Lecture 3

Statistical Multipath Channel Models

(Small-Scale Fading)

Review of Lecture 2

- Path loss models simplify Maxwell's equations
- Models vary in complexity and accuracy
- Power falloff with distance is proportional to d^2 in free space, d^4 in two path model
- Main characteristics of path loss captured in simple model $P_r = P_t K[d_0/d]^\gamma$

经验模型

- Empirical models used in simulations
  - Low accuracy (15-20 dB std)
  - Capture phenomena missing from formulas
  - Can be awkward to use in analysis

Review of Lecture 2

- Random attenuation due to shadowing modeled as log-normal (empirical parameters)
- Combined path loss and shadowing leads to outage and amoeba-like cell shapes
- Cellular coverage area dictates the percentage of locations within a cell that are not in outage

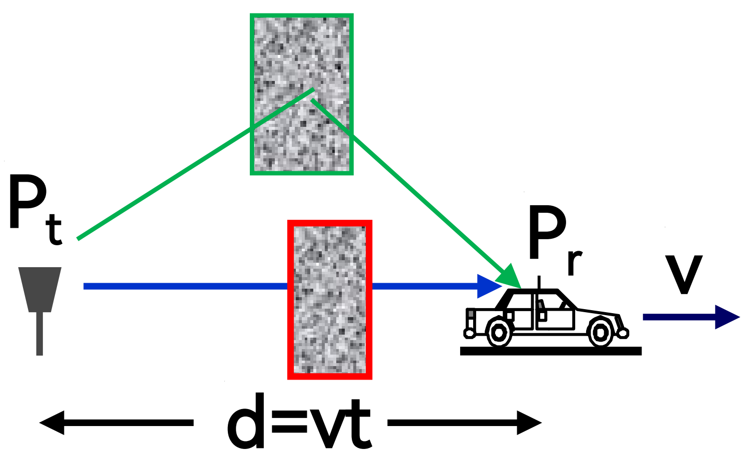
Recall the Propagation Effect

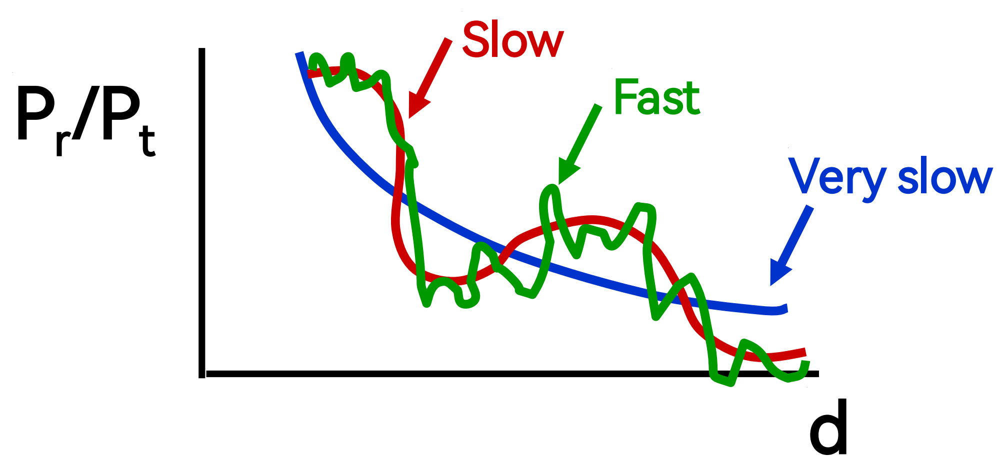
- Path Loss (includes average shadowing)
- Shadowing (due to obstructions)

large-scale propagation effect

- Multipath Fading -- small-scale propagation effect

average power 不足以描述





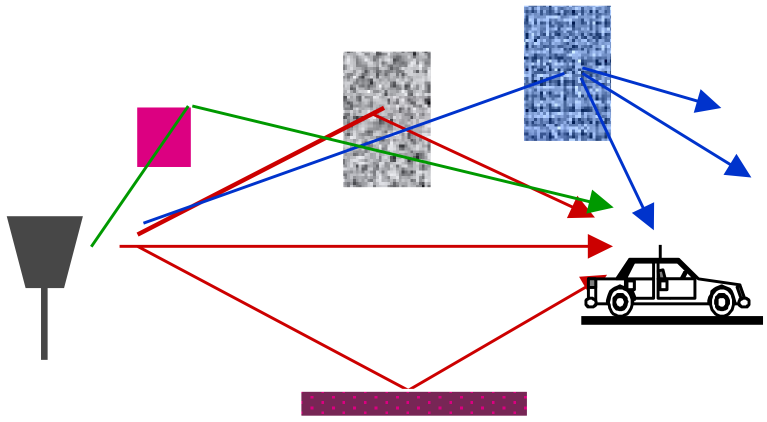
Lecture 3 Outline

- Time Varying Channel Impulse Response
- Narrowband Fading Model
- Flat Fading vs. Frequency Selective Fading
- Effect of Flat and Frequency Selective Fading
- Signal Envelope Distributions (Rayleigh and Rician)
- Average Fade Duration

Lecture 3 Outline

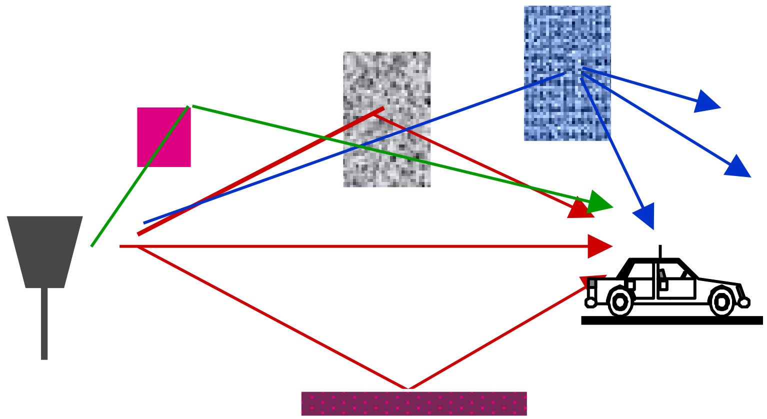
- Time Varying Nature of Channel Fading
- Scattering Function
- Multipath Intensity Profile
- Doppler Power Spectrum

Multipath Fading



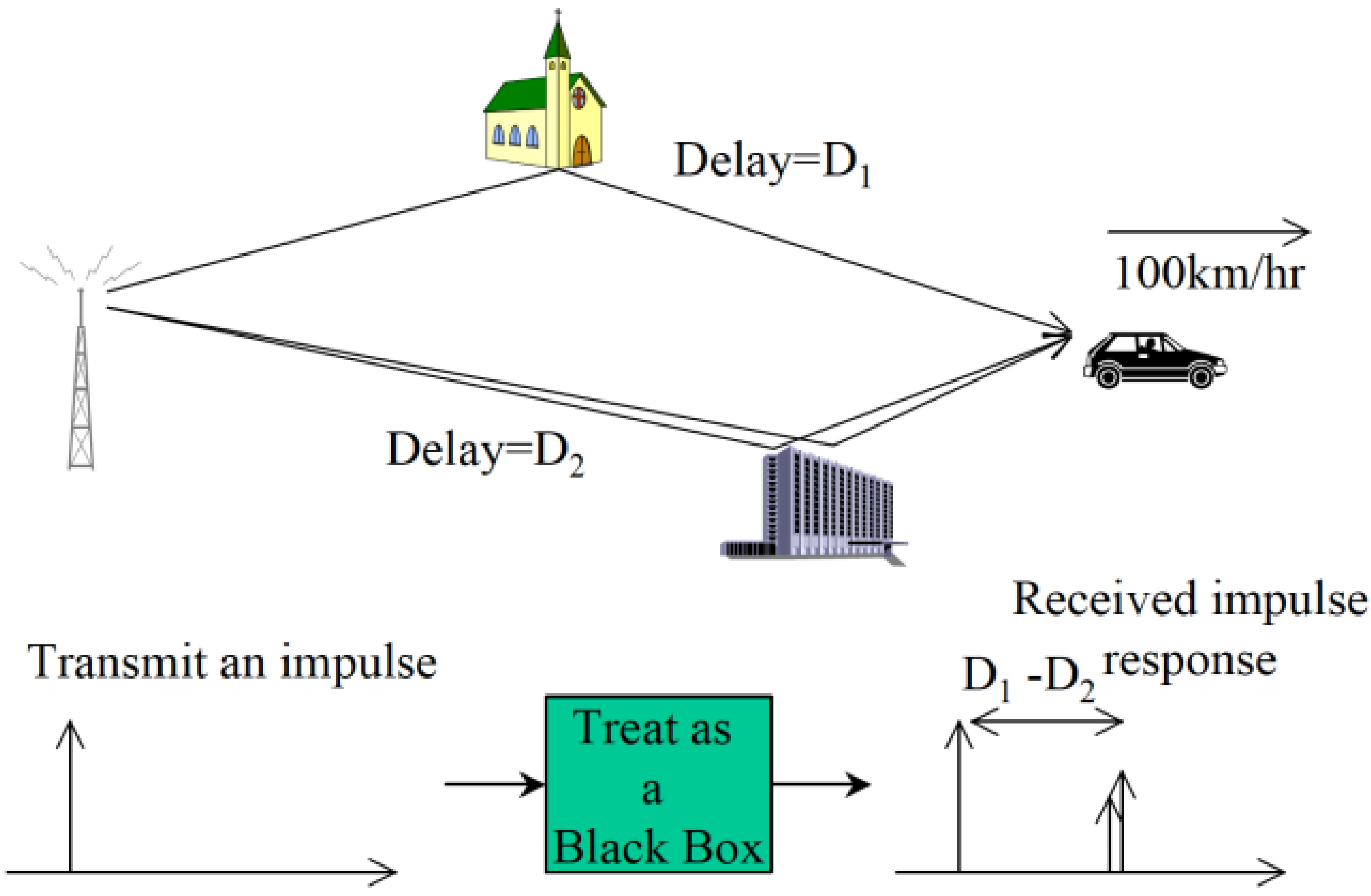
- Results from
  - Interference between multiple paths with different path lengths
  - Movement of the mobile or environment makes this effect time varying
  - Rate of variation of the channel characteristics is measured by Doppler Frequency

