1. A triangular signal $m(t) = 4\Lambda[(t - 6)/2]$ frequency modulates a carrier signal $f(t) = 100 \cos(2\pi f_c t)$ with $k_f = 30$ Hz/volt.
   a. Sketch the instantaneous frequency deviation in hertz for the obtained FM signal.
   b. Sketch the instantaneous phase deviation in radian for this FM signal.

Solution:

a) For a FM signal

$$f_i(t) = f_c + \boxed{k_f m(t)}$$

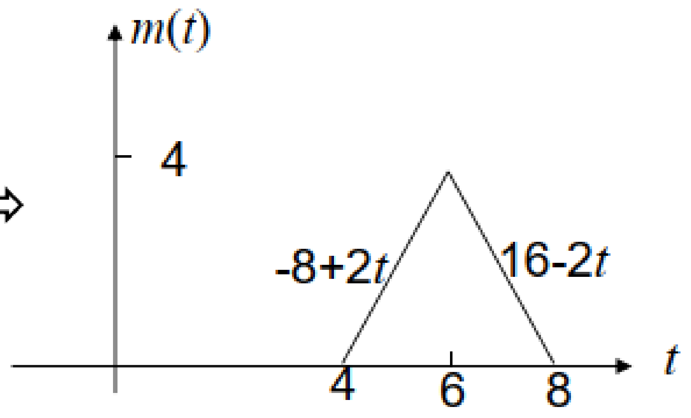
Instantaneous carrier frequency deviation, $\Delta f_i\,(t)$

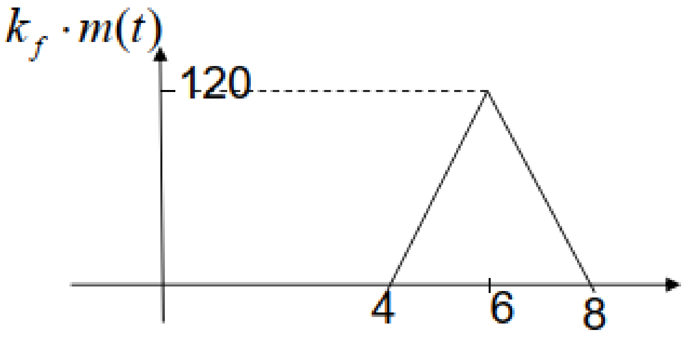
f_c : carrier frequency, a constant

k_f : frequency sensitivity, a constant (Hz/volt).

Instantaneous frequency deviation:

$$m(t) = 4\Lambda[(t-6)/2] \Longrightarrow$$

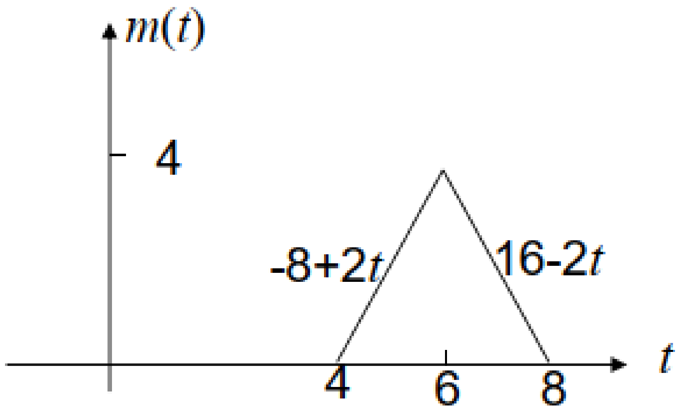




Solution:

$f_i(t) = f_c + k_f m_t$

$\theta = \int_0^t 2\pi f_i(\tau) d\tau = 2\pi \int_0^t (f_c + k_f m_\tau) d\tau = 2\pi f_c t + \boxed{2\pi \int_0^t k_f m_\tau d\tau}$



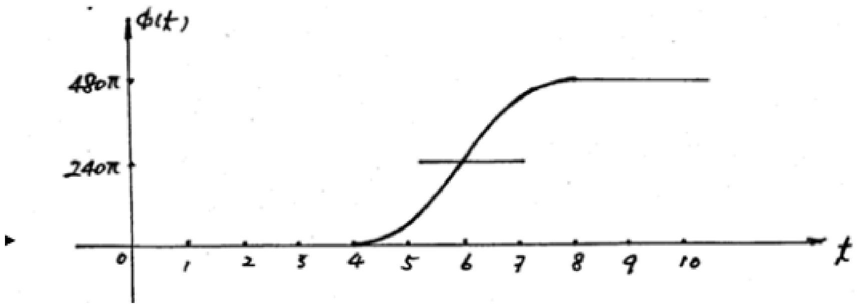
b) The phase deviation in radian is obtained by $\Phi(t) = 2\pi k_f \cdot \int_0^t m(\tau) d\tau$.

