Chapter 8:
Baseband Transmission of Digital Signals

Energy Signal

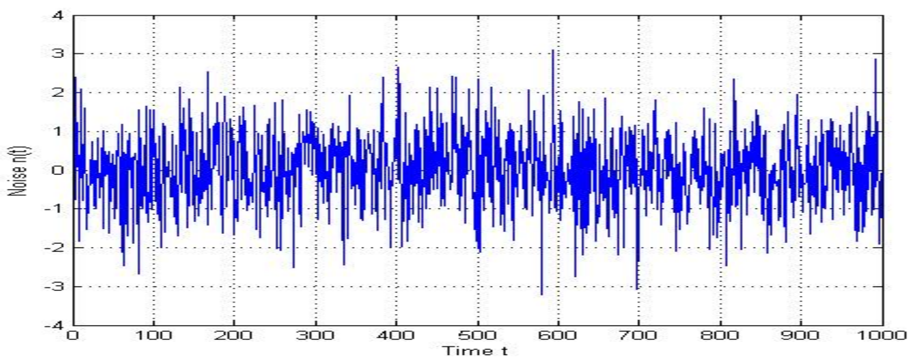
- Energy signal: signal with finite energy
  - $E = \int_{-\infty}^{\infty} |g(t)|^2 dt < \infty$
- Energy spectral density: energy distribution in frequency domain
  - Fourier transform of signal: $G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t} dt$
  - $\int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df = \mathrm{E}$
  - Definition: $|G(f)|^2$
  - Signal energy within [f1,f2]=$\int_{f_1}^{f_2} |G(f)|^2 df$

Power Signal

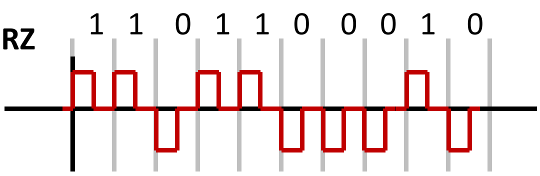
- Power signal: signal with finite average power
  - $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 dt < \infty$
  - Energy signals are power signals
  - Power signal may not be energy signal
- Power spectral density (PSD): power distribution in frequency domain
  - Definition: $S_g(f) = \lim_{T\to\infty} \frac{|\int_{-T/2}^{T/2} g(t) e^{-j2\pi f t} dt|^2}{T}$
  - Average signal power within [f1,f2]=$\int_{f_1}^{f_2} S_g(f) df$
- Examples
  - Noise

Random Process

- Random (stochastic) process
  - g(t): for any time t, g(t) is a random variable
  - Example: noise, line code, temperature of next 24 hours



Polar RZ



Gaussian Process

- Gaussian distribution
  - $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
  - Mean μ, variance σ^2 方差
- Gaussian process
  - A random process $g(t)$ where $\forall m, t_1, t_2, \ldots, t_m\ and\ \forall a_1, a_2, \ldots, a_m, \sum_{i=1}^{m} a_m g(t_m)$ is a Gaussian random variable.
- If g(t) is a Gaussian process, the following RVs are Gaussian
  - $g(t_1)$
  - $a_1 g(t_1) + a_2 g(t_2)$
  - $\int_{t_1}^{t_2} f(t) g(t) dt$

PSD of Random Process

- Autocorrelation 自相关 function of a stationary 平稳的 process $g(t)$
  - $R_g(\tau) = E[g(t+\tau)g(t)]$ for all t 统计平均

确定信号不存在自相关函数

- Power spectral density of random process
  - $S_g(f) = \int_{-\infty}^{\infty} R_g(\tau) e^{-j2\pi f\tau} d\tau$
- Summary
  - **Deterministic energy signal** (确定的): energy spectral density $|G(f)|^2$
  - **Deterministic power signal:** power spectral density

$$S_g(f) = \lim_{T\to\infty} \frac{|\int_{-T/2}^{T/2} g(t) e^{-j2\pi f t} dt|^2}{T}$$

  - **Random power signal :** power spectral density

$$S_g(f) = \int_{-\infty}^{\infty} R_g(\tau) e^{-j2\pi f\tau} d\tau$$

Additive White Gaussian Noise n(t)