Multipath Fading



- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse

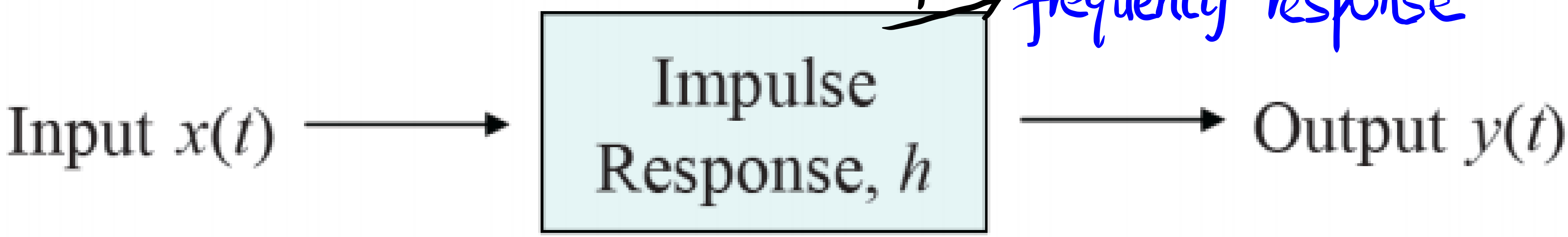
Physics of Multipath Fading



Review of Impulse Response of a Linear System

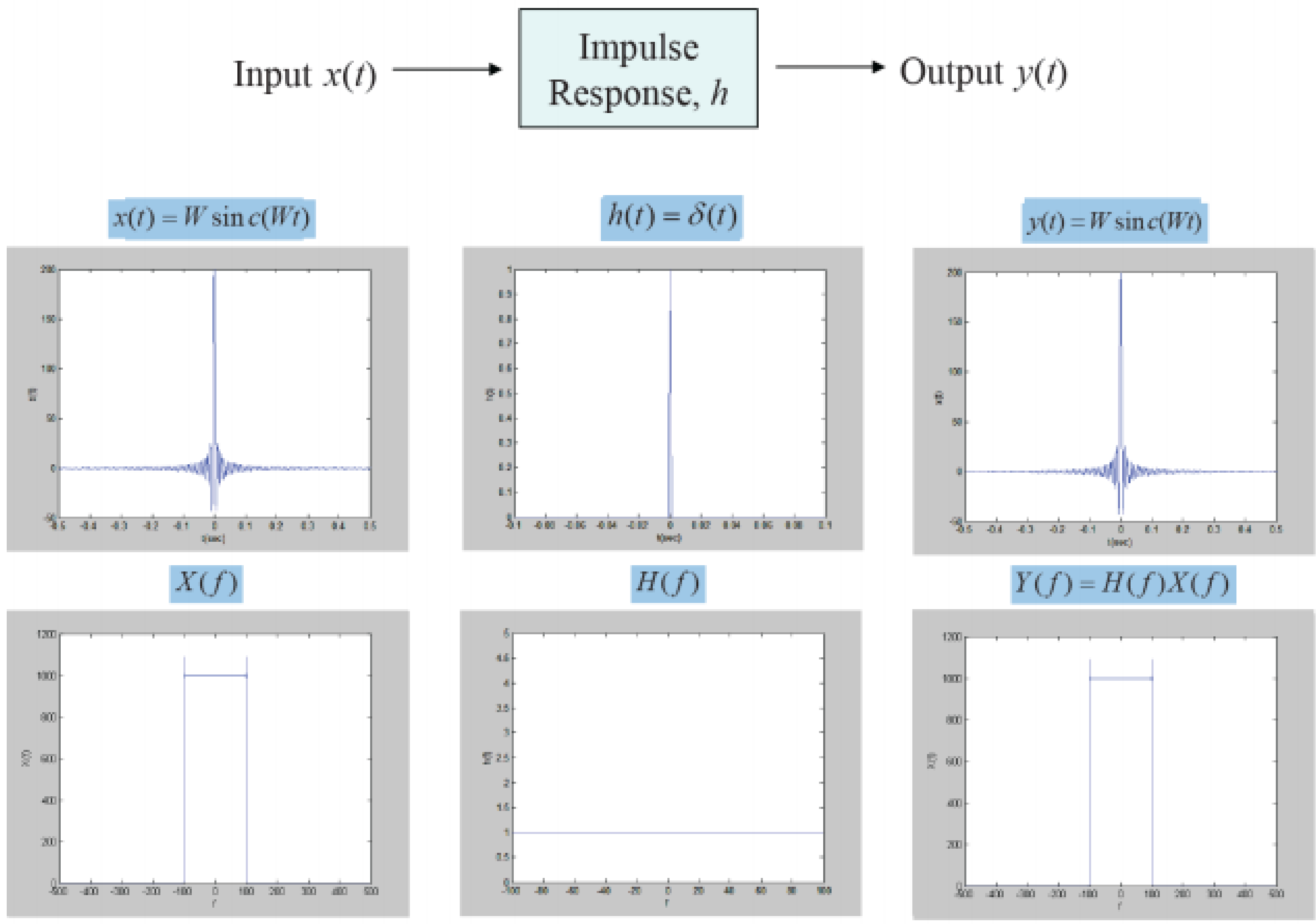
LTI

频率响应 frequency response



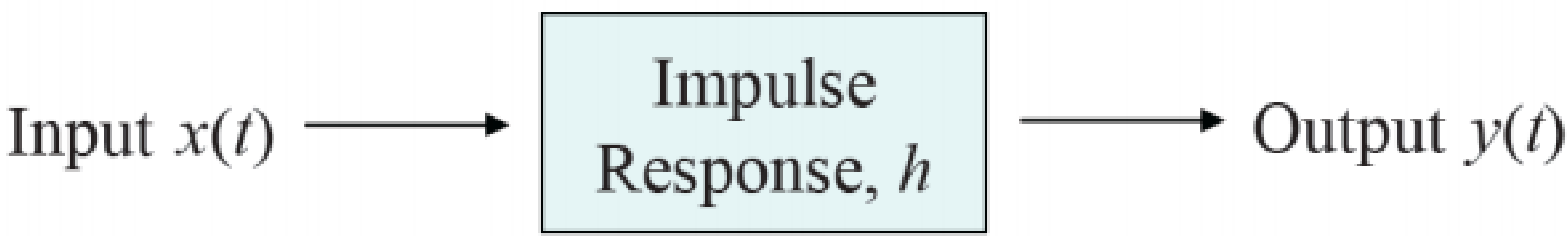
$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau = x(t) * h(t)$$

Example



Input $x(t)$
Impulse Response, h
Output $y(t)$
$x(t) = W \operatorname{sinc}(Wt)$
$h(t) = \delta(t)$
$y(t) = W \operatorname{sinc}(Wt)$
$X(f)$
$H(f)$
$Y(f) = H(f)X(f)$

Time-Varying Linear System

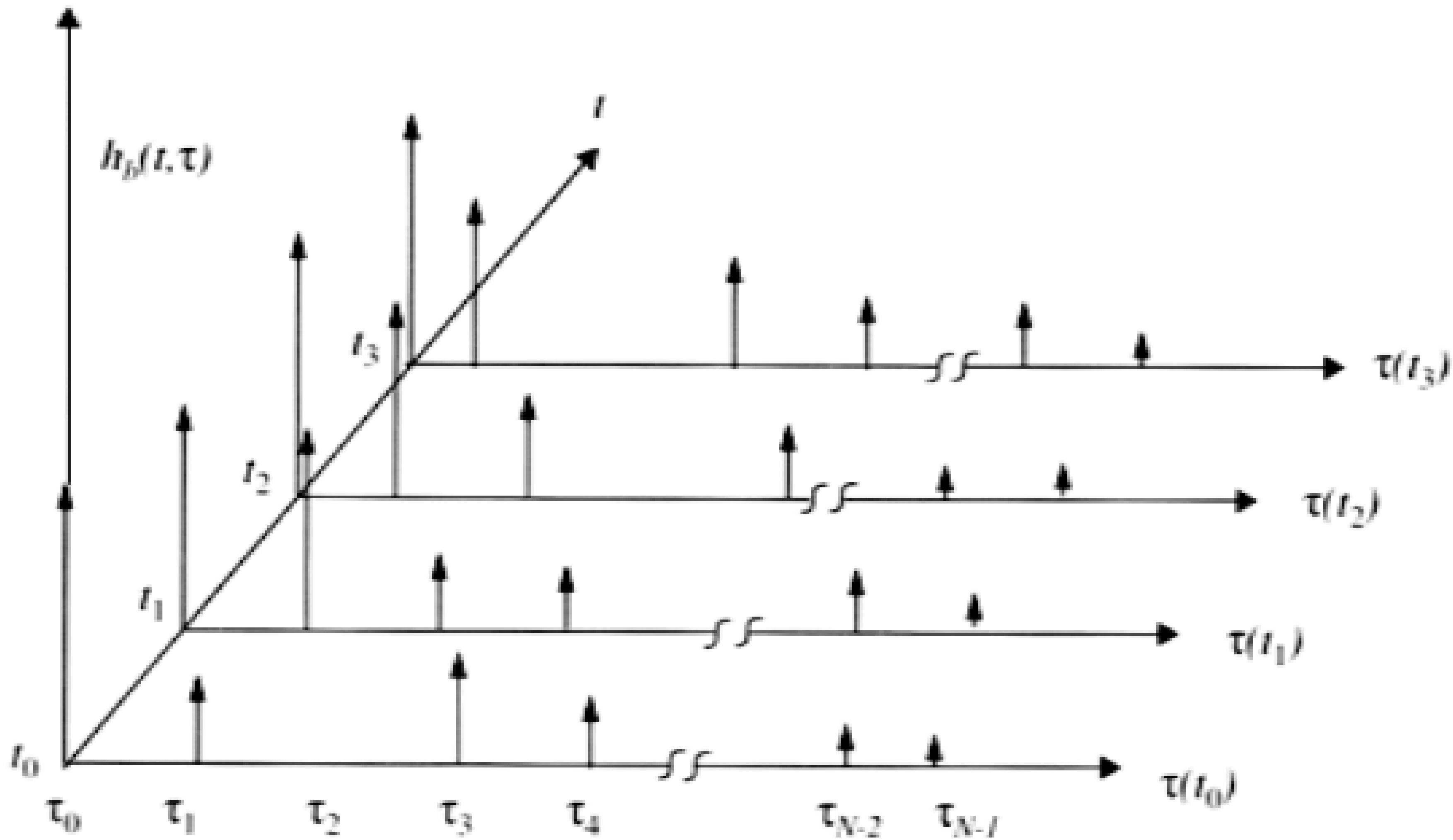


$$y(t) = \int_{-\infty}^{\infty} h(t,\tau)x(t-\tau)\,d\tau$$

t代表 instant

τ代表 delay

Time-Varying Multipath Channel



Time Varying Impulse Response

- Response of channel at time t to impulse input at time t−τ:

