通信原理
习题课

Assignment No. 3

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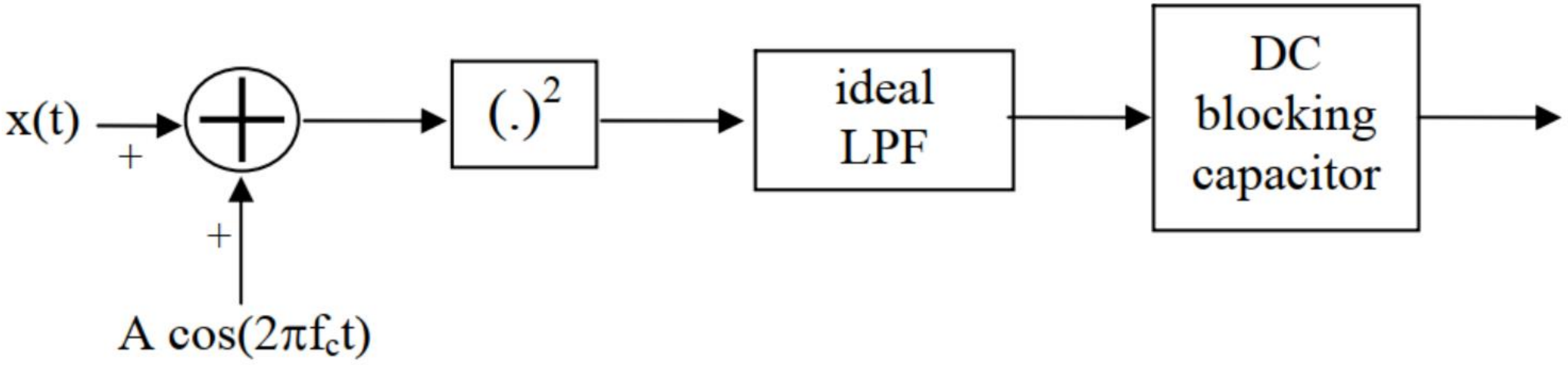
1. A DSBSC-AM signal $x(t) = \text{sinc}(1000t)\cos(2\pi f_c t)$ is demodulated using the system shown below. The box marked $(.)^2$ is a square-law device that produces an output equal to the square of its input. The DC blocking capacitor removes all DC components at its input.

(a) Show that the demodulated output contains distortion.

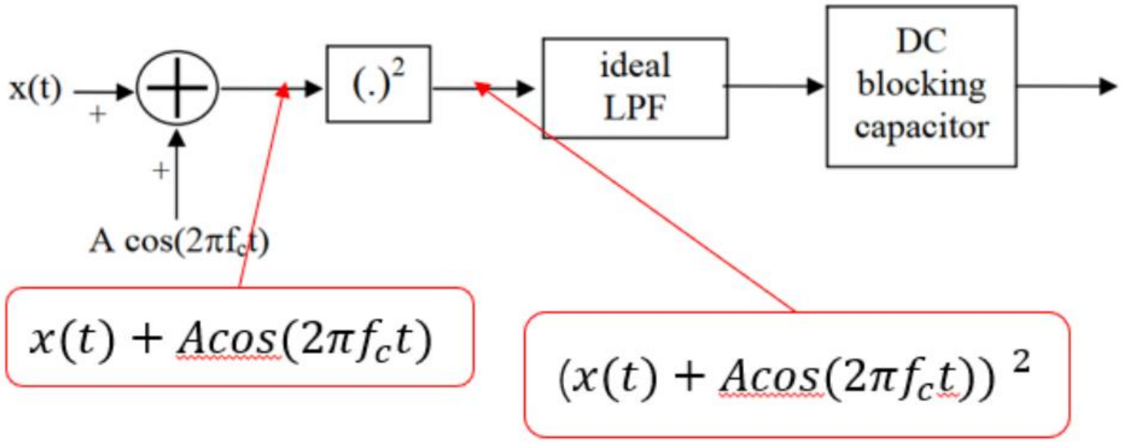
(b) How should the lowpass filter (LPF) be designed to minimize this distortion?

(c) What is the minimum carrier frequency f_c permitted for this demodulator?



