

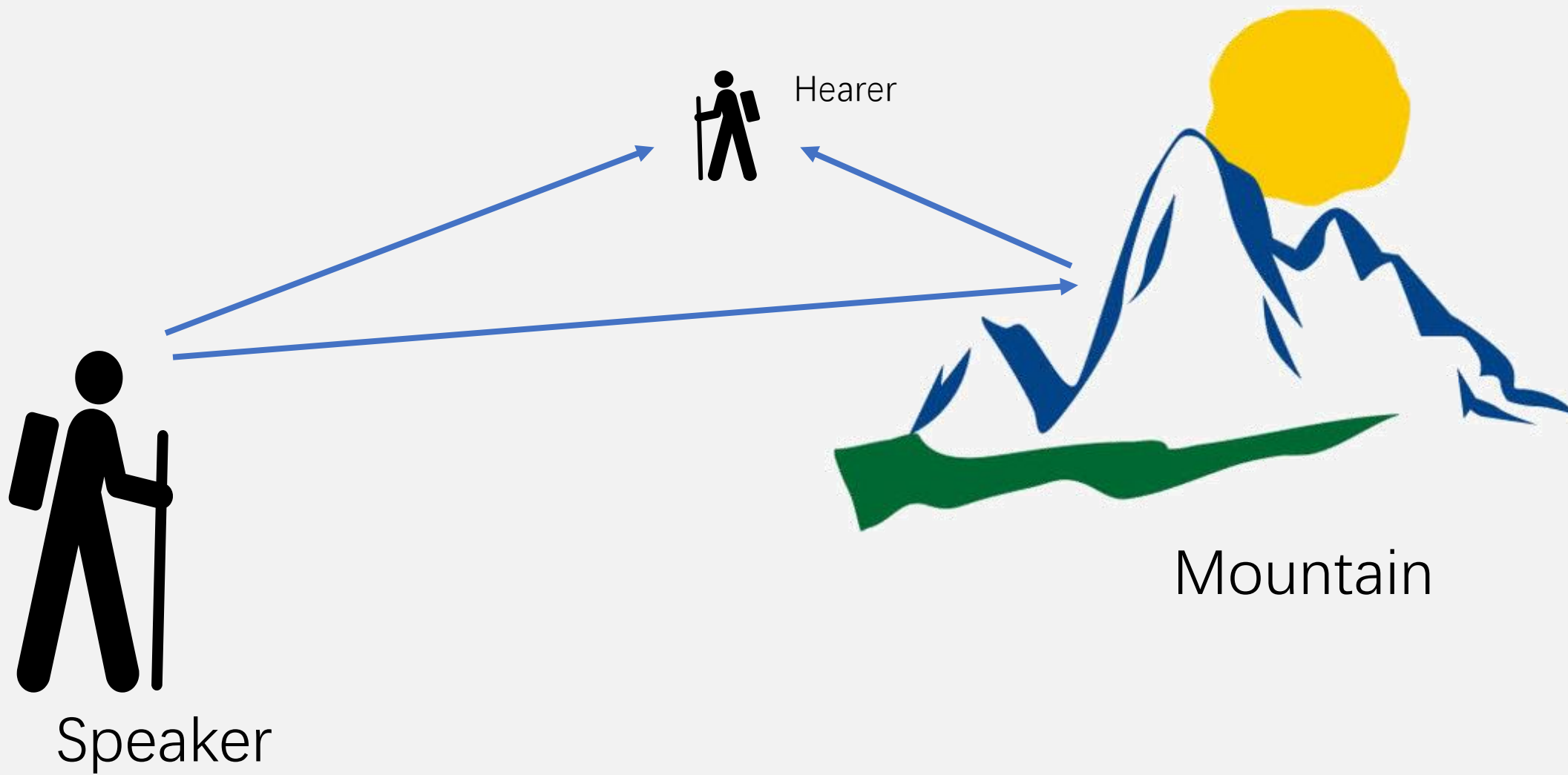
# 无线通信实验在线开放课程

主讲人：吴光 博士

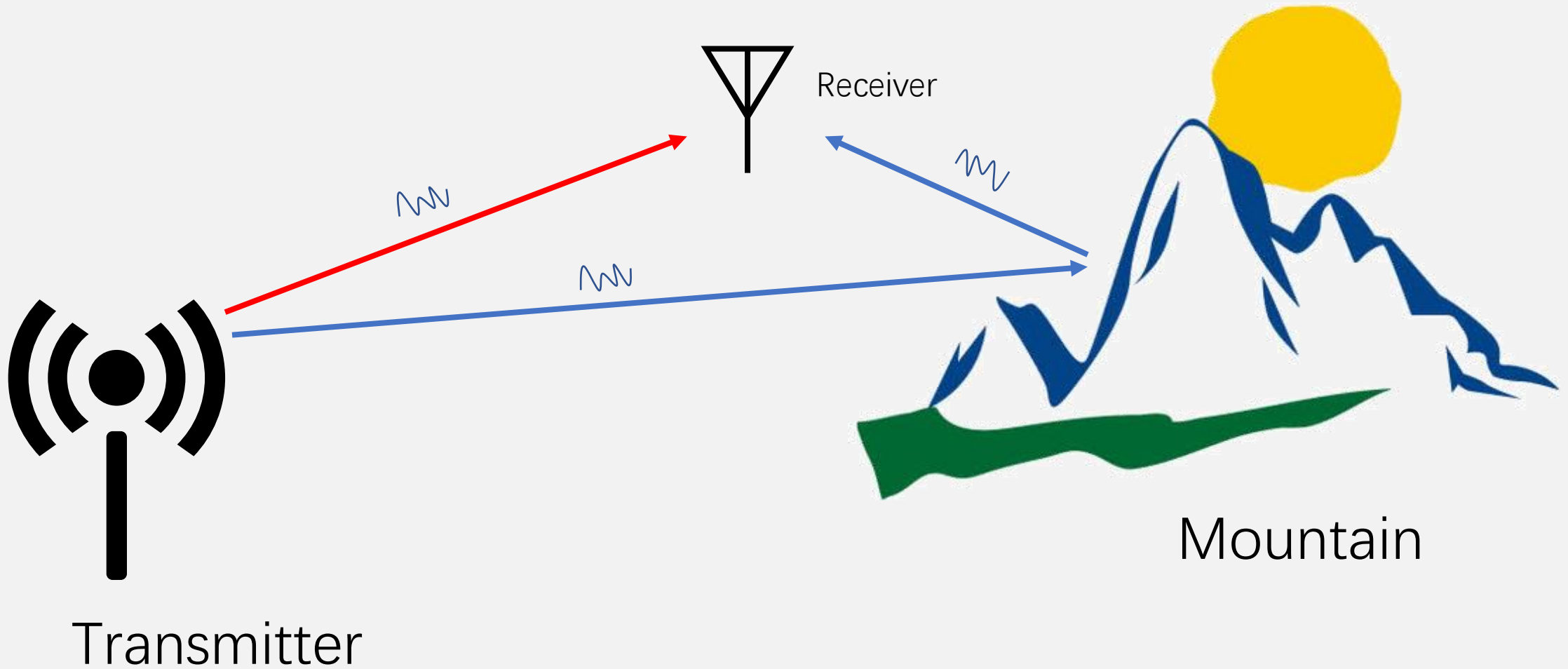
广东省教学质量工程建设项目



# Echo



# Multipath Propagation



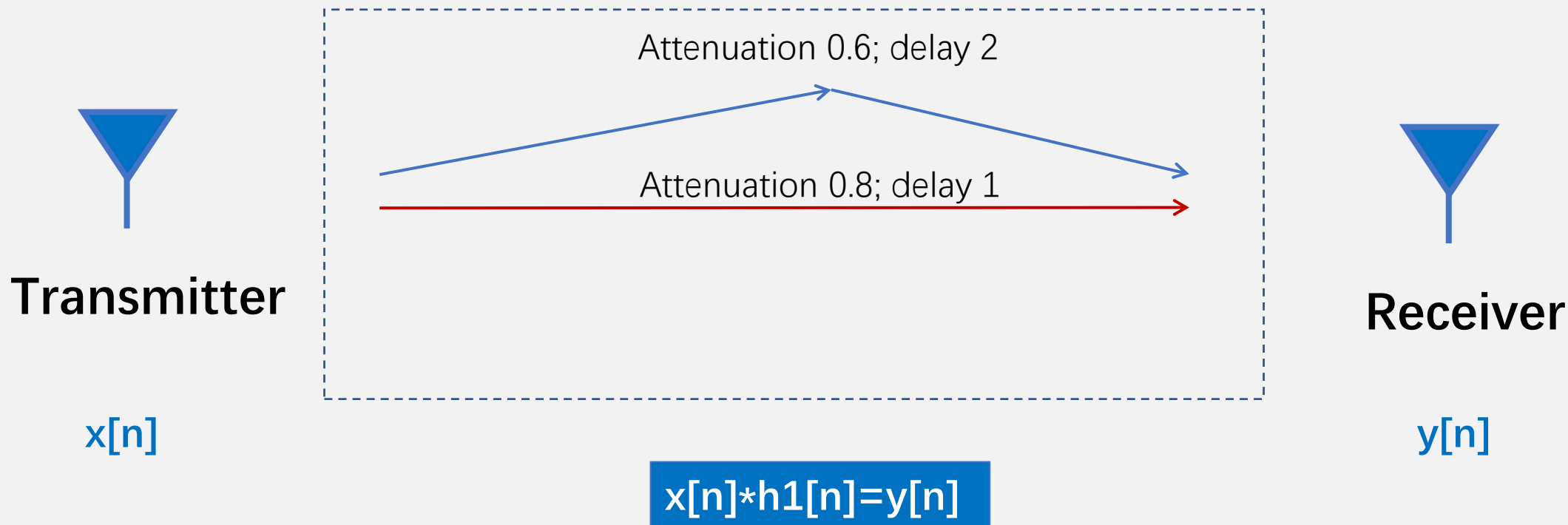


# Introduction



$$y[n] = 0.8 x[n-1] + 0.6 x[n-2]$$

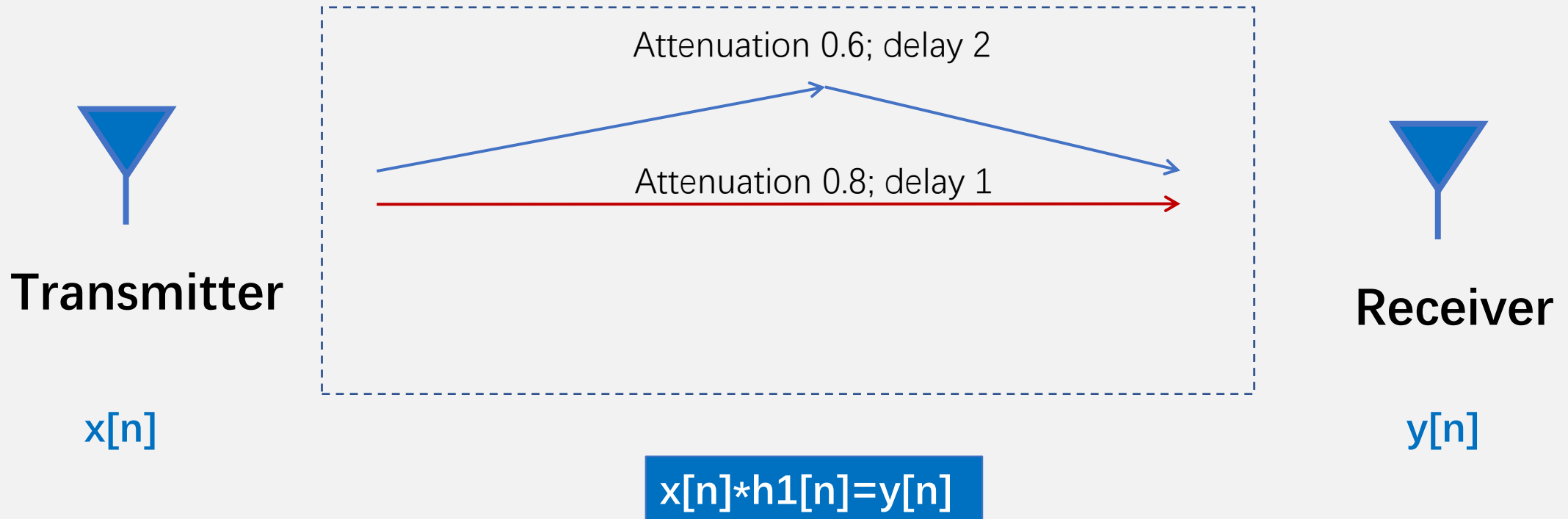
$$h1[n] = 0.8 \delta[n-1] + 0.6 \delta[n-2]$$