记

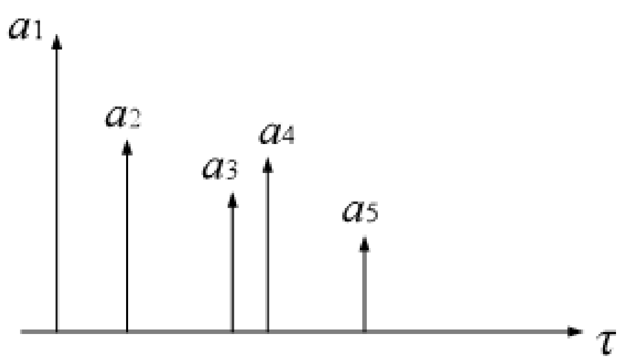
delay

多普勒效应造成的频移

$$h(t,\tau) = c(\tau, t) = \sum_{n=1}^{N(t)} a_n(t)\, e^{-j\phi_n(t)}\, \delta(\tau - \tau_n(t))$$

$$\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{Dn} - \phi_0 \rightarrow \text{相移}$$

  - $c(\tau, t)$: impulse response of the channel at time t to the impulse input at time t−τ
  - $a_n(t)$: random amplitude
  - $\varphi_n(t)$: random phase
  - $\tau_n(t)$: random delay



let $s(t)=s_I(t)\cdot\cos(2\pi f_c t)-s_Q(t)\sin(2\pi f_c t) \Rightarrow u(t)=s_I(t)+js_Q(t)$

bandpass　　　　　　　　　　　　　　　　baseband

假设t时刻 $N(t)$ paths

$\Longrightarrow$ n-th path: delay $\tau_n(t)$, ϕ_{D_n}

$\alpha_n(t)$随t变化而很慢，所以$\alpha_n(t)$近似不变

The received signal of the n-th path

bandpass: $r_n(t)=\alpha_n(t)\{s_I[t-\tau_n(t)]\cos[2\pi f_c[t-\tau_n(t)]+\phi_{D_n}]-s_Q[t-\tau_n(t)]\sin[2\pi f_c[t-\tau_n(t)]+\phi_{D_n}]\}$

$u_n(t)=\alpha_n(t)\,u[t-\tau_n(t)]\,e^{j[-2\pi f_c\tau_n(t)+\phi_{D_n}]} \longrightarrow u(t)$

$r_n(t)=\mathrm{Re}\{u_n(t)e^{j2\pi f_c t}\}$

为什么是$\delta[\tau-\tau_n(t)]$而不是$\delta[t-\tau_n(t)]$

Impluse Response of nth path: $c_n(\tau,t)=\delta[\tau-\tau_n(t)]\cdot\alpha_n(t)e^{j[-2\pi f_c\tau_n(t)+\phi_{D_n}]}$

Total Impluse Response: $\sum_{n=1}^{N(t)} c_n(\tau,t)=\sum_{n=1}^{N(t)}\alpha_n(t)e^{j[-2\pi f_c\tau_n(t)+\phi_{D_n}]}\delta[\tau-\tau_n(t)]$

Total Rx Signal: $U_r(t) = \int_{-\infty}^{+\infty} c(\tau, t)\, u(t-\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} \sum_{n=1}^{N(t)} \alpha_n(t)\, e^{j[-2\pi f_c \tau_n(t) + \phi_{D_n}]}\, \delta[\tau - \tau_n(t)]\, u(t-\tau)\, d\tau$$

?

$$= \sum_{n=1}^{N(t)} \alpha_n(t)\, e^{j[-2\pi f_c \tau_n(t) + \phi_{D_n}]} \int_{-\infty}^{\infty} \delta[\tau - \tau_n(t)]\, u(t-\tau)\, d\tau$$

$\rightarrow u[t - \tau_n(t)]$

$$= \sum_{n=1}^{N(t)} \alpha_n(t)\, e^{j[-2\pi f_c \tau_n(t) + \phi_{D_n}]}\, u[t - \tau_n(t)]$$

Received Signal Characteristics

- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly $\varphi_n(t) = 2\pi f_c \tau_n(t) - \varphi_{D_n} - \varphi_0$
  - Constructive and destructive addition of signal components
  - Amplitude fading of received signal (both wideband and narrowband signals)

Narrowband Model

当该条件满足时，称为narrowband

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)|<<1/B$

$\tau_n(t)\approx\tau_0$

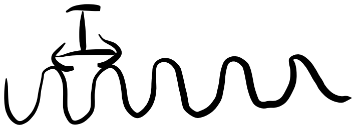
- Then $u(t)\approx u(t-\tau)$.
- Received signal given by

$$r(t)=\mathrm{R}\left\{u(t)e^{j2\pi f_c t}\left[\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\right]\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.

Info  bandwidth $B \approx \frac{1}{T_s}$ → 记

x

carrier  $T_s \gg T$

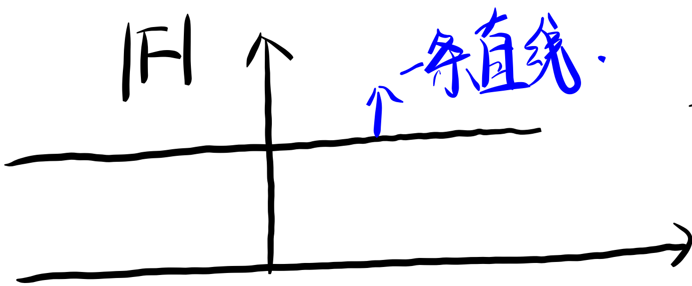
$|\alpha^2(t)|$ = channel power gain

$$u_r(t) \approx u(t-\tau_0)\sum_{n=1}^{N(t)} \alpha_n(t)\, e^{j[2\pi f_c \tau_n(t)+\phi_{D_n}]}$$

$$c(\tau,t) \approx \delta(\tau-\tau_0)\sum_{n=1}^{N(t)} \underbrace{\alpha_n(t)\, e^{j[-2\pi f_c \tau_n(t)+\phi_{D_n}]}}_{\alpha(t)} = \alpha(t)\,\delta(\tau-\tau_0)$$

↑ channel gain

$\xleftrightarrow{F}$ (对τ) $\left|e^{-j2\pi f\tau_0}\sum_{n=1}^{N(t)} - - - - - - -\right|, f$



flat fading ⟺ narrowband

the channel amplitude gain is constant 在某t时刻

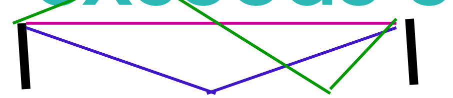
in frequency domain

$$\alpha(t)\,\delta(\tau-\tau_0) \xleftrightarrow[\tau]{F} e^{-j2\pi f\tau_0}\cdot\alpha(t)$$

$$\left|e^{-j2\pi f\tau_0}\cdot\alpha(t)\right| = |\alpha(t)|$$

Wideband Channels

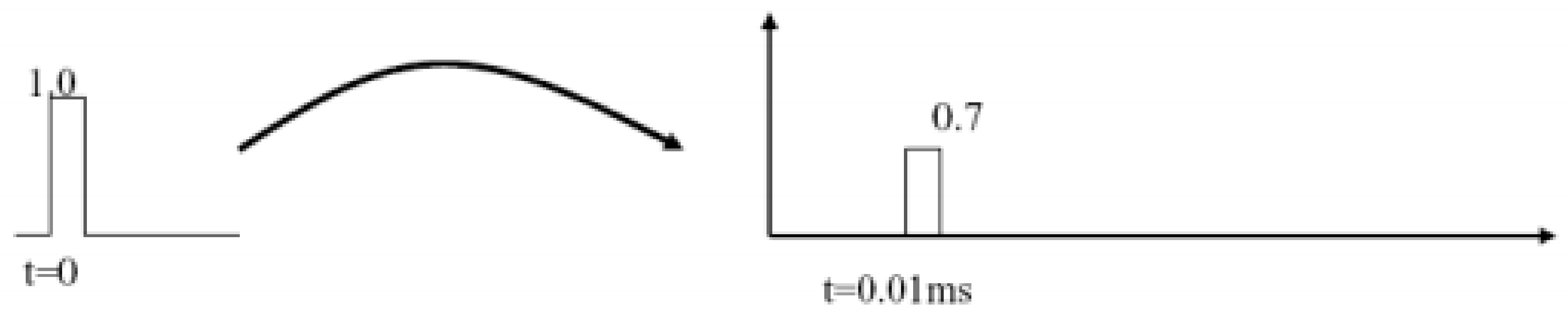
- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth



Flat Fading vs. Frequency Selective Fading

发出的信号在各个频段功率相等，那么收到的信号在各个频段功率也相等

- Flat fading: $y(t) = \alpha s(t-\tau) + n(t)$
  - N(t)=1, i.e. no delay spread
  - Channel impulse response is one single impulse
  - The frequency response is a constant (over the communication bandwidth)
  - Usually occurs for narrowband communication, i.e., low symbol rates



Channel Gain in Narrowband.

$$\alpha(t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} = \underbrace{\underbrace{\sum_{n=1}^{N(t)} \alpha_n(t)\cos[\phi_n(t)]}_{N(0,\frac{\sigma^2}{2})} - j \underbrace{\sum_{n=1}^{N(t)} \alpha_n(t)\sin[\phi_n(t)]}_{N(0,\frac{\sigma^2}{2})}}_{CN(0,\sigma^2)}$$

$N(0,\frac{\sigma^2}{2}) \xleftrightarrow{\text{independent}} N(0,\frac{\sigma^2}{2})$ (高斯随机变量)

$CN(0,\sigma^2)$ → 复高斯变量

CLT + NLOS → none LOS

CLT → 中心极限定理

相位

$\angle \alpha(t)$: uniform distribution $[0, 2\pi)$

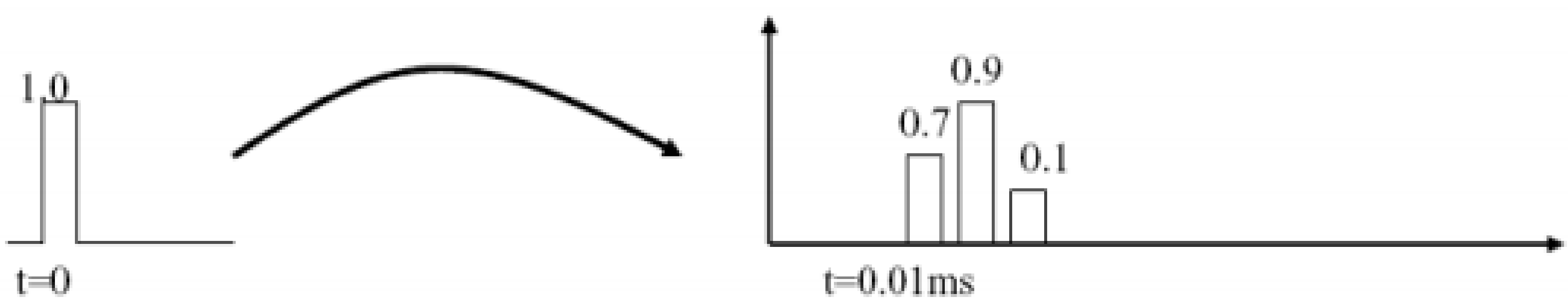
$|\alpha(t)|$: Rayleigh distribution

} Rayleigh channel.

