Symbol Tables
Binary Search Trees

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Email Subject： (L1- | L2- | L3-) + *last 4 digits of ID* + *Name*: *TOPIC*

Your Lab Class

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数据结构与算法分析B

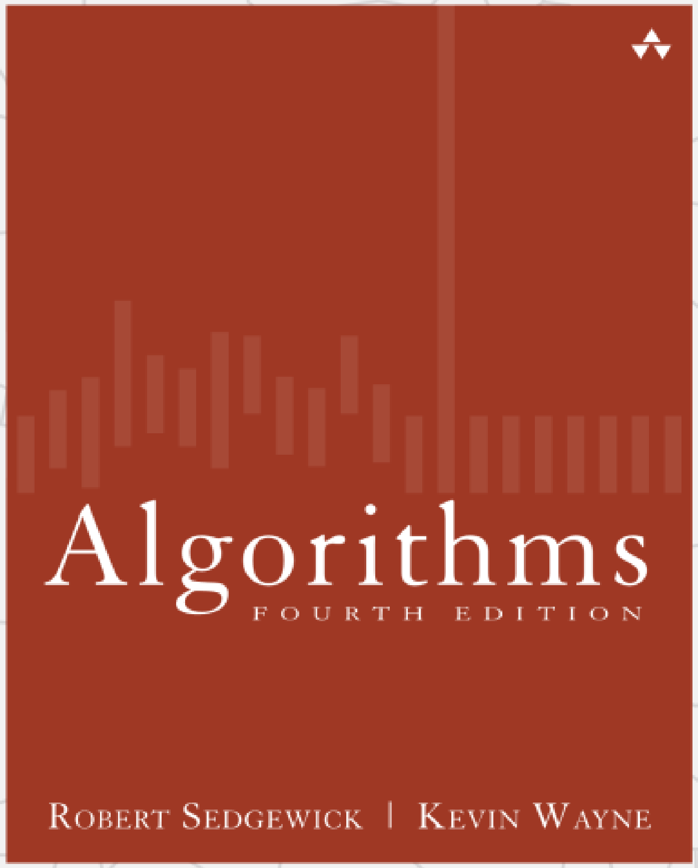
Data Structures and Algorithm Analysis

Lecture 10

- Symbol Tables (3.1 of Text A)
- Binary Search Trees (3.2 of Text A)

To be discussed in Lecture 11:

- Balanced Search Trees (3.3 of Text A)
- Hash Tables (3.4 of Text A)



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3.1 Symbol Tables

- API
- elementary implementations
- ordered operations

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3.1 Symbol Tables

- API
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Symbol tables

Key-value pair abstraction.

- Insert a value with specified key.
- Given a key, search for the corresponding value.

Ex. DNS lookup.

- Insert domain name with specified IP address.
- Given domain name, find corresponding IP address.

domain name	IP address
www.cs.princeton.edu	128.112.136.11
www.princeton.edu	128.112.128.15
www.yale.edu	130.132.143.21
www.harvard.edu	128.103.060.55
www.simpsons.com	209.052.165.60

key value

Symbol table applications

application	purpose of search	key	value
dictionary	find definition	word	definition
book index	find relevant pages	term	list of page numbers
file share	find song to download	name of song	computer ID
financial account	process transactions	account number	transaction details
web search	find relevant web pages	keyword	list of page names
compiler	find properties of variables	variable name	type and value
routing table	route Internet packets	destination	best route
DNS	find IP address	domain name	IP address
reverse DNS	find domain name	IP address	domain name
genomics	find markers	DNA string	known positions
file system	find file on disk	filename	location on disk

Symbol tables: context

Also known as: maps, dictionaries, associative arrays.

Generalizes arrays. Keys need not be between 0 and $N-1$. 不限于

Language support.

- External libraries: C, VisualBasic, Standard ML, bash, ...
- Built-in libraries: Java, C#, C++, Scala, ...
- Built-in to language: Awk, Perl, PHP, Tcl, JavaScript, Python, Ruby, Lua.

every array is an associative array

every object is an associative array

table is the only primitive data structure

```
hasNiceSyntaxForAssociativeArrays["Python"] = true
hasNiceSyntaxForAssociativeArrays["Java"]   = false
```

legal Python code

Basic symbol table API

Associative array abstraction. Associate one value with each key.