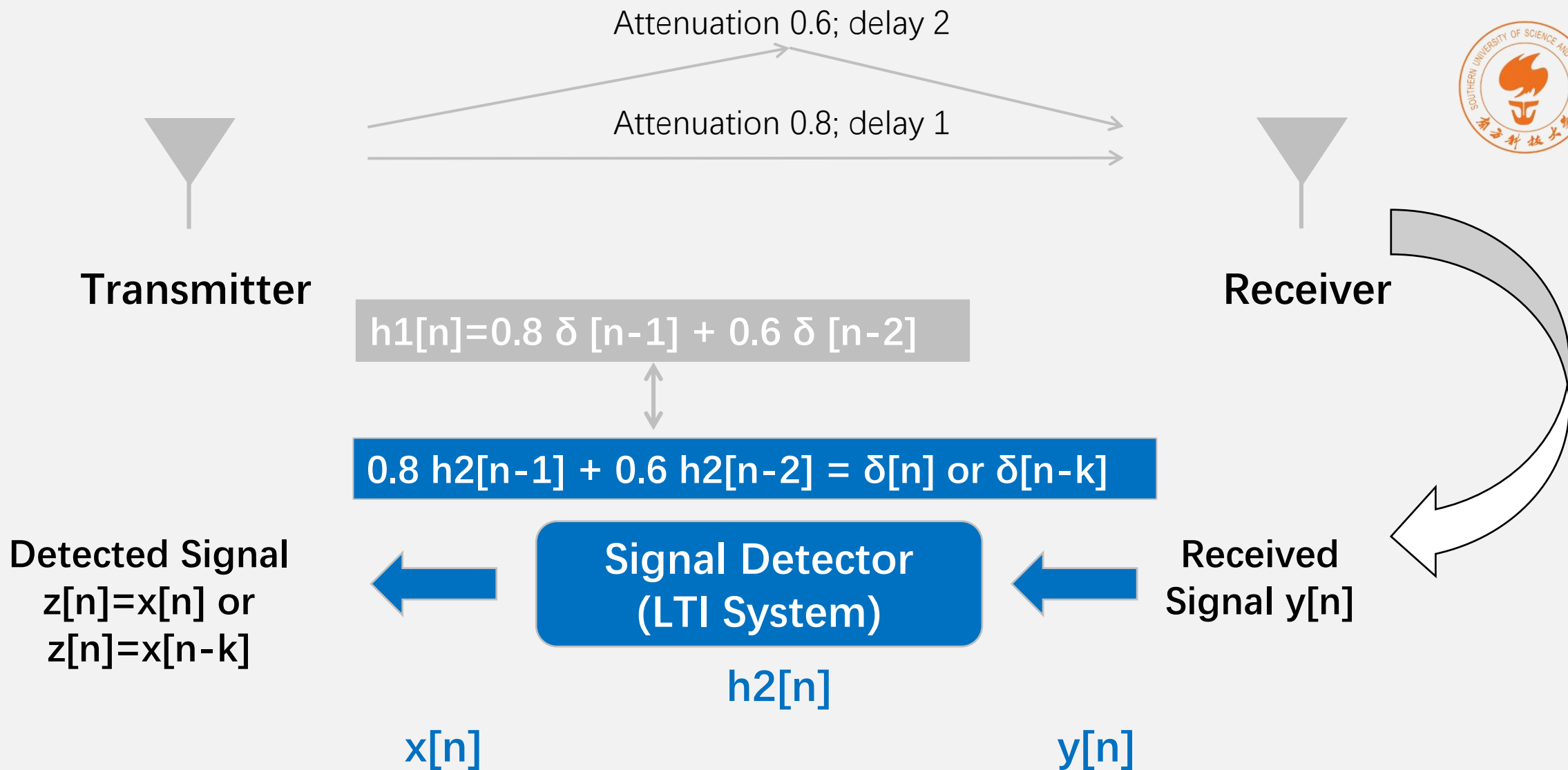


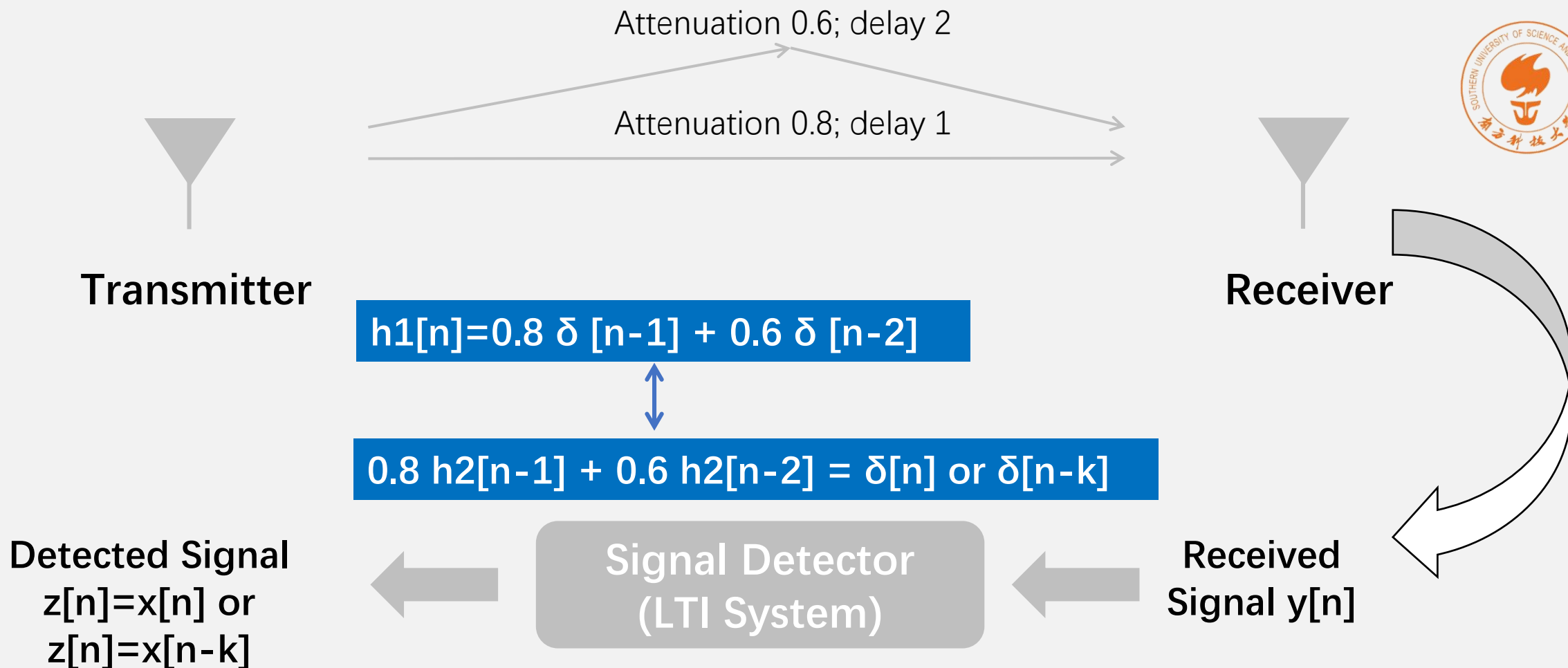


$$y[n] = 0.8 x[n-1] + 0.6 x[n-2]$$

$$h1[n] = 0.8 \delta[n-1] + 0.6 \delta[n-2]$$







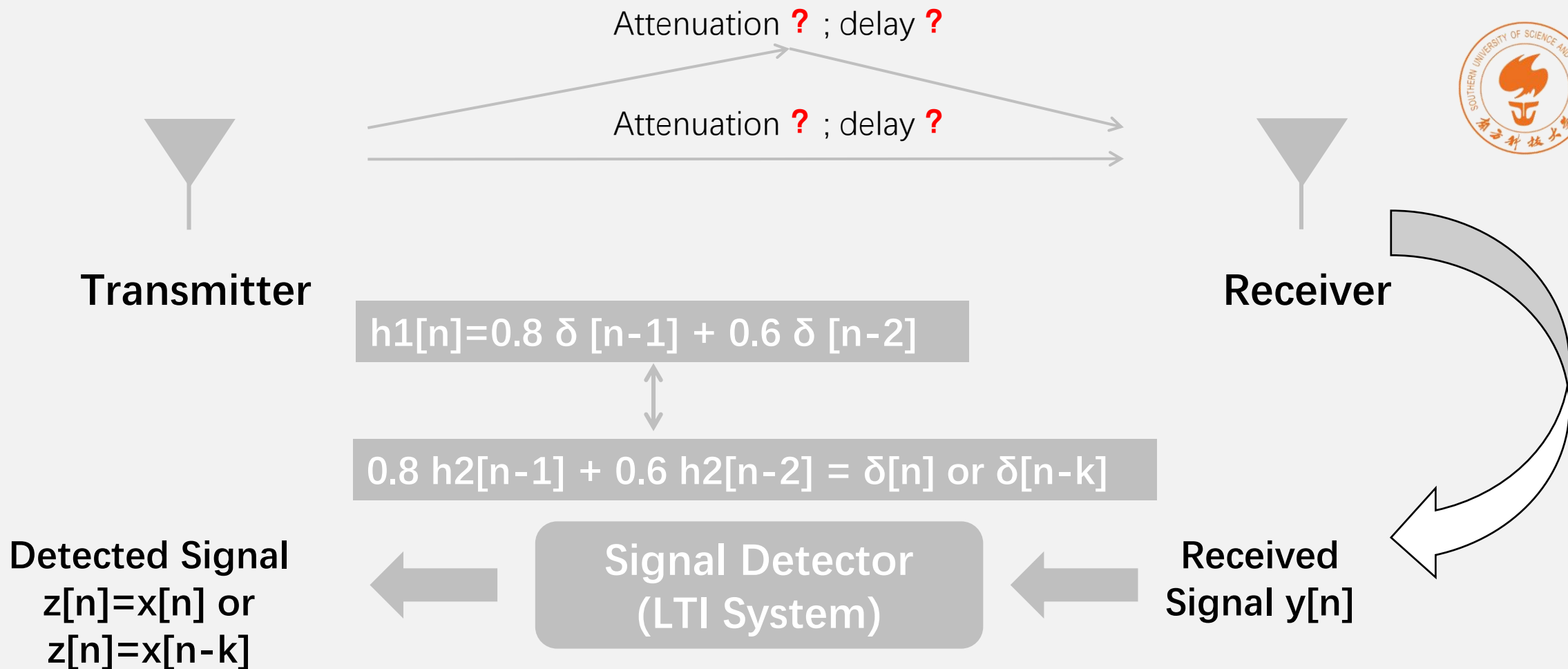
Impulse response:

$$h2[n] * h1[n] = \delta[n] \text{ or } \delta[n-k]$$

Difference Equation:

