chapter-5&6 习题课（hw10&11）

2022.12.12

5-10. Show that the ML receiver of Figure 5.4 is equivalent to the matched filter receiver of Figure 5.7.

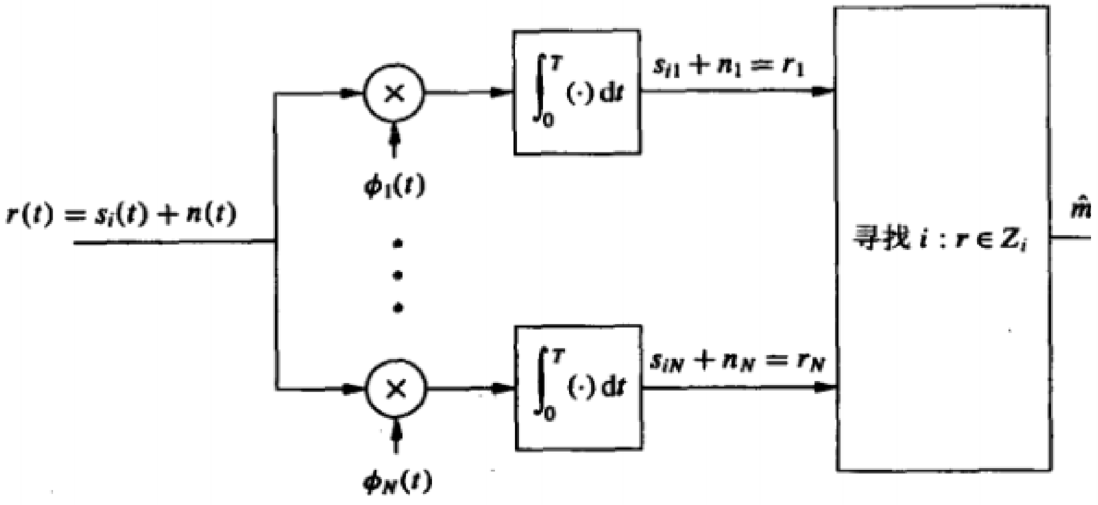


图5-4 AWGN信道下信号检测的接收机结构

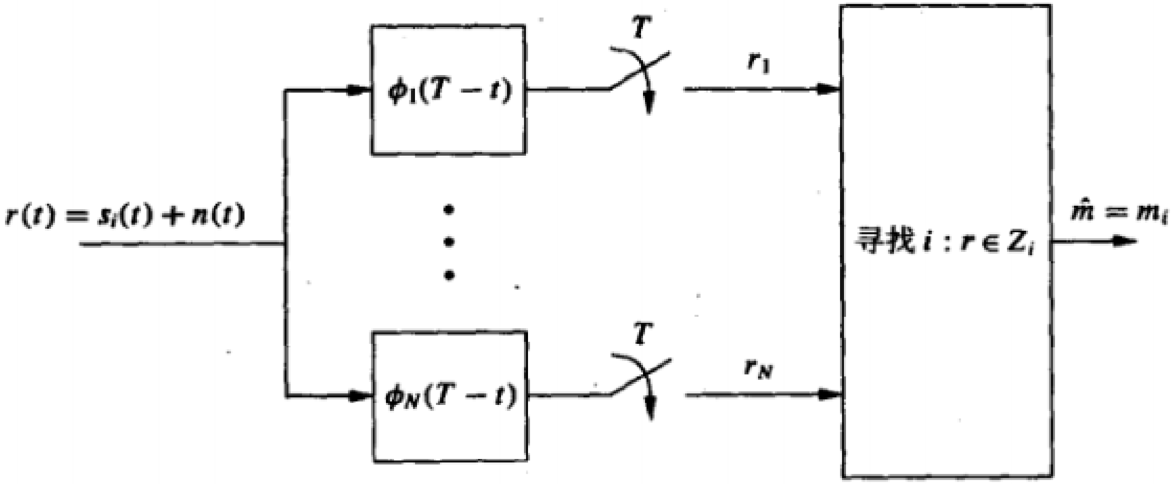


图5-7 匹配滤波接收机结构

For Fig 5.4 $\gamma_k = \int_0^T \gamma(\tau)\phi_k(\tau)d\tau$

For Fig 5. $\gamma_k = \int_0^T \gamma(\tau)\phi_k(T-(T-\tau))d\tau = \int_0^T \gamma(\tau)\phi_k(\tau)d\tau$ which is the same as above.