$$\Phi(t) = 0 \quad for \quad t < 4$$

$$\Phi(t) = 2\pi k_f \int_4^t (-8 + 2\tau) d\tau = 60\pi(t^2 - 8t + 16) \quad for \quad 4 \le t \le 6$$

$$\Phi(t) = \Phi(6) + 2\pi k_f \int_6^t (16 - 2\tau) d\tau = 240\pi - 60\pi(t^2 - 16t + 60) \quad for \quad 6 \le t \le 8$$

$$\Phi(t) = 480\pi \quad for \quad t > 8$$



2. An 18 MHz carrier is frequency modulated by a 400 Hz cosine waveform. If the FM signal has an amplitude of 5 volts and a peak frequency deviation of 30 KHz.

   a. Write the expression for the obtained FM signal
   b. Calculate the peak phase deviation in radian for this FM signal
   c. Calculate the peak frequency deviation and the peak phase deviation if the frequency of the modulating signal is tripled.

Solution:

a)

$$f_i(t) = f_c + k_f A\cos(2\pi f t)$$

$$\theta = \int_0^t 2\pi f_i(\tau)d\tau = 2\pi \int_0^t (f_c + k_f A\cos(2\pi f\tau))d\tau = 2\pi f_c t + 2\pi \int_0^t k_f A\cos(2\pi f t)\, d\tau = 2\pi f_c t + \frac{Ak_f}{f}\sin(2\pi f t)$$

$f_c = 18MHz$
$f = 400Hz$
Peak frequency deviation: $k_f A = 30KHz$
$A_{FM} = 5$

$$f_{FM} = A_{FM}\cos\left(2\pi f_c t + \frac{Ak_f}{f}\sin(2\pi f t)\right)$$

$$= 5\cos(36\pi * 10^6 t + 75\sin(800\pi t))$$

b)

$$\theta = 36\pi * 10^6 t + \boxed{75\sin(2\pi f t)}$$

The maximum phase deviation is 75 rad

c)

$f = 1200Hz$
Peak frequency deviation: $k_f A = 30KHz$
The peck phase deviation: $\frac{Ak_f}{f} = 25$ rad

3. A message signal $m(t) = 0.5\cos(2\pi 1000t)$ phase modulates a carrier signal $f(t) = 10\cos(2\pi\, 10^6 t)$ with modulation phase sensitivity $k_p = 0.3$ rad/V.

   a. Write the expression of the obtained PM signal.

   b. Construct a phasor diagram for this PM signal.

   c. Re-construct the phasor diagram if $m(t) = 0.5\sin(2\pi 1000t)$.

Solution:

a)

$$f_{PM}(t) = A_c cos\left(2\pi f_c t + k_p m(t)\right) = 10cos\left(2\pi 10^6 t + 0.15cos(2\pi 1000t)\right)$$

$$= 10cos\left(2\pi 10^6 t\right)cos(0.15cos(2\pi 1000t)) - 10\sin\left(2\pi 10^6 t\right)sin(0.15cos(2\pi 1000t))$$

b)