$|\alpha(t)|^2$: Exponential distribution.

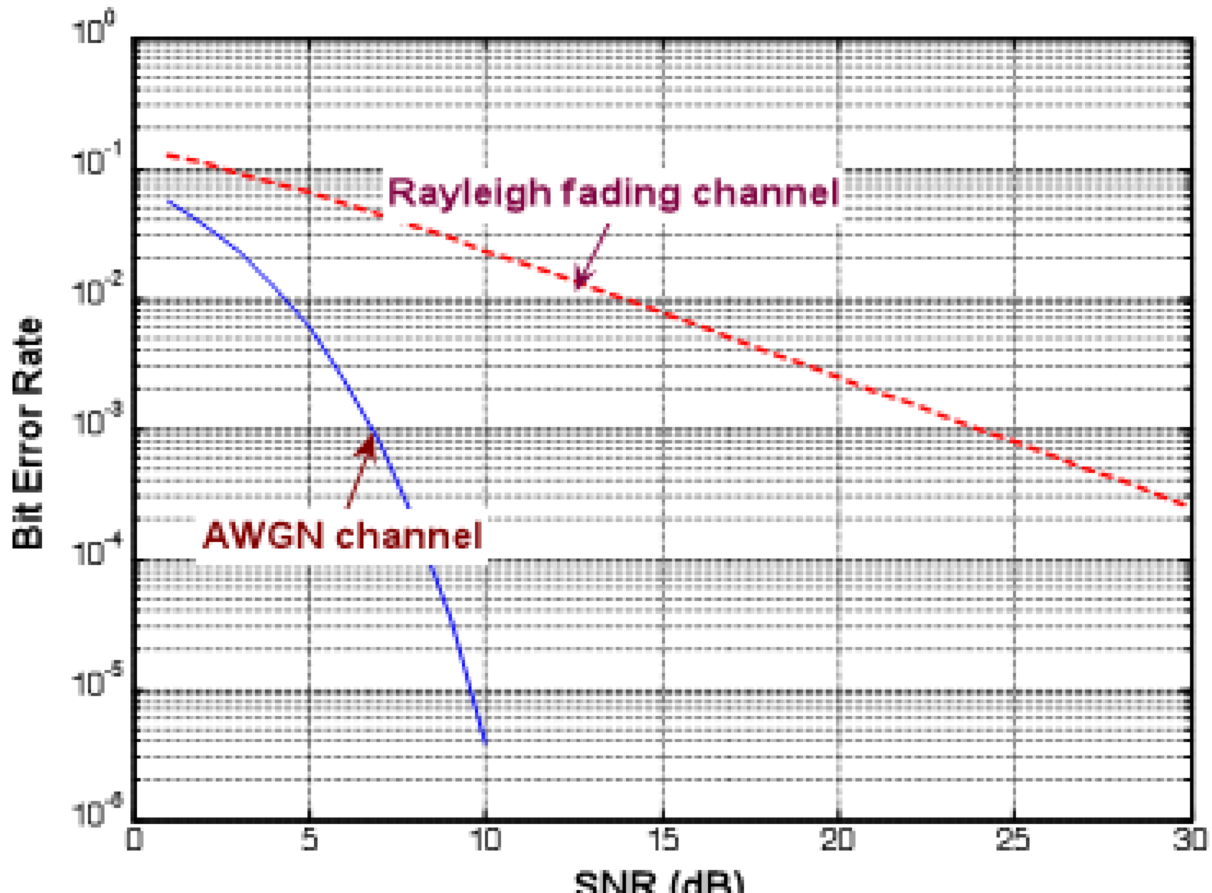
Flat Fading vs. Frequency Selective Fading

- Frequency selective fading:
  - N(t)>1 i.e. with delay spread
  - Channel impulse response consists of multiple impulses
  - The frequency response varies over the communication bandwidth
  - Usually occurs for wideband communication, i.e., high symbol rates



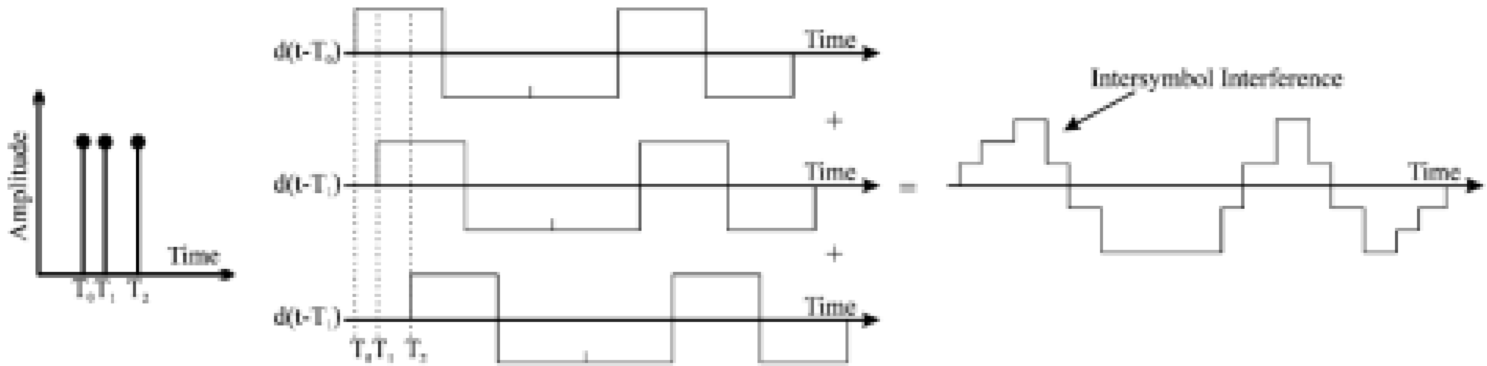
Effect of Flat Fading

- Flatten BER curves
- Around 15dB penalty when BER is 10^{-3}

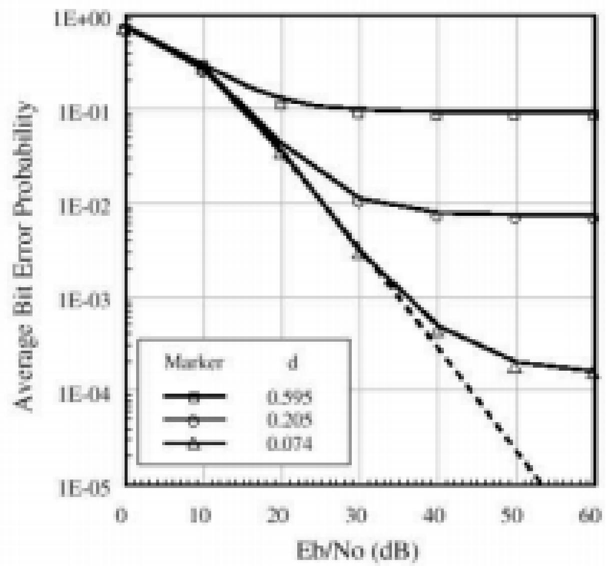


Effect of Frequency Selective Fading

- Multipath → Inter-symbol interference (ISI)



- Result in irreducible error floor



Signal Envelope Distribution for Narrow Band Model

个中心极限定理

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models "worse than Rayleigh"
  - Lends itself better to closed form BER expressions

Rayleigh Channel

$f_{\alpha_I,\alpha_Q}(\underset{\sim}{\alpha_I,\alpha_Q})$: Independent Gaussian with variance σ^2

$f_\Phi(\varphi) = \dfrac{1}{2\pi}$ if $\varphi \in [0, 2\pi)$: Uniform Phase

$f_R(r) = \dfrac{r}{\sigma^2}\exp(-\dfrac{r^2}{2\sigma^2})$ if $r > 0$: Rayleigh Amplitude

$f_P(p) = \dfrac{1}{P_o}\exp(-\dfrac{p}{P_o})$ if $p > 0$: Exponential Channel Power Gain

where $p = r^2$ and $P_o = 2\sigma^2$ is mean channel power gain

Note: $a = a_I + j a_Q$

Rician Channel

LOS channels:

$$p_Z(z) = \frac{z}{\sigma^2} \exp\left[\frac{-(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad z \geq 0,$$

记

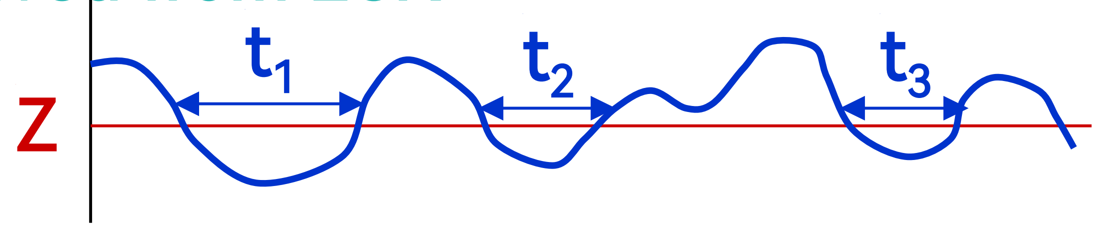
$$\bar{P}_r = \int_0^\infty z^2 p_Z(z)\, dz = \underset{\text{(LOS)}}{s^2} + \underset{\text{(non-LOS)}}{2\sigma^2} = \alpha_0^2 + 2\sigma^2$$

$$p_Z(z) = \frac{2z(K+1)}{\bar{P}_r} \exp\left[-K - \frac{(K+1)z^2}{\bar{P}_r}\right] I_0\left(2z\sqrt{\frac{K(K+1)}{\bar{P}_r}}\right)$$

$$K = \frac{s^2}{2\sigma^2} = \frac{S^2}{2\sigma^2}$$

Level Crossing Rate and Average Fade Duration 了解.

