通信原理
习题课

Assignment No. 5

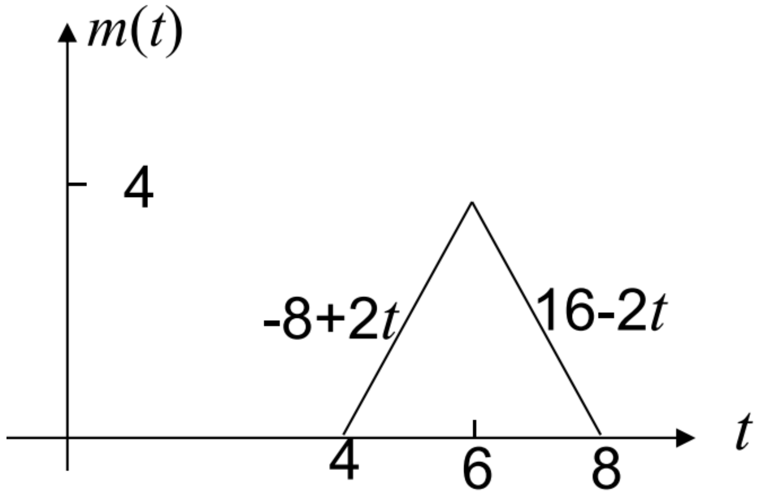
TA 周梓钦

2021/3/24

1. A triangular signal $m(t) = 4\Lambda[(t-6)/2]$ frequency modulates a carrier signal $f(t) = 100\cos(2\pi f_c t)$ with $k_f = 30$ Hz/volt.

   a. Sketch the instantaneous frequency deviation in hertz for the obtained FM signal.

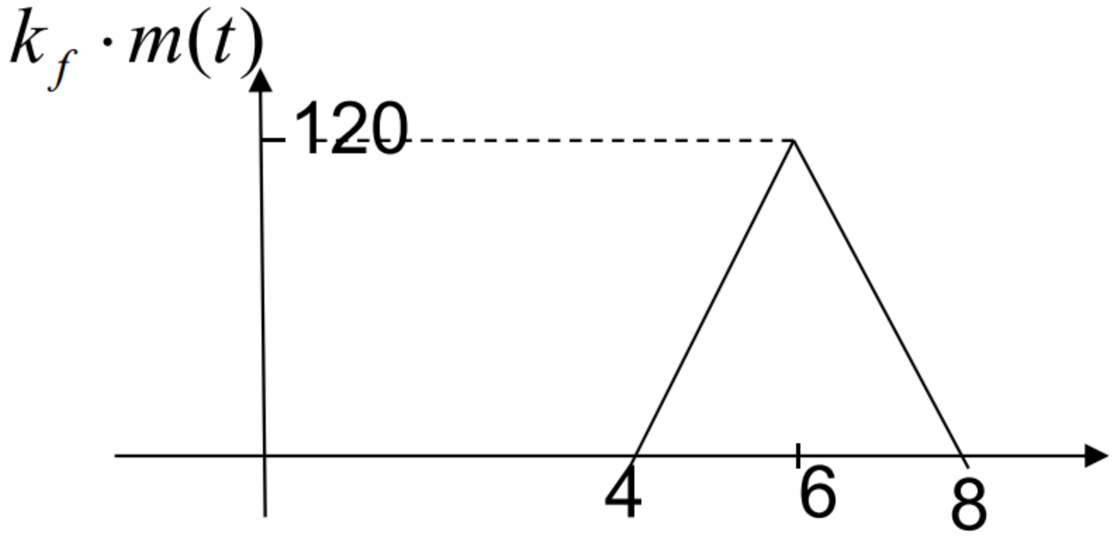
- $m(t) = 4\,\Lambda[(\mathrm{t}-6)/2]$ $\Longrightarrow$



Solution of (a)

1. A triangular signal $m(t) = 4\Lambda[(t - 6)/2]$ frequency modulates a carrier signal $f(t) = 100\cos(2\pi f_c t)$ with $k_f = 30$ Hz/volt.
   a. Sketch the instantaneous frequency deviation in hertz for the obtained FM signal.

