△ Channel Impulse Response:

$$c(\tau,t)=\sum_{n=1}^{N(t)}\alpha_n(t)\cdot e^{-j\phi_n(t)}\,\delta(\tau-\tau_n(t))$$

$$\phi_n(t)=2\pi f_c\tau_n(t)-\phi_{Dn}-\phi_0$$

$\alpha_n(t)$, $\tau_n(t)$, ϕ_{Dn}, ϕ_0 are all random.

△ Narrowband Model.

$$T=1/B$$

$$\max_{m,n}|\tau_n(t)-\tau_m(t)|\ll T=1/B$$

$$c(\tau,t)=\delta(\tau-\tau_0)\sum_{n=1}^{N(t)}\alpha_n(t)\cdot e^{-j\phi_n(t)}$$

$$=\underbrace{\alpha(t)}_{\text{Channel Gain.}}\cdot\delta(\tau-\tau_0)$$

$$\alpha(t)\delta(\tau-\tau_0)\xrightarrow[\tau]{\text{Fourier Transform}}\underbrace{e^{-j2\pi f\tau_0}\alpha(t)}_{\text{Frequency Response at } t}$$

$$|e^{-j2\pi f\tau_0}\alpha(t)|=|\alpha(t)|\Rightarrow \text{Flat Fading}$$

Narrowband $\Leftrightarrow$ Flat Fading

$\downarrow$ Not

Wideband $\Leftrightarrow$ Frequency Selective Fading

△ Channel Gain of Narrowband

$$\alpha(t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)}$$

$$= \underbrace{\sum_{n=1}^{N(t)} \alpha_n(t) \cos\phi_n(t)}_{\text{NLoS + CLT: Gaussian } N(0, \sigma^2/2)} - \underbrace{\sum_{n=1}^{N(t)} j\alpha_n(t) \sin\phi_n(t)}_{\text{Gaussian } N(0, \sigma^2/2)}$$

Independent

$\mathcal{CN}(0, \sigma^2)$

$\angle\alpha(t)$: uniform in $[0, 2\pi)$

$|\alpha(t)|$: Rayleigh

$|\alpha(t)|^2$: Exponential.

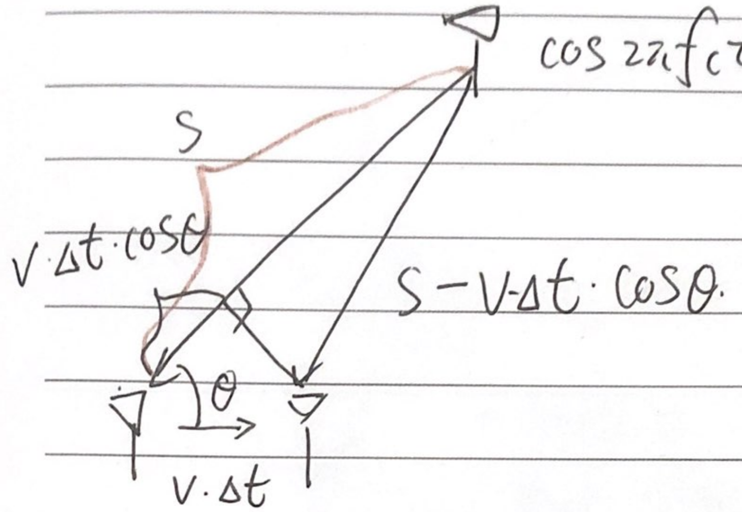
Rayleigh Channel

If there is dominant LoS, ⇒ Rician Channel.

2022-10-11

Δ Doppler Shift.

$\cos 2\pi f_c t \rightarrow \cos 2\pi f_c (t+\Delta t)$



$\cos 2\pi f_c (t-\frac{S}{c}) \rightarrow \cos 2\pi f_c (t+\Delta t - \frac{S - v\Delta t \cos\theta}{C})$

$= \cos(2\pi f_c t - 2\pi f_c \frac{S}{C})$ $= \cos\{\underbrace{2\pi f_c t - 2\pi f_c \frac{S}{c}} + 2\pi f_c \Delta t + 2\pi f_c \frac{v \cdot \Delta t \cdot \cos\theta}{C}\}$

Phase difference $= 2\pi f_c \Delta t + 2\pi f_c \frac{v\cos\theta \cdot \Delta t}{C}$.

