EE206 2022 Spring

通信原理
习题课

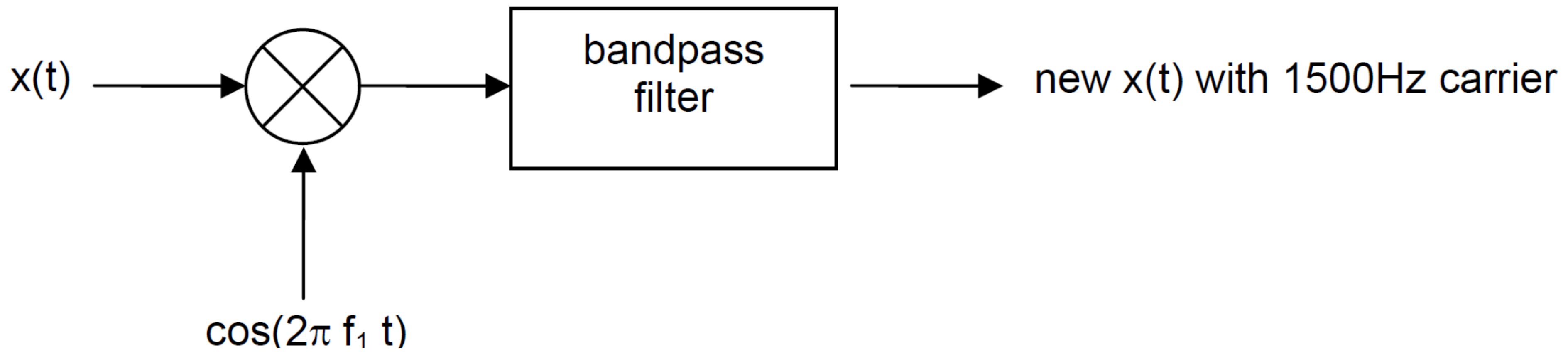
Assignment No. 4

TA 郑沛聪

1. The signal $s(t) = \sin(200\pi t + \pi/3) + \cos(200\pi t + \pi/3)$ is modulated using a cosine carrier signal with carrier frequency 500Hz and zero phase to generate a Suppressed-Carrier AM signal $x(t)$.

(a) Write $s(t)$ as a single cosine term. Then find $x(t)$.

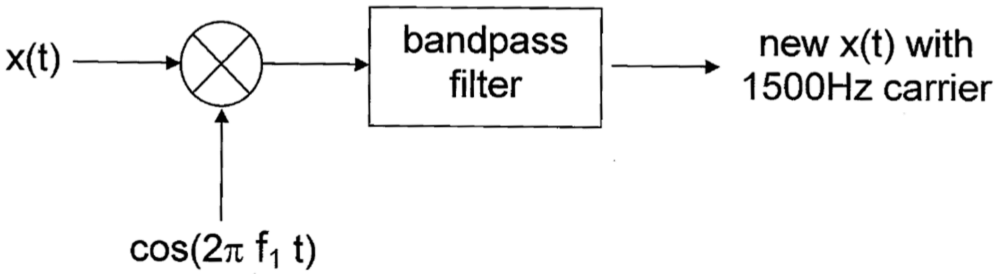
(b) Use the mixer below to shift the carrier frequency of $x(t)$ to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.



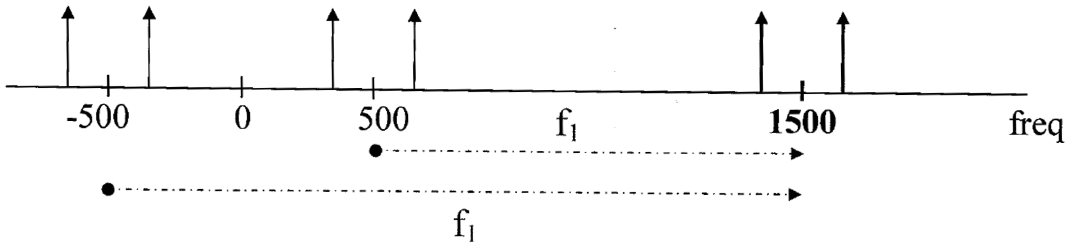
Solution of (a)

$$s(t) = \sin(200\pi t + \pi/3) + \cos(200\pi t + \pi/3)$$
$$= \sqrt{1+1}\ \cos(200\pi t + \pi/3 - \tan^{-1}(1/1))$$
$$= \sqrt{2}\cos(200\pi t + \pi/12)$$

AM signal $x(t) = \sqrt{2}\cos(200\pi t + \pi/12)\cos(1000\pi t)$



Solution of (b)



$f_1 = 1500 - 500 = 1000$ Hz, or $1500 - (-500) = 2000$ Hz

filter centre freq = 1500 Hz

filter BW = $2 \times 100 = 200$ Hz

2. a) A received signal a(t) has SNR 13dB and **noise** power 64 μW (μW = 10^{-6} Watt). Another received signal b(t) also has SNR 13dB but **total** (signal+noise) power of 64 μW. Determine the useful signal power in mW in each of these signals.