The frequency deviation in Hz is $km(t)$



Solution of (b)

1. A triangular signal $m(t) = 4\Lambda[(t - 6)/2]$ frequency modulates a carrier signal $f(t) = 100\cos(2\pi f_c t)$ with $k_f = 30$ Hz/volt.
   b. Sketch the instantaneous phase deviation in radian for this FM signal.

The phase deviation in radian is obtained by $\phi(t) = 2\pi k_f \cdot \int_0^t m(\tau)\, d\tau$

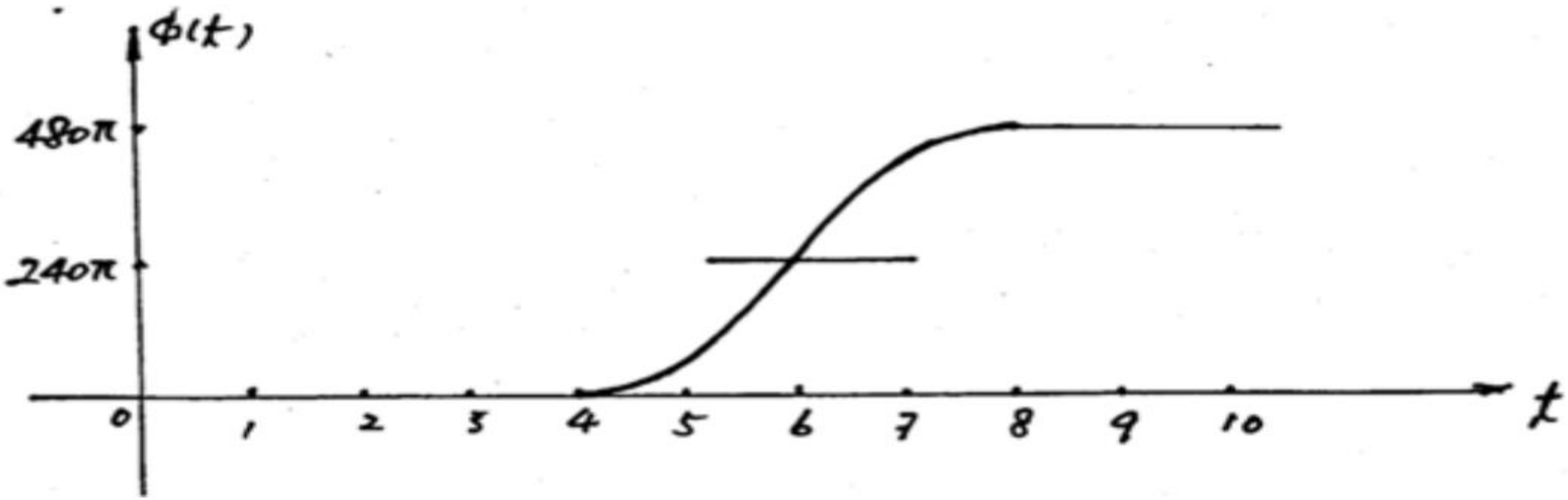
Solution of (b)

$$\Phi(t) = 0 \quad for \quad t < 4$$

$$\Phi(t) = 2\pi k_f \int_4^t (-8 + 2\tau)d\tau = 60\pi(t^2 - 8t + 16) \quad for \quad 4 \le t \le 6$$

$$\Phi(t) = \Phi(6) + 2\pi k_f \int_6^t (16 - 2\tau)d\tau = 240\pi - 60\pi(t^2 - 16t + 60) \quad for \quad 6 \le t \le 8$$

$$\Phi(t) = 480\pi \quad for \quad t > 8$$



FM: (Single tone modulation)

$$f_i(t) = f_c + k_f A_m \cos 2\pi f_m t = f_c + \Delta f \cos 2\pi f_m t$$

where $\Delta f = k_f A_m$ is the "peak frequency deviation", i.e., the maximum departure of $f_i(t)$, the instantaneous freq. of FM signal, from f_c .