5-11. Compute the three bounds (5.40), (5.43), (5.44) as well as the approximation (5.45) for an asymmetric signal constellation $s_1 = (A_c, 0)$, $s_2 = (0, 2A_c)$, $s_3 = (-2A_c, 0)$, and $s_4 = (0, -A_c)$, assuming that $A_c/\sqrt{N_0} = 4$.

(5.40) gives $\frac{1}{4}\sum_{i=1}^{4}\sum_{j=1,j\neq i}^{4} Q\left(\frac{d_{ij}}{\sqrt{2N_0}}\right) = 4.1 \times 10^{-9}$

(5.43) gives $(4-1)Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2.3 \times 10^{-8}$

(5.44) gives $\frac{(4-1)}{\sqrt{\pi}} \exp\left(-\frac{d_{min}^2}{4N_0}\right) = 1.9 \times 10^{-7}$

(5.45) gives $M_{d_{min}} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 1.5 \times 10^{-8}$

$$P_e = \sum_{i=1}^{M} p(m_i) P_e(m_i \text{ sent}) \leqslant \frac{1}{M}\sum_{i=1}^{M}\sum_{\substack{k=1\\k\neq i}}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right) \quad (5\text{-}40)$$

$$P_e \leqslant (M-1) Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) \quad (5\text{-}43)$$

$$P_e \leqslant \frac{M-1}{d_{\min}\sqrt{\pi/N_0}} \exp\left[\frac{-d_{\min}^2}{4N_0}\right] \quad (5\text{-}44)$$

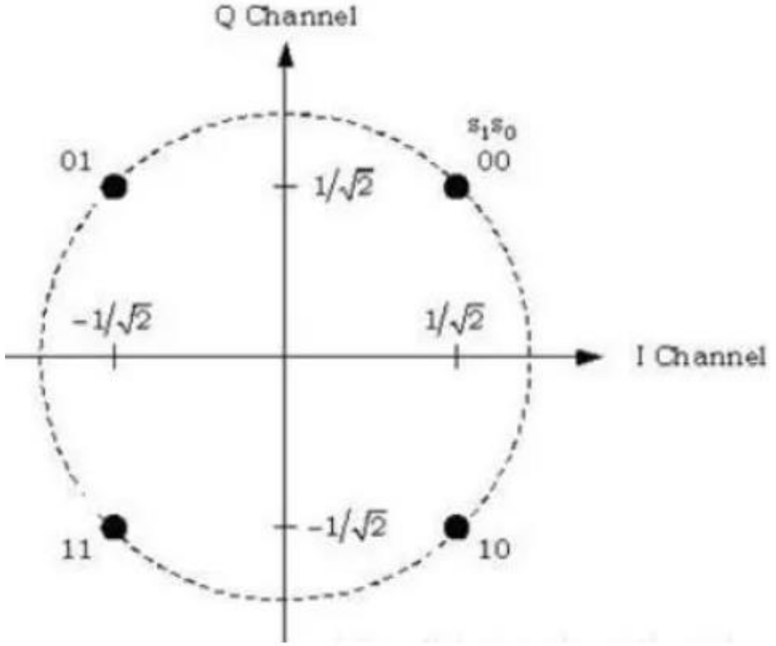
$$P_e \approx M_{d_{\min}} Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) \quad (5\text{-}45)$$

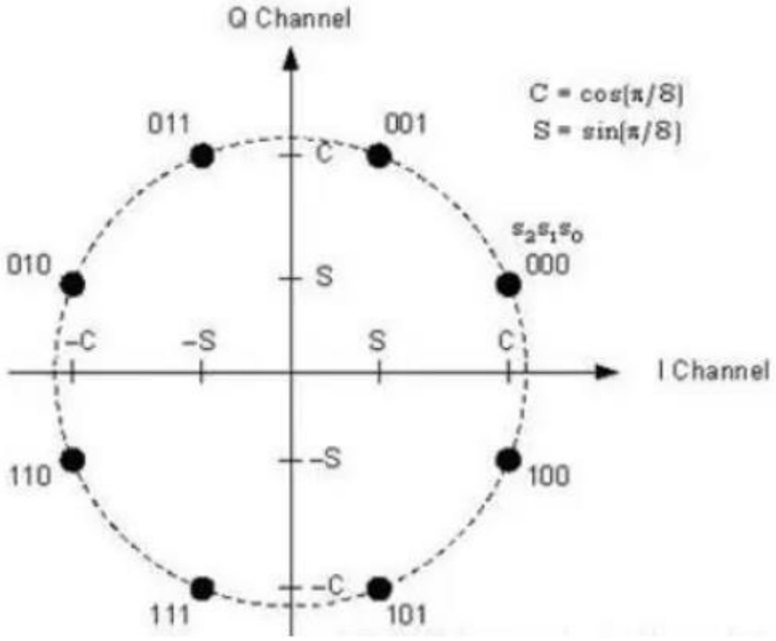
Solving Q function： search table or MATLAB

