

发射机和接收机都知道 H.

review: $\vec{y} = H\vec{x} + \vec{n}$ $H = U\Lambda V^H$ $\vec{x} = V\vec{\tilde{x}}$ $\vec{y} = U^H\vec{\tilde{y}}$

$\vec{y} = \Lambda\vec{\tilde{x}} + \vec{\tilde{n}}$ $\vec{\tilde{n}} = U^H\vec{n}$

$\vec{y}_i = \sigma_i \tilde{x}_i + \tilde{n}_i, i=1,2,\dots, R_H, R_H = \text{rank}(H).$

$E[\tilde{n}\tilde{n}^H] = R_n = \sigma^2 I = E[\tilde{n}\tilde{n}^H] = R_n, E[\tilde{n}_i^2] = \sigma^2$

总发射功率: $\text{Tr } E[\vec{\tilde{x}}\vec{\tilde{x}}^H] \leq P$

$\Rightarrow \text{Tr } E[\vec{x}\vec{x}^H] \leq P$

$\sum_{i=1}^{R_H} E|\tilde{x}_i|^2 \leq P \quad E|\tilde{x}_i|^2 \triangleq P_i \Rightarrow \sum_{i=1}^{R_H} P_i \leq P$

$C = \sum_{i=1}^{R_H} B \log_2(1 + \sigma_i^2 \frac{P_i}{\sigma^2})$

$C^* = \max \sum_{i=1}^{R_H} B \log_2(1 + \sigma_i^2 \frac{P_i}{\sigma^2})$

s.t. $\sum_{i=1}^{R_H} P_i \leq P$

water-filling \rightarrow 课本附录.

Lagrangian $J(\lambda, \{P_i\}) = \sum_{i=1}^{R_H} B \log_2(1 + \sigma_i^2 \frac{P_i}{\sigma^2}) - \lambda (\sum_{i=1}^{R_H} P_i - P)$

$C^* = \min_{\lambda} (\max_{\{P_i\}} J(\lambda, \{P_i\})) \quad \frac{\partial J(\lambda, \{P_i\})}{\partial P_i} = 0$

$\Rightarrow P_i = (\frac{B}{\lambda \ln 2} - \frac{\sigma^2}{\sigma_i^2})^+ = \max\{0, \frac{B}{\lambda \ln 2} - \frac{\sigma^2}{\sigma_i^2}\}$

$\Rightarrow \frac{P_i}{P} = (\frac{B}{P \lambda \ln 2} - \frac{\sigma^2}{P \sigma_i^2})^+, \text{ Let } \gamma_0 = \frac{P \lambda \ln 2}{B} \quad \gamma_i = \frac{P \sigma_i^2}{\sigma^2}$

$\Rightarrow \frac{P_i}{P} = (\frac{1}{\gamma_0} - \frac{1}{\gamma_i})^+, \sum_{i=1}^{R_H} P_i \leq P, (\sum_{i=1}^{R_H} \frac{P_i}{P} = 1) \Rightarrow \text{求 } \lambda$

$\sum_{i=1}^{R_H} (\frac{1}{\gamma_0} - \frac{1}{\gamma_i})^+ = 1 \Rightarrow \lambda = ?$

γ_i 一般从大到小排列. $\gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_{R_H-1} \geq \gamma_{R_H}$

假设 $0 \leq \gamma_1 \leq \dots \leq \gamma_{R_H} \leq \gamma_0$

$\sum_{i=1}^{R_H} (\frac{1}{\gamma_0} - \frac{1}{\gamma_i}) = 1 \Rightarrow \frac{R_H}{\gamma_0} = 1 + \sum_{i=1}^{R_H} \frac{1}{\gamma_i} \Rightarrow \gamma_0 = \frac{R_H}{1 + \sum_{i=1}^{R_H} \frac{1}{\gamma_i}}$

check if $\gamma_0 \leq \gamma_{R_H}$ yes $\gamma_0 \Rightarrow \lambda = ?$

② if no, suppose $\gamma_1 \geq \dots \geq \gamma_{R_H-1} \geq \gamma_0 > \gamma_{R_H} \Rightarrow \sum_{i=1}^{R_H-1} \frac{1}{\gamma_0} - \frac{1}{\gamma_i} = 1$

$\Rightarrow \frac{R_H-1}{\gamma_0} = 1 + \sum_{i=1}^{R_H-1} \frac{1}{\gamma_i} \Rightarrow \gamma_0 = \frac{R_H-1}{1 + \sum_{i=1}^{R_H-1} \frac{1}{\gamma_i}}, \text{ check if } \gamma_{R_H} < \gamma_0 \leq \gamma_{R_H-1}$

③ if no, suppose $\gamma_1 \geq \dots \geq \gamma_{R_H-2} \geq \gamma_0 > \gamma_{R_H-1} \geq \gamma_{R_H}$

Δ Space-Time coding. (CSIR only)

$\vec{y} = H\vec{x} + \vec{n}$

$\vec{y}_1, \vec{y}_2, \dots, \vec{y}_T \leftarrow \vec{x}_1, \vec{x}_2, \dots, \vec{x}_T$

$\vec{n}_1, \vec{n}_2, \dots, \vec{n}_T$

$Y \sim M_T \times T \quad X \sim M_T \times T$

$Y = HX + N$

QPSK: $\mathcal{X} = \{\pm 1 \pm j\} \quad |\mathcal{X}| = 4 \Rightarrow X \sim 4^{M_T \times T}$

Receiver: $H, Y \Rightarrow X = ? \quad \|X\|_F^2$

ML Detector: $\arg \min_X \|Y - HX\|_F^2 = \arg \min_X \sum_{i=1}^T \|\vec{y}_i - H\vec{x}_i\|^2$

Alamouti Code - 2 Tx antennas

So S_1 (发射机有两根天线, 发送两个信号).

