$$\vec{y} = H\vec{x} + \vec{n} \qquad H = U\Lambda V^H \qquad \vec{x} = V\tilde{x} \qquad \tilde{y} = U^H\vec{y}$$

$M_r \times 1 \quad M_t \times 1$

$$\tilde{y} = \Lambda\hat{x} + \tilde{n} \qquad \tilde{n} = U^H\vec{n} \qquad R_H = \mathrm{Rank}(H)$$

$$\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{n}_i \qquad i = 1, 2, \cdots, R_H$$

$$E[\vec{n}\vec{n}^H] = R_n = \sigma^2 I = E[\tilde{n}\tilde{n}^H] = R_{\tilde{n}} \qquad E|\tilde{n}_i|^2 = \sigma^2$$

$$\mathrm{Tr}\ E[\vec{x}\vec{x}^H] \le P$$

$$\Rightarrow \mathrm{Tr}\ E[\tilde{x}\tilde{x}^H] \le P$$

$$\sum_{i=1}^{R_H} E|\tilde{x}_i|^2 \le P \qquad E|\tilde{x}_i|^2 \triangleq P_i \Rightarrow \sum_{i=1}^{R_H} P_i \le P$$

$$C = \sum_{i=1}^{R_H} B \log_2\left(1 + \sigma_i^2 \frac{P_i}{\sigma^2}\right)$$

$$\boxed{C^* = \max \sum_{i=1}^{R_H} B\log_2\left(1+\sigma_i^2\frac{P_i}{\sigma^2}\right) \quad \text{s.t.} \sum_{i=1}^{R_H} P_i \le P}$$

water-filling

Lagrangian $J(\lambda, \{P_i\}) = \sum_{i=1}^{R_H} B\log_2\left(1+\sigma_i^2\frac{P_i}{\sigma^2}\right) - \lambda\left(\sum_{i=1}^{R_H} P_i - P\right)$

$$C^* = \min_{\lambda} \boxed{\max_{\{P_i\}} J(\lambda, \{P_i\})} \qquad \frac{\partial J(\lambda, \{P_i\})}{\partial P_i} = 0$$

$$\Rightarrow P_i = \left(\frac{B}{\lambda \cdot \ln 2} - \frac{\sigma^2}{\sigma_i^2}\right)^+ = \max\left\{0, \frac{B}{\lambda \ln 2} - \frac{\sigma^2}{\sigma_i^2}\right\}$$

$$\Rightarrow \frac{P_i}{P} = \left(\frac{B}{P\lambda\ln 2} - \frac{\sigma^2}{P\sigma_i^2}\right)^+ \qquad \text{Let } \gamma_0 = \frac{P\lambda\ln 2}{B} \quad \gamma_i = \frac{P\sigma_i^2}{\sigma^2}$$

$$\Rightarrow \frac{P_i}{P} = \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right)^+ \qquad \sum_{i=1}^{R_H} P_i = P \qquad \sum_{i=1}^{R_H} \frac{P_i}{P} = 1$$

$$\sum_{i=1}^{R_H}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma_i}\right)^+ = 1 \Rightarrow \lambda = ?$$

$$\gamma_i = P\frac{\sigma_i^2}{\sigma^2}$$

$$\boxed{\frac{P_i}{P} = \left(\frac{1}{\gamma_0}-\frac{1}{\gamma_i}\right)^+}$$

$\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_{R_H-1} \geq \gamma_{R_H}$

① $\gamma_1 \geq \cdots \geq \gamma_{R_H} \geq \gamma_0$

$$\sum_{i=1}^{R_H} \frac{1}{\gamma_0}-\frac{1}{\gamma_i} = 1 \Rightarrow \frac{R_H}{\gamma_0} = 1+\sum_{i=1}^{R_H}\frac{1}{\gamma_i} \Rightarrow \gamma_0 = \frac{R_H}{1+\sum_{i=1}^{R_H}\frac{1}{\gamma_i}}$$

check if $\gamma_0 \leq \gamma_{R_H}$ yes $\gamma_0 \Rightarrow \lambda = ?$

② No

suppose $\gamma_{R_H} < \gamma_0 \leq \gamma_{R_H-1} \Rightarrow \sum_{i=1}^{R_H-1} \frac{1}{\gamma_0}-\frac{1}{\gamma_i} = 1$

