Summary of Angle Modulation

FM & PM

Definitions

- The general expression for an FM/PM signal:

  $f(t) = A_c \cos\theta_i(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$

  where $\phi(t)$ denotes the instantaneous phase.

- The instantaneous freq. $f_i(t) = \frac{1}{2\pi}\cdot\frac{d\theta_i(t)}{dt} = f_c + \frac{1}{2\pi}\cdot\frac{d\phi(t)}{dt}$

- The instantaneous freq. deviation $\Delta f_i(t) = \frac{1}{2\pi}\cdot\frac{d\phi(t)}{dt}$

  - PM: $\phi(t) = 0 + k_p \cdot m(t)$
  - FM: $f_i(t) = f_c + k_f \cdot m(t)$, or equivalently, $\Delta f_i(t) = k_f \cdot m(t)$

Single Tone Modulation

- Single tone message: $m(t) = A_m \cos[2\pi f_m t]$

- Single-tone PM signal:

  $f_{PM}(t) = A_c \cos[2\pi f_c t + \beta_p \cos 2\pi f_m t]$

  where PM modulation index $\beta_p = k_p A_m$.

- Single-tone FM signal:

  $f_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$

  where FM modulation index $\beta \triangleq \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$

  Here, $\Delta f = k_f A_m$ is the peak frequency deviation.

Narrow-Band FM

- Narrow-band FM: $\beta \leq 0.2$

$$f_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$\approx A_c \cos[\omega_c t] + \frac{\beta A_c}{2} \cos[(\omega_c + \omega_m)t] - \frac{\beta A_c}{2} \cos[(\omega_c - \omega_m)t]$$

✓Compare with conventional AM signals.

- Phasor diagram.

✓Indicate all necessary amplitudes and angles.

Wide-Band FM

- For single-tone message and arbitrary β value:

$$f_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[(\omega_c + n\omega_m)t\right]$$

- So, the amplitude spectrum is

$$\left|F_{FM}(f)\right| = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} \left|J_n(\beta)\right| [\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

- Bandwidth estimation: 1% rule and Carson's rule.

✓Power calculation (before & after a BPF, within some BW, etc).

Generation of Wide-Band FM — Indirect Method

- Message signal $m(t)$ first modulates the carrier to an NBFM signal (with $\beta_1 \leq 0.2$).
- The frequency multiplier and frequency converter (i.e., mixer) are used next to generate a WBFM signal with desired carrier frequency and modulation index.

✓Design of the frequency multiplier and mixer: Order of frequency multiplication, mixer design (filter bandwidth, filter centre frequency, etc).

Demodulation of FM Signals

- What is a frequency discriminator? It consists of Differentiator, Envelope Detector and DC Blocking.
- Understand the step-by-step demodulation procedures.

- Understand the FM receiver model.

✓What is purpose of each block in this model?

✓How to demodulate a PM signal?

SNRs in FM Demodulator

- Input SNR: $\frac{S_i}{N_i} = \frac{A_c^2/2}{\eta \cdot B_{FM}}$
- Output SNR: $\frac{S_0}{N_0} = \frac{3A_c^2\beta^2}{4\eta f_m}$ (for single-tone message)
- Using Carson's rule for B_{FM}, it can be easily shown that $\frac{S_0}{N_0} = 3\beta^2(\beta+1)\frac{S_i}{N_i}$.

✓FM demodulation threshold occurs when $\frac{S_i}{N_i} = 10$ dB. This is due to the "capture effect".