- LCR: rate at which the signal crosses a fade value Z in the downward direction
  - $L_Z = \sqrt{2\pi} f_D \rho e^{-\rho^2}$ where $\rho = Z/\sqrt{\bar{P}_r}$
- AFD: How long a signal stays below target Z
  - Derived from LCR



- For Rayleigh fading

$$\bar{t}_Z = (e^{\rho^2} - 1) / (\rho f_D \sqrt{2\pi})$$

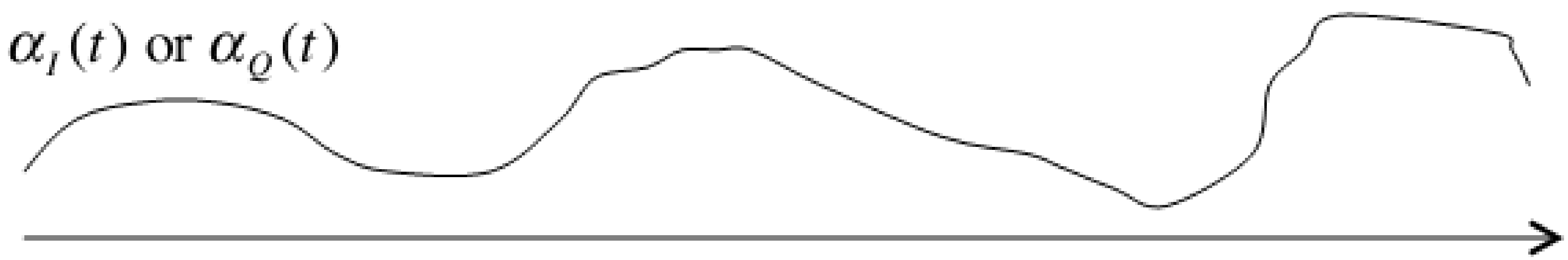
  - Depends on ratio of target to average level (ρ)
  - Inversely proportional to Doppler frequency

Main Points

- Time Varying Channel Impulse Response
- Narrowband Fading Model
- Flat Fading vs. Frequency Selective Fading
- Effect of Flat and Frequency Selective Fading
- Signal Envelope Distributions (Rayleigh and Rician)
- Average Fade Duration

Time Varying Nature of Wireless Channel

- Cause for time-varying nature
  - movements of mobile or objects in the environment



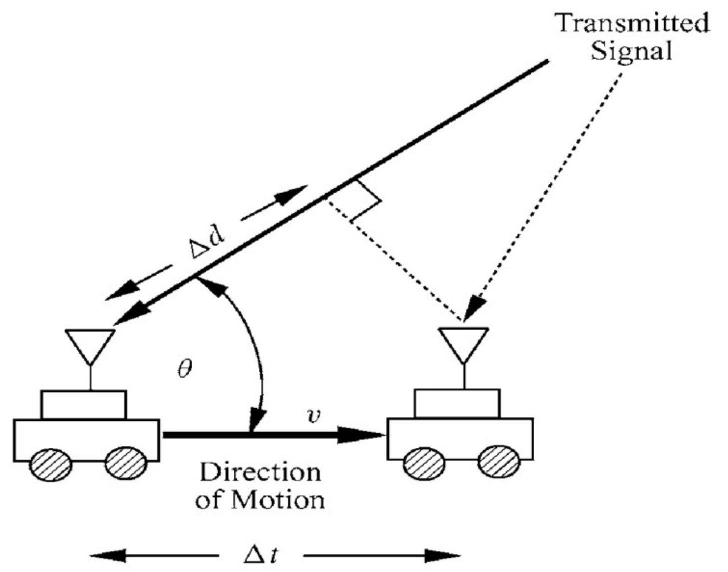
- On each path, $\alpha_n(t)$ is a random process
- This random process is correlated in time
- The faster the autocorrelation function decays with the time difference, the faster the channel varies
- To measure how fast the channel varies: Doppler spread and coherence time.

Time-Varying Linear System 特有的一种现象.

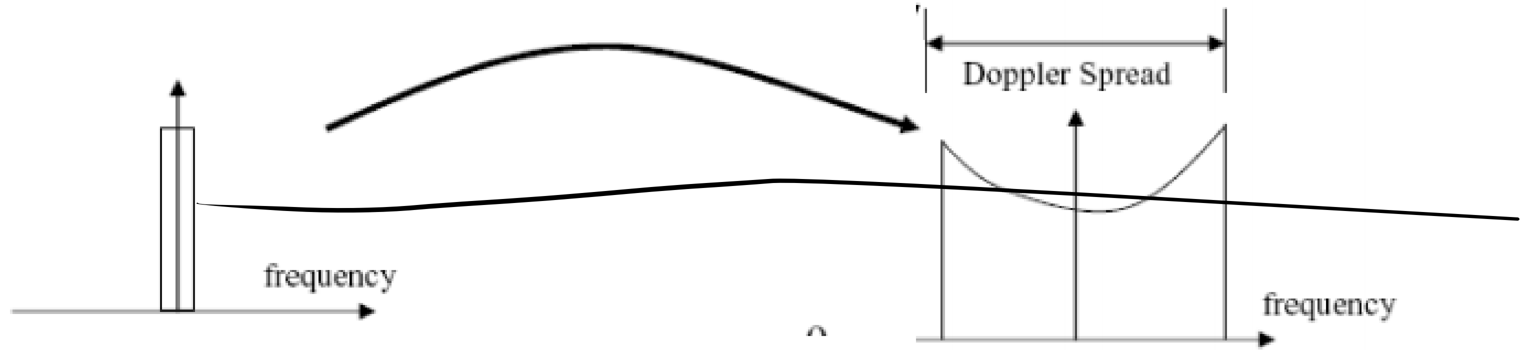
Doppler Shift

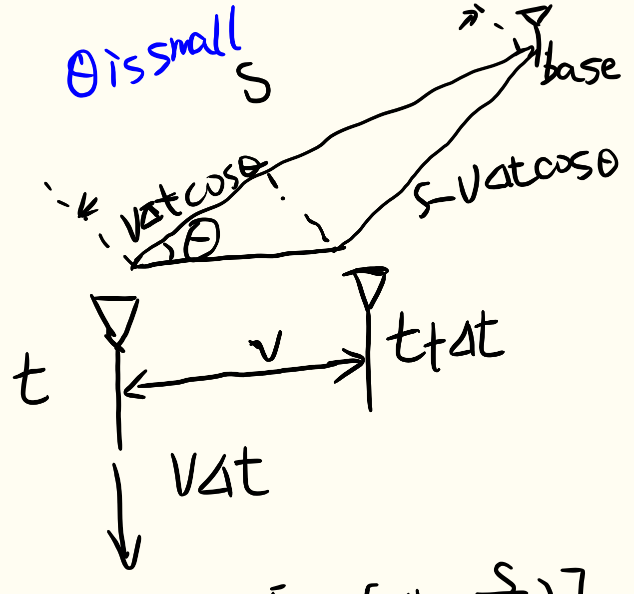
- Doppler frequency shift

$$f_d = \frac{v}{\lambda}\cos\theta$$



- Doppler spread: the maximum Doppler shift





$\cos 2\pi f_c t$

$r(t) = \cos[2\pi f_c(t - \frac{S}{c})]$

t+Δt 时刻:

发射信号 $s(t+\Delta t) = \cos[2\pi f_c(t+\Delta t)]$

接收信号 $r(t+\Delta t) = \cos[2\pi f_c(t+\Delta t - \frac{S-v\Delta t\cos\theta}{c})]$

phase difference $= 2\pi f_c \Delta t + 2\pi f_c \frac{v\Delta t\cos\theta}{c}$

frequency of receiver : $f_r = f_c + f_c\frac{v\cos\theta}{c}$ (Hz)

$= f_c + \underbrace{\boxed{\frac{v\cos\theta}{\lambda}}}_{f_d}$ called Doppler { shift / freq

△ channel Inpluse Response. Doppler 造成的

$$c(\tau,t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{j[-2\pi f_c \tau_n(t) + \phi_{D_n}]}\delta[\tau - \tau_n(t)] \qquad \phi_{D_n} = 2\pi f_{D_n}(t)\,t$$

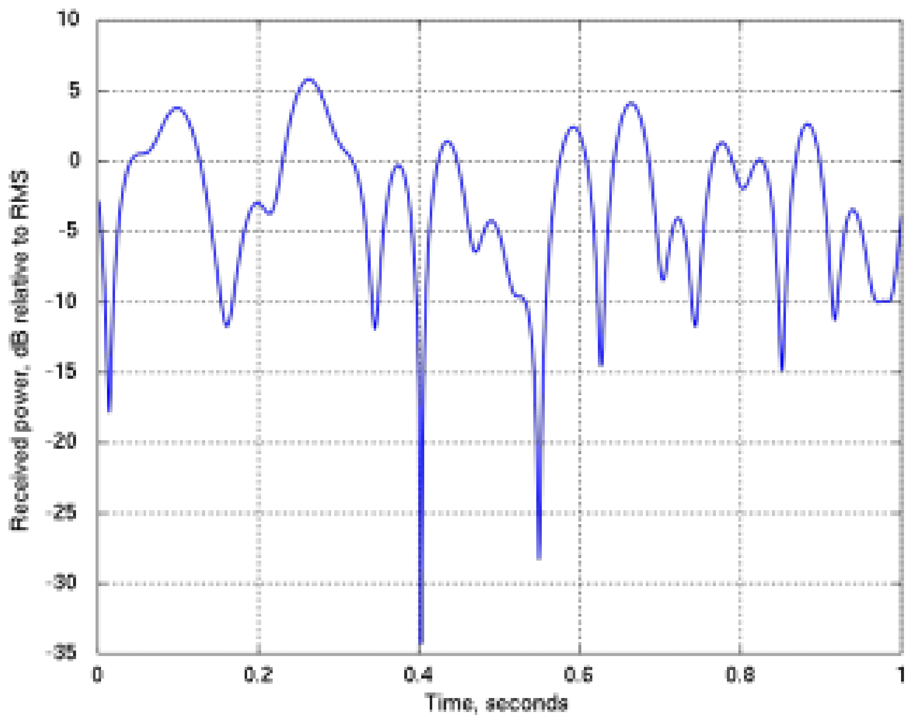
$$= \sum_{n=1}^{N(t)} \alpha_n(t) e^{j[-2\pi f_c \tau_n(t) + 2\pi f_{D_n}(t)\,t]}\delta[\tau - \tau_n(t)]$$