x	Q(x)	x	Q(x)	x	Q(x)
3.00	$1.35*10^{-3}$	4.00	$3.17*10^{-5}$	5.00	$2.87*10^{-7}$
3.05	$0.14*10^{-3}$	4.05	$2.56*10^{-5}$	5.05	$2.21*10^{-7}$
3.10	$9.68*10^{-4}$	4.10	$2.07*10^{-5}$	5.10	$1.70*10^{-7}$
3.15	$8.16*10^{-4}$	4.15	$1.66*10^{-5}$	5.15	$1.30*10^{-7}$
3.20	$6.87*10^{-4}$	4.20	$1.33*10^{-5}$	5.20	$9.96*10^{-8}$
3.25	$5.77*10^{-4}$	4.25	$1.07*10^{-5}$	5.25	$7.61*10^{-8}$
3.30	$4.83*10^{-4}$	4.30	$8.54*10^{-6}$	5.30	$5.79*10^{-8}$
3.35	$4.04*10^{-4}$	4.35	$6.81*10^{-6}$	5.35	$4.40*10^{-8}$
3.40	$3.37*10^{-4}$	4.40	$5.41*10^{-6}$	5.40	$3.33*10^{-8}$

5-13. Consider a 4-PSK constellation with $d_{\min} = \sqrt{2}$. What is the additional energy required to send one extra bit (8-PSK) while keeping the same minimum distance (and thus with the same bit error probability)?

For 4PSK $d_{min} = \sqrt{2\varepsilon} \Rightarrow \varepsilon_{4PSK} = 1$
For 8PSK $d_{min} = \sqrt{\varepsilon + \varepsilon - 2\varepsilon\cos(\pi/4)} \Rightarrow \varepsilon_{8PSK} = \frac{1}{1-\cos(\pi/4)} = 3.4142$
extra energy factor $= 3.4142 = 5.33$dB





5-17. Consider the octal signal point constellation shown in Figure 5.34.

(a) The nearest neighbor signal points in the 8-QAM signal constellation are separated by a distance of A. Determine the radii a and b of the inner and outer circles.

(b) The adjacent signal points in the 8-PSK are separated by a distance of A. Determine the radius r of the circle.

(c) Determine the average transmitter powers for the two signal constellations and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume that all signal points are equally probable.)

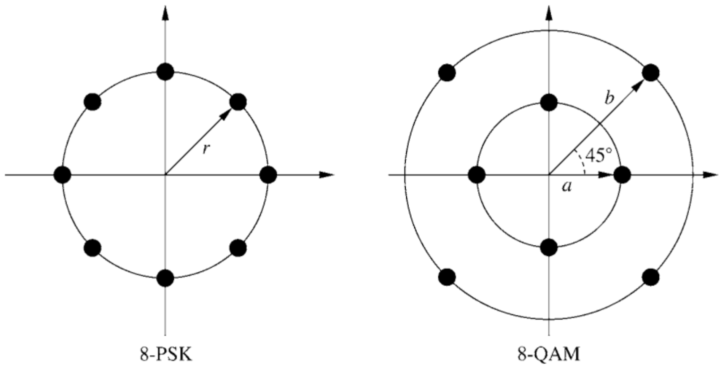


Figure 5.34: Octal signal point constellation for Problem 5-17.

