Review:

$\Phi = \{\phi_1(t), \phi_2(t), \cdots, \phi_N(t)\}$

$t \in [0, T)$

25 十三
Monday / 星期一

$<\phi_i(t), \phi_j(t)> = \int_0^T \phi_i(t) \cdot \phi_j(t) dt = \begin{cases} 1 \\ 0 \end{cases}$

$\{s_1(t), s_2(t), \cdots, s_m(t)\}$

$s_i(t) = \sum_{j=1}^{N} s_{ij} \phi_j(t)$

$s_{ij} = < s_i(t), \phi_j(t) >$

26 十四
Tuesday / 星期二

$\vec{s_i} = (s_{i1}, s_{i2}, \cdots, s_{iN})$ signal space

$\|\vec{s_i}\|$, $\|\vec{s_i} - \vec{s_k}\|$

Example: QPSK, BPSK

BPSK $\phi = \{\sqrt{\frac{2}{T}} \cos 2\pi f_c t\}$

$\{\vec{s_1} = \alpha, \vec{s_2} = -\alpha\}$

27 十五
Wednesday / 星期三

QPSK $\phi = \{\sqrt{\frac{2}{T}} \cos 2\pi f_c t, \sqrt{\frac{2}{T}} \sin 2\pi f_c t\}$

$\{\vec{s_1} = (\alpha, \alpha), \vec{s_2} = (\alpha, -\alpha), \vec{s_3} = (-\alpha, \alpha), \vec{s_4} = (-\alpha, -\alpha)\}$

<Move the discussion on AWGN here>

Δ Receiver Structure

<Receiver Picture from Slides>

Suppose message m_i is sent

$r(t) = s_i(t) + n(t)$ $\quad$ $n(t)$: Gaussian Process, $E[n(t)]=0$, $E[n(t)n(\tau)] = \frac{N_0}{2}\delta(t-\tau)$

$$r_j = \int_0^T r(t)\cdot\phi_j(t)\,dt$$

$$= \int_0^T s_i(t)\phi_j(t)\,dt + \int_0^T n(t)\cdot\phi_j(t)\,dt$$

$$= s_{ij} + n_j$$

where $n_j = \int_0^T n(t)\cdot\phi_j(t)\,dt = \langle n(t), \phi_j(t)\rangle$

rewrite $n(t)$ as $n(t) = n_r(t) + \sum_{j=1}^{N} n_j\phi_j(t)$

$\Rightarrow \int_0^T n_r(t)\cdot\phi_j(t)\,dt + n_j = n_j$

$\Rightarrow \int_0^T n_r(t)\cdot\phi_j(t)\,dt = 0$

$\Rightarrow$ $n_r(t)$ and $\phi_j(t)$ $(\forall j)$ are $\perp$

$\Rightarrow$ noise without impact on signal receiving

$r(t) = s_i(t) + n(t)$

$= \sum_{j=1}^{N} (s_{ij} \cdot \phi_j(t) + n_j \phi_j(t)) + n_r(t)$

$= \sum_{j=1}^{N} r_j \phi_j(t) + n_r(t)$

28 十六
Thursday / 星期四

$\vec{r} \triangleq (r_1, r_2, \dots, r_N)$

$\Rightarrow \vec{r}$ is a sufficient statistic for $r(t)$ in optimal. signal detection.

$r_j = s_{ij} + n_j$

29 十七
Friday / 星期五

$\because n_j$ is Gaussian.

$E[n_j] = E\int_0^T n(t) \cdot \phi_j(t) dt$

$= 0$

30 十八
Saturday / 星期六

31 十九
Sunday / 星期日

n_j : Gaussian. $N(0, \frac{N_0}{2})$

$$E[n_j^2] = E\left[\int_0^T \int_0^T n(t) \cdot n(\tau) \cdot \phi_j(t) \cdot \phi_j(\tau)\, dt\, d\tau\right]$$

$$= \int_0^T \int_0^T E[n(t)\, n(\tau)] \cdot \phi_j(t) \phi_j(\tau)\, dt\, d\tau.$$

$$= \int_0^T \int_0^T \frac{N_0}{2} \delta(t-\tau) \phi_j(t) \phi_j(\tau)\, dt \cdot d\tau.$$

$$= \int_0^T \frac{N_0}{2} \phi_j^2(\tau)\, d\tau$$

$$= \frac{N_0}{2}.$$

故 $p(\vec{r}\,|\,m_i) = \prod_{j=1}^{N} p(r_j\,|\,m_i)$

$$= \prod_{j=1}^{N} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(r_j - s_{ij})^2}$$

$$= \frac{1}{(\pi N_0)^{N/2}} \cdot e^{-\frac{1}{N_0}\underbrace{\sum_{j=1}^{N}(r_j - s_{ij})^2}_{\|\vec{r}-\vec{s_i}\|^2}}$$

$p(m_i \text{ is sent} \,|\, \vec{r})$

$$= p[m_i \text{ is sent}, \vec{r}\,] / p[\vec{r}\,]$$

$$= \frac{p(\vec{S_i}, \vec{r}) \cdot p(\vec{S_i})}{p(\vec{r}) \quad p(\vec{S_i})} \qquad \vec{S_i} = (S_{i1}, S_{i2}, \cdots S_{iN})$$

$$= p(\vec{r}\,|\,\vec{S_i}) \cdot p(\vec{S_i}) / p(\vec{r})$$

Best estimation of message =

$$\underset{i}{\operatorname{argmax}}\ p(m_i\,|\,\vec{r})$$

$$= \underset{i}{\operatorname{argmax}}\ \frac{p(\vec{r}\,|\,\vec{S_i}) \cdot p(\vec{S_i})}{p(\vec{r})}$$

if all the messages are transmitted equally likely,

$$= \arg\max_i p(\vec{r} \mid \vec{s}_i) = \arg\max_i p(\vec{r} \mid m_i)$$

Liklihood function

$$L(\vec{s}_i) = p(\vec{r} \mid \vec{s}_i)$$

$$= \frac{1}{(\pi N_0)^{N/2}} e^{-\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2}$$

☆ $\arg\max_i L(\vec{s}_i)$: maximum liklihood detection (ML)

☆ If all the messages are transmitted with equal prob., ML is optimal.

$\mathrm{Cov}[n_k, n_j]$

$= E[n_k n_j] \quad (k \neq j)$

$= E\left\{ \int_0^T n(t)\phi_k(t)dt \cdot \int_0^T n(\tau)\phi_j(\tau)d\tau \right.$

$= \int_0^T \int_0^T E[n(t) \cdot n(\tau)] \cdot \phi_k(t) \cdot \phi_j(\tau)\, dt\, d\tau.$

$= \int_0^T \frac{N_0}{2} \phi_k(\tau)\phi_j(\tau)\, d\tau = 0$

n_k, n_j are independent.

$E[r_j | m_i] = s_{ij} \quad \forall j = 1, \cdots, N$

$\cancel{E}\,\mathrm{Var}[r_j | m_i] = \mathrm{Var}[n_j] = N_0/2.$

$\mathrm{Cov}[r_j r_k | m_i] = E[(r_j - s_{ij})(r_k - s_{ik}) | m_i]$

$= E[n_j \cdot n_k | m_i]$

$= \begin{cases} N_0/2, & j = k \\ 0, & j \neq k \end{cases}$

Given m_i, $\begin{cases} r_j \sim N(\cancel{0}\, s_{ij}, \frac{N_0}{2}). \quad \forall j \\ r_j \text{ and } r_k \text{ are independent.} \quad \forall j \neq k \end{cases}$