- Additive: received signal = transmitted signal + noise
- Gaussian: noise is a Gaussian process
  - $\int_{t_1}^{t_2} \mathrm{n(t)}dt$ is a Gaussian RV
- White
  - power spectral density is a constant $N_0/2$
  - Autocorrelation function is $\mathrm{E}[\mathrm{n}(\mathrm{t}+\tau)n(t)] = \frac{N_0}{2}\delta(\tau)$ → 怎么得到的？
- Properties

1. PSD of white noise after a filter H(f): $\frac{|H(f)|^2 N_0}{2}$

2. $$E\left[\int_{t_1}^{t_2} \mathrm{n(t)}dt\right]^2 = E\left[\int_{t_1}^{t_2} \mathrm{n(x)}dx \int_{t_1}^{t_2} \mathrm{n(y)}dy\right] = \int_{t_1}^{t_2}\int_{t_1}^{t_2} E[n(x)n(y)]dx\,dy$$

$$= \int_{t_1}^{t_2}\int_{t_1}^{t_2} \frac{N_0}{2}\delta(x-y)dx\,dy = \int_{t_1}^{t_2} \frac{N_0}{2}dy = \frac{N_0}{2}(t_2 - t_1)$$

Read by Yourself

- Textbook 5.5-5.11
- Problem 8.3: PSD of line code?
  - Useful resource: Http://www.utdallas.edu/~torlak/courses/ee4367/lectures/CodingI.pdf

Baseband Digital Communications

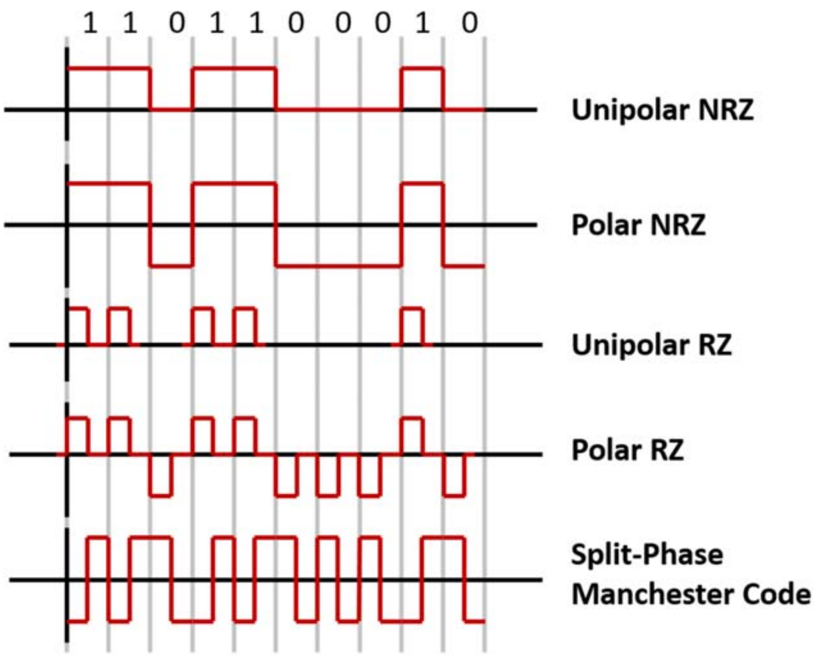
- Digital Signal: discrete in time; discrete and finite in signal scale
  - Can always be represented by a bit sequence
- Digital Modulation: approaches to transmission of binary bits via analog waves
  - E.g., line code
- Baseband Digital Modulation : center frequency of signal spectrum is smaller than signal bandwidth
- Band-pass Digital Modulation : center frequency of signal spectrum is much larger than signal bandwidth

Baseband Digital Modulation

- Example: line code

2-ary Modulation:

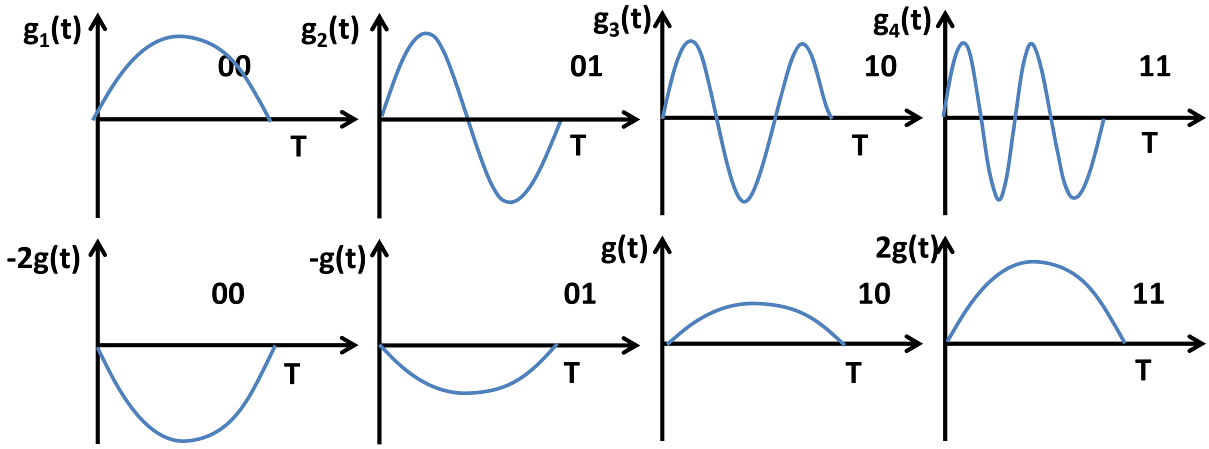
- The transmitter has two different waveforms $g_0(t)$ and $g_1(t)$,
- $1 \rightarrow g_1(t)$
- $0 \rightarrow g_0(t)$
- 1 bit per transmission period



M-ary Modulation

Message: 1 0 1 1 1 0 0 0 1 1 0 1

4-ary Modulation



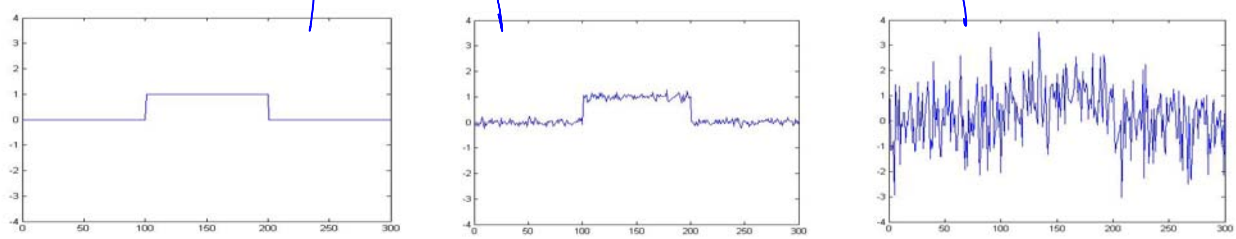
General Description of M-ary Modulation:

1. Take $N=\log_2 M$ bits each time for transmission (N=1 for line code)
2. Use $M=2^N$ different pulses to represent the N bits