`public class ST<Key, Value>`

	`ST()`	*create an empty symbol table*	
`void`	`put(Key key, Value val)`	*put key-value pair into the table*	`a[key] = val;`
`Value`	`get(Key key)`	*value paired with key*	`a[key]`
`boolean`	`contains(Key key)`	*is there a value paired with key?*	
`void`	`delete(Key key)`	*remove key (and its value) from table*	
`boolean`	`isEmpty()`	*is the table empty?*	
`int`	`size()`	*number of key-value pairs in the table*	
`Iterable<Key>`	`keys()`	*all the keys in the table*	

Conventions (惯例)

- Values are not `null`. ← Java allows null value
- Method `get()` returns `null` if key not present.
- Method `put()` overwrites old value with new value.

Intended consequences.

- Easy to implement `contains()`.

```
public boolean contains(Key key)
{  return get(key) != null;  }
```

- Can implement lazy version of `delete()`.

```
public void delete(Key key)
{  put(key, null);  }
```

将空值作为put( )方法的第二个参数存入表中来实现删除.

Keys and values

Value type. Any generic type.

specify Comparable in API.

Key type: several natural assumptions.

- Assume keys are `Comparable`, use `compareTo()`.
- Assume keys are any generic type, use `equals()` to test equality.
- Assume keys are any generic type, use `equals()` to test equality; use `hashCode()` to scramble key.

built-in to Java
(stay tuned)

Best practices. Use immutable types for symbol table keys.

- Immutable in Java: `Integer`, `Double`, `String`, `java.io.File`, …
- Mutable in Java: `StringBuilder`, `java.net.URL`, arrays, ...

Equality test

All Java classes inherit a method `equals()`.

Java requirements. For any references `x`, `y` and `z`:

- Reflexive: `x.equals(x)` is `true`.
- Symmetric: `x.equals(y)` iff `y.equals(x)`.
- Transitive: if `x.equals(y)` and `y.equals(z)`, then `x.equals(z)`.
- Non-null: `x.equals(null)` is `false`.

equivalence relation

do x and y refer to the same object?

Default implementation. `(x == y)`

Customized implementations. `Integer`, `Double`, `String`, `java.io.File`, ...

User-defined implementations. Some care needed.

Implementing equals for user-defined types

Seems easy.

```
public      class Date implements Comparable<Date> {

   private final int month;
   private final int day;
   private final int year;
   ...

   public boolean equals (Date that) {









      if (this.day   != that.day  ) return false;
      if (this.month != that.month) return false;
      if (this.year  != that.year ) return false;
      return true;
   }
}
```

check that all significant fields are the same

Implementing equals for user-defined types

Seems easy, but requires some care.

```
public final class Date implements Comparable<Date> {

   private final int month;
   private final int day;
   private final int year;
   ...

   public boolean equals (Object y) {

      if (y == this) return true;

      if (y == null) return false;

      if (y.getClass() != this.getClass())
         return false;

      Date that = (Date) y;
      if (this.day   != that.day  ) return false;
      if (this.month != that.month) return false;
      if (this.year  != that.year ) return false;
      return true;
   }
}
```

typically unsafe to use equals() with inheritance (would violate symmetry)

must be Object. Why? Experts still debate.

optimize for true object equality

check for null

objects must be in the same class (religion: getClass() vs. instanceof)

cast is guaranteed to succeed

check that all significant fields are the same

Equals design

"Standard" recipe for user-defined types.

- Optimization for reference equality.
- Check against `null`.
- Check that two objects are of the same type and cast.
- Compare each significant field:
  - if field is a primitive type, use == ← but use Double.compare() with double (or otherwise deal with -0.0 and NaN)
  - if field is an object, use `equals()` ← apply rule recursively
  - if field is an array, apply to each entry ← can use `Arrays.deepEquals(a, b)` but not `a.equals(b)`

Best practices.

e.g., cached Manhattan distance

- No need to use calculated fields that depend on other fields.
- Compare fields mostly likely to differ first.
- Make `compareTo()` consistent with `equals()`.