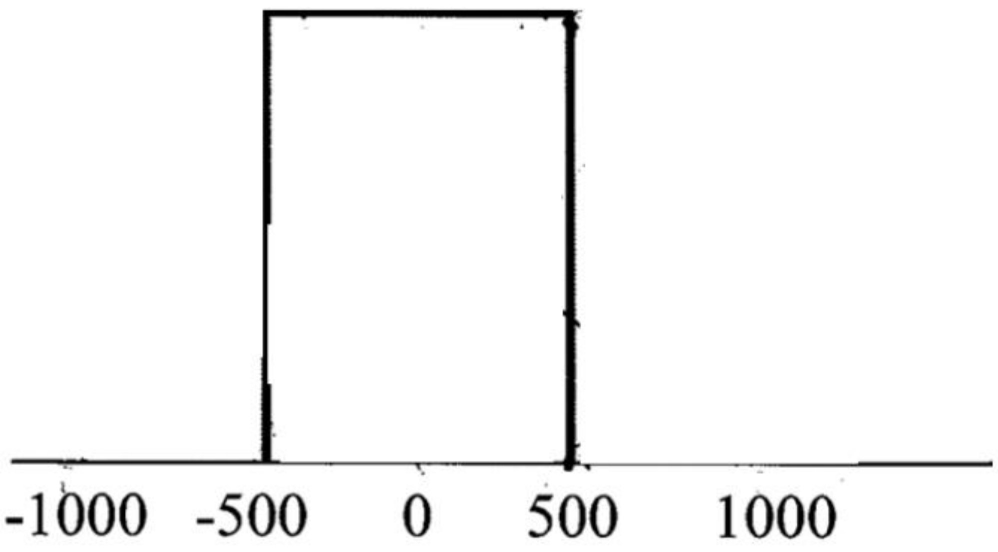
Solution of (a)

$$[x(t) + Acos(2\pi f_c t)]^2 = [sinc(1000t)cos(2\pi f_c t) + Acos(2\pi f_c t)]^2$$
$$= [sinc(1000t) + A]^2 cos^2(2\pi f_c t)$$
$$= \frac{1}{2}[sinc^2(1000t) + 2Asinc(1000t) + A^2] + \frac{1}{2}cos(4\pi f_c t)(sinc^2(1000t) + 2Asinc(1000t) + A^2$$

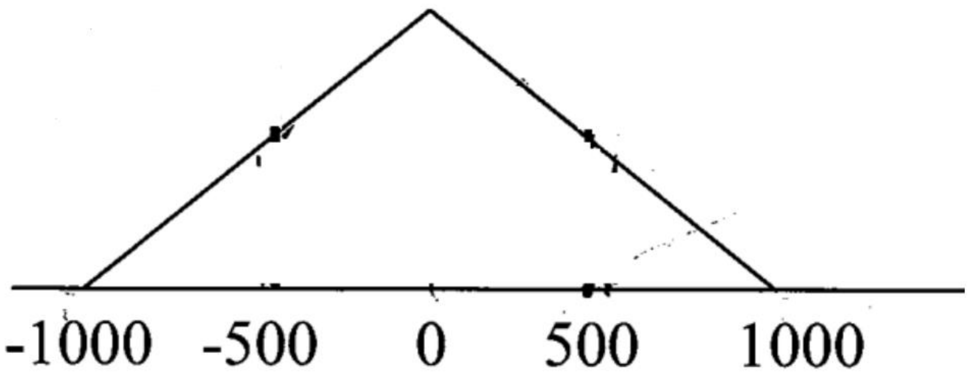


Solution of (a)