(a)

$$a = A * cos(\frac{\pi}{4}) = 0.7071A$$

$$a^2 + b^2 - 2abcos(\frac{\pi}{4}) = A^2, b = \frac{1+\sqrt{3}}{2}A = 1.366A$$

(b) $A^2 = r^2(2 - 2cos(\frac{\pi}{4})) = 1.3066A$

(c) for 8PSK:

$$P = \sum_{1}^{8} \frac{1}{8} r^2 = r^2 = 1.7071A^2$$

for 8QAM:

$$P = \sum_{1}^{4} \frac{1}{8} a^2 + \sum_{1}^{4} \frac{1}{8} b^2 = \frac{a^2+b^2}{2} = 1.1830A^2$$

Comment:

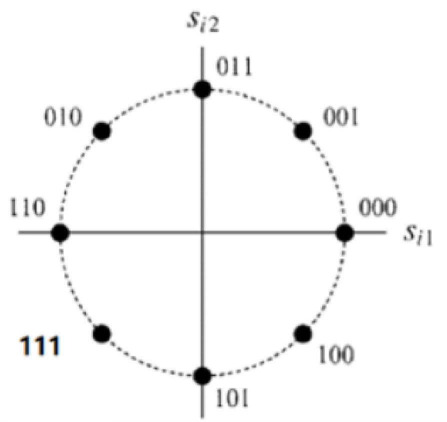
8-QAM has relative power advantage, with required transmitted power lower than that of 8-PSK by 1.59dB. ($10\log_{10}(1.707/1.183)$)

(d) Is it possible to assign three data bits to each point of the signal constellation such that nearest neighbor (adjacent) points differ in only one bit position?

(e) Determine the symbol rate if the desired bit rate is 90 Mbps.

(d)

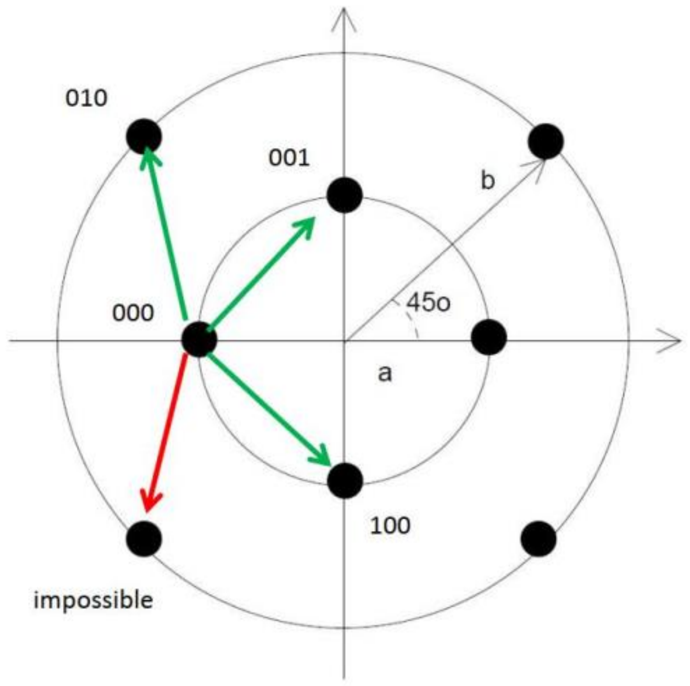
Only possible in 8-PSK. An example for 8-PSK bits assignment is the 3-bit Gray code in below:



(e) symbol rate = bit rate/ $\log_2 8$= 30M symbols per second

For 8-QAM, this is impossible. The reasons are that:

Because for the constellation points in the inner circle, each point has four nearest neighbor points. Assume that 000 is assigned to one of the 4 points, there are at most three 3-bit codes which differs in only one bit: 001,010,100, there will be one neighbor point left which has no bit codes to be assigned.



6-2. Consider BPSK modulation where the a priori probability of 0 and 1 is not the same. Specifically, $p(s_n = 0) = 0.3$ and $p(s_n = 1) = 0.7$.

(a) Find the probability of bit error P_b in AWGN assuming we encode a 1 as $s_1(t) = A\cos(2\pi f_c t)$ and a 0 as $s_2(t) = -A\cos(2\pi f_c t)$ for $A > 0$, assuming the receiver structure is as shown in Figure 5.17.

(b) Suppose you can change the threshold value in the receiver of Figure 5.17. Find the threshold value that yields equal error probability regardless of which bit is transmitted – that is, the threshold value that yields $p(\hat{m} = 0 \mid m = 1)p(m = 1) = p(\hat{m} = 1 \mid m = 0)p(m = 0)$.