$$0.8z[n-1] + 0.6z[n-2] = y[n]$$



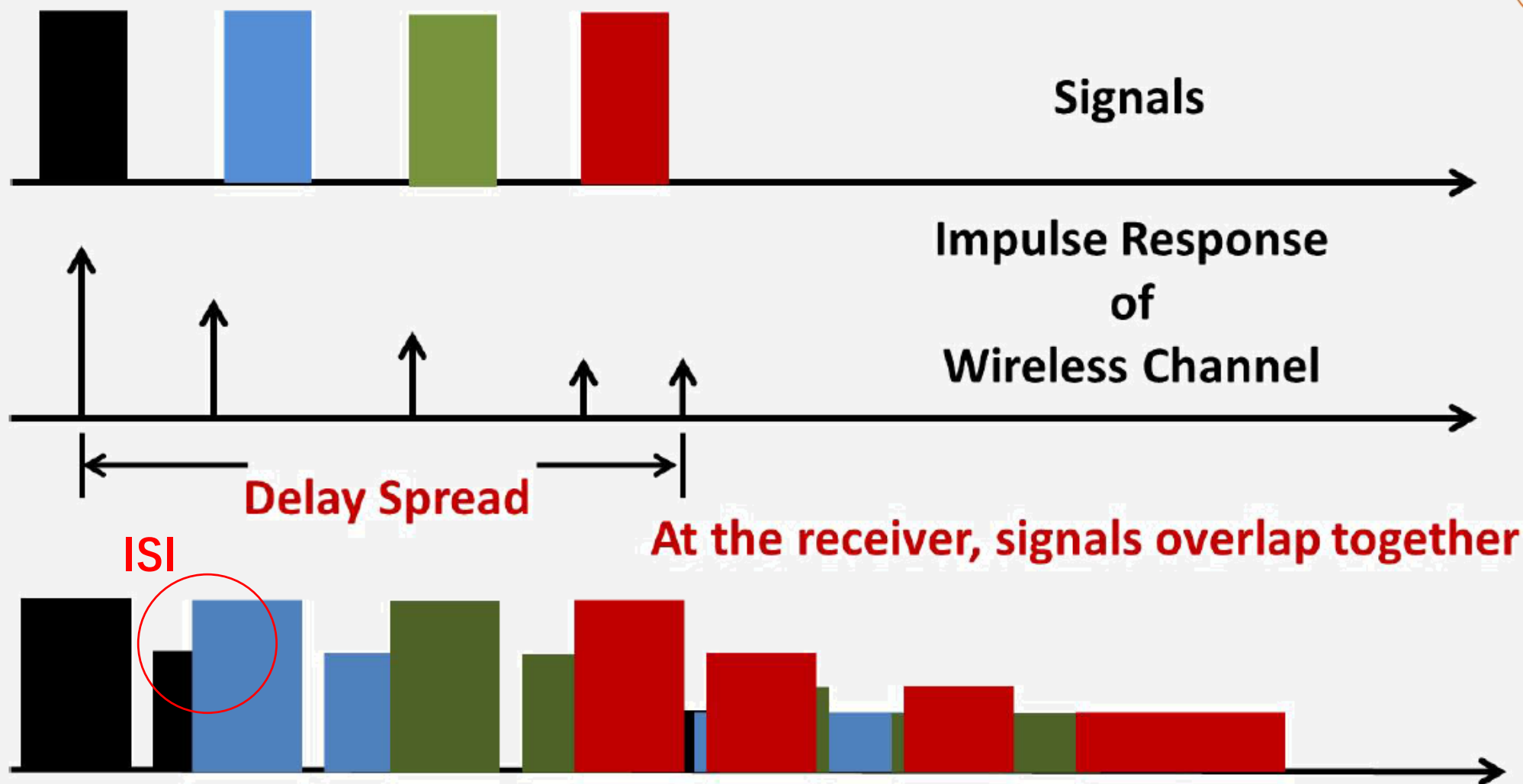




$$x[n]*h_1[n]=t[n]$$



# Frequency Selective Channel



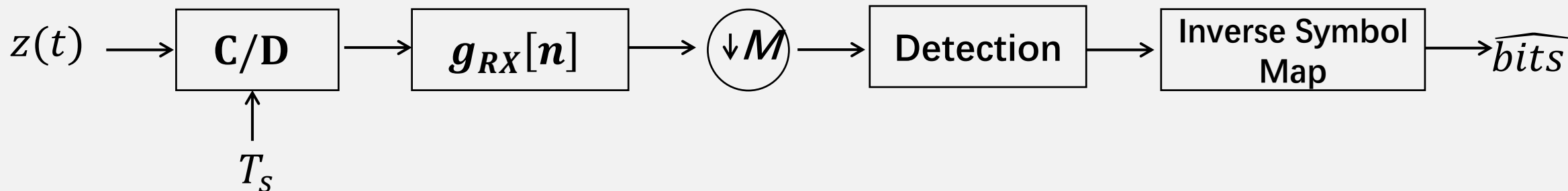


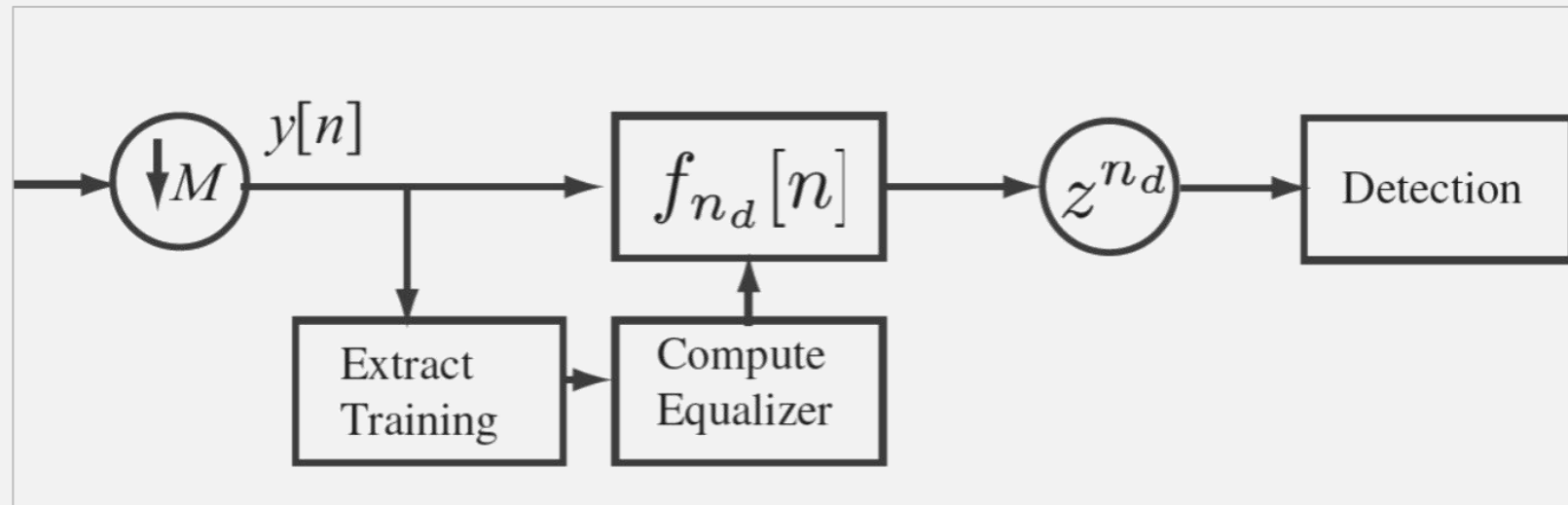
# Channel Model

AWGN **Frequency Selective** Channel

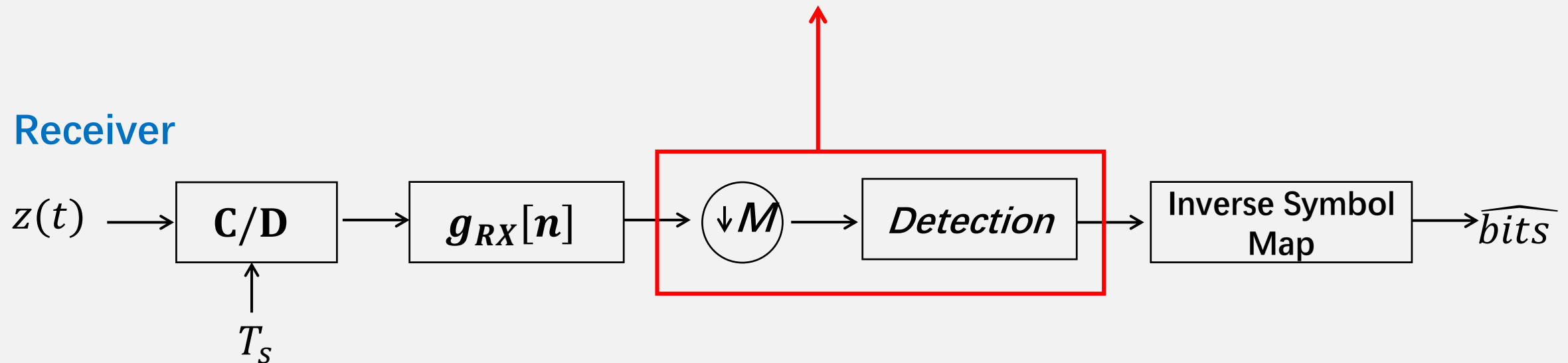
$$z(t) = \alpha_0 e^{j\varphi_0} x(t - \tau_0) + \alpha_1 e^{j\varphi_1} x(t - \tau_1) + v(t)$$

*Receiver*





## Receiver





## Channel estimate

## Equalization

Frequency Offset Correction

Zero Pad    Frame Synchronization    Payload    Zero Pad



Step1:  $h1[n]$

Step2:  $h2[n]$

↓  
fnd.

Step3:  $y[n] * h2[n] = x[n] * h1[n] * h2[n]$

↙  $\delta[n]$  or  $\delta[n-k]$   
Equalization.

# Lab 13 : Channel Estimation and Equalization

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Email: [wug@sustech.edu.cn](mailto:wug@sustech.edu.cn)





## Algorithm Analysis and Verification

1 Channel Estimation

2 Indirect Equalizer Analysis

3 Algorithm Simulation

4 USRP Experiments

5 Discussion and Conclusion



# Channel Estimation



$$\mathbf{Ax} = \mathbf{b}$$



$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{N} = \mathbf{M}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$



$$\min ||\mathbf{Ax} - \mathbf{b}||^2$$

$$N > M$$

$$\mathbf{x}_{LS} = (\mathbf{A}^* \mathbf{A})^{-1} \mathbf{A}^* \mathbf{b}$$



$$s[n] = t[n], n = 0, 1, \dots, N_t - 1$$

$$y[n] = \sum_{l=0}^L s[l]h[n-l] + v[n]$$

$$y[0] = s[0]h[0] + v[0]$$

$$y[1] = s[0]h[1] + s[1]h[0] + v[1]$$

...



$$s[n] = t[n], n = 0, 1, \dots, N_t - 1$$

$$y[n] = \sum_{l=0}^L s[l]h[n-l] + v[n]$$

$$y[n] = \sum_{l=0}^L h[l]t[n-l] + v[n]$$



$$s[n] = t[n], n = 0, 1, \dots, N_t - 1$$

$$\{\hat{h}[0], \hat{h}[1], \dots, \hat{h}[L]\}$$

$$= \underset{\{h[0], h[1], \dots, h[L]\}}{\operatorname{argmin}} \sum_{n=L}^{N_t-1} ||y[n] - \sum_{l=0}^L h[l]t[n-l] ||^2$$





$$\underbrace{\begin{bmatrix} y[L] \\ y[L+1] \\ \vdots \\ y[N_t-1] \end{bmatrix}}_y = \underbrace{\begin{bmatrix} t[L] & \cdots & t[0] \\ t[L+1] & \cdots & t[1] \\ \vdots & \ddots & \vdots \\ t[N_t-1] & \cdots & t[N_t-1-L] \end{bmatrix}}_T \underbrace{\begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[L] \end{bmatrix}}_h + \underbrace{\begin{bmatrix} v[L] \\ v[L+1] \\ \vdots \\ v[N_t-1] \end{bmatrix}}_v$$

