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4.1. Solution: $C = B\log_2(1+\frac{P}{N_0B}) = \frac{\log_2(1+\frac{P}{N_0B})}{\frac{1}{B}}$

令 $f(x) = \log_2(1+\frac{P}{N_0B})$, $g(x) = \frac{1}{B}$, 当 $B\to\infty$, $f(x)\to 0$, $g(x)\to 0$

$\cancel{f'(x)} = \frac{df(x)}{dB} = \frac{1}{(1+\frac{P}{N_0B})\ln 2}\cdot\frac{P}{N_0}\cdot\frac{-1}{B^2} = -\frac{P}{(1+\frac{P}{N_0B})\ln 2\, N_0 B^2}$

$\frac{dg(x)}{dB} = \frac{-1}{B^2}$

$\lim_{B\to\infty} C = \lim_{B\to\infty}\frac{f(x)}{g(x)} = \lim_{B\to\infty}\frac{f'(x)}{g'(x)} = \frac{P}{N_0(1+\frac{P}{N_0B})\ln 2} = \frac{P}{N_0\ln 2}$.

4.2 Solution: $C_0 = B\log_2(1+P/N_0B) = 50\times10^6\log_2(1+\frac{10\times10^{-3}}{2\times10^{-9}\times50\times10^6})$

$\approx 6.87\,\text{Mbps}$

doubling the received power: $P_1 = 20\,\text{mW}$.

$C_1 = B\log_2(1+P_1/N_0B) = 50\times10^6\log_2(1+\frac{20\times10^{-3}}{2\times10^{-9}\times50\times10^6}) \approx 13.15\,\text{Mbps}$

$\Delta C = C_1 - C_0 \approx 6.28\,\text{Mbps}$.

doubling the channel bandwidth: $B_2 = 100\,\text{MHz}$

$C_2 = B_2\log_2(1+P/N_0B) = 100\times10^6\log_2(1+\frac{10\times10^{-3}}{2\times10^{-9}\times100\times10^6}) \approx 7\,\text{Mbps}$.

$\Delta C = C_2 - C_1 \approx 0.13\,\text{Mbps}$.

4. Solution: (a) $C = \int_0^{+\infty} B\log_2(1+\gamma)p(\gamma)\,d\gamma$

$= B[\sum_{i=1}^{6}\log_2(1+\gamma_i)p(\gamma_i)] \approx 2.8831\times B = 57.66\,\text{Mbps}$.

(b) $P_{out} = P(\gamma < \gamma_{min})$, $C_0 = (1-P_{out})B\log_2(1+\gamma_{min})$,

For $\gamma_{min} > 20\,\text{dB}$, $P_{out} = 1$, $C_0 = 0$.

For $15\,\text{dB} < \gamma_{min} < 20\,\text{dB}$, $P_{out} = 0.9$, $C_0 = 0.1\times20\times10^6\log_2(1+\gamma_{min})$;
$\max C_0$ at $\gamma_{min} \approx 20\,\text{dB}$.

Hint: C_0中γ_{min}中的单位不能dB,需要进行换算

$10\,\text{dB} < \gamma_{min} < 15\,\text{dB}$, $P_{out} = 0.75$, $C_0 = 0.25B\log_2(1+\gamma_{min})$,
$\max C_0$ at $\gamma_{min} \approx 15\,\text{dB}$.

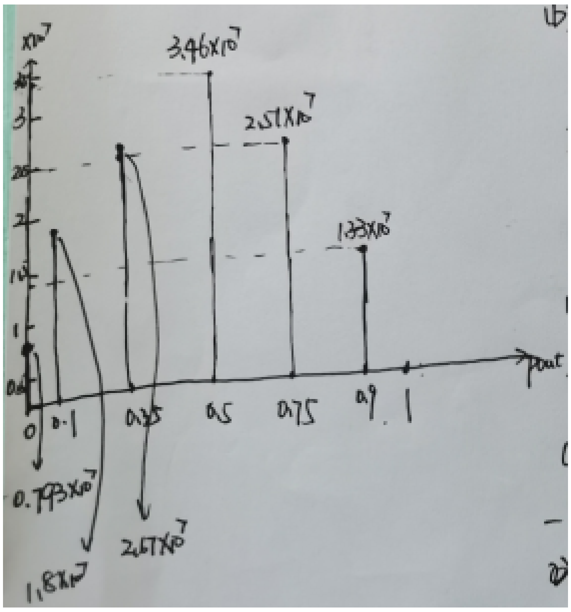
P_{out} $5\,\text{dB} < \gamma_{min} < 10\,\text{dB}$, $P_{out} = 0.5$. $C_0 = 0.5B\log_2(1+\gamma_{min})$ $\max C_0$ at $\gamma_{min} \approx 10\,\text{dB}$

$0\,\text{dB} < \gamma_{min} < 5\,\text{dB}$, $P_{out} = 0.35$, $C_0 = 0.65B\log_2(1+\gamma_{min})$ $\max C_0$ at $\gamma_{min} \approx 5\,\text{dB}$

$-5\,\text{dB} < \gamma_{min} < 0\,\text{dB}$, $P_{out} = 0.1$, $C_0 = 0.9B\log_2(1+\gamma_{min})$, $\max C_0$ at $\gamma_{min} \approx 0\,\text{dB}$

$\gamma_{min} < -5\,\text{dB}$, $P_{out} = 0$, $C_0 = B\log_2(1+\gamma_{min})$, $\max C_0$ at $\gamma_{min} \approx -5\,\text{dB}$.

Maximum at $\gamma_{min} = 10\,\text{dB}$, $P_{out} = 0.5$, $C_0 \approx 3.46\times10\,\text{Mbps}$.



3.46x10^7
2.51x10^7
1.73x10^7
0.793x10^7
2.67x10^7
1.8x10^7
pout
0
0.1
0.25
0.5
0.75
0.9
1