$\beta_p = k_p A_m = 0.15 < 0.2$ => **NBPM**

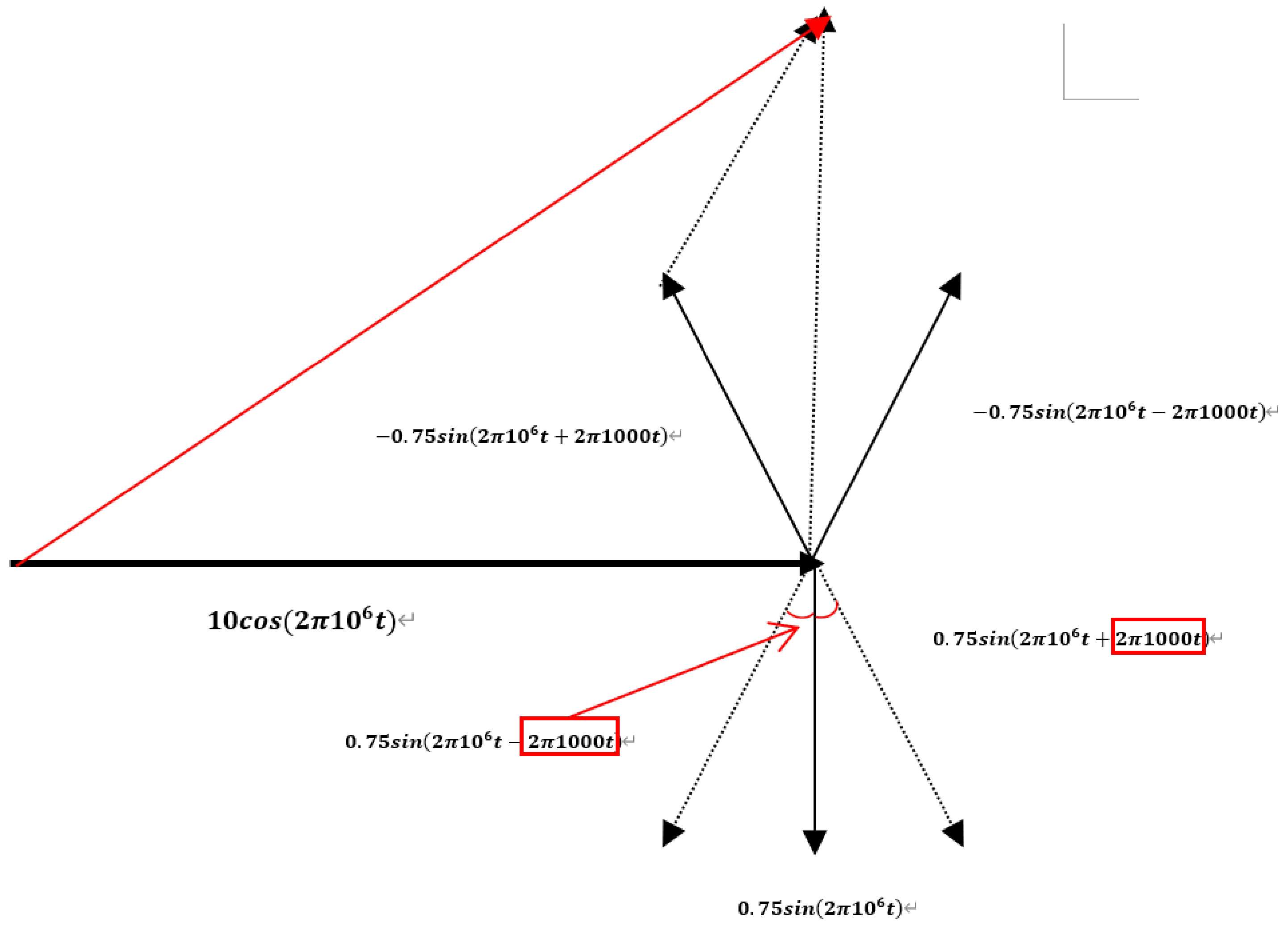
$$sin(0.15cos(2\pi 1000t)) \approx 0.15cos(2\pi 1000t))$$

$$cos(0.15cos(2\pi 1000t)) \approx 1$$

$$f_{NBPM}(t) = 10cos\left(2\pi 10^6 t\right) - 1.5\sin\left(2\pi 10^6 t\right)cos(2\pi 1000t))$$

$$= 10cos\left(2\pi 10^6 t\right) - 0.75\sin\left(2\pi 10^6 t - 2\pi 1000t\right) - 0.75\sin\left(2\pi 10^6 + 2\pi 1000t\right)$$

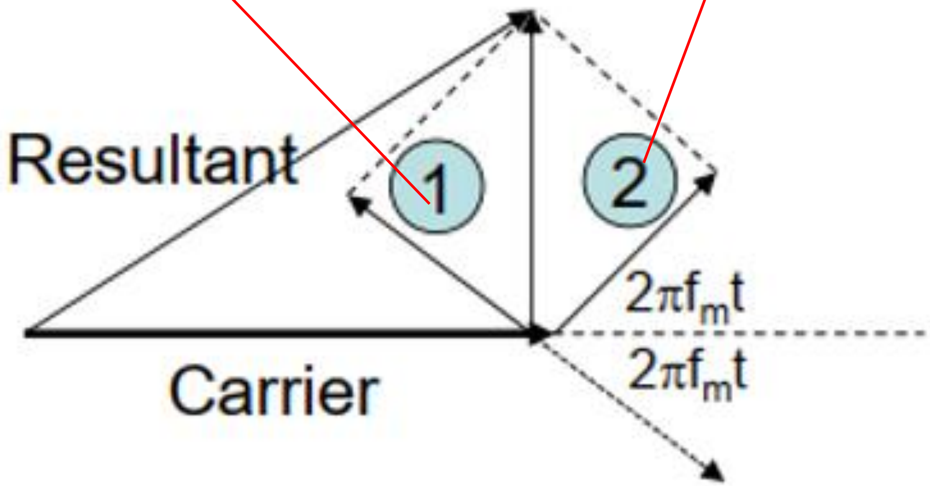
$$f_{NBPM}(t) = 10cos(2\pi 10^6 t) - 0.75\sin(2\pi 10^6 t - 2\pi 1000t) - 0.75\sin(2\pi 10^6 + 2\pi 1000t)$$



$$\sin(x) = \cos(x - \frac{\pi}{2})$$

c)

$$f_{NBPM}(t) = 10cos(2\pi 10^6 t) \; - 1.5\sin(2\pi 10^6 t)sin(2\pi 1000t))$$
$$= 10cos(2\pi 10^6 t) \boxed{-0.75cos(2\pi 10^6 t - 2\pi 1000t)} \boxed{+0.75cos(2\pi 10^6 + 2\pi 1000t)}$$



Phase diagram