↓  
满秩, 列向量线性无关



$$\underbrace{\begin{bmatrix} y[L] \\ y[L+1] \\ \vdots \\ y[N_t-1] \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} t[L] & \cdots & t[0] \\ t[L+1] & \cdots & t[1] \\ \vdots & \ddots & \vdots \\ t[N_t-1] & \cdots & t[N_t-1-L] \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[L] \end{bmatrix}}_{\mathbf{h}} + \underbrace{\begin{bmatrix} v[L] \\ v[L+1] \\ \vdots \\ v[N_t-1] \end{bmatrix}}_{\mathbf{v}}$$

$$\mathbf{y} = \mathbf{T}\mathbf{h} + \mathbf{v}$$

$$\mathbf{h}_{LS} = (\mathbf{T}^*\mathbf{T})^{-1}\mathbf{T}^*\mathbf{y}$$



$$\underbrace{\begin{bmatrix} y[L] \\ y[L+1] \\ \vdots \\ y[N_t-1] \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} t[L] & \cdots & t[0] \\ t[L+1] & \cdots & t[1] \\ \vdots & \ddots & \vdots \\ t[N_t-1] & \cdots & t[N_t-1-L] \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[L] \end{bmatrix}}_{\mathbf{h}} + \underbrace{\begin{bmatrix} v[L] \\ v[L+1] \\ \vdots \\ v[N_t-1] \end{bmatrix}}_{\mathbf{v}}$$

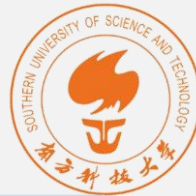
$$\mathbf{y} = \mathbf{T}\mathbf{h} + \mathbf{v}$$

$$\mathbf{h}_{LS} = (\mathbf{T}^* \mathbf{T})^{-1} \mathbf{T}^* \mathbf{y}$$

列向量线性无关满足的条件.

$$N_t - L \geq L \overset{\uparrow}{+} 1$$

$$N_t \geq 2L + 1$$



## Algorithm Analysis and Verification

1 Channel Estimation

2 Indirect Equalizer Analysis

3 Algorithm Simulation

4 USRP Experiments

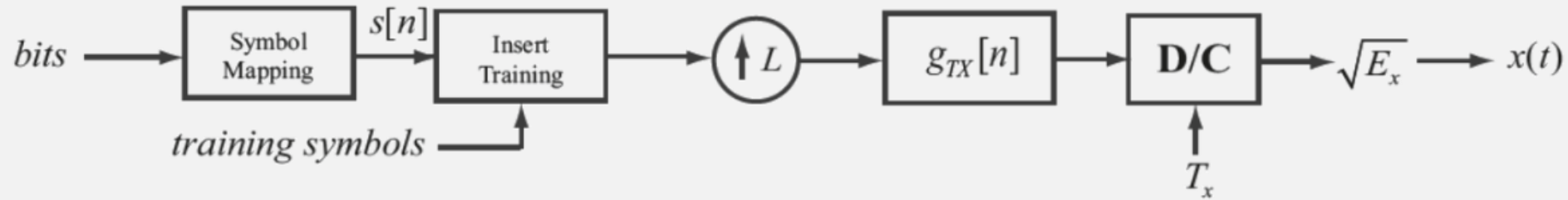
5 Discussion and Conclusion



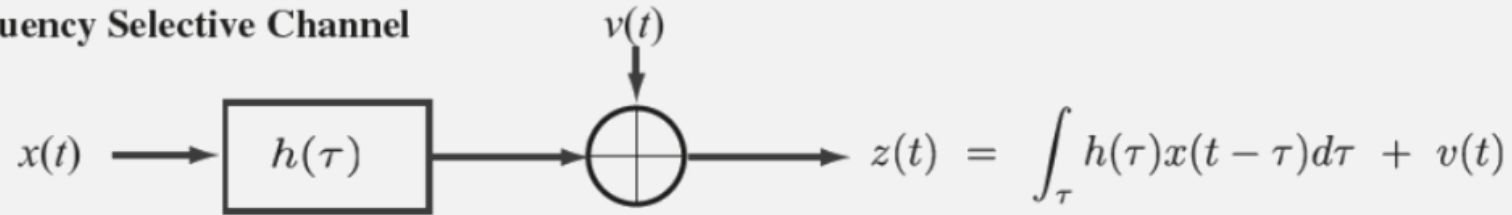
# Programming for Channel Estimation



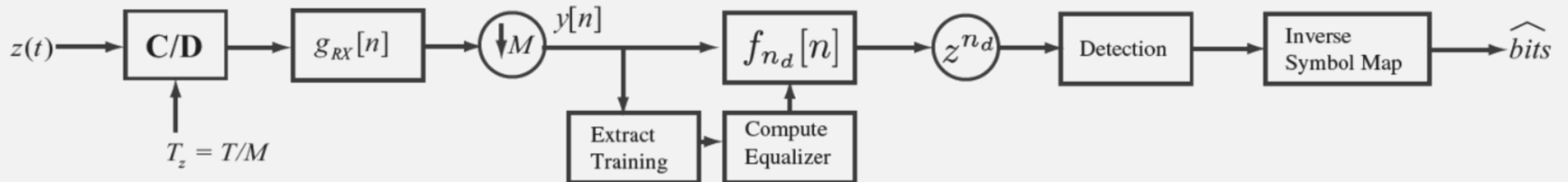
## Transmitter



## AWGN Frequency Selective Channel



## Receiver







student\_LLSE 1.vi Block Diagram \*

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17pt Application Font

Search

Student MMSE  $A\hat{x}=b$

**Functions**

- Programming
- Measurement I/O
- Instrument I/O
- Vision and Motion
- Mathematics
  - Numeric
  - Elementary
  - Linear Alge...
  - Fitting
  - Interp & Ex...
  - Integ & Diff
  - Prob & Stat
  - Optimization
  - Differential ...
  - Geometry
  - Polynomial
  - Script & Fo...
- Signal Processing
- Data Communication
- Connectivity
- Control Design & Simulation
- Express