`x.equals(y)` if and only if `(x.compareTo(y) == 0)`

ST test client for traces

Build ST by associating value i with i^{th} string from standard input.

键值对

```
public static void main(String[] args) {
   ST<String, Integer> st = new ST<String, Integer>();

   for (int i = 0; !StdIn.isEmpty(); i++) {
     String key = StdIn.readString();
     st.put(key, i);
   }
   for (String s : st.keys())
      StdOut.println( s + " " + st.get(s));
}
```

keys	S	E	A	R	C	H	E	X	A	M	P	L	E
values	0	1	2	3	4	5	6	7	8	9	10	11	12

output

```
A  8
C  4
E  12
H  5
L  11
M  9
P  10
R  3
S  0
X  7
```

ST test client for analysis

Frequency counter. Read a sequence of strings from standard input and print out one that occurs with highest frequency.

```
% more tinyTale.txt
it was the best of times
it was the worst of times
it was the age of wisdom
it was the age of foolishness
it was the epoch of belief
it was the epoch of incredulity
it was the season of light
it was the season of darkness
it was the spring of hope
it was the winter of despair

% java FrequencyCounter 1 < tinyTale.txt
it 10

% java FrequencyCounter 8 < tale.txt
business 122

% java FrequencyCounter 10 < leipzig1M.txt
government 24763
```

tiny example (60 words, 20 distinct)

real example (135,635 words, 10,769 distinct)

real example (21,191,455 words, 534,580 distinct)

Frequency counter implementation

```
public class FrequencyCounter {

   public static void main (String[] args) {

      int minlen = Integer.parseInt( args[0]);
      ST<String, Integer> st = new ST<String, Integer>();

      while (!StdIn.isEmpty()) {
         String word = StdIn.readString();
         if (word.length() < minlen) continue;
         if (!st.contains(word)) st.put( word, 1);
         else                    st.put( word, st.get(word) + 1);
      }
      String max = "";
      st.put(max, 0);
      for (String word : st.keys())
         if (st.get(word) > st.get(max))
            max = word;
      StdOut.println( max + " " + st.get(max));
   }
}
```

create ST

ignore short strings

read string and update frequency

print a string with max freq

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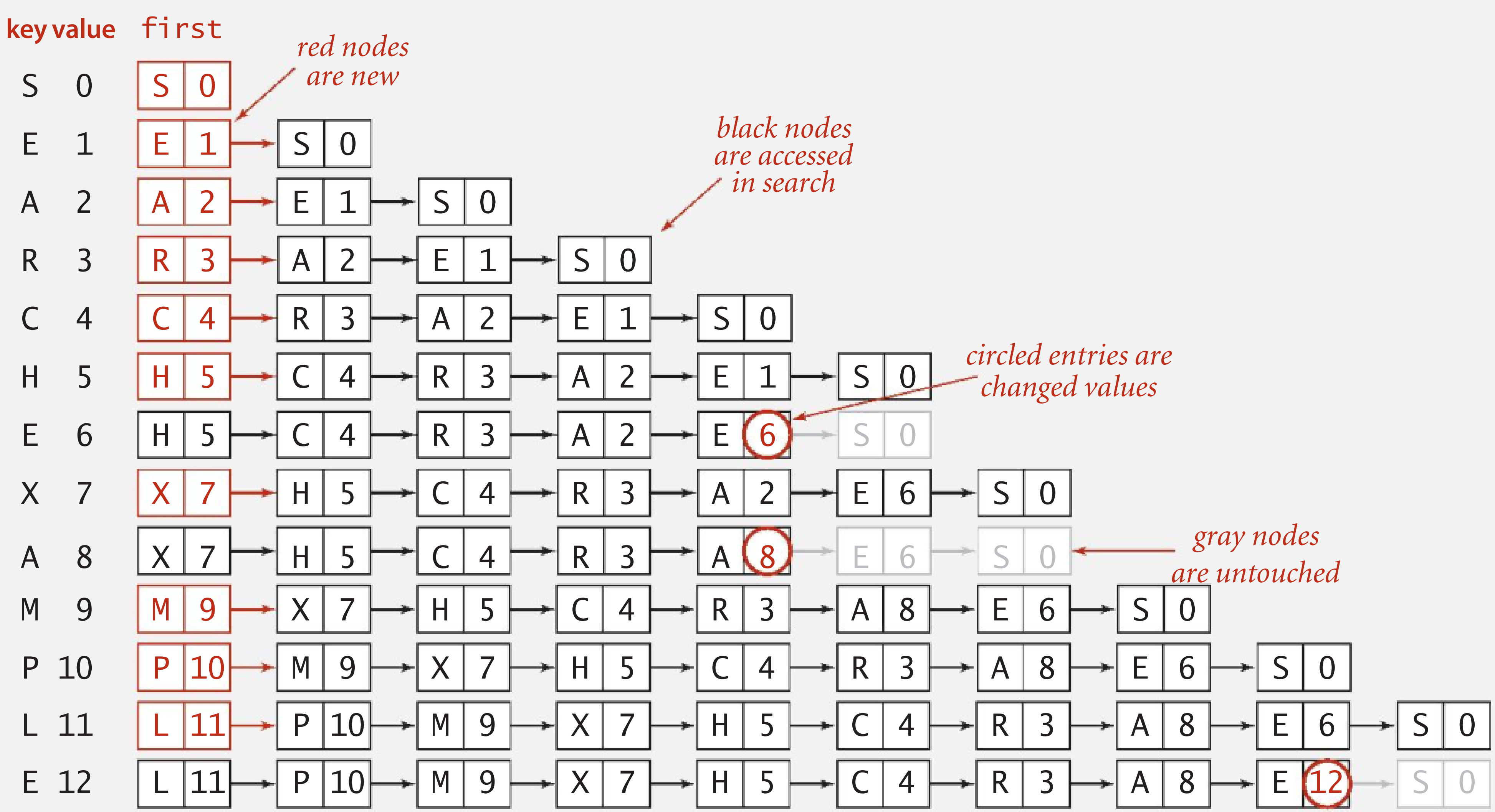
Sequential search in a linked list