Thus, $\theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau = 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t = 2\pi f_c t + \beta \sin 2\pi f_m t$

where $\boxed{\beta \triangleq \frac{\Delta f}{f_m}}$ is referred to as **modulation index** of FM.

β also denotes the peak phase deviation of the FM signal.

The single tone FM Signal is given by

$$\boxed{f_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]}$$

2. An 18 MHz carrier is frequency modulated by a 400 Hz cosine waveform. If the FM signal has an amplitude of 5 volts and a peak frequency deviation of 30 KHz.
    a. Write the expression for the obtained FM signal
    b. Calculate the peak phase deviation in radian for this FM signal

Solution of (a)

$$f_c = 18 \times 10^6 \, Hz$$

$$f_m = 400 Hz$$

$$\beta = \frac{\Delta f}{f_m} = \frac{30K}{400} = 75$$

$$\therefore f_{FM}(t) = 5\cos[36\pi \times 10^6 t + 75\sin(800\pi t)]$$

b. Calculate the peak phase deviation in radian for this FM signal

Solution of (b) The maximum phase deviation is 75 radians

c. Calculate the peak frequency deviation and the peak phase deviation if the frequency of the modulating signal is tripled.

Solution of (c) When the frequency of modulating signal is tripled, that is

$$f_m = 1200 Hz$$

$$\beta = \frac{\Delta f}{f_m} = \frac{30K}{1200} = 25$$

$$\therefore f_{FM}(t) = 5\cos[36\pi \times 10^6 t + 25\sin(2400\pi t)]$$

The maximum phase deviation is 25 radians but the maximum frequency deviation is unchanged. $(\Delta f = k_f A_m = 30kHz)$

3. A message signal $m(t) = 0.5 \cos(2\pi 1000t)$ phase modulates a carrier signal $f(t)= 10 \cos(2\pi\, 10^6 t)$ with modulation phase sensitivity $k_p = 0.3$ rad/V.

   a. Write the expression of the obtained PM signal.

   b. Construct a phasor diagram for this PM signal.

   c. Re-construct the phasor diagram if $m(t) = 0.5 \sin(2\pi 1000t)$.

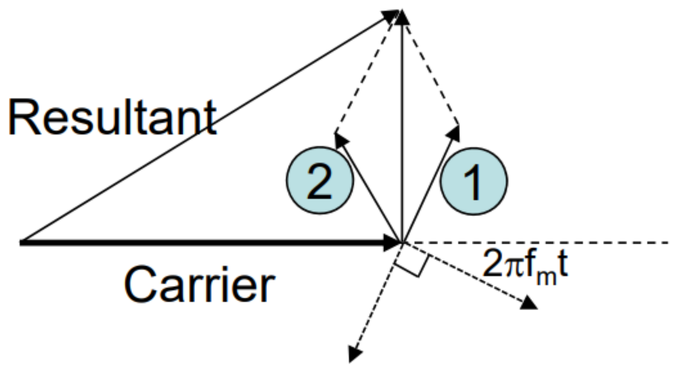
Solution of (a&b)