**Linear Algebra**

$A \times B$

$A \times B$

**Inverse Matrix VI**

mean square error

DBL

x.estimate

DBL

error in (no error)

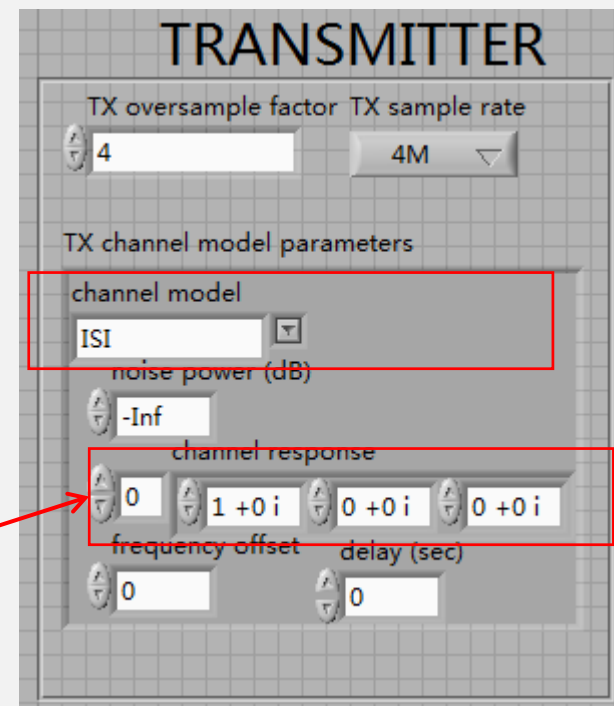
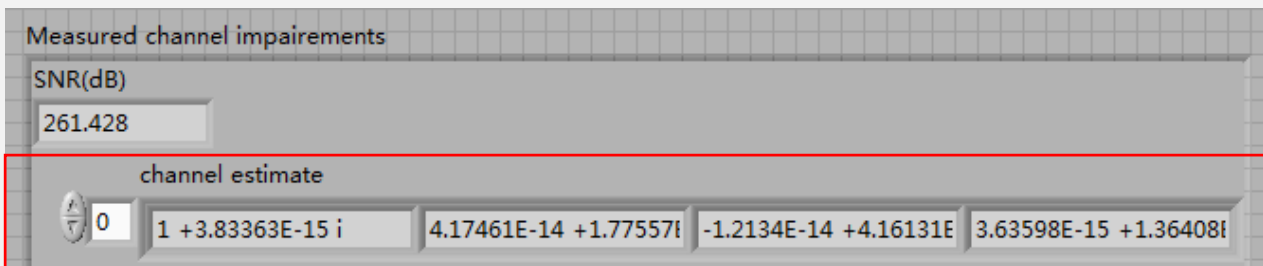
error out



# Simulation setup

Test your block diagram with these parameters!

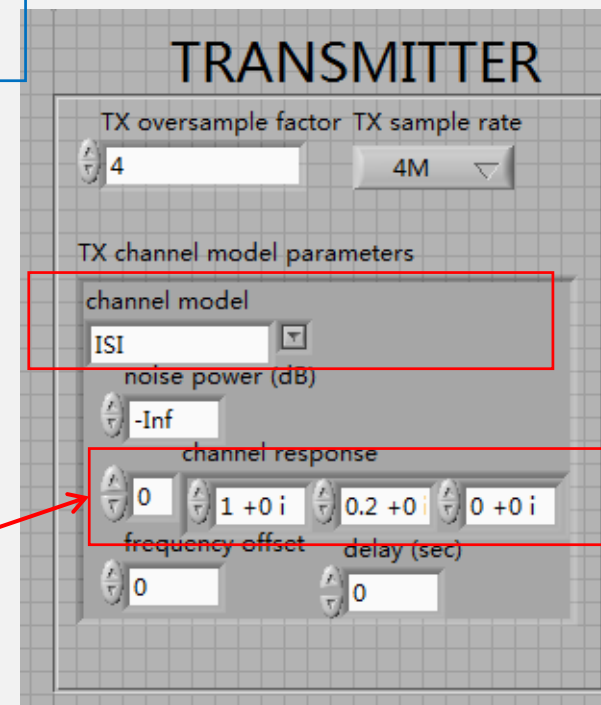
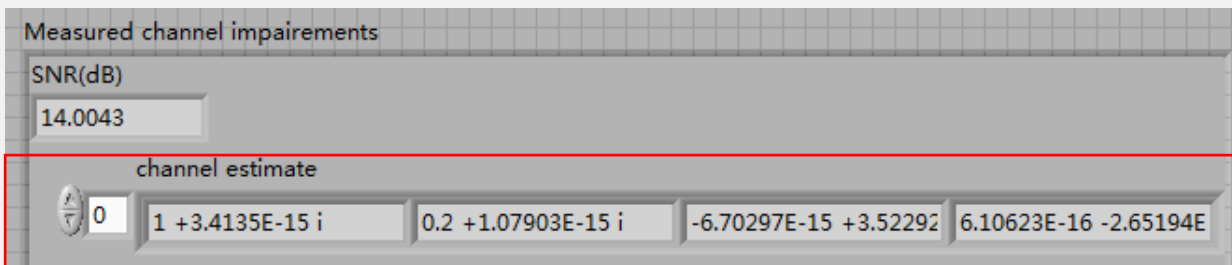
Set the channel model: ISI



# Simulation setup

Test your block diagram with these parameters!

Configure the channel model: ISI





# Demo: Linear Equalization



$$z[n] = \sum_{l=0}^{L_f} f_{n_d}[l]y[n-l] \approx \hat{s}[n-n_d]$$

$$f_{n_d}(n) * \hat{h}_c(n) = \delta(n-n_d)$$

$$\sum_{l=0}^{L_f} f_{n_d}[l]\hat{h}[n-l] \approx \delta[n-n_d]$$



$$z[n] = \sum_{l=0}^{L_f} f_{n_d}[l]y[n-l] \approx \hat{s}[n-n_d]$$

$$f_{n_d}(n) * \hat{h}_c(n) = \delta(n-n_d)$$

$$\sum_{l=0}^{L_f} f_{n_d}[l]\hat{h}[n-l] \approx \delta[n-n_d]$$



$$\underbrace{\begin{bmatrix} \hat{h}[0] & 0 & \dots & \dots \\ \hat{h}[1] & \hat{h}[0] & 0 & \dots \\ \vdots & \ddots & \ddots & \vdots \\ \hat{h}[L] & & & \\ 0 & \hat{h}[L] & \dots & \dots \\ \vdots & & & \end{bmatrix}}_{\hat{H}}$$



$$\underbrace{\begin{bmatrix} \hat{h}[0] & 0 & \cdots & \cdots \\ \hat{h}[1] & \hat{h}[0] & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \hat{h}[L] & \hat{h}[L] & \cdots & \cdots \\ 0 & \hat{h}[L] & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}}_{\hat{H}} \underbrace{\begin{bmatrix} f[0] \\ f[1] \\ \vdots \\ \vdots \\ f[L_f] \end{bmatrix}}_f = \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}}_{e_{n_d}}$$

$n_d+1$



$$\underbrace{\begin{bmatrix} \hat{h}[0] & 0 & \cdots & \cdots \\ \hat{h}[1] & \hat{h}[0] & 0 & \cdots \\ \vdots & \ddots & \ddots & \vdots \\ \hat{h}[L] & & & \\ 0 & \hat{h}[L] & \cdots & \cdots \\ \vdots & & & \end{bmatrix}}_{\hat{H}} \underbrace{\begin{bmatrix} f[0] \\ f[1] \\ \vdots \\ f[L_f] \end{bmatrix}}_f = \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}}_{e_{n_d}} \quad n_d+1$$

$$\hat{f}_{n_d} = (\hat{H}^* \hat{H})^{-1} \hat{H}^* e_{n_d}$$

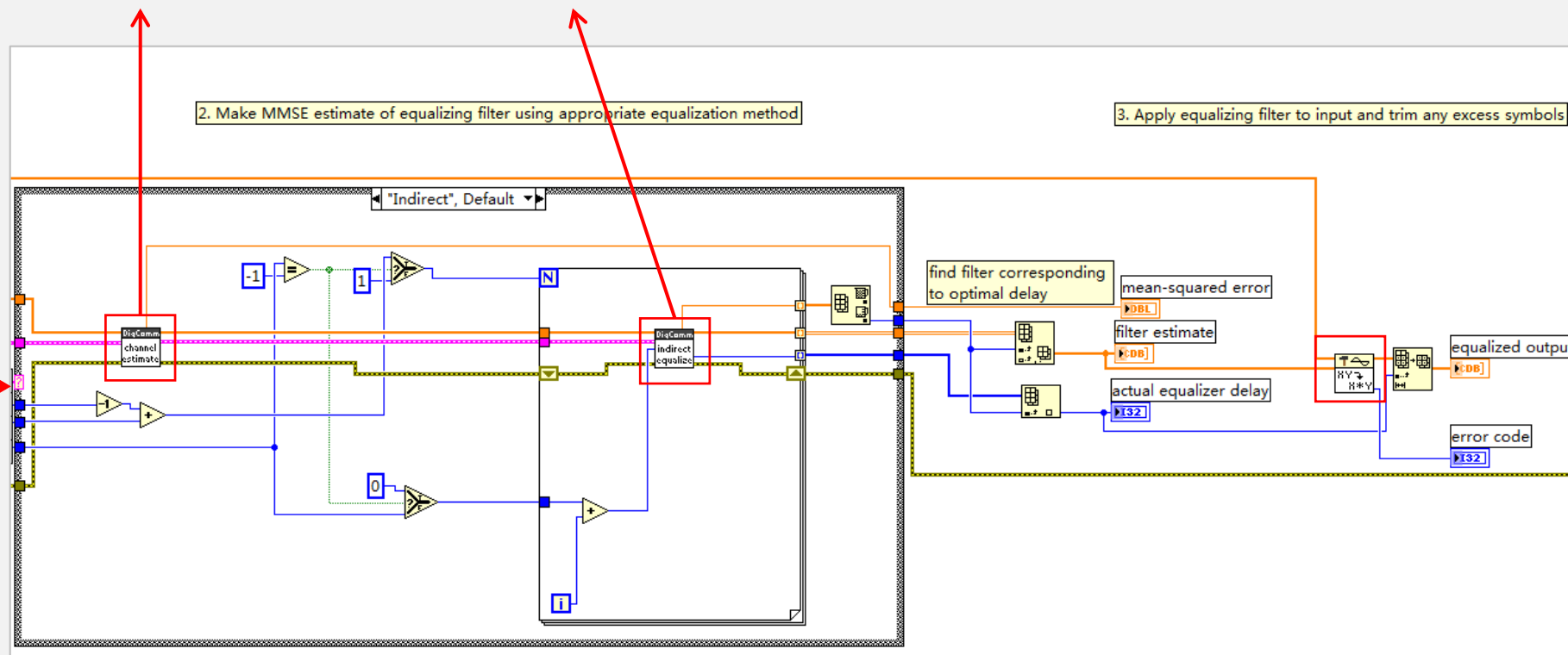
$$\overset{\substack{\uparrow \\ \text{残差}}}{J_f[n_d]} = || \hat{H} \hat{f} - e_{n_d} ||^2$$