Data structure. Maintain an (unordered) linked list of key-value pairs.

插入链表开头.

Search. Scan through all keys until find a match.

Insert. Scan through all keys until find a match; if no match add $\boxed{\text{to front.}}$



Trace of linked-list ST implementation for standard indexing client

Elementary ST implementations: summary

ST implementation	guarantee		average case		key interface
	search	insert	search hit	insert	
sequential search (unordered list)	N	N	$N/2$	N	`equals()`

向一个空表中插入N个不同的键需要 ~$\frac{N^2}{2}$ 次比较.

Challenge. Efficient implementations of both search and insert.

Binary search in an ordered array

Data structure. Maintain an ordered array of key-value pairs.

Rank helper function. How many keys < k?

keys[]

0	1	2	3	4	5	6	7	8	9
A	C	E	H	L	M	P	R	S	X

successful search for P

lo	hi	m										
0	9	4	A	C	E	H	L	M	P	R	S	X
5	9	7	A	C	E	H	L	M	P	R	S	X
5	6	5	A	C	E	H	L	M	P	R	S	X
6	6	6	A	C	E	H	L	M	P	R	S	X

entries in black are `a[lo..hi]`

entry in red is `a[m]`

loop exits with `keys[m] = P`: *return* 6

unsuccessful search for Q

unsuccessful search for Q

lo	hi	m										
0	9	4	A	C	E	H	L	M	P	R	S	X
5	9	7	A	C	E	H	L	M	P	R	S	X
5	6	5	A	C	E	H	L	M	P	R	S	X
7	6	6	A	C	E	H	L	M	P	R	S	X

loop exits with `lo > hi`: *return* 7

Binary search: Java implementation

```
public Value get (Key key) {
   if (isEmpty()) return null;
   int i = rank(key);
   if (i < N && keys[i].compareTo(key) == 0) return vals[i];
   else return null;
}
```

```
private int rank (Key key) {

   int lo = 0, hi = N-1;

   while (lo <= hi) {
       int mid = lo + (hi - lo) / 2;
       int cmp = key.compareTo( keys[mid]);
       if      (cmp  < 0) hi = mid - 1;
       else if (cmp  > 0) lo = mid + 1;
       else if (cmp == 0) return mid;
   }
   return lo;
}
```

number of keys < key

不管存不存在该键,都返回表中小于它的键的数量.