Amplitude spectrum of $sinc(1000t) = \frac{1}{1000} rect(\frac{1}{1000} f)$

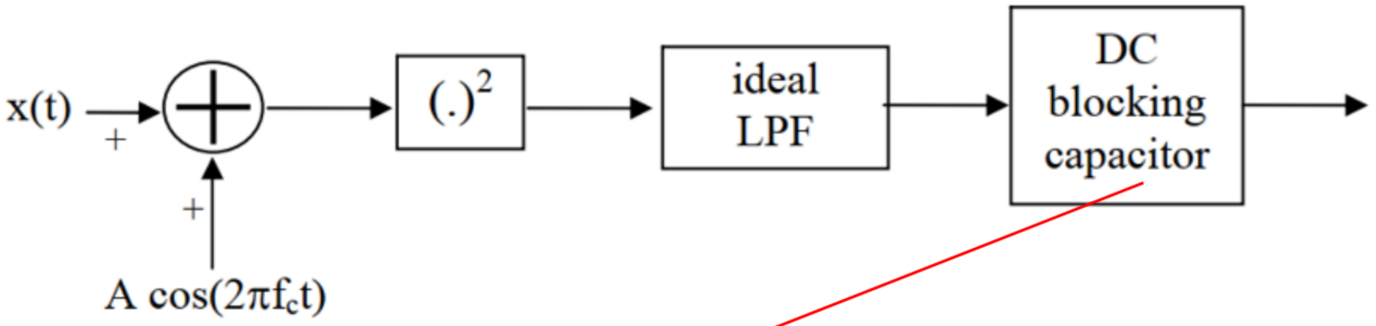


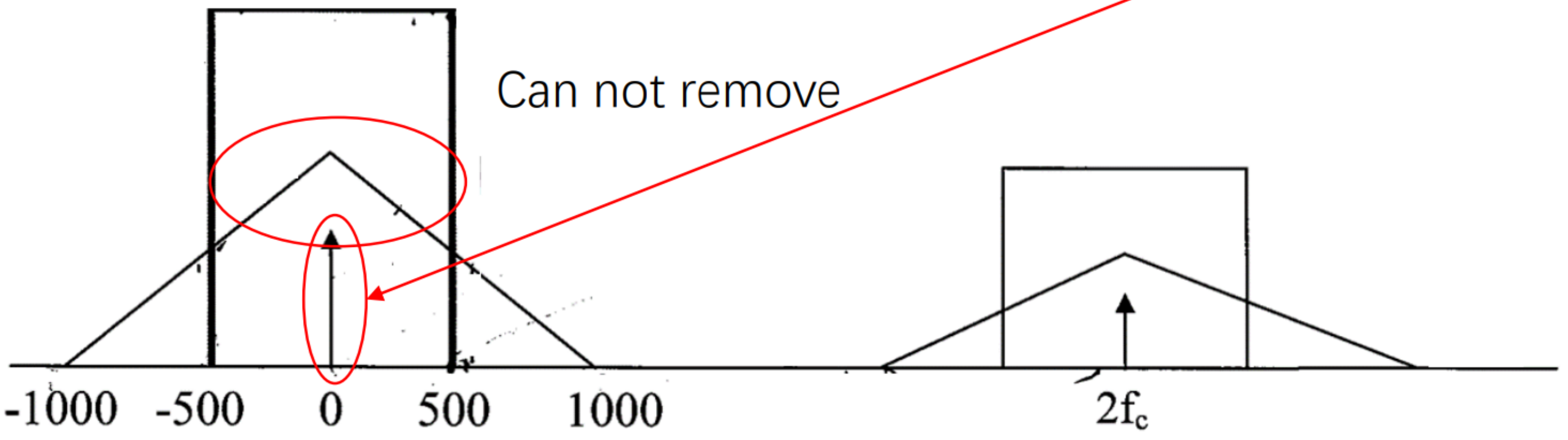
Amplitude spectrum of $sinc^2(1000\text{t}) = \frac{1}{1000} rect(\frac{1}{1000} f) \otimes \frac{1}{1000} rect(\frac{1}{1000} f)$



Solution of (a)

Amplitude spectrum:





Output = $A sinc(1000t)$+distortion

Solution of (b)

