How to measure the Doppler frequency of a channel $c(\tau, t)$? $\Rightarrow$ Autocorrelation Function and related functions

$\triangle$ Autocorrelation function.

$$A_c(\tau_1, \tau_2; t; t+\Delta t) = E\left[c^*(\tau_1, t)\, c(\tau_2, t+\Delta t)\right]$$

$\Downarrow$ Wide sense stationary:

$A_c(\tau_1, \tau_2; t, t+\Delta t)$ is independent of t

$$A_c(\tau_1, \tau_2; \Delta t) = E\left[c^*(\tau_1, t)\, c(\tau_2, t+\Delta t)\right]$$

10
OCT

$\Downarrow$ uncorrelated scattering:

$$E\left[\alpha_m^*(t_1)\, \alpha_n(t_2)\right] = E\left[\alpha_m^*(t_1)\right] \cdot E\left[\alpha_n(t_2)\right] \quad m \neq n.$$

$$= 0$$

$A_c(\tau_1, \tau_2; \Delta t)$

$$= \cancel{\text{Autocorr}}\, E\left[C^*(\tau_1, t)\; C(\tau_2, t+\Delta t) \right]$$

$$= E\left[\sum_{n=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \delta(\tau_1 - \tau_n(t)) \sum_{m=1}^{N(t)} \alpha_m(t+\Delta t) e^{-j\phi_m(t+\Delta t)} \delta(\tau_2 - \tau_m(t+\Delta t)) \right]$$

$$= E\left[\sum_{m,n=1}^{N(t)} \alpha_n(t) \alpha_m(t+\Delta t) e^{j[\phi_n(t) - \phi_m(t+\Delta t)]} \delta(\tau_1 - \tau_n(t)) \cdot \delta(\tau_2 - \tau_m(t+\Delta t)) \right]$$

$$= E\left[\sum_{n=1}^{N(t)} \alpha_n(t) \alpha_n(t+\Delta t) e^{j[\phi_n(t) - \phi_n(t+\Delta t)]} \delta(\tau_1 - \tau_n(t)) \delta(\tau_2 - \tau_n(t+\Delta t)) \right]$$

$$\approx E\left[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j[\phi_n(t) - \phi_n(t+\Delta t)]} \delta(\tau_1 - \tau_n(t)) \delta(\tau_2 - \tau_n(t)) \right]$$

$$= E\left[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j[\phi_n(t) - \phi_n(t+\Delta t)]} \delta(\tau_1 - \tau_n(t)) \right] \delta[\tau_1 - \tau_2]$$

$\Rightarrow A_c(\tau_1, \tau_2; \Delta t)$ is non-zero only when $\tau_1 = \tau_2$

$$A_c(\tau_1; \Delta t) = A_c(\tau_1, \tau_1; \Delta t) / \delta(0)$$

or $$A_c(\tau; \Delta t) = A_c(\tau, \tau; \Delta t) / \delta(0)$$

$$A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1; \Delta t) \cdot \delta(\tau_1 - \tau_2)$$

Note that $\phi_n(t) = 2\pi f_c \tau_n(t) - 2\pi f_{D,n}(t)\, t$

$$\phi_n(t) - \phi_n(t+\Delta t) = 2\pi f_c \tau_n(t) - 2\pi f_{D,n}(t)\cdot t$$
$$- 2\pi f_c \tau_n(t+\Delta t) + 2\pi f_{D,n}(t+\Delta t)(t+\Delta t)$$
$$\approx 2\pi f_{D,n} \Delta t$$

Suppose constant Doppler

$$A_c(\tau; \Delta t) = E\left[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j[\phi_n(t) - \phi_n(t+\Delta t)]} \delta(\tau - \tau_n(t))\right]$$

$$= E\left[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j2\pi f_{D,n} \cdot \Delta t} \cdot \delta(\tau - \tau_n(t))\right]$$

$$\cancel{= \sum_{n=1}^{N(t)} E[\alpha_n^2(t)]\, e^{j2\pi f_{D,n} \cdot \Delta t} \cdot \delta(\tau - \tau_n(t))}$$

$$= \sum_{n=1}^{N(t)} E[\alpha_n^2(t) \cdot \delta(\tau - \tau_n(t))]\, e^{j2\pi f_{D,n} \cdot \Delta t}$$

Observation:

Given τ, treat Δt as independent variable, the frequencies of $A_c(\tau; \Delta t)$'s frequency components are the Doppler frequencies.

$\Rightarrow$ Taking Fourier Transform w.r.t. Δt, we get Doppler frequencies.

Scattering function

$$S_c(\tau, \rho) = \int_{-\infty}^{+\infty} A_c(\tau, \Delta t) e^{-j2\pi \rho \cdot \Delta t} d\Delta t$$

$S_c(\tau, \rho)$ shows the Dopple frequencies in the paths with delay τ.