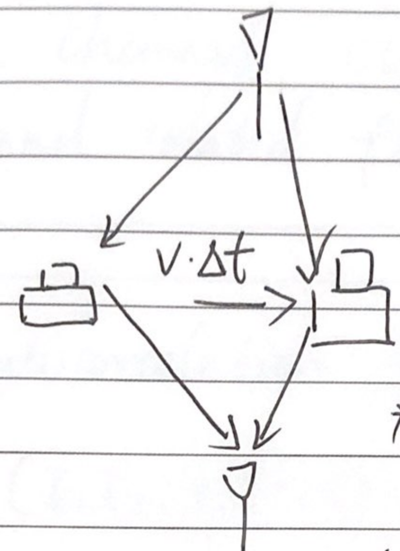
Frequency of received ~~st~~ signal

$= f_c + f_c \frac{v\cos\theta}{C}$.

$= f_c + \underbrace{\frac{v}{\lambda}\cos\theta}_{f_d}$ λ: wavelength.

f_d: Doppler Frequency Shift.

Another Scenario of Doppler Shift.



$f_d = ?$

* There is no Doppler effect in LTI.
But time-varying linear system has.

Doppler spread: maximum Doppler shift.

△ Channel Impulse Response with Doppler.

$$c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{j(-2\pi f_c \tau_n(t) + \phi_{Dn})} \delta(\tau - \tau_n(t))$$

$$= \sum_{n=1}^{N(t)} \alpha_n(t) e^{j(-2\pi f_c \tau_n(t) + \underbrace{2\pi f_{D,n}(t) t}_{\text{Doppler Effect.}})} \delta(\tau - \tau_n(t))$$

~~Single~~

$\alpha_n(t)$ and $\tau_n(t)$ do not change fast w.r.t. t usually.

=> Doppler Effect is the main reason for fast-time-varying channel.

How to measure the Doppler frequency of a channel $c(\tau, t)$? $\Rightarrow$ Autocorrelation Function and related functions

△ Autocorrelation function.

$$A_c(\tau_1, \tau_2; t, t+\Delta t) = E[c^*(\tau_1, t)\, c(\tau_2, t+\Delta t)]$$

$\Downarrow$ Wide sense stationary:

$A_c(\tau_1, \tau_2; t, t+\Delta t)$ is independent of t

$$A_c(\tau_1, \tau_2; \Delta t) = E[c^*(\tau_1, t)\, c(\tau_2, t+\Delta t)]$$

$\Downarrow$ uncorrelated scattering:

$$E[\alpha_m^*(t_1)\alpha_n(t_2)] = E[\alpha_m^*(t_1)] \cdot E[\alpha_n(t_2)] \quad m \neq n.$$
$$= 0$$

$$E[C^*(\tau_1,t)\, C(\tau_2,t+\Delta t)]$$

$$= E\Big[\sum_{n=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \delta(\tau_1-\tau_n(t)) \sum_{m=1}^{N(t)} \alpha_m(t+\Delta t) e^{-j\phi_m(t+\Delta t)} \delta(\tau_2-\tau_m(t+\Delta t))\Big]$$

$$= E\Big[\sum_{m,n=1}^{N(t)} \alpha_n(t)\alpha_m(t+\Delta t) e^{j[\phi_n(t)-\phi_m(t+\Delta t)]} \delta(\tau_1-\tau_n(t))\cdot\delta(\tau_2-\tau_m(t+\Delta t))\Big]$$

$$= E\Big[\sum_{n=1}^{N(t)} \alpha_n(t)\alpha_n(t+\Delta t) e^{j[\phi_n(t)-\phi_n(t+\Delta t)]} \delta(\tau_1-\tau_n(t))\,\delta(\tau_2-\tau_n(t+\Delta t))\Big]$$

$$\approx E\Big[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j[\phi_n(t)-\phi_n(t+\Delta t)]} \delta(\tau_1-\tau_n(t))\,\delta(\tau_2-\tau_n(t))\Big]$$

$$= E\Big[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j[\phi_n(t)-\phi_n(t+\Delta t)]} \delta(\tau_1-\tau_n(t))\Big]$$

$$\delta[\tau_1-\tau_2]$$

$\Rightarrow A_c(\tau_1,\tau_2;\Delta t)$ is non-zero only when $\tau_1=\tau_2$.

$$A_c(\tau_1;\Delta t) = A_c(\tau_1,\tau_1;\Delta t)$$

or $$A_c(\tau;\Delta t) = A_c(\tau,\tau;\Delta t)$$