→ { time-varying, 画线部分与t有关 / change fast

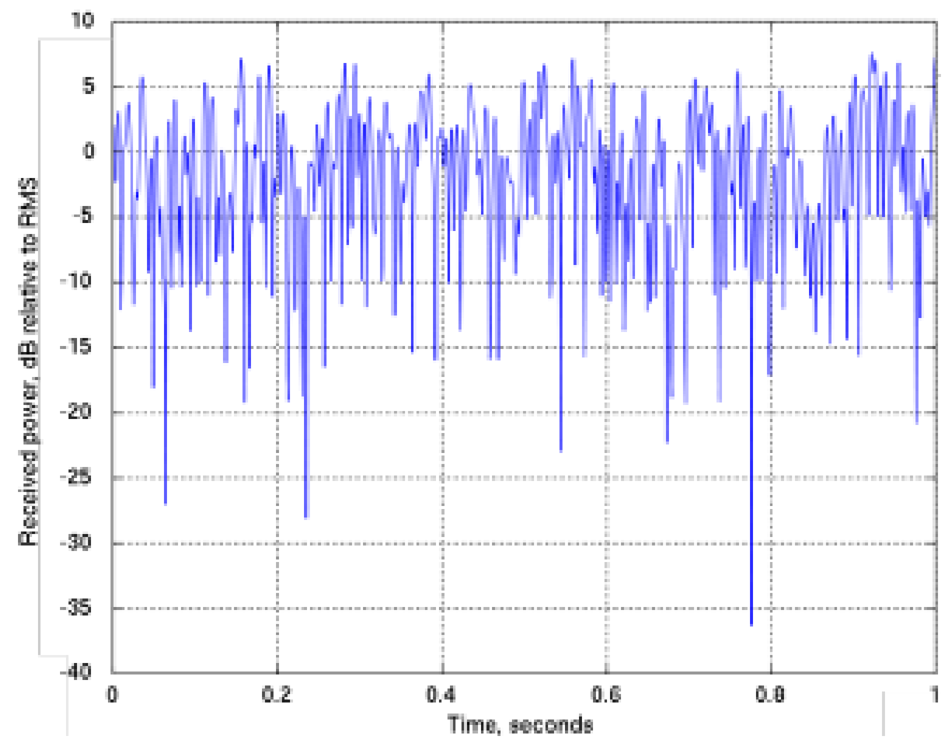
$\alpha_n(t), \tau_n(t), f_{D_n}(t)$ change slowly with t.

but △ makes $c(\tau,t)$ change fast.

Doppler Shift



One second of Rayleigh fading with a Doppler spread of 10Hz



One second of Rayleigh fading with a Doppler spread of 100Hz

Assignment

- Read Chapter 3
- Homework: 3-3, 3-6,3-8,3-12

Review of Last Lecture

- Time Varying Channel Impulse Response
- Narrowband Fading Model
- Flat Fading vs. Frequency Selective Fading
- Effect of Flat and Frequency Selective Fading
- Signal Envelope Distributions (Rayleigh and Rician)
- Average Fade Duration
- Time Varying Nature of Channel Fading

从 $c(\tau, t)$ 中提取信道相关信息.

已知.

Autocorrelation Function

- Channel impulse response is a random process

$$c(\tau, t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t))$$

- $c(\tau, t)$ is a complex Gaussian process, is characterized by mean, autocorrelation and cross correlation.
- Autocorrelation:

conjugate.

$$A_c(\tau_1, \tau_2; t, t + \Delta t) = \mathbf{E}[c^*(\tau_1; t) c(\tau_2; t + \Delta t)]$$

- Mean = 0, crosscorrelation = 0

从这开始讲

Scattering Function

意义：信道随着时间变化，但信道的统计特征不随着时间变化。

- Wide sense stationary: (WSS)

$$A_c(\tau_1, \tau_2; \Delta t) = \mathbf{E}[c^*(\tau_1; t)c(\tau_2; t + \Delta t)] = A_c(\tau_1, \tau_2, \Delta t)$$

在WSS的条件下

- Uncorrelated scattering → 有N个信道，每个信道造成的响应都不相同，则认为不相关性。

$$\mathbf{E}[c^*(\tau_1; t)c(\tau_2; t + \Delta t)] = A_c(\tau_1; \Delta t)\delta[\tau_1 - \tau_2] \triangleq A_c(\tau; \Delta t)$$

- Scattering function is Fourier trans. of $A_c(\tau, \Delta t)$ (with respect to Δt)

$$S_c(\tau, \rho) = \int_{-\infty}^{\infty} A_c(\tau, \Delta t) e^{-j2\pi\rho\Delta t}\, d\Delta t$$

τ: multipath delay

ρ: Doppler frequency

uncorrelated scattering

$E[\alpha_m(t_1), \alpha_n(t_2)] = E[\alpha_m(t)]E[\alpha_n(t)] = 0, \; m \neq n.$

$$\begin{cases} A_c(\tau_1, \tau_2; \Delta t) = f(\tau_1, \Delta t)\,\delta(\tau_1 - \tau_2) \\ A_c(\tau, \Delta t) = f(\tau_1, \Delta t) \end{cases}$$

S

$\therefore A_c(\tau_1, \tau_2, \Delta t) = E[c^*(\tau_1, t) \cdot c(\tau_2, t+\Delta t)]$

$= E\left\{ \sum_{n=1}^{N(t)} \sum_{m=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \delta[\tau_1 - \tau_n(t)] \alpha_m(t+\Delta t) e^{-j\phi_m(t+\Delta t)} \delta[\tau_2 - \tau_m(t+\Delta t)] \right\}$

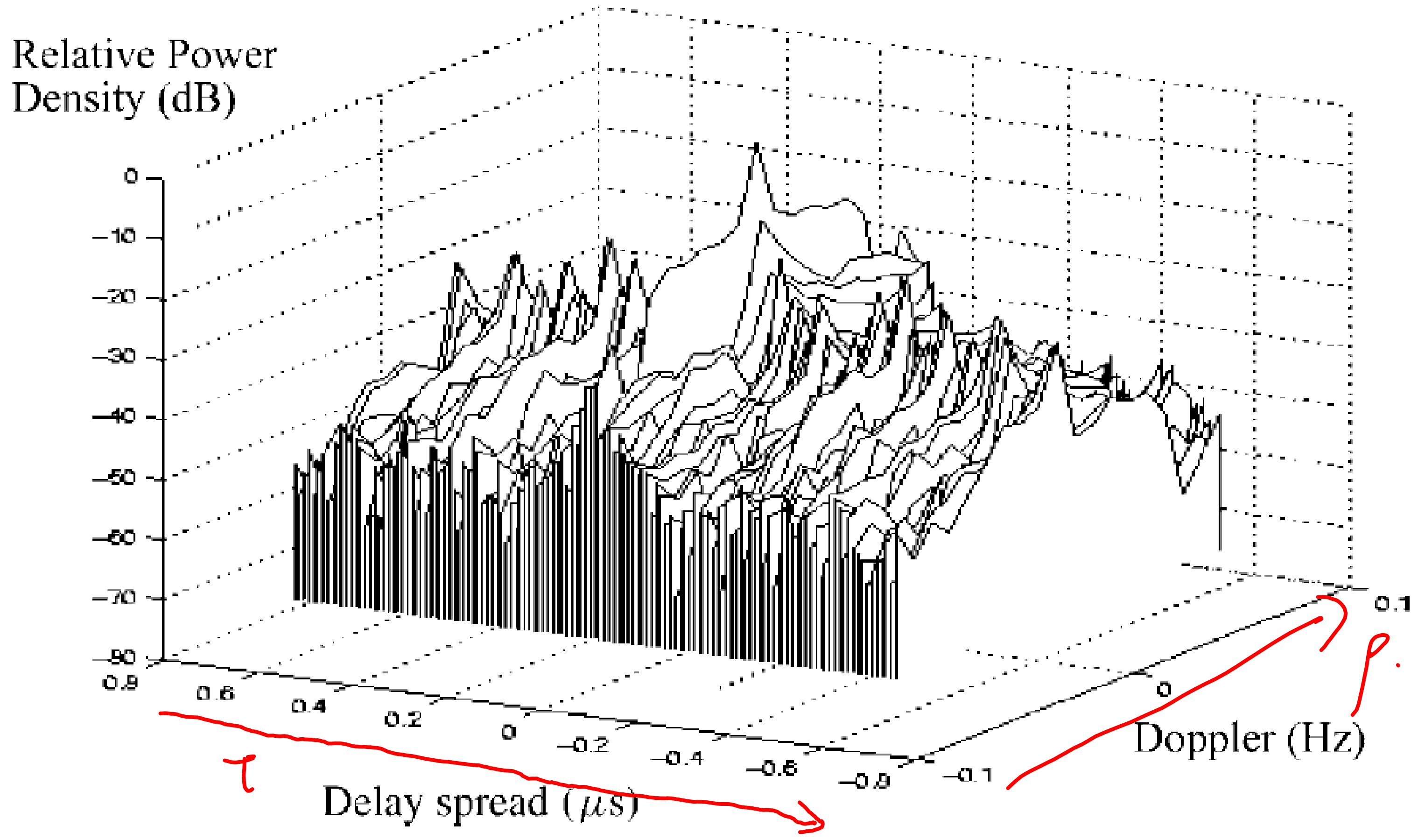
U.S.