(a) The bit error probability is

$$P_b = p(\hat{m}=1 \mid m=0)\,p(m=0) + p(\hat{m}=0 \mid m=1)\,p(m=1)$$

$$= 0.3Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) + 0.7Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) = Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2}{N_0}}A\right)$$

(b) Assume the threshold $t = -a$, then the distance to each symbols are $s_1 = A + a$, and $s_2 = A - a$ respectively, thus

$$p(\hat{m}=0 \mid m=1) = p(n > A+a)$$

$$= \int_{A+a}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-v^2/2} dv = \int_{\sqrt{\frac{2}{N_0}}(A+a)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-v^2/2} dv = Q\left(\sqrt{\frac{2}{N_0}}(A+a)\right)$$

Similarly,

$$p(\hat{m}=0 \mid m=1) = p(n > A - a) = Q\left(\sqrt{\frac{2}{N_0}}(A - a)\right)$$

Thus the threshold should satisfy the following equation

$$0.7Q\left(\sqrt{\frac{2}{N_0}}(A+a)\right) = 0.3Q\left(\sqrt{\frac{2}{N_0}}(A-a)\right)$$

(c) Now suppose we change the modulation so that $s_1(t) = A\cos(2\pi f_c t)$ and $s_2(t) = -B\cos(2\pi f_c t)$. Find $A > 0$ and $B > 0$ so that the receiver of Figure 5.17 with threshold at zero has $p(\hat{m} = 0 \mid m = 1)p(m = 1) = p(\hat{m} = 1 \mid m = 0)p(m = 0)$.

(d) Compute and compare the expression for P_b in parts (a), (b), and (c) assuming $E_b/N_0 = 10$ dB and $N_0 = .1$. For which system is P_b minimized?

(c) Since $s_1 = A$, $s_2 = B$, we have

$$p(\hat{m}=0\,|\,m=1) = p(n > A) = Q\left(\sqrt{\frac{2}{N_0}}A\right)$$

$$p(\hat{m}=1\,|\,m=0) = p(n > B) = Q\left(\sqrt{\frac{2}{N_0}}B\right)$$

Thus the amplitude should satisfy the following equation

$$0.7Q\left(\sqrt{\frac{2}{N_0}}A\right) = 0.3Q\left(\sqrt{\frac{2}{N_0}}B\right)$$

(d) Take $\frac{E_b}{N_0} = \frac{A^2}{N_0} = 10$

In part a) $P_e = 3.87 \times 10^{-6}$

In part b) a=0.0203 $P_e = 3.53 \times 10^{-6}$

In part c) B=0.9587 $P_e = 5.42 \times 10^{-6}$

Clearly part (b) is the best way to decode.

Using MATLAB, taking part(b) as an example

```
a = [0:0.00001:1];
for i = 1:length(a)
    q1(i) = 0.7*qfunc(sqrt(2)*(A+a(i))/sqrt(N));
    q2(i) = 0.3*qfunc(sqrt(2)*(A-a(i))/sqrt(N));
    diff(i) = abs(q1(i) - q2(i));
end
[c,d] = min(diff);
P2 = q1(d)+q2(d);
```

6-5. Find an approximation to P_s for the signal constellations shown in Figure 6.7.

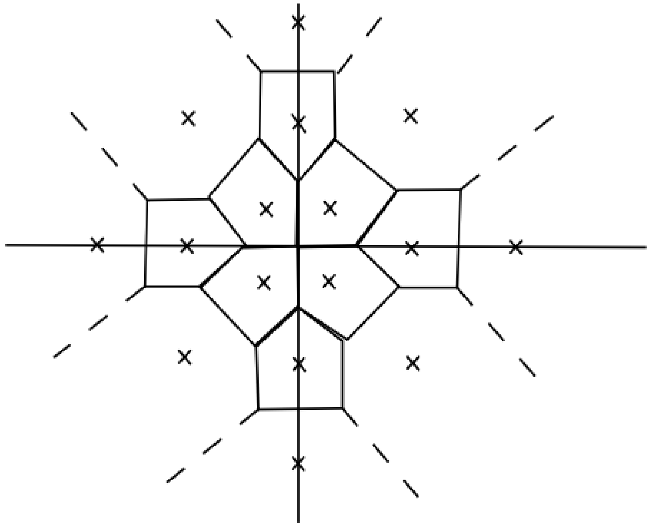
(a) 12 inner points have 5 neighbors
4 outer points have 3 neighbors
avg number of neighbors = 4.5
$$P_e = 4.5Q\left(\frac{2a}{\sqrt{2N_0}}\right)$$