△ Power Delay Profile

Let $\Delta t = 0$ in $A_c(\tau; \Delta t)$, denote as $A_c(\tau)$

Thus, $A_c(\tau) = A_c(\tau; 0)$ Power Delay Profile

$$A_c(\tau) = E[C^*(\tau, t)\, C(\tau, t)]$$

$$= \sum_{n=1}^{N(t)} E[\alpha_n^2(t)\, \delta(\tau - \tau_n(t))]$$

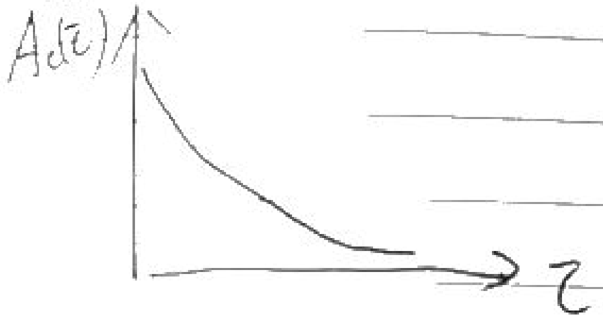
$$= \sum_{n=1}^{N(t)} \underbrace{E[\alpha_n^2(t)] \cdot E[\delta(\tau - \tau_n(t))]}$$

gain

Average power of nth path ; P Chance of delay $= \tau$.

Average channel power gain at delay τ.

Typical $A_c(\tau)$ curve.



Average delay spread.

$$\mu_{Tm} = \frac{\int_0^\infty \tau A_c(\tau) d\tau}{\int_0^\infty A_c(\tau) d\tau} \quad (\text{like mean})$$

RMS delay spread

$$\sigma_{Tm} = \sqrt{\frac{\int_0^\infty (\tau - \mu_{Tm})^2 A_c(\tau) d\tau}{\int_0^\infty A_c(\tau) d\tau}} \quad (\sigma_{Tm}^2 \text{ like variance})$$

SUSTech

△ Coherent Bandwidth:

How flat of the frequency selective fading

$$C(f,t) = \int_{-\infty}^{+\infty} c(\tau,t) e^{-j2\pi f\tau} d\tau.$$

Frequency domain autocorrelation function.

$$A_{\tilde{C}}(f_1, f_2; t, t+\Delta t)$$

WSS

$$= A_{\tilde{C}}(f_1, f_2; \Delta t)$$

$$= E[C^*(f_1, t)\, C(f_2, t+\Delta t)]$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E[c^*(\tau_1, t)\, c(\tau_2, t+\Delta t)] e^{j2\pi f_1 \tau_1} e^{-j2\pi f_2 \tau_2} d\tau_1 d\tau_2$$

11 NOV

uncorrelated scattering

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_c(\tau_1, \Delta t) \cdot \delta[\tau_1 - \tau_2] e^{-j2\pi(f_2 \tau_2 - f_1 \tau_1)} d\tau_1 d\tau_2$$

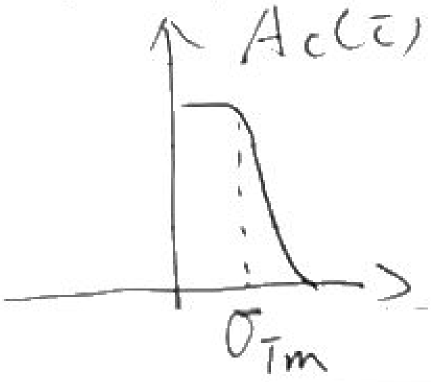
$$= \int_{-\infty}^{+\infty} A_c(\tau_1, \Delta t) e^{-j2\pi \underbrace{(f_2 - f_1)}_{\Delta f} \tau_1} d\tau_1 \triangleq A_{\tilde{C}}(\Delta f, \Delta t)$$

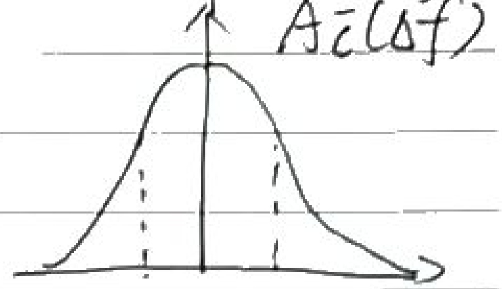
$A_{\tilde{C}}(\Delta f, \Delta t)$: Fourier Transform of $A_c(\tau, \Delta t)$ w.r.t. τ

Let $A_{\bar{c}}(\Delta f) = A_{\bar{c}}(\Delta f, 0)$

$A_c(\tau) = A_c(\tau, 0)$.

$A_{\bar{c}}(\Delta f)$ is the Fourier transform of $A_c(\tau)$





B_c : coherent bandwidth.

$$B_c = 1/\sigma_{T_m}$$

Narrowband : $B \ll B_c$

$\Rightarrow 1/T \ll 1/\sigma_{T_m}$

$\Rightarrow T \gg \sigma_{T_m}$.

- Doppler Power Spectrum

Channel coherent time can be observed from $A_{\bar{c}}(\Delta f = 0, \Delta t) \triangleq A_{\bar{c}}(\Delta t)$