$$\beta_p = k_p A_m = 0.15 < 0.2$$

$$f_{PM}(t) = A_c \cos[2\pi f_c t + \beta_p \cos(2\pi f_m t)]$$

$$= A_c \cos(2\pi f_c t)\cos[\beta_p \cos(2\pi f_m t)]$$

$$- A_c \sin(2\pi f_c t)\sin[\beta_p \cos(2\pi f_m t)]$$



Phase diagram

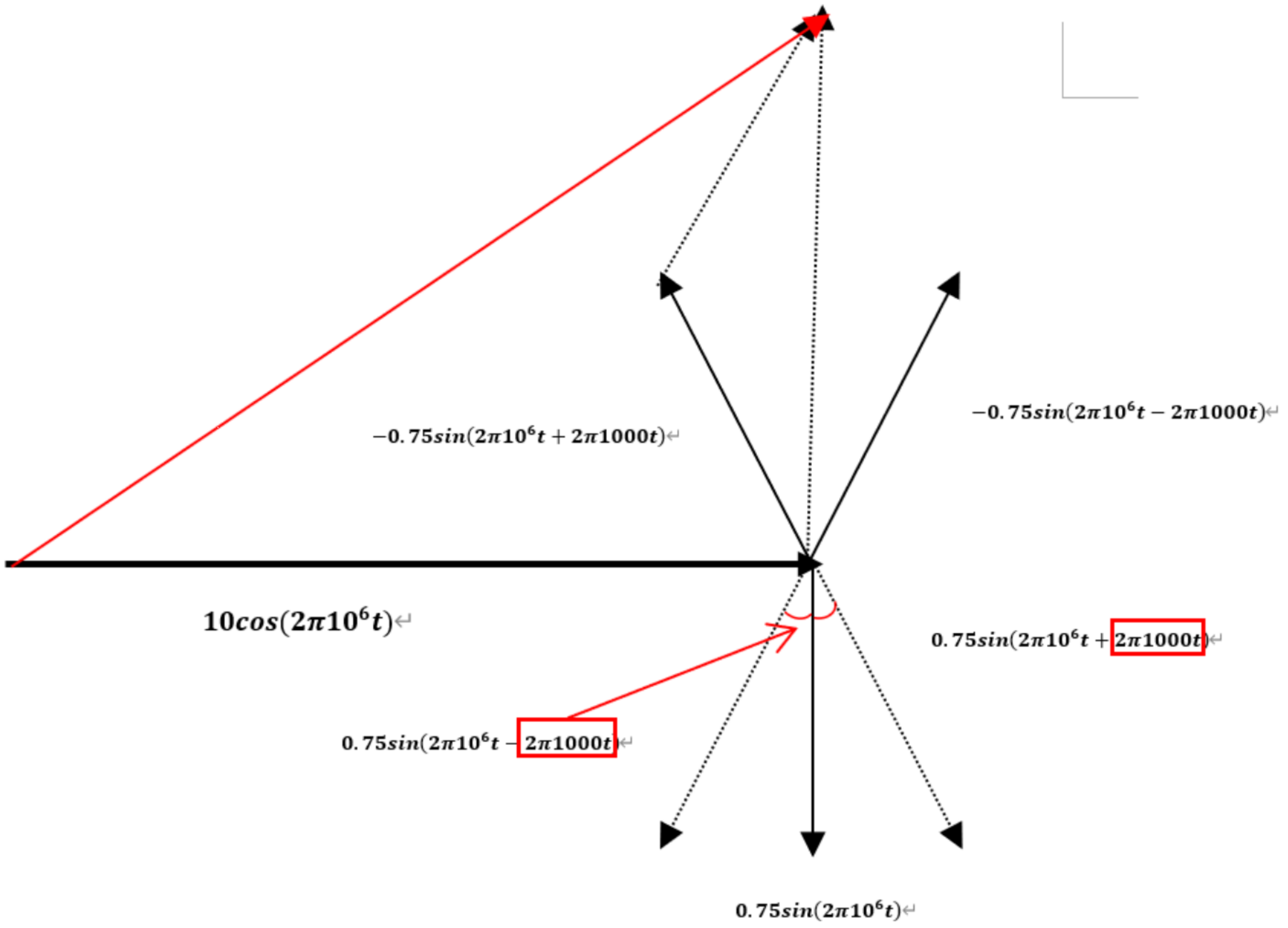
$$\because \beta_p = 0.15 < 0.2,$$

$$\therefore \cos[\beta_p \cos(2\pi f_m t)] \approx 1, \sin[\beta_p \cos(2\pi f_m t)] \approx \beta_p \cos(2\pi f_m t)$$

$$\therefore f_{NBPM}(t) = A_c \cos(2\pi f_c t) - A_c \beta_p \sin(2\pi f_c t)\cos(2\pi f_m t)$$