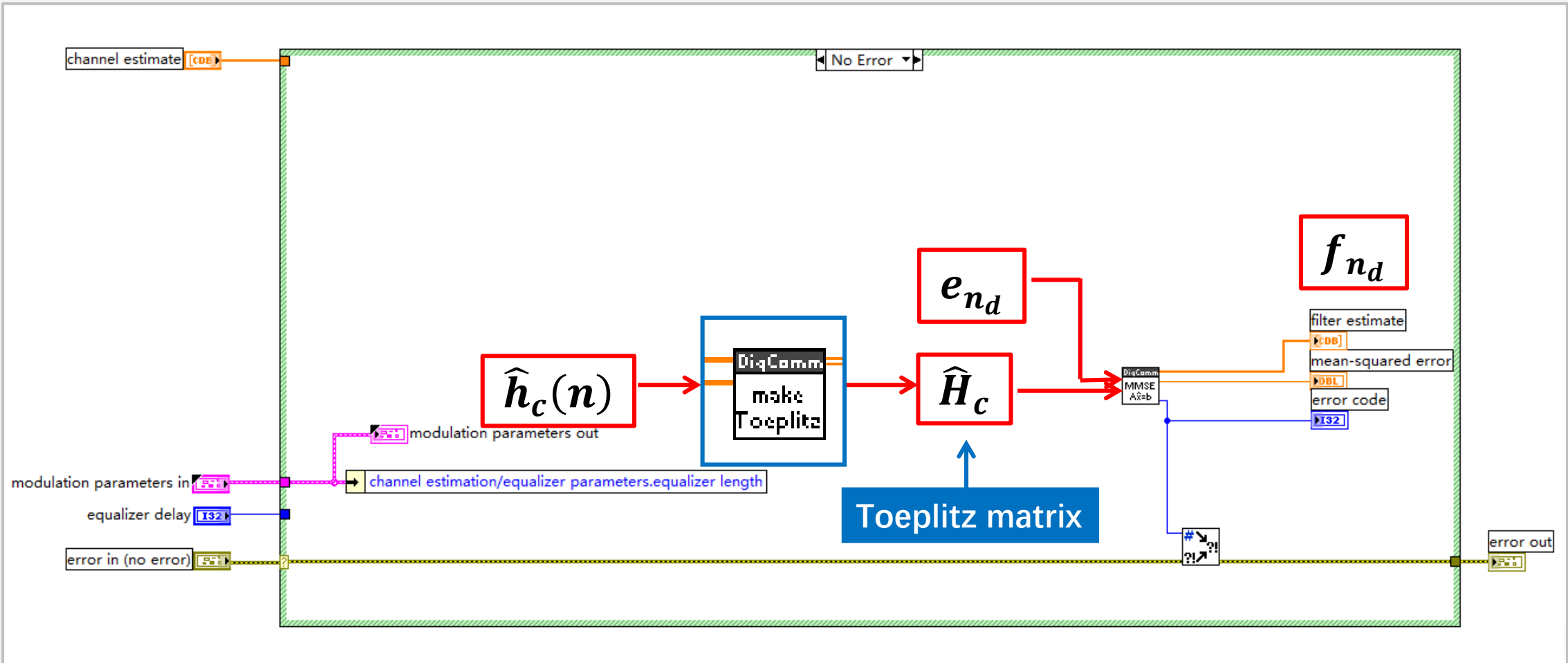


# Programming for Equalization

7、均衡编程



# Indirect\_equalizer.vi





*Channel response:*  $h[0] = 1, h[1] = 0.35e^{j\pi/4}$   
*Sample rate:*  $2M$       *Oversample factor:* 20  
*Noise power:*  $-\text{Inf}$       *Channel est. length:* 4

信号长度

↑

1 -5.39022E-15 i	0 +0 i	0 +0 i	0 +0 i
0.24745 +0.24745 i	1 -5.39022E-15 i	0 +0 i	0 +0 i
-3.51021E-15 +4.06793E-15 i	0.24745 +0.24745 i	1 -5.39022E-15 i	0 +0 i
-8.36831E-15 +1.00614E-16 i	-3.51021E-15 +4.06793E-15 i	0.24745 +0.24745 i	1 -5.39022E-15 i
0 +0 i	-8.36831E-15 +1.00614E-16 i	-3.51021E-15 +4.06793E-15 i	0.24745 +0.24745 i
0 +0 i	0 +0 i	-8.36831E-15 +1.00614E-16 i	-3.51021E-15 +4.06793E-15 i
0 +0 i	0 +0 i	0 +0 i	-8.36831E-15 +1.00614E-16 i

row

1 -5.39022E-15 i	0 +0 i	0 +0 i	0 +0 i
------------------	--------	--------	--------

column

1 -5.39022E-15	0.24745 +0.247	-3.51021E-15 +	-8.36831E-15 +	0 +0 i	0 +0 i	0 +0 i
----------------	----------------	----------------	----------------	--------	--------	--------

Toeplitz matrix



### TRANSMITTER

TX oversample factor TX sample rate  
20 2M

TX channel model parameters

channel model  
ISI ☐

noise power (dB)  
-Inf

channel response  
0 1 + 0 i 0.2474s 0 + 0 i  
frequency offset delay (sec)