(b) 16QAM, $P_e = 4\left(1-\frac{1}{4}\right)Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 3Q\left(\frac{2a}{\sqrt{2N_0}}\right)$

(c) $P_e \sim \frac{2\times3+3\times2}{5}Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 2.4Q\left(\frac{2a}{\sqrt{2N_0}}\right)$

(d) $P_e \sim \frac{1\times4+4\times3+4\times2}{9}Q\left(\frac{3a}{\sqrt{2N_0}}\right) = 2.67Q\left(\frac{3a}{\sqrt{2N_0}}\right)$

Attention: “**approximation**”



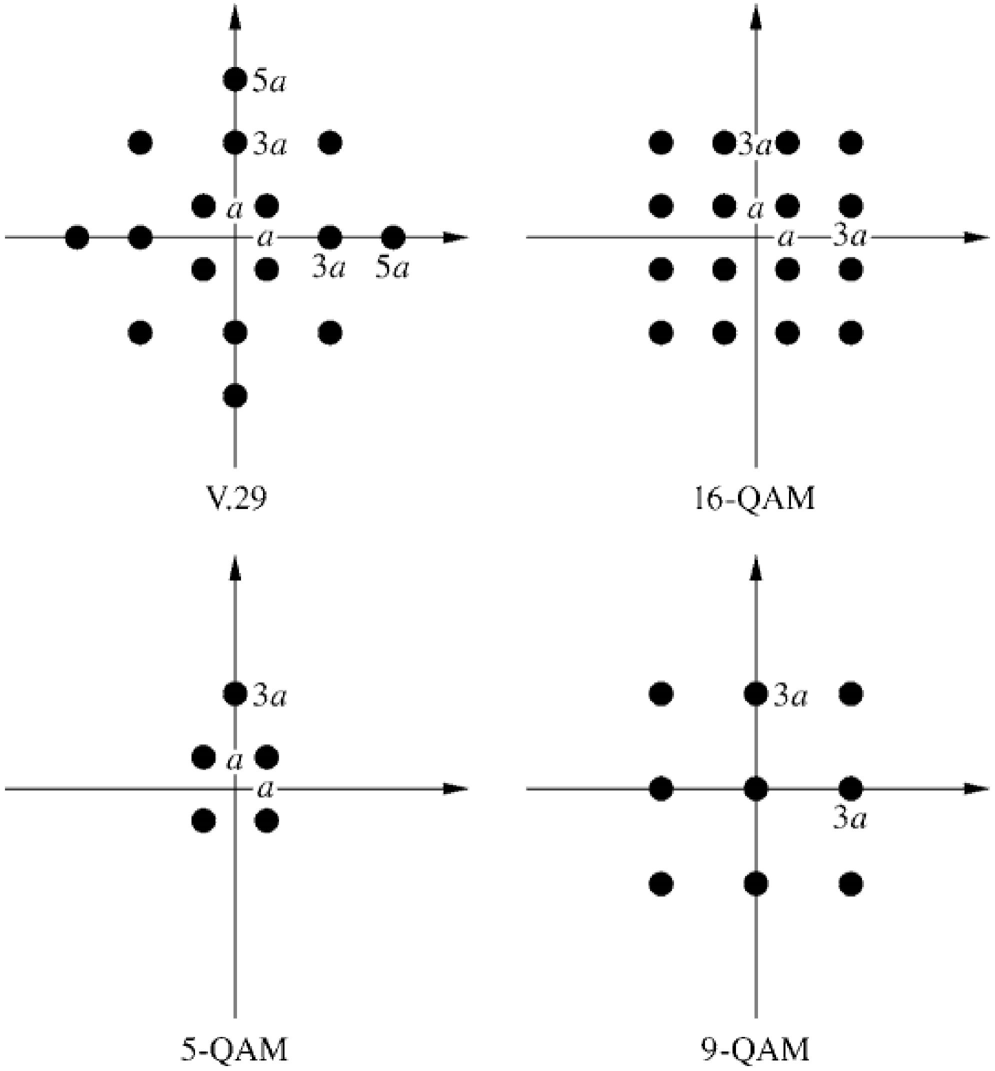
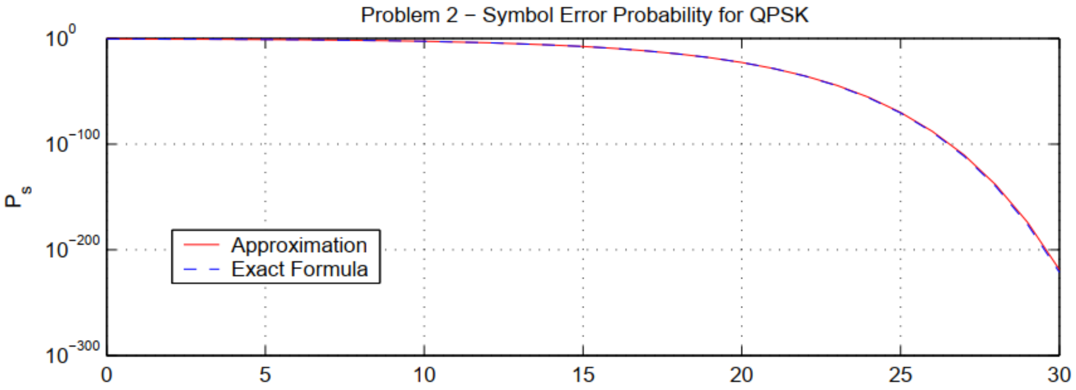