$\approx E\left\{ \sum_{n=1}^{N(t)} \alpha_n(t)\alpha_n(t+\Delta t) e^{j[\phi_n(t) - \phi_n(t+\Delta t)]} \delta[\tau_1 - \tau_n(t)]\, \delta[\tau_2 - \tau_n(t+\Delta t)] \right\}$

Assume $\alpha_n(t)$ changes slowly with t, → $\alpha_n(t) \approx \alpha_n(t+\Delta t)$, $\tau_n(t+\Delta t) \approx \tau_n(t)$

Scatter Function

- Scattering function is the average output power of the channel as a function of τ, ρ



$$\approx \underbrace{E\left\{\sum_{n=1}^{N(t)} \alpha_n^2(t)\, e^{j[\phi_n(t)-\phi_n(t+\Delta t)]}\, \delta[\tau_1-\tau_n(t)]\right\}}_{A_c(\tau_1,\Delta t)} \cdot \delta(\tau_1-\tau_2)$$

WSSUS $\rightarrow$ $A(\tau_1,\tau_2,t,t+\Delta t)=A_c(\tau,\Delta t)\cdot\delta(\tau_1-\tau_2)$

$$A_c(\tau,\Delta t)= E\left\{\sum_{n=1}^{N(t)} \alpha_n^2(t)\, e^{j2\pi f_{D_n}(t)\cdot\Delta t}\cdot\delta[\tau-\tau_n(t)]\right\}$$

$$= \sum_n \underbrace{E\{\alpha_n^2(t)\cdot\delta[\tau-\tau_n(t)]\}}_{\text{constant.}}\cdot e^{j2\pi f_{D_n}\Delta t}$$

$\phi_n(t)=2\pi f_c\tau_n(t)-2\pi f_{D_n}(t)\,t$

$\phi_n(t+\Delta t)=2\pi f_c\tau_n(t+\Delta t)-2\pi f_{D_n}(t+\Delta t)\cdot(t+\Delta t)$

$\tau_n(t)\approx\tau_n(t+\Delta t)$ $f_{D_n}(t)\approx f_{D_n}(t+\Delta t)$

$\phi_n(t)-\phi_n(t+\Delta t)\approx 2\pi f_{D_n}\Delta t.$

Scattering function $S_c(\tau,\rho)=\int_{-\infty}^{+\infty} A_c(\tau,\Delta t)\, e^{-j2\pi\rho\cdot\Delta t}\, d\Delta t.$

Power Delay Profile

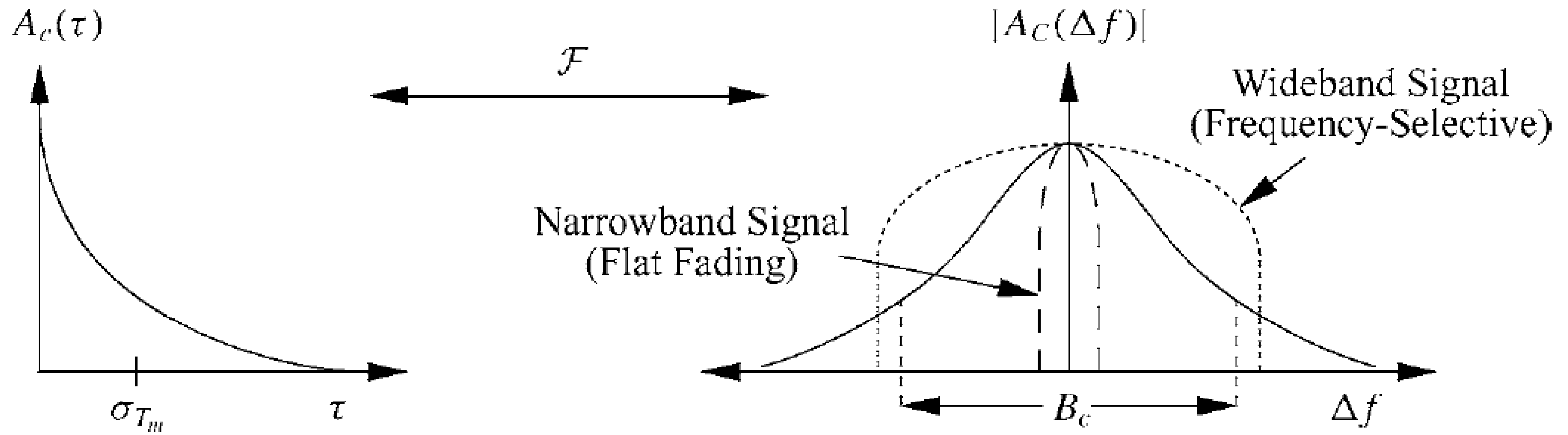
- Power Delay Profile (multipath intensity profile)
  - Let Δt=0 in $A_c(\tau, \Delta t)$, written as $A_c(\tau)$
  - Average power associated with a given multipath delay
  - Average and rms delay spread

$$\mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau} \qquad \sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau-\mu_{T_m})^2 A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau}}$$

$= E(\tau)$; $=1$; average delay ; 标准差

- Coherence Bandwidth

$B_c = 1/\sigma_{T_m}$ Signals separated in frequency by Bc is uncorrelated



$A_c(\tau,\Delta t) \xrightarrow[\Delta t=0]{} A_c(\tau,0) \triangleq A_c(\tau)$

$A_c(\tau) = \sum_n E\{\alpha_n^2(t)\,\delta[\tau-\tau_n(t)]\} = \sum_n E[\alpha_n^2(t)]\, E[\delta[\tau-\tau_n(t)]]$

$E[\alpha_n^2(t)]$: nth path's power gain

$E[\delta[\tau-\tau_n(t)]]$: chance of nth path's delay $=\tau$

Average power gain at delay τ. ⇐

$A_c(\tau,\Delta t) \xleftrightarrow[\tau]{F} A_{\tilde{c}}(\Delta f,\Delta t)$

$A_c(\tau)=A_c(\tau,0) \xleftrightarrow{F} A_{\tilde{c}}(\Delta f,0)=A_{\tilde{c}}(\Delta f$

△ Coherent Bandwidth

$c(\tau,t) \xleftrightarrow[\tau]{\text{Fourier}} C(f,t) = \int_{-\infty}^{+\infty} c(\tau,t)\,e^{-j2\pi f\tau}\,d\tau$

$C(f,t)$: frequency response at time t.

Narrowband

$B \ll B_c$

$\rightarrow \frac{1}{T} \ll \frac{1}{\sigma_{T_m}}$

$\rightarrow \sigma_{T_m} \ll T.$

Autocorrelation function. in frequency domain.

$A_{\tilde{c}}(f_1,f_2,t,t+\Delta t) = E[\tilde{C}^*(f_1,t)\,\tilde{C}(f_2,t+\Delta t)] \overset{}{\underset{\text{WSS}}{=\!=}} A_{\tilde{c}}(f_1,f_2,\Delta t)$

$= E\left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c^*(\tau_1,t)\,e^{j2\pi f_1\tau_1}d\tau_1\; c(\tau_2,t+\Delta t)\,e^{-j2\pi f_2\tau_2}\,d\tau_2\right]$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{E[c^*(\tau_1,t)\,c(\tau_2,t+\Delta t)]}_{A_c(\tau_1,\tau_2,t,t+\Delta t)} e^{j2\pi(f_1\tau_1-f_2\tau_2)}\,d\tau_1 d\tau_2$

$\xrightarrow{\text{WSSTUS}} A_c(\tau_1,\Delta t)\,\delta(\tau_1-\tau_2)$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_c(\tau_1,\Delta t)\,e^{j2\pi(f_1\tau_1-f_2\tau_2)}\,\delta(\tau_1-\tau_2)\,d\tau_1 d\tau_2$

$= \int_{-\infty}^{\infty} A_c(\tau_1,\Delta t)\,e^{j2\pi\overbrace{(f_1-f_2)}^{\Delta f}\tau_1}\,d\tau_1$

$= A_{\tilde{c}}(\Delta f,\Delta t)$

从史

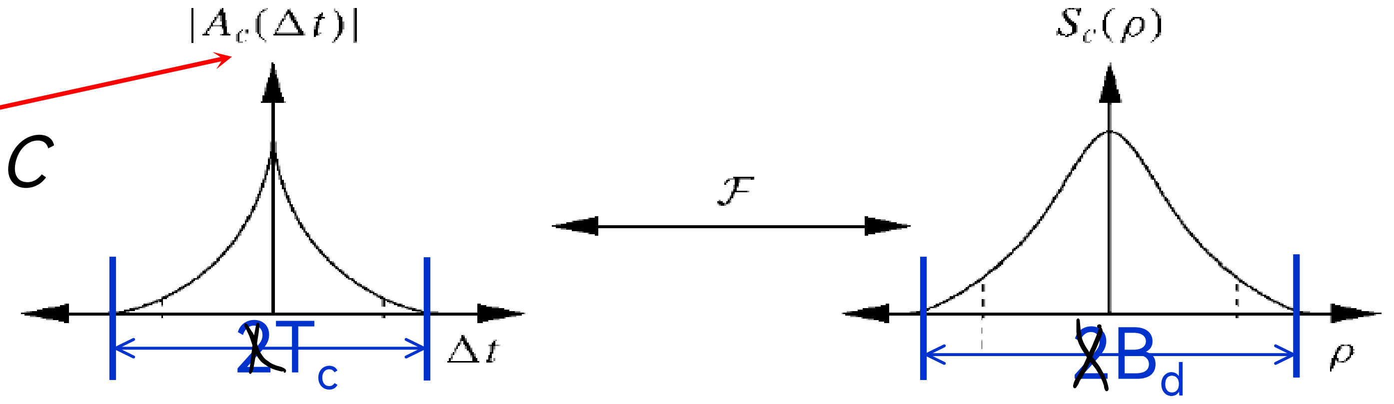
Doppler Power Spectrum

- Doppler Power Spectrum
  - Let $\Delta f = 0$ in $A_C(\Delta f, \Delta t) \rightarrow A_C(\Delta t)$
  - DPS: Fourier Trans. of $A_C(\Delta t)$ with respective of Δt

$$S_C(\rho) = \int_{-\infty}^{\infty} A_C(\Delta t) e^{-j2\pi\rho\Delta t}\, d\Delta t$$

  - Determines Doppler spread: B_d is maximum doppler ρ for which $S_c(\rho) \Rightarrow > 0$
- Coherence Time: $T_c = 1/B_d$
  - Signals separated in time by T_c is uncorrelated

captial C



$A_{\bar{c}}(\Delta f, \Delta t) \xrightarrow{\text{let } \Delta f = 0} A_{\bar{c}}(\Delta t)$: 频率响应的自相关系数 随着$\Delta t$时

if $A_{\bar{c}}(\Delta t) = 0$, 信道随时间剧烈变化.

Channel Coherent Time. $\triangleq T_c$: 认为 channel 在$(0, T_c)$内保持不变

Fourier Transform of $A_{\bar{c}}(\Delta f, \Delta t)$ w.r.t. Δt. = Fourier Transform of $S_c(\tau, \rho)$ w.r.t τ.