Distortion in AWGN Channel

- Channel Noise

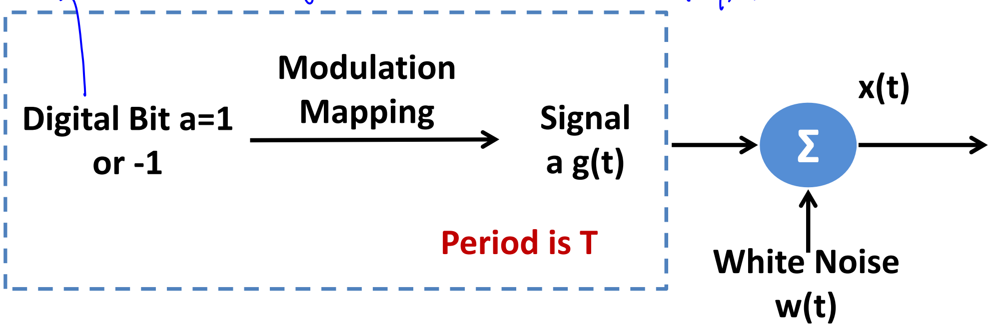
噪声几乎为0　噪声相对信号较小　噪声相对较强

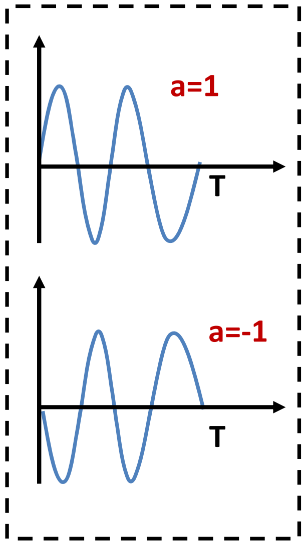


- Detection approach
  - Measure the similarity between the receive wave and standard transmission wave
  - Make a decision according to the similarity coefficient

Digital Transmitter with 2-ary Modulation

用1,-1表示 binary bit (1,0) → 发射机

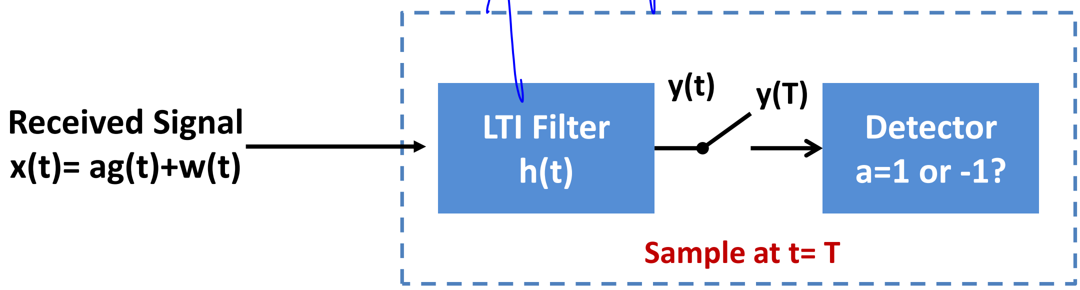




- a_i =(1,-1) is the information bit
- g(t) is the pulse used by modulation (with duration T)
- 2-ary modulation: $a_i \rightarrow a_i g(t)$
- $\sum_i a_i g(t-iT)$ is the modulated signal

Receiver Structure

匹配滤波器 接收机



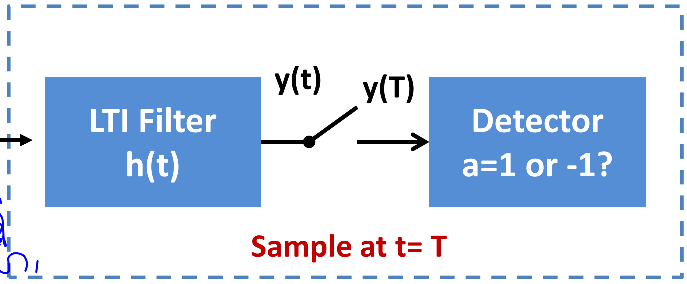
- LTI Filter (match filter): to differentiate g(t) and –g(t)
  - Positive for a=1
  - Negative for a=-1
- Sample with period T: to obtain the output of LTI filter
- Detector: to make a good guess on a

Problem Formulation for h(t)

Received Signal
x(t)=a g(t)+w(t)

g(t)本身不携带信息，与a相乘时携带信息 w,

a: information bit 1 or -1
g(t): pulse to carry information



What is the best choice of h(t)?

$$x(t) = a\,g(t) + w(t) \quad 0 \le t \le T$$

$$y(t) = h(t) * x(t) = \underbrace{h(t) * ag(t)}_{g_0(t)} + \underbrace{h(t) * w(t)}_{n(t)}$$

After sampling at time T: $y(T) = g_0(T) + n(T)$

SNR of y(T) : $\eta = |g_0(T)|^2 / E[n^2(T)]$

Choose h(t) to maximize η

According to the property of AWGN, the PSD of n(t) is

$$\frac{|H(f)|^2 N_0}{2}$$

and the average power of n(t) is

$$E[n^2(t)] = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df$$

Therefore,

$$E[n^2(T)] = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df,$$

Comparison with the average power of w(t)

$$\int_{-\infty}^{+\infty} \mathrm{sinc}(x)\,dx = \pi \qquad \int_{-\infty}^{+\infty} \mathrm{sinc}^2(x)\,dx = \pi$$