$\begin{pmatrix} S_0 \\ S_1 \end{pmatrix} \begin{pmatrix} -S_1^* \\ S_0^* \end{pmatrix} \rightarrow \text{两次传输.}$

Example: 1 Rx antenna.

$y_1 = (h_1 \ h_2) \begin{pmatrix} S_0 \\ S_1 \end{pmatrix} + n_1$

$y_2 = (h_1 \ h_2) \begin{pmatrix} -S_1^* \\ S_0^* \end{pmatrix} + n_2 \Rightarrow y_2^* = -h_1^* S_1 + h_2^* S_0 + n_2^*$

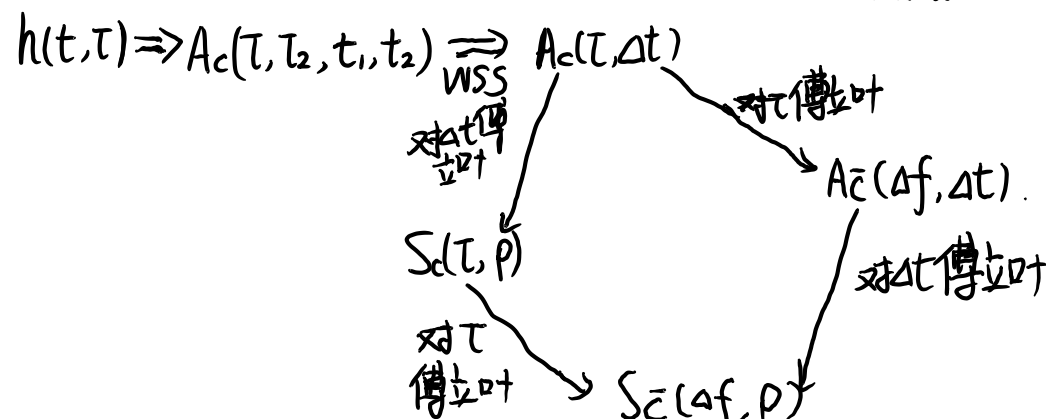
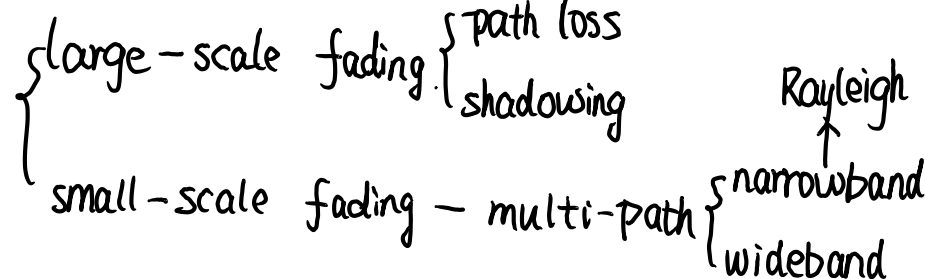
$\Rightarrow y_2^* = (h_2^* \ -h_1^*) \begin{pmatrix} S_0 \\ S_1 \end{pmatrix} + n_2^* \Rightarrow \begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} = \underbrace{\begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix}}_H \begin{pmatrix} S_0 \\ S_1 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix}$

$H H^H = (|h_1|^2 + |h_2|^2) I$

$\vec{y} = H \begin{pmatrix} S_0 \\ S_1 \end{pmatrix} + \vec{n}$

$H^H \vec{y} = (|h_1|^2 + |h_2|^2) \begin{pmatrix} S_0 \\ S_1 \end{pmatrix} + H^H \vec{n} \rightarrow \text{估计 } S_0, S_1 \quad \frac{H^H \vec{y}}{|h_1|^2 + |h_2|^2}$

复习: Δ wireless channel.



Δ Channel capacity

AWGN $\left\{ \begin{array}{l} \text{CSIR} \\ \text{flat fading} \end{array} \right\} \left\{ \begin{array}{l} \text{CSIR} \\ \text{CSIR + CSIT} \end{array} \right.$

Δ Digital Modulation

Signal Space

constellation.

AWGN Receiver

ML detection \Rightarrow minimum distance \Rightarrow decision region.

Error Probability \Rightarrow union bound.

Δ MIMO.

narrowband MIMO model $\vec{y} = H\vec{x} + \vec{n}$

$\left\{ \begin{array}{l} \text{CSIT + CSIR} \Rightarrow \text{SVD} \\ \text{CSIR} \Rightarrow \text{space time coding.} \end{array} \right.$

ML detection.