Binary search: trace of standard indexing client

Problem. To insert, need to shift all greater keys over.

		keys[]											vals[]									
key	value	0	1	2	3	4	5	6	7	8	9	N	0	1	2	3	4	5	6	7	8	9
S	0	S										1	0									
E	1	E	S									2	1	0								
A	2	A	E	S								3	2	1	0							
R	3	A	E	R	S							4	2	1	3	0						
C	4	A	C	E	R	S						5	2	4	1	3	0					
H	5	A	C	E	H	R	S					6	2	4	1	5	3	0				
E	6	A	C	E	H	R	S					6	2	4	6	5	3	0				
X	7	A	C	E	H	R	S	X				7	2	4	6	5	3	0	7			
A	8	A	C	E	H	R	S	X				7	8	4	6	5	3	0	7			
M	9	A	C	E	H	M	R	S	X			8	8	4	6	5	9	3	0	7		
P	10	A	C	E	H	M	P	R	S	X		9	8	4	6	5	9	10	3	0	7	
L	11	A	C	E	H	L	M	P	R	S	X	10	8	4	6	5	11	9	10	3	0	7
E	12	A	C	E	H	L	M	P	R	S	X	10	8	4	12	5	11	9	10	3	0	7
		A	C	E	H	L	M	P	R	S	X		8	4	12	5	11	9	10	3	0	7

entries in red were inserted

entries in gray did not move

entries in black moved to the right

circled entries are changed values

Elementary ST implementations: summary

在N个键的有序数组中进行二分查找最多需要(lgN+1)次比较.

向一个空符号表中插入N个元素在最坏情况下需要访问~N^2次数组.

ST implementation	guarantee		average case		key interface
	search	insert	search hit	insert	
sequential search (unordered list)	N	N	$N / 2$	N	`equals()`
binary search (ordered array)	$\log N$	N	$\log N$	$N / 2$	`compareTo()`

Challenge. Efficient implementations of both search and insert.

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Examples of ordered symbol table API

	keys	*values*
min() →	09:00:00	Chicago
	09:00:03	Phoenix
	09:00:13	Houston
get(09:00:13) →	09:00:59	Chicago
	09:01:10	Houston
floor(09:05:00) →	09:03:13	Chicago
	09:10:11	Seattle
select(7) →	09:10:25	Seattle
	09:14:25	Phoenix
	09:19:32	Chicago
	09:19:46	Chicago
keys(09:15:00, 09:25:00) →	09:21:05	Chicago
	09:22:43	Seattle
	09:22:54	Seattle
	09:25:52	Chicago
ceiling(09:30:00) →	09:35:21	Chicago
	09:36:14	Seattle
max() →	09:37:44	Phoenix

rank(k=i)

size(09:15:00, 09:25:00) *is* 5

rank(09:10:25) *is* 7 → 比 09:10:25 小的有几个.

该值可在也可不在.

Ordered symbol table API

public class ST<Key extends Comparable<Key>, Value>		
	...	
Key	min()	*smallest key*
Key	max()	*largest key*
Key	floor(Key key)	*largest key less than or equal to key*
Key	ceiling(Key key)	*smallest key greater than or equal to key*
int	rank(Key key)	*number of keys less than key*
Key	select(int k)	*key of rank k*
void	deleteMin()	*delete smallest key*
void	deleteMax()	*delete largest key*
int	size(Key lo, Key hi)	*number of keys between lo and hi*
Iterable<Key>	keys()	*all keys, in sorted order*
Iterable<Key>	keys(Key lo, Key hi)	*keys between lo and hi, in sorted order*

Binary search: ordered symbol table operations summary

	sequential search	binary search
search	N	$\log N$
insert / delete	N	N
min / max	N	1
floor / ceiling	N	$\log N$
rank	N	$\log N$
select	N	1
ordered iteration	$N \log N$	N

order of growth of the running time for ordered symbol table operations

Summary

- Symbol Tables (3.1 of Text A)
- Binary Search Trees (3.2 of Text A)

To be discussed in Lecture 11:

- Balanced Search Trees (3.3 of Text A)
- Hash Tables (3.4 of Text A)