Review:

Narrowband MIMO

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_r} \end{pmatrix} = H \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{M_t} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ \vdots \\ n_{M_r} \end{pmatrix}$$

29 十七 Monday / 星期一

$$\vec{y} = \underset{M_r \times M_t}{H} \vec{x} + \vec{n}$$

$n_i \sim CN(0, \sigma^2)$

$R_n = E[\vec{n}\,\vec{n}^H] = \sigma^2 I$

transmit power of the i-th antenna.

30 十八 Tuesday / 星期二

$E[x_i x_i^*] = E[|x_i|^2]$

Total transmit power $\sum_{i=1}^{M_t} E[|x_i|^2] = P$

Let $R_x = E[\vec{x}\,\vec{x}^H] \Rightarrow tr(R_x) = P$

$R_n = E[\vec{n}\,\vec{n}^H] = \cancel{\frac{N_0}{2}} = \sigma^2 I.$

$\sum_{i=1}^{M_t} E[x_i x_i^*] = P$ total transmit signal energy / power

$\Rightarrow$ Let $R_x = E[\vec{x}\vec{x}^H]$, $tr(R_x) = P$

Δ Parallel Decomposition of MIMO.

Suppose $H \sim M_r \times M_t$, with $Rank(H) = R_H$

SVD: $H = U \Lambda V^H$ $\quad Rank(H) = R_H$

$U \sim M_r \times M_r$, $UU^H = U^H U = I$

$V \sim M_t \times M_t$, $VV^H = V^H V = I$

$\Lambda \sim M_r \times M_t$, $[\Lambda]_{i,i} = \sigma_i > 0$. $i = 1, \cdots, R_H$

W.L.O.G, $\cancel{\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m > 0}$

$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{R_H} > 0$

σ_i: singular value of H

σ_i^2: i-th largest eigenvalue of HH^H

$HH^H = U \Lambda \Lambda^H U^H$

$$\vec{y} = H\vec{x} + \vec{n}$$

$$U^H \vec{y} = \Lambda V^H \vec{x} + U^H \vec{n}$$

Let $\underset{M_r \times 1}{\tilde{y}} = U^H \vec{y}$, $\cancel{\vec{x} = V}$ $\underset{M_t \times 1}{\tilde{x}} = V^H \vec{x}$

$$\tilde{y} = \Lambda \tilde{x} + U^H \vec{n}$$

$$= \Lambda \tilde{x} + \tilde{n}$$

$$\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{n}_i \qquad i = 1, \dots, R_H$$

$$\vec{x} \Rightarrow \boxed{\begin{matrix}\text{MIMO} \\ \text{Channel}\end{matrix}} \Rightarrow \vec{y} = H\vec{x} + \vec{n}$$

$$\vec{y} = HV\tilde{x} + \vec{n}$$

$$\tilde{x} \Rightarrow \boxed{V} \Rightarrow \vec{x} = V\tilde{x} \Rightarrow \boxed{CH} \Rightarrow \boxed{U^H} \Rightarrow$$

$$\tilde{y} = \Lambda \tilde{x} + \vec{n}$$

transmit precoding
precoder

receiver shaping

△ MIMO Channel Capacity with CSIT & CSIR.

$$\vec{Y} = H\vec{X} + \vec{n}$$

$$\Rightarrow \tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{n}_i \quad i = 1, 2, \cdots, R_H$$

$$P = tr(R_x) = tr(E[\vec{x}\vec{x}^H])$$

$$= tr(E[V\tilde{x}\tilde{x}^H V^H])$$

$$= tr(V\, E[\tilde{x}\tilde{x}^H]\, V^H)$$

$$= tr(E[\tilde{x}\tilde{x}^H])$$

$$= E\left[\sum_{i=1}^{R_H} |\tilde{x}_i|^2\right] = \sum_{i=1}^{R_H} E|\tilde{x}_i|^2.$$

$$R_{\tilde{n}} = E[U^H \vec{n}\vec{n}^H U]$$

$$= U^H \cdot \sigma^2 I \cdot U$$

$$= \sigma^2 I. \quad \Rightarrow E[\tilde{n}_i \tilde{n}_j] = \begin{cases} 0 & i \neq j \\ \sigma^2 & i = j \end{cases}$$

$$\tilde{n}_i \sim CN(0, \sigma^2)$$

$$C = \sum_{i=1}^{R_H} B \cdot \log_2\left(1 + \frac{\sigma_i^2 E|\tilde{x}_i|^2}{E|\tilde{n}_i|^2}\right)$$

$$= \sum_{i=1}^{R_H} B \log_2\left(1 + \sigma_i^2 \frac{P_i}{\sigma^2}\right)$$

$$= \sum_{i=1}^{R_H} B \cdot \log_2 (1 + \sigma_i^2 \rho_i) \qquad \rho_i \triangleq \frac{P_i}{\sigma^2}$$

water-filling power allocation

$$\max \sum_{i=1}^{R_H} B \cdot \log_2 \left(1 + \sigma_i^2 \frac{P_i}{\sigma^2}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{R_H} P_i \le P$$

Lagrangian

$$J(\lambda) = \sum_{i=1}^{R_H} B \cdot \log_2 \left(1 + \frac{\sigma_i^2}{\sigma^2} P_i\right) - \lambda \left(\sum_{i=1}^{R_H} P_i - P\right)$$

$$C = \min_{\lambda} \max_{\{P_i\}} J(\lambda)$$

$$\frac{\partial J(\lambda)}{\partial P_i} = \frac{B}{\ln 2} \frac{\sigma^2}{\sigma^2 + P_i \sigma_i^2} \frac{\sigma_i^2}{\sigma^2} - \lambda.$$

$$= \frac{B}{\ln 2} \frac{1}{\sigma^2/\sigma_i^2 + P_i} - \lambda. = 0$$

$$\Rightarrow P_i = \frac{B}{\lambda \cdot \ln 2} - \frac{\sigma^2}{\sigma_i^2}$$

$$\Rightarrow \frac{P_i}{P} = \left(\frac{B}{P \cdot \lambda \cdot \ln 2} - \frac{\sigma^2}{P \cdot \sigma_i^2}\right)^+$$