$$\Rightarrow \frac{R_H-1}{\gamma_0} = 1+\sum_{i=1}^{R_H-1}\frac{1}{\gamma_i} \Rightarrow \gamma_0 = \frac{R_H-1}{1+\sum_{i=1}^{R_H-1}\frac{1}{\gamma_i}}$$

check if $\gamma_{R_H} < \gamma_0 \leq \gamma_{R_H-1}$

③ No $\Rightarrow \gamma_{R_H-1} < \gamma_0 \leq \gamma_{R_H-2}$

△ Space-Time Coding (CSIR only)

$$\vec{y} = H\vec{x}+\vec{n}$$

$$\underbrace{\vec{y}_1\,\vec{y}_2\cdots\vec{y}_T}_{Y\sim M_r\times T} \Leftarrow \underbrace{\vec{x}_1\,\vec{x}_2\cdots\vec{x}_T}_{X\sim M_t\times T} + \underbrace{\vec{n}_1\,\vec{n}_2\cdots\vec{n}_T}_{N\sim M_r\times T}$$

$$Y = HX+N$$

QPSK: $\mathcal{X} = \{\pm1\pm i\}$ $|\mathcal{X}| = 4 \Rightarrow X \sim 4^{M_t\times T}$

Receiver H. $Y \Rightarrow X = ?$

ML Detector: $\underset{X}{\mathrm{argmin}}\, \|Y-HX\|_F^2 = \underset{X}{\mathrm{argmin}} \sum_{i=1}^{T}\|\vec{y}_i - H\vec{x}_i\|^2$

$\|X\|_F^2$

Alamouti Code — 2 Tx antennas 10.1 10.2 10.3.1 10.6.1

$s_0 \quad s_1$

$$\begin{pmatrix} s_0 \\ s_1 \end{pmatrix} \begin{pmatrix} -s_1^* \\ s_0^* \end{pmatrix}$$
$$\vec{x}_1 \qquad \vec{x}_2$$

Example 1 Rx antenna

$$y_1 = (h_1 \ h_2) \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + n_1$$

$$y_2 = (h_1 \ h_2) \begin{pmatrix} -s_1^* \\ s_0^* \end{pmatrix} + n_2 \Rightarrow y_2^* = -h_1^* s_1 + h_2^* s_0 + n_2^*$$

$$\Rightarrow y_2^* = (h_2^* \ \ -h_1^*) \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + n_2^* \Rightarrow \begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} = \underbrace{\begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix}}_{H} \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix}$$

$$\boxed{\begin{aligned} &H H^H = (|h_1|^2 + |h_2|^2) \cdot I \\ &\vec{y} = H \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + \vec{n} \\ &H^H \vec{y} = (|h_1|^2 + |h_2|^2) \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + H^H \vec{n} \\ &H^H \vec{y} / (|h_1|^2 + |h_2|^2) \end{aligned}}$$

- Wireless Channel

$$\begin{cases} \text{large-scale fading} & \begin{cases} \text{path loss} \\ \text{shadowing} \end{cases} \\ \text{small-scale fading} \sim \text{multi-path} & \begin{cases} \text{narrowband} \\ \text{wideband} \end{cases} \end{cases}$$

Rayleigh

$$h(t, \tau) \Rightarrow A_c(\tau_1, \tau_2, t_1, t_2) \Rightarrow A_c(\tau, \Delta t)$$

WSS

$$S_c(\tau, \rho) \qquad A_{\tilde{c}}(\Delta f, \Delta t)$$

$$S_{\tilde{c}}(\Delta f, \rho)$$

△ Channel Capacity

AWGN

Flat fading $\begin{cases} \text{CSIR} \\ \text{CSIR} + \text{CSIT} \end{cases}$

△ Digital Modulation

Signal Space

constellation.

AWGN Receiver

ML detection ⇒ minimum distance ⇒ decision Region

Error Probability ⇒ union bound

△ MIMO

narrowband MIMO model $\vec{y} = H\vec{x} + \vec{n}$

$\begin{cases} \text{CSIT} + \text{CSIR} \Rightarrow \text{SVD} \quad ✓ \\ \text{CSIR} \Rightarrow \text{space time coding} \\ \qquad\qquad \text{ML detection} \end{cases}$