$$|g_0(T)|^2 = \left| h(t) * ag(t) \Big|_{t=T} \right|^2$$

Let H(f) and G(f) be the Fourier transform of h(t) and g(t) respectively, we have

$$h(t) * ag(t) \xleftrightarrow{Fourier\ Transform} aH(f)G(f)$$

or

$$h(t) * ag(t) = \int_{-\infty}^{\infty} H(f)aG(f)e^{j2\pi ft}\, df$$

$$g_0(T) = a \int_{-\infty}^{\infty} H(f)G(f)e^{j2\pi fT} df$$

$$|g_0(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)G(f)e^{j2\pi fT} df \right|^2$$

Problem Formulation for h(t) --- Cont'd

- The SNR $\eta = \frac{\left|\int_{-\infty}^{\infty} H(f)G(f)e^{j2\pi fT}\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$
- We can control the impulse response h(t) or H(f) such that η is maximized
- Thus, the receiver design is formulated as the following problem

用柯西不等式求解. $(ab+cd)^2 \le (a^2+c^2)(b^2+d^2)$

$$\max_{h(t)} \boldsymbol{\eta} = \max_{h(t)\ or\ H(f)} \frac{\left|\int_{-\infty}^{\infty} H(f)G(f)e^{j2\pi fT} df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

解法: $h^*(t) = \arg\max_{h(t)} \frac{\bigcirc}{\bigcirc}$

Homework #D4

Fig. 1

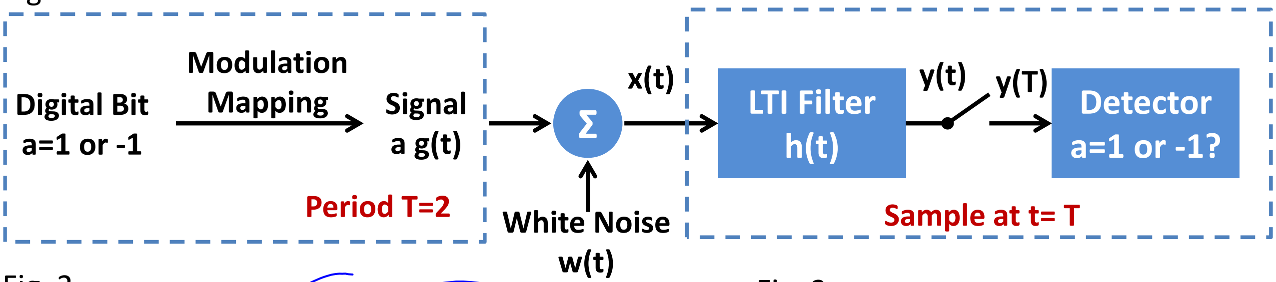


Fig. 2

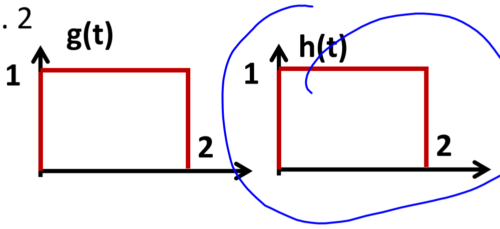
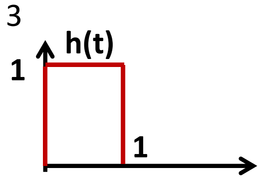


Fig. 3



- D4.1

Consider the baseband transceiver in Fig. 1, where g(t) and h(t) are given by Fig. 2,

(a) Please sketch the PSD of noise in y(t).

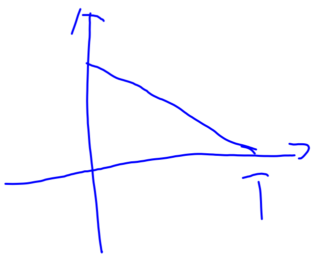
(b) What is the signal power in y(T)? What is the noise power in y(T)? What is the SNR of y(T)?

(c) If h(t) is given by Fig. 3, what is your answer of question (b)?

(d) Compare the SNR of question (b) and (c), which impulse response h(t) is better for receiver?

最佳

若 g(t)



則 h(t) 最佳解