Figure 6.7: Signal constellations for Problem 6-5.

6-7. Plot the symbol error probability P_s for QPSK using the approximation in Table 6.1 and Craig's exact result for $0 \le \gamma_s \le 30$ dB. Does the error in the approximation increase or decrease with γ_s? Why?

See figure. The approximation error decreases with SNR because the approximate formula is based on nearest neighbor approximation which becomes more realistic at higher SNR. The nearest neighbor bound over-estimates the error rate because it over-counts the probability that the transmitted signal is mistaken for something other than its nearest neighbors. At high SNR, this is very unlikely and this over-counting becomes negligible.



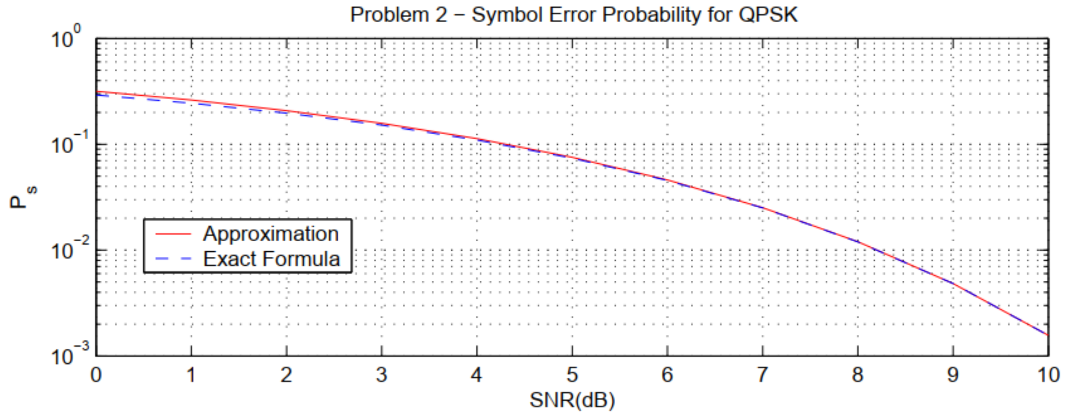


表6-1 相干解调时的近似误码率及误比特率

调制	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BFSK		$P_b = Q(\sqrt{\gamma_b})$
BPSK		$P_b = Q(\sqrt{2\gamma_b})$
QPSK, 4-QAM	$P_s \approx 2Q(\sqrt{\gamma_s})$	$P_b \approx Q(\sqrt{2\gamma_b})$
MPAM	$P_s = \frac{2(M-1)}{M}Q\left(\sqrt{\frac{6\bar{\gamma}_s}{M^2-1}}\right)$	$P_b \approx \frac{2(M-1)}{M\log_2 M}Q\left(\sqrt{\frac{6\bar{\gamma}_b\log_2 M}{M^2-1}}\right)$
MPSK	$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)\right)$	$P_b \approx \frac{2}{\log_2 M}Q\left(\sqrt{2\gamma_b\log_2 M}\sin\left(\frac{\pi}{M}\right)\right)$
矩形MQAM	$P_s \approx 4Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M}Q\left(\sqrt{\frac{3\bar{\gamma}_b\log_2 M}{M-1}}\right)$
非矩形MQAM	$P_s \approx 4Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M}Q\left(\sqrt{\frac{3\bar{\gamma}_b\log_2 M}{M-1}}\right)$