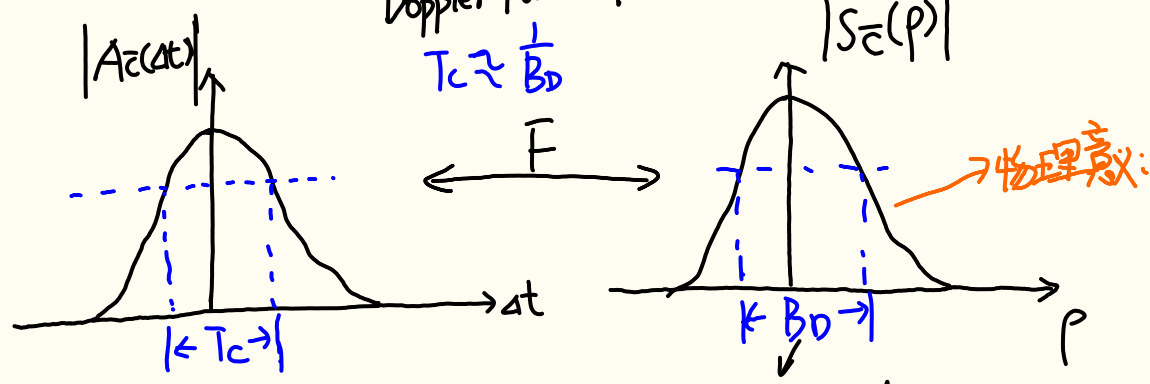
$$\int_{-\infty}^{+\infty} A_{\bar{c}}(\Delta f, \Delta t) e^{-j2\pi\rho\Delta t} d(\Delta t)$$
$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{\infty} A_c(\tau, \Delta t) e^{-j2\pi\Delta f \cdot \tau} d\tau \cdot e^{-j2\pi\rho\Delta t} d(\Delta t)$$
$$= \int_{-\infty}^{+\infty} S_c(\tau, \rho) e^{-j2\pi\Delta f\tau} d\tau.$$
$$= S_{\bar{c}}(\Delta f, \rho)$$

$$A_{\bar{c}}(\Delta f, \Delta t) \overset{F}{\underset{\Delta t}{\longleftrightarrow}} S_{\bar{c}}(\Delta f, \rho).$$

$$A_{\bar{c}}(\Delta t) \overset{\Delta f = 0}{\underset{F}{\longleftrightarrow}} S_{\bar{c}}(\rho)$$

Doppler Power Spectrum.

$T_c \approx \frac{1}{B_D}$



认为在T_c内 channel 不会发生改变.

Doppler spread.

→物理意义: channel power gain 随多普勒效应的变化.

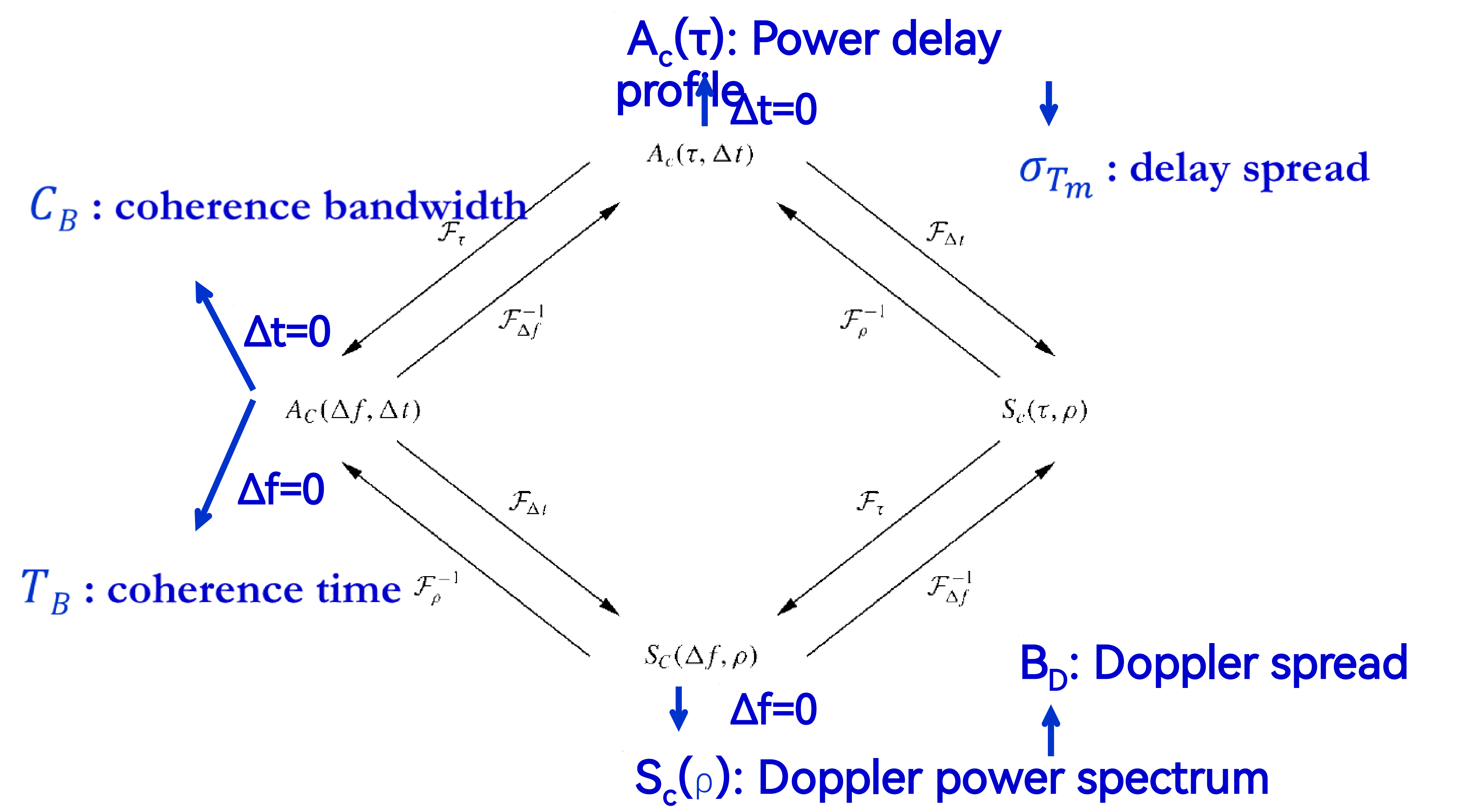
$$S_{\bar{c}}(\rho) = S_{\bar{c}}(\Delta f, \rho)|_{\Delta f=0} = F_\tau[S_c(\tau, \rho)]|_{\Delta f=0}$$
$$= \int_{-\infty}^{+\infty} S_c(\tau, \rho) e^{-j2\pi\Delta f\tau} d\tau |_{\Delta f=0}$$
$$= \int_{-\infty}^{\infty} S_c(\tau, \rho) d\tau = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_c(\tau, \Delta t) e^{-j2\pi\Delta t\rho} d(\Delta t) d\tau$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_c(\tau, \Delta t) d\tau\, e^{-j2\pi\Delta t\rho} d(\Delta t).$$

$$\int_{-\infty}^{\infty} A_c(\tau, \Delta t) d\tau = \int_{-\infty}^{\infty} \sum_{n=1}^{N(t)} E[\alpha_n^2(t) \cdot \delta[\tau - \tau_n(t)] \cdot e^{j2\pi f_{D_n}\cdot\Delta t}] d\tau$$

$$= \int_{-\infty}^{\infty} \boxed{\sum_{n=1}^{N(t)} E[\alpha_n^2(t)] e^{j2\pi f_{D_n}\cdot\Delta t}} e^{-j2\pi\Delta t\rho} \cdot d(\Delta t)$$

单一多普勒效应频率分量

Summary



Review:

$A_c(\tau_1,\tau_2,t,t+\Delta t)$

$\Downarrow$ WSSUS

$A_c(\tau,\Delta t) \xleftarrow{\text{对}\Delta t,F} S(\tau,\rho)$

$S(\tau,\rho) \xrightarrow{\text{对}\tau,F} S_{\tilde{c}}(\Delta f,\rho)$

$A_c(\tau,\Delta t) \xrightarrow{\text{对}\tau,F} A_{\tilde{c}}(\Delta f,\Delta t)$

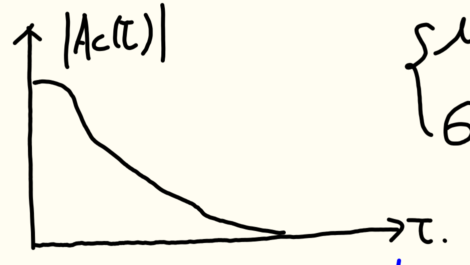
$A_{\tilde{c}}(\Delta f,\Delta t) \xrightarrow{\text{对}\Delta t,F} S_{\tilde{c}}(\Delta f,\rho)$

$\tilde{c}$: 表示大写

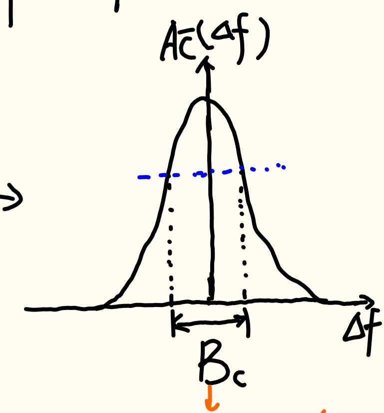
当$\Delta t=0$时, $A_{\tilde{c}}(\Delta f,\Delta t)=A_{\tilde{c}}(\Delta f) \xleftrightarrow{F} A_c(\tau)$

当$\Delta t=0$, $A_c(\tau,\Delta t)=A_c(\tau)\rightarrow$ power delay profile.

物理意义: 当delay=τ时, the total power gain of all path.



$\begin{cases} \mu_{T_m} \\ \sigma_{T_m} \end{cases} \xleftrightarrow{F}$



$\frac{1}{B_c} \approx \sigma_{T_m}$

可认为frequency response在B_c范围保持不变.

if frequency response in B_c is not smooth, σ_{T_m} is large.

Narrowband: $B \ll B_c$.

Summary

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading

1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading

1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading

1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than base-band signal variations

Slow Fading

1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Main Points

- Scattering Function
- Power Delay Profile → Coherence Bandwidth
- Doppler Power Spectrum → Coherence Time
- Flat/Frequency Selective Fading
- Fast/Slow Fading