Solution

(a) For the signal a(t),

SNR = Signal power / Noise power = 13dB = $10^{13/10} \approx 20$

Signal power = 20×Noise power = 20×64 μW = 1280 μW.

For b(t), total power = Signal power + Noise power

= S + N = 64 μW ---------- (1)

From 1(a), S / N = 20 ---------------- (2)

Put (2) into (1), 20N + N = 64 → N = 64/21 ≈ 3 μW

Put N into (1), S = 64 - 3 = 61 μW.

b) An AM signal x(t) is received with 6mW signal power, 20KHz bandwidth and carrier freq 100MHz. Another AM signal y(t) is received with 100mW signal power, 3MHz bandwidth and carrier freq 500MHz. The channel contains white noise. Which signal has better quality?

<u>Solution</u>

(b) To compare received signal quality, compare their SNR.

Since white noise PSD is not mentioned, assume it is $N_o/2$.

SNR of x(t) = S / N

$$= 6\text{mW} / (20\times10^3 \times N_o/2 \times 2) = 3/(10^7\, N_o)$$

SNR of y(t) = $100\text{mW} / (3\times10^6 \times N_o/2 \times 2)$

$$= 1/(3\times10^7\, N_o) < \text{SNR of } x(t)$$

So, x(t) has better quality (even though it has smaller signal power of 6mW).

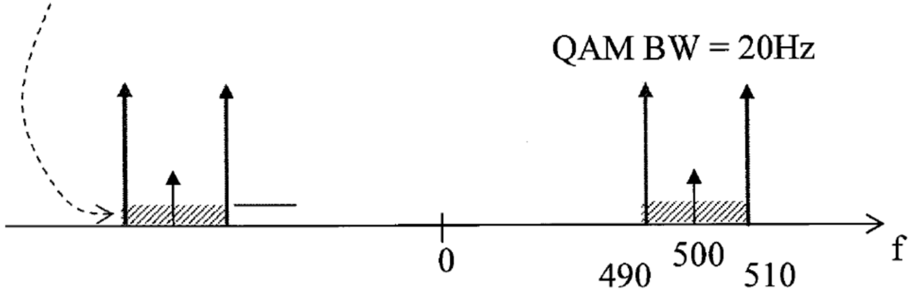
<u>Note</u>: The carrier freq info in the question is not needed

3. Two message signals, $s_1(t) = 2$ and $s_2(t) = 10\sin(20\pi t)$, are modulated to form a QAM signal x(t) with carrier frequency 500Hz. $s_1(t)$ is modulated onto the I-phase, $s_2(t)$ onto the Q-phase. During transmission, x(t) is corrupted by white noise with 2-sided PSD of 10^{-5} Watt/Hz. At the receiver, it is demodulated using a coherent demodulator. Determine the SNR of the Q-branch output signal in dB.

Solution

QAM signal $x(t) = 2\cos(1000\pi t) + 10\sin(20\pi t)\sin(1000\pi t)$

$|X(f)|$ + bandpass noise in freq domain:



Q-branch demodulator output signal

$= [\,(2\cos x + 10\sin 20\pi t\,\sin x)\sin x\,]_{LPF}$ where $x = 1000\pi t$

$= [\,2(\tfrac{1}{2}\sin 2x) + 10\sin 20\pi t\,(\tfrac{1}{2} - \tfrac{1}{2}\cos 2x)\,]_{LPF}$

$= 5\sin 20\pi t$ ➔ power $S_o = 5^2/2 = 12.5$

Q-branch demodulator output noise

$= [\,(n_c(t)\cos x - n_s(t)\sin x)\sin x\,]_{LPF}$

$= -\tfrac{1}{2}\, n_s(t)$

➔ power $N_o = \tfrac{1}{4}\overline{n_s^2(t)} = \tfrac{1}{4}\overline{n^2(t)} = \tfrac{1}{4}\times 2\times\left(10^{-5}\times 20\right) = 10^{-4}$

Q-branch output SNR = $S_O/N_O = 12.5/10^{-4} = 50.97$ dB