} J_n^2(\beta) = \frac{6^2}{2}\{(0.3971)^2 + 2[(0.06604)^2 + (0.3641)^2 + (0.4302)^2$$

$$+(0.2811)^2 + (0.1321)^2]\} = \frac{6^2}{2}\times\boxed{0.99464} = 17.90352$$

$\boxed{99.464\%}$ of the total signal power is included in the bandwidth.

4. A message signal $m(t) = 5\sin(2000\pi t)$ phase modulates a cosine wave of 100 MHz. The PM signal has peak-phase deviation of $\pi/2$ and amplitude $A_c = 100$ volts.

   a. Determine the amplitude spectrum of the PM signal.

   b. Determine the approximate bandwidth which contains 99% of total power of the PM signal.

   c. Determine the approximate bandwidth using Carson's rule and compare the results with the analytical result obtained in part (b).

Given: $J_0(\pi/2)=0.4720$, $J_1(\pi/2)=0.5668$, $J_2(\pi/2)=0.2497$, $J_3(\pi/2)=0.0690$, $J_4(\pi/2)=0.0140$.

Solution

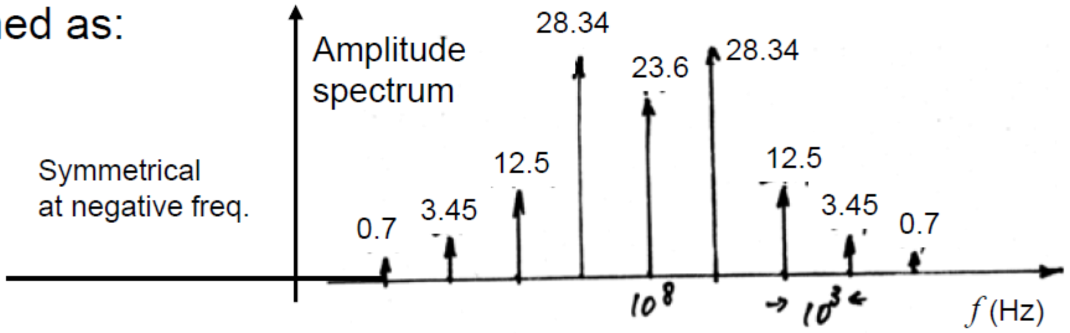
Q4. (a) $f_c = 10^8 Hz, \quad f_m = 1000 Hz, \quad \beta_p = \frac{\pi}{2}, \quad A_c = 100V$

$$f_{PM}(t) = 100\cos[2\pi 10^8 t + \frac{\pi}{2}\sin 2000\pi t] = 100\sum_{n=-\infty}^{\infty} J_n(\frac{\pi}{2})\cos[2\pi(10^8 + n10^3)t]$$

Using $J_0(\pi/2)$=0.4720, $J_1(\pi/2)$=0.5668, $J_2(\pi/2)$=0.2497, $J_3(\pi/2)$=0.0690, $J_4(\pi/2)$=0.0140,

$|J_n(\pi/2)|>0.01 \rightarrow n'=4$

The amplitude spectrum is obtained as:



(b) The total signal power is

$$\frac{A_c^2}{2} = \frac{100^2}{2} = 5000 \qquad (W)$$

To find the BW (containing 99% of total power), we need to find the minimum integer K with

$$\frac{100^2}{2}\sum_{n=-K}^{K} J_n{}^2(\frac{\pi}{2}) \geq 0.99\times 5000$$

From the given Bessel function values, we can find K = 2.

$$BW_{effective} = 2Kf_m = 4000 \quad (Hz)$$

(C) Using Carson's rule, $BW = 2(\frac{\pi}{2}+1)f_m \approx 5140 \quad (Hz)$

Bessel Function Table

Values of the Bessel Functions $J_n(\beta)$

n \ β	0.5	1	2	3	4	5	6	7	8	9	10
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	−0.1776	0.1506	0.3001	0.1717	+0.09033	−0.2459
1	0.2423	0.4401	0.5767	0.3391	−0.06604	−0.3276	−0.2767	−0.004683	0.2346	0.2453	0.04347
2	0.03060	0.1149	0.3528	0.4861	0.3641	0.04657	−0.2429	−0.3014	−0.1130	0.1448	0.2546
3	0.002564	0.01956	0.1289	0.3091	0.4302	0.3648	0.1148	−0.1676	−0.2911	−0.1809	0.05838
4		0.002477	0.03400	0.1320	0.2811	0.3912	0.3576	0.1578	−0.1054	−0.2655	−0.2196
5			0.007040	0.04303	0.1321	0.2611	0.3621	0.3479	0.1858	−0.05504	−0.2341
6			0.001202	0.01139	0.04909	0.1310	0.2458	0.3392	0.3376	0.2043	−0.01446
7				0.002547	0.01518	0.05338	0.1296	0.2336	0.3206	0.3275	0.2167
8					0.004029	0.01841	0.05653	0.1280	0.2235	0.3051	0.3179
9						0.005520	0.02117	0.05892	0.1263	0.2149	0.2919
10						0.001468	0.006964	0.02354	0.06077	0.1247	0.2075
11							0.002048	0.008335	0.02560	0.06222	0.1231
12								0.002656	0.009624	0.02739	0.06337
13									0.003275	0.01083	0.02897
14									0.001019	0.003895	0.01196
15										0.001286	0.004508
16											0.001567