0 0

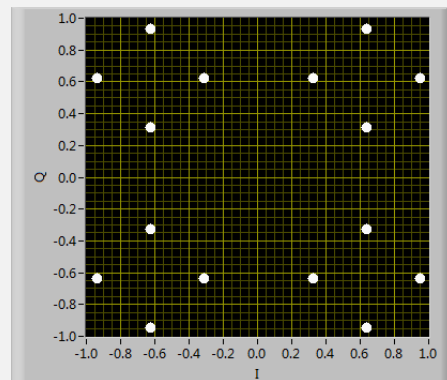
channel estimation/equalizer parameters

Equalization Method  
Indirect

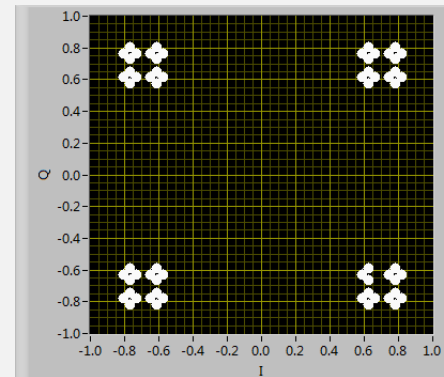
channel estimate length  
2

equalizer length  
1

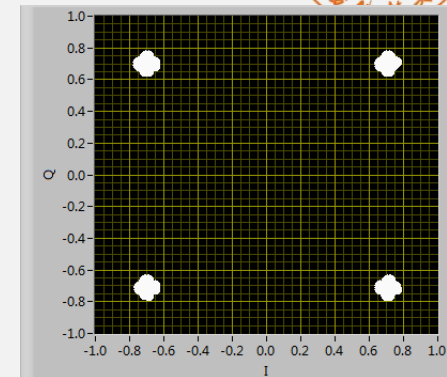
equalizer delay  
-1



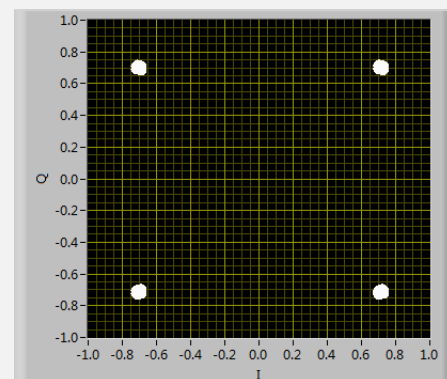
Length=1



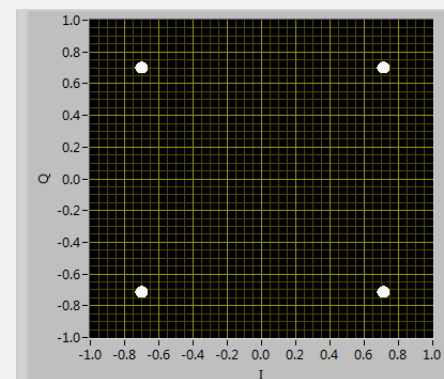
Length=2



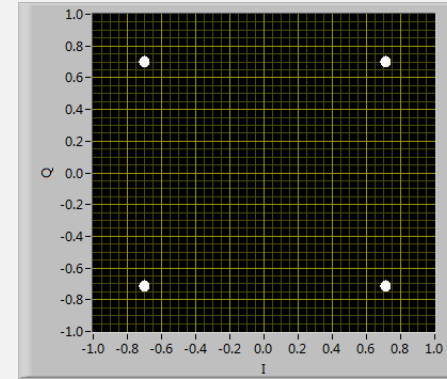
Length=3



Length=4



Length=5



Length=6



## Algorithm Analysis and Verification

1 Channel Estimation

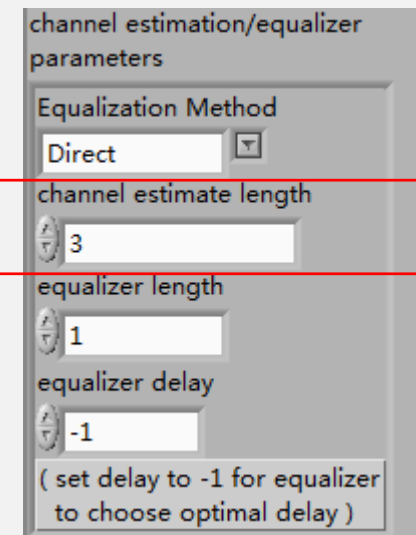
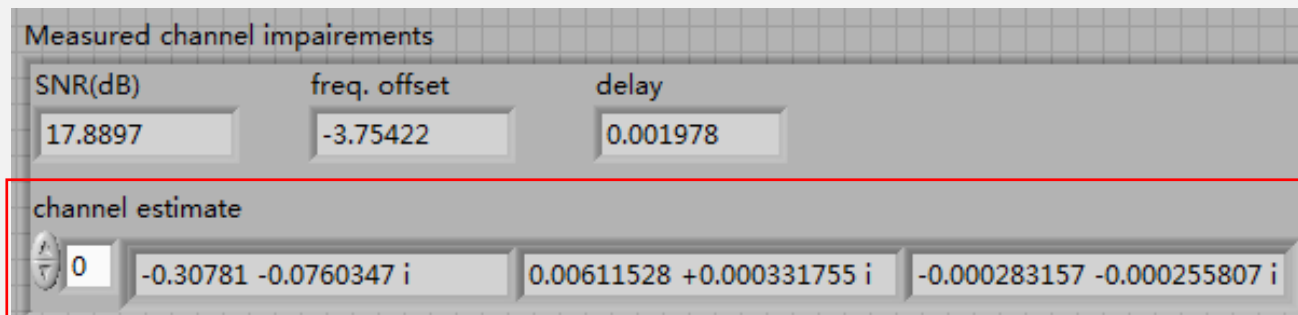
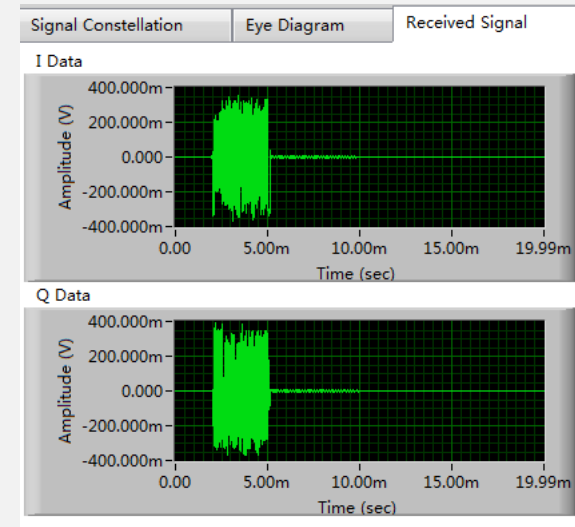
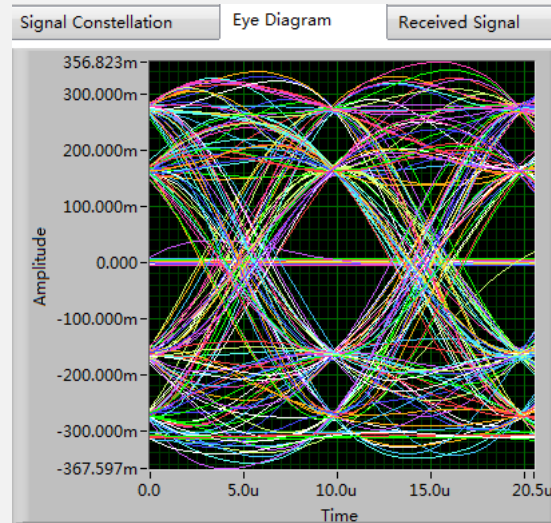
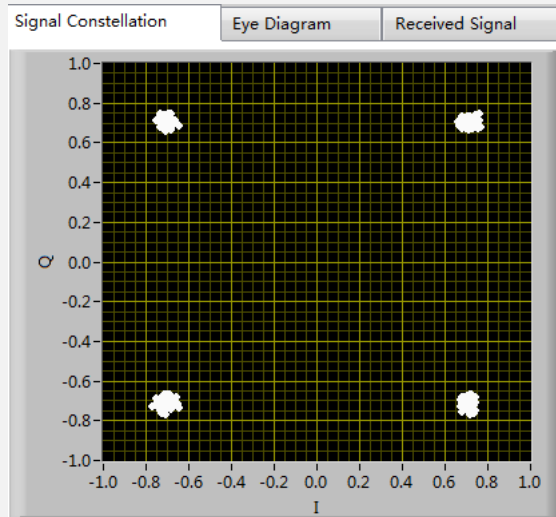
2 Indirect Equalizer Analysis

3 Algorithm Simulation

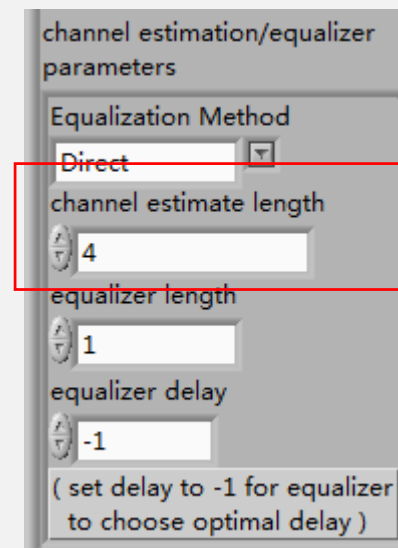
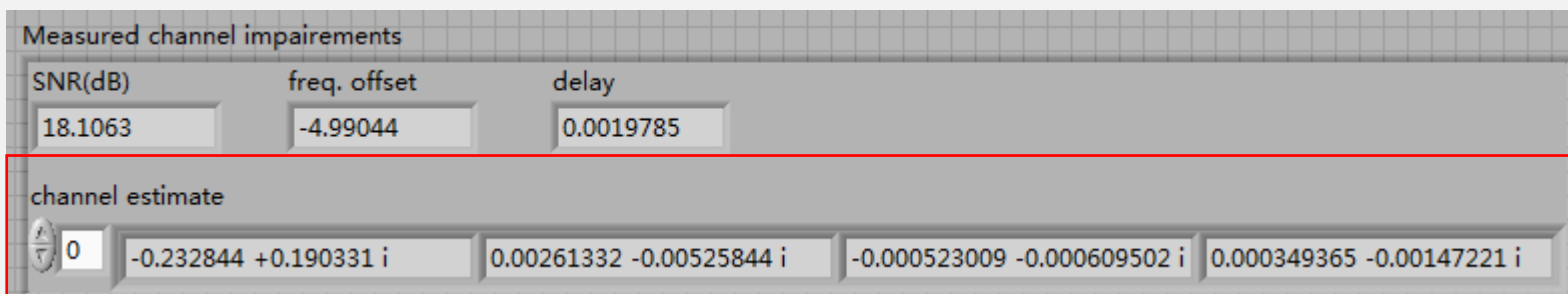
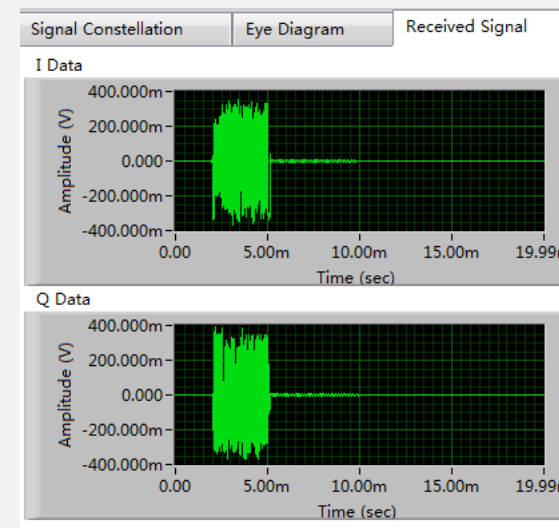
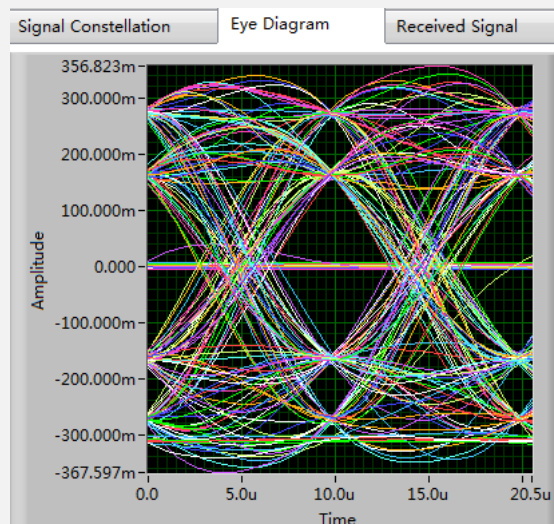
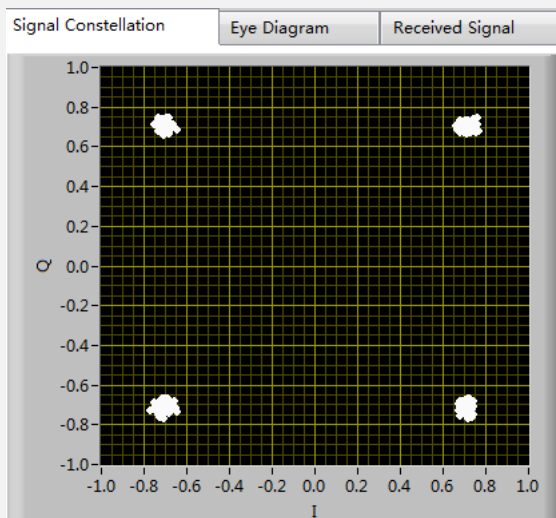
4 USRP Experiments

