△ Narrowband MIMO

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_r} \end{pmatrix} = \underset{\downarrow \; M_r \times M_t}{\underline{H}} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{M_t} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ \vdots \\ n_{M_r} \end{pmatrix}$$

$\rightarrow \vec{y} = H\vec{x} + \vec{n}$

$n_i \sim \underbrace{\mathcal{CN}}_{\text{复高斯分布}}(0, \sigma^2)$

$\underbrace{R_n}_{\text{噪声的协方差噪声}} = E[\vec{n}\,\vec{n}^H] = \sigma^2 I$.

$E[x_i x_i^*] = E|x_i|^2 \rightarrow$ 一个符号的 power.

$P = \sum_{i=1}^{M_t} E|x_i|^2 \rightarrow$ total transmit power

let $R_x = E[\vec{x}\,\vec{x}^H] = E\left[\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{M_t} \end{pmatrix} (x_1^* \; x_2^* \cdots x_{M_t}^*)\right]$

对角线上的每个元素代表每个天线上的平均功率

$\underbrace{\mathrm{tr}\{R_x\}}_{\text{对角线上元素相加之和即为发射符号的总能量}} = \sum_i E[x_i x_i^*] = P$

△ Decomposition of MIMO

Assume $H \sim M_r \times M_t$ $\quad Rank(H) = R_H$

SVD (single value decomposition)

↓

any matrix can be written as $H = U \Lambda V^H$

$U \sim M_r \times M_r \quad UU^H = U^H U = I$

$V \sim M_t \times M_t \quad VV^H = V^H V = I$

$\Lambda \sim M_r \times M_t \quad [\Lambda]_{i,i} = \underset{\uparrow}{\sigma_i} > 0 \quad , i = 1, 2, \cdots R_H$

singular value of matrix

不失一般性, Assume $\sigma_1 > \sigma_2 > \cdots > \sigma_{R_H}$

σ_i^2: i-th largest eigenvalue of HH^H

Case: $H \sim 2\times2 \quad R_H = 2 \rightarrow H$ is full rank matrix

$\Lambda = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$

$H \sim 2\times3 \quad R_H = 2$

$\Lambda = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix}$

$H \sim 3\times2 \quad R_H = 2$

$\Lambda = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix}$

$H \sim 2\times2 \quad R_H = 1$

$\Lambda = \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix}$

$\vec{y} = H\vec{x} + \vec{n} = U\Lambda V^H \vec{x} + \vec{n}$

if $H = \Lambda$, 则代表 y_i 收到 x_i, 即彼此信号互不干扰

$\rightarrow U^H \vec{y} = \Lambda V^H \vec{x} + U^H \vec{n}$

let $\tilde{y} = U^H \vec{y} \quad \tilde{x} = V^H \vec{x} \quad \tilde{n} = U^H \vec{n}$

$\Rightarrow \tilde{y} = \Lambda \tilde{x} + \tilde{n}$

$$\begin{pmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_{M_r} \end{pmatrix} = \underbrace{\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \ddots \\ & & & \sigma_{R_H} \end{bmatrix}}_{\text{传输的符号数由H的rank来决定}} \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_{M_t} \end{bmatrix} + \tilde{n}$$

$\tilde{y}_1 = \sigma_1 \tilde{x}_1 + \tilde{n}_1$

$\rightarrow \underbrace{\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{n}_i}_{\text{flat fading}} \; , i = 1, 2, \cdots R_H$

$$\vec{x} \Rightarrow \boxed{\overset{H}{\text{MIMO}}} \Rightarrow \vec{y} = H\vec{x} + \vec{n}$$

$$\vec{y} = H\vec{x} + \vec{n}$$

$$\tilde{x} \Rightarrow \boxed{V} \xrightarrow{\vec{x}} \boxed{\text{MIMO}} \Rightarrow \boxed{U^H} \Rightarrow \tilde{y}$$

$$\vec{x} = V\tilde{x} \quad \tilde{y} = U^H \vec{y} = U^H H \vec{x} + U^H \vec{n}$$

$$= \underline{U^H U} \Lambda \underline{V^H \vec{x}} + U^H \vec{n} = \Lambda \tilde{x} + U^H \vec{n}$$

△ MIMO Capacity with CSIT & CSIR

$$C = \sum_{i=1}^{R_H} B \log_2 \left(1 + \frac{\sigma_i^2 E|\tilde{x}_i|^2}{\underbrace{E|\tilde{n}_i|^2}_{=\sigma^2}} \right)$$

$n_i \sim CN(0, \sigma^2)$

$\vec{n} \qquad \tilde{n} = U^H \vec{n}$

$$R_n = E[\vec{n}\,\vec{n}^H] = \sigma^2 I$$

$$E[\tilde{n}] = E[U^H \vec{n}] = 0$$

$$R_{\tilde{n}} = E[\tilde{n}\,\tilde{n}^H] = E[U^H \vec{n}\,\vec{n}^H U] = U^H \sigma^2 I U = \sigma^2 I$$

$$C = \sum_{i=1}^{R_H} B \log \left(1 + \frac{\sigma_i^2}{\sigma^2} P_i \right) \quad P_i = E|\tilde{x}_i|^2$$

total transmit power

约束条件

$$\underset{\sim}{P} \geq tr\left(E[\vec{x}\,\vec{x}^H]\right) = tr\left(E[V\tilde{x}\,\tilde{x}^H V^H]\right)$$

$$= tr\left(V E[\tilde{x}\tilde{x}^H] V^H\right) = tr\left(E[\tilde{x}\tilde{x}^H] V V^H\right) = tr\left(E[\tilde{x}\tilde{x}^H]\right)$$

$$= \sum_{i=1}^{M_t} E(\tilde{x}_i \tilde{x}_i^*) = \sum_{i=1}^{M_t} P_i$$