$$= A_c \cos(2\pi f_c t) - \frac{1}{2} A_c \beta_p \underbrace{\sin 2\pi(f_c - f_m)t}_{1} - \frac{1}{2} A_c \beta_p \underbrace{\sin 2\pi(f_c + f_m)t}_{2}$$

$$f_{NBPM}(t) = 10cos(2\pi 10^6 t) - 0.75\sin(2\pi 10^6 t - 2\pi 1000t) - 0.75\sin(2\pi 10^6 + 2\pi 1000t)$$



$$\sin(x) = \cos(x - \frac{\pi}{2})$$

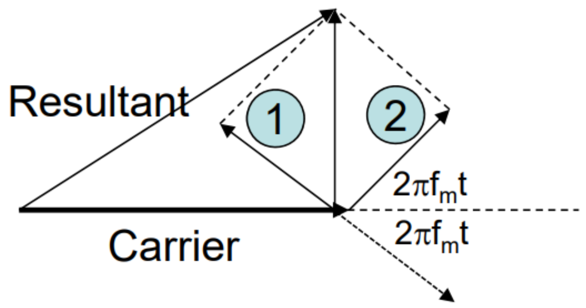
3. A message signal $m(t) = 0.5\cos(2\pi 1000t)$ phase modulates a carrier signal $f(t) = 10\cos(2\pi\, 10^6 t)$ with modulation phase sensitivity $k_p = 0.3$ rad/V.

c. Re-construct the phasor diagram if $m(t) = 0.5\sin(2\pi 1000t)$.

Solution of (c)

$$\therefore f_{NBPM}(t) = A_c\cos(2\pi f_c t) - A_c\beta_p\sin(2\pi f_c t)\sin(2\pi f_m t)$$

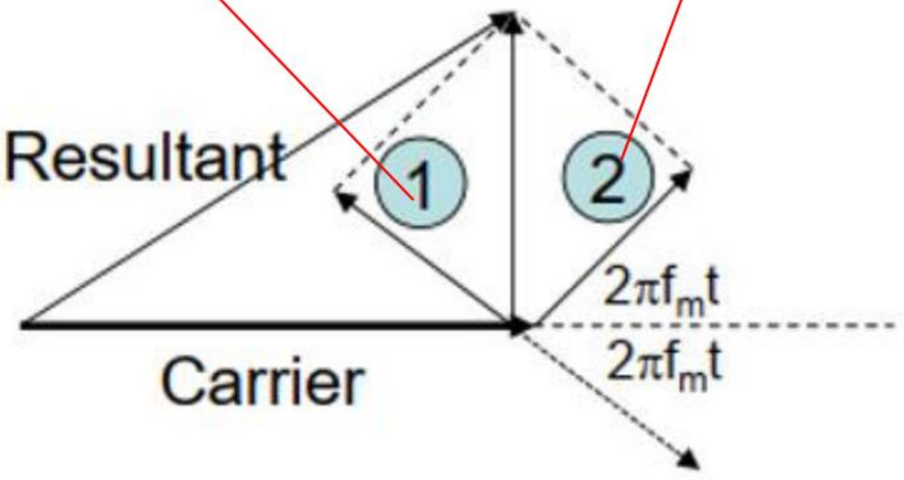
$$= A_c\cos(2\pi f_c t) - \frac{1}{2}A_c\beta_p\underbrace{\cos 2\pi(f_c - f_m)t}_{1} + \frac{1}{2}A_c\beta_p\underbrace{\cos 2\pi(f_c + f_m)t}_{2}$$



Phase diagram

c)

$$f_{NBPM}(t) = 10cos(2\pi 10^6 t) \ - 1.5\sin(2\pi 10^6 t)sin(2\pi 1000t))$$
$$= 10cos(2\pi 10^6 t) \boxed{- 0.75cos(2\pi 10^6 t - 2\pi 1000t)} \boxed{+ 0.75cos(2\pi 10^6 + 2\pi 1000t)}$$



Phase diagram