Notes for Lecture 3

September 26, 2022

1 Multipath Fading

Path loss and shadowing refer to the average power loss, multipath fading captures the complete feature of wireless channel.

Let

$$s(t) = s_I(t)cos(2\pi f_c t) - s_Q(t)sin(2\pi f_c t)$$

be the transmission signal with baseband expression

$$u(t) = s_I(t) + js_Q(t).$$

Suppose there are $N(t)$ paths from the transmitter to the receiver at the time instance t, the delay and Doppler phase shift of the n-th path are $\tau_n(t)$ and ϕ_{D_n} respectively.

Then the received signal component of the n-th path can be written as

$$r_n(t) = \alpha_n(t)\left[s_I(t-\tau_n(t))cos(2\pi f_c(t-\tau_n(t)) + \phi_{D_n}) - s_Q(t-\tau_n(t))sin(2\pi f_c(t-\tau_n(t)) + \phi_{D_n})\right]. \tag{1}$$

Its baseband expression can be written as

$$u_n(t) = \alpha_n(t)u(t-\tau_n(t))e^{j(-2\pi f_c\tau_n(t)+\phi_{D_n})}. \tag{2}$$

This is because

$$Re\{u_n(t)e^{j2\pi f_c t}\} = r_n(t). \tag{3}$$

Hence, the impulse response of the n-th path in baseband can be written as

$$\alpha_n(t)\delta(\tau-\tau_n(t))e^{j(-2\pi f_c\tau_n(t)+\phi_{D_n})}.$$

As a result, the time-varying baseband channel impulse response is

$$c(\tau,t) = \sum_{n=1}^{N(t)} \alpha_n(t)e^{j(-2\pi f_c\tau_n(t)+\phi_{D_n})}\delta(\tau-\tau_n(t)), \tag{4}$$

and the baseband received signal is

$$\begin{aligned} u_r(t) &= \int_{-\infty}^{\infty} c(\tau,t)u(t-\tau)d\tau \\ &= \sum_{n=1}^{N(t)} \alpha_n(t)u(t-\tau_n(t))e^{j(-2\pi f_c\tau_n(t)+\phi_{D_n})}. \end{aligned} \tag{5}$$

1.1 Narrowband Model

When the signal duration $T = 1/B$ is much larger than the path delay, the communication system is named as narrowband system. Thus,

$$\max_{m,n} |\tau_m(t) - \tau_n(t)| << T = 1/B. \tag{6}$$

In narrowband system, the baseband received signal can be approximated as

$$\begin{aligned} u_r(t) &\approx \sum_{n=1}^{N(t)} \alpha_n(t) u(t-\tau_0) e^{-j2\pi f_c \tau_n(t) + \phi_{D_n}} \\ &= u(t-\tau_0) \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j2\pi f_c \tau_n(t) + \phi_{D_n}} \end{aligned} \tag{7}$$

Hence, $\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j2\pi f_c \tau_n(t) + \phi_{D_n}}$ is the complex channel gain in narrowband case, and the channel impulse response is

$$c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j2\pi f_c \tau_n(t) + \phi_{D_n}} \delta(\tau - \tau_0). \tag{8}$$

The corresponding frequency response at time t is

$$\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j2\pi f_c \tau_n(t) + \phi_{D_n}} \delta(\tau - \tau_0) \leftrightarrow e^{-j2\pi f \tau_0} \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j2\pi f_c \tau_n(t) + \phi_{D_n}} \tag{9}$$

Flat fading: the channel amplitude gain is constant in frequency domain. Otherwise, the multipath fading is referred to as the **frequency selective fading**.

1.2 Stochastic Fading Models

In narrowband situation, the channel gain is

$$\sum_{n=1}^{N(t)} \alpha_n(t) cos(-j2\pi f_c \tau_n(t) + \phi_{D_n}) + j\alpha_n(t) sin(-j2\pi f_c \tau_n(t) + \phi_{D_n}). \tag{10}$$

Rayleigh Fading: if there is no LoS path, both real and imaginary parts consist of approximately i.i.d. components. According to the central limit theorem, the real and imaginary parts can be treated as Gaussian distributed random variables.

Rician Fading: if there is LoS path.