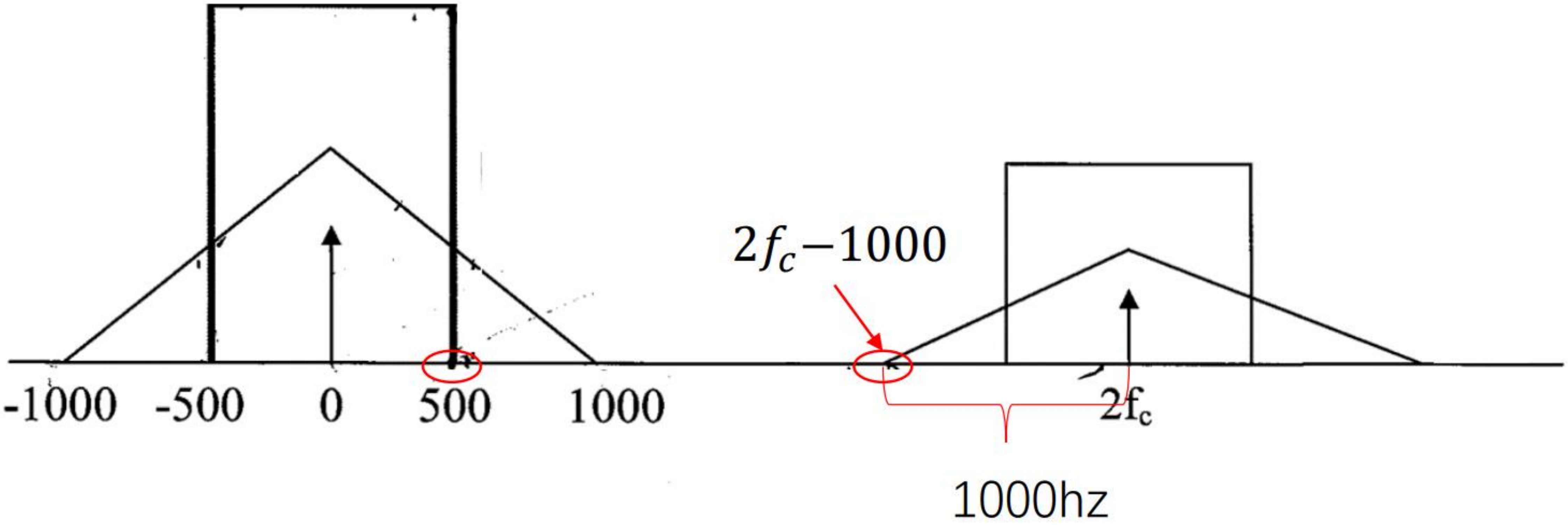
$$BW = 500 \text{ HZ}$$

$$\frac{1}{2}[sinc^2(1000t) + 2Asinc(1000t) + A^2] + \frac{1}{2}cos(4\pi f_c t)(sinc^2(1000t) + 2Asinc(1000t) + A^2$$

Solution of (c)

$$2f_c - 1000 > 500$$

$$f_c > 750Hz$$



2. A QAM signal with a carrier frequency of 4KHz is formed by modulating a message signal $s_1(t) = 1$ volt onto the in-phase carrier and another message signal $s_2(t) = -1$ volt onto the quadrature-phase carrier.

   (a) Determine the time-domain expression of the QAM signal. Write your answer as a single cosine term.

   (b) Demodulate the QAM signal obtained in Part (a) using a coherent detector.

<u>Solution of (a)</u>

$$\text{QAM signal} = s_1(\mathrm{t})\cos(2\pi f_c t) + s_2(\mathrm{t})\sin(2\pi f_c t)$$
$$= \cos(8000\pi t) - \sin(8000\pi t)$$
$$= \sqrt{2}\cos\left(8000\pi t + \frac{\pi}{4}\right)$$

Solution of (b)

1. In- phase demodulator output:

$$= [\sqrt{2}\ \cos\left(8000\pi t + \frac{\pi}{4}\right) * \cos(8000\pi t)]_{LPF}$$

$$= \frac{\sqrt{2}}{2}[\cos\left(\frac{\pi}{4}\right) + \cos(16000\pi t + \frac{\pi}{4})]_{LPF}$$

$$= \frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

2. Q- phase demodulator output:

$$= [\sqrt{2}\ \cos\left(8000\pi t + \frac{\pi}{4}\right) * \sin(8000\pi t)]_{LPF}$$

$$= \frac{\sqrt{2}}{2}[-\sin\left(\frac{\pi}{4}\right) + \sin(16000\pi t + \frac{\pi}{4})]_{LPF}$$

$$= -\frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{4}\right) = -\frac{1}{2}$$

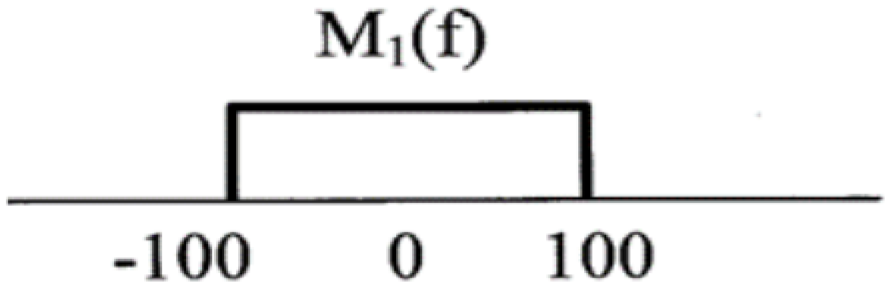
3. Given two message signals $m_1(t) = \mathrm{sinc}(200t)$ and $m_2(t) = 2\cos(2\pi f_0 t)$ where f_0 can range from 0Hz to 120Hz. Compare the minimum amount of bandwidth required to transmit them using