5 Discussion and Conclusion

# USRP experiment



# USRP experiment





channel estimation/equalizer parameters

Equalization Method  
Indirect

channel estimate length  
3

equalizer length  
1

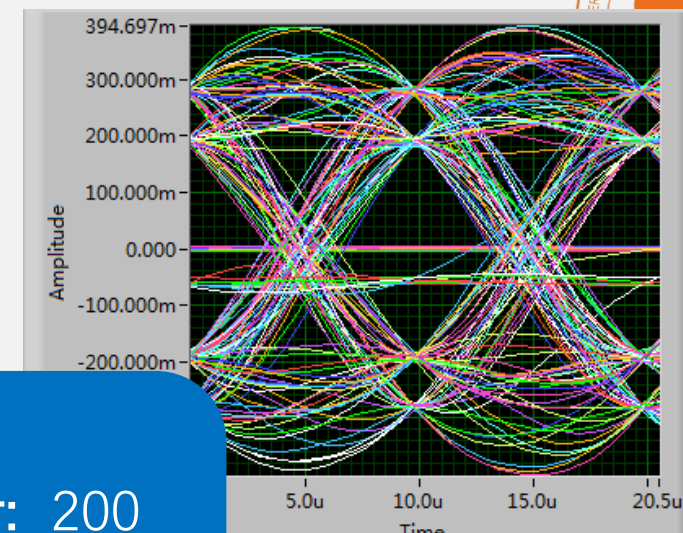
equalizer delay  
-1  
( set delay to -1 for equalizer to choose optimal delay )

Measured channel impairments

SNR(dB)  
25.8855

channel estimate  
0 0.25697 0.00534 -0.0013

freq. offset  
-1.85903



TX Sample rate: 20M  
TX Oversample factor: 200  
RX Sample rate: 2M  
RX Oversample factor: 20  
Symbol rate: 100k

channel estimation/equalizer parameters

Equalization Method  
Indirect

channel estimate length  
3

equalizer length  
6

equalizer delay  
-1  
( set delay to -1 for equalizer to choose optimal delay )

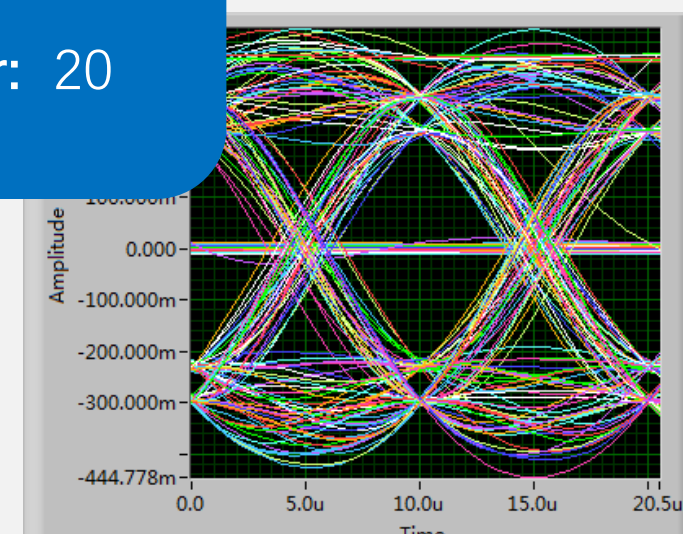
Measured channel

SNR(dB)  
30.0754

channel estimate  
0 0.37046 0.00184 -0.0002

freq. offset  
1.53805

delay  
0.000978



channel estimation/equalizer parameters

Equalization Method  
Indirect ☐

channel estimate length  
3

equalizer length  
1

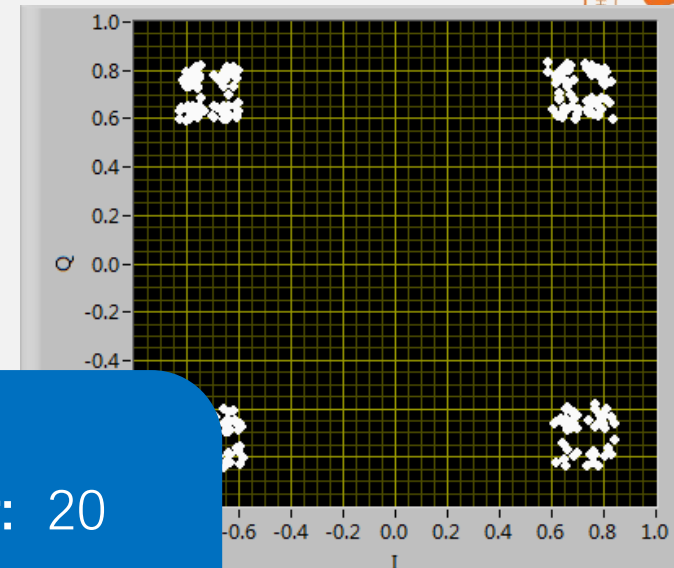
equalizer delay  
-1  
( set delay to -1 for equalizer to choose optimal delay )

Measured channel impairments

SNR(dB)  
14.0143

channel estimate  
0 -0.0718 0.00250 -0.0007

freq. offset  
185.505



TX Sample rate: 20M  
TX Oversample factor: 20  
RX Sample rate: 2M  
RX Oversample factor: 2  
Symbol rate: 1M

channel estimation/equalizer parameters

Equalization Method  
Indirect ☐

channel estimate length  
3

equalizer length  
6

equalizer delay  
-1  
( set delay to -1 for equalizer to choose optimal delay )

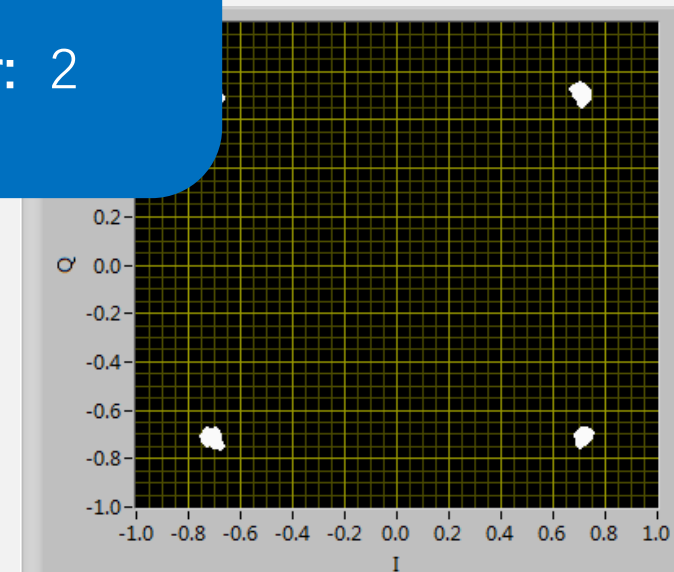
Measured channel impairments

SNR(dB)  
27.1681

channel estimate  
0 -0.1076 -0.0031 0.00195

freq. offset  
-19.3333

delay  
0.002997



channel estimation/equalizer parameters

Equalization Method  
Indirect

channel estimate length  
3

equalizer length  
1

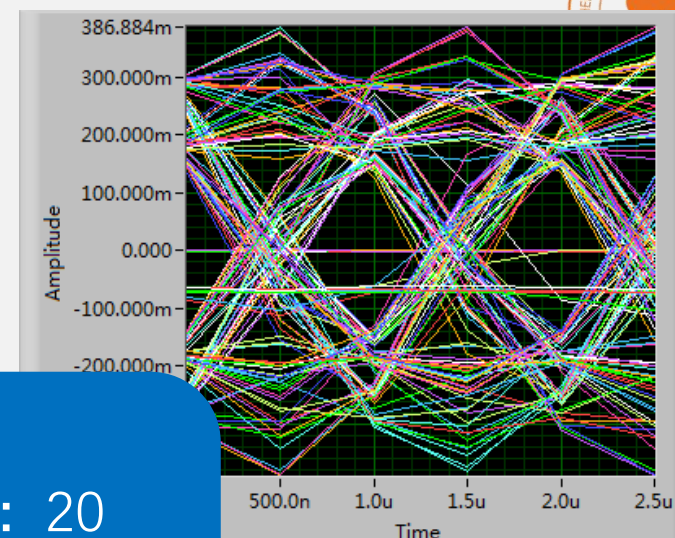
equalizer delay  
-1  
( set delay to -1 for equalizer to choose optimal delay )

Measured channel impairments

SNR(dB)  
14.0143

channel estimate  
0 -0.0718 0.00250 -0.0007

freq. offset  
185.505



TX Sample rate: 20M  
TX Oversample factor: 20  
RX Sample rate: 2M  
RX Oversample factor: 2  
Symbol rate: 1M

channel estimation/equalizer parameters

Equalization Method  
Indirect

channel estimate length  
3

equalizer length  
6

equalizer delay  
-1  
( set delay to -1 for equalizer to choose optimal delay )

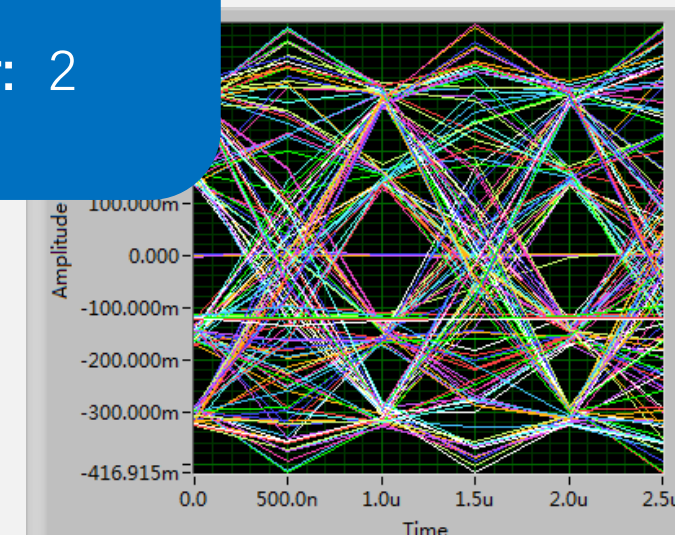
Measured channel

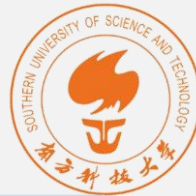
SNR(dB)  
27.1681

channel estimate  
0 -0.1076 -0.0031 0.00195

freq. offset  
-19.3333

delay  
0.002997





## Algorithm Analysis and Verification

1 Channel Estimation

2 Indirect Equalizer Analysis

3 Algorithm Simulation

4 USRP Experiments

5 Discussion and Conclusion



- Question ?