(a) DSBSC-AM and frequency division multiplexing (FDM)

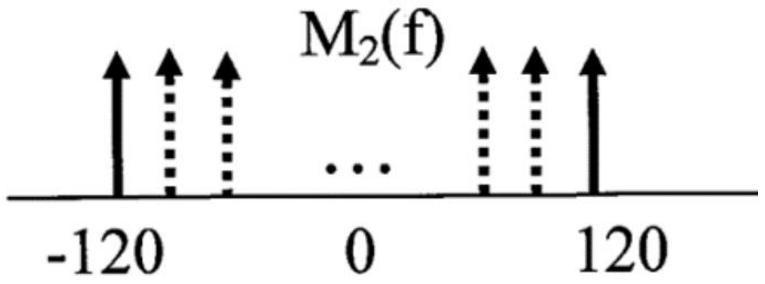
(b) QAM

Solution of (a)

Amplitude spectrum of $sinc(200t) = 1/200\, rect(1/200\, f)$

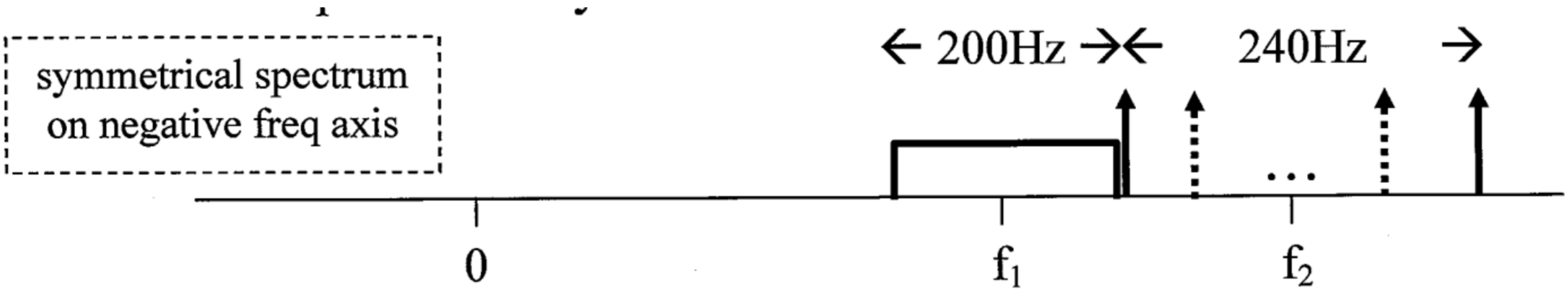


Amplitude spectrum of $2\cos(2\pi f_0 t) = \delta(f - f_c) + \delta(f + f_c)$



Solution of (a)

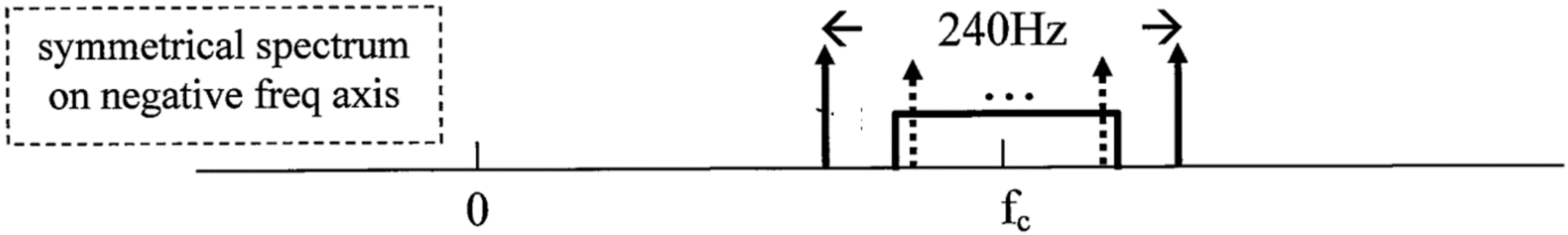
FDM signal = $m_1(t)\cos(2\pi f_1 t) + m_2(t)\cos(2\pi f_2 t)$



BW = 200 + 240 ≥ 440 Hz

Solution of (b)

b) QAM signal = $m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t))$



BW = 240 ≥ 240 Hz