AWGN Channel.

$$X \longrightarrow \boxed{AWGN} \longrightarrow Y$$

$\uparrow$ Noise

(X, Y) are correlated. R.V.s $\sim p(x, y) = p(x)\,p(y|x)$

$$Y = X + N$$

$\uparrow$ Gaussian Noise.

$$N(0, \frac{N_0 B}{2})$$

Information from X to Y without error.

$$I(X;Y) \triangleq \sum_{x,y} p(x,y) \cdot \log \frac{p(x,y)}{p(x)p(y)}$$

$$= H(Y) - H(Y|X)$$

$$H(Y) \triangleq -\sum_y p(y) \log p(y)$$

$$H(Y|X) \triangleq -\sum_{x,y} p(x,y) \cdot \log p(y|x)$$

Capacity : maximum amount of information from X to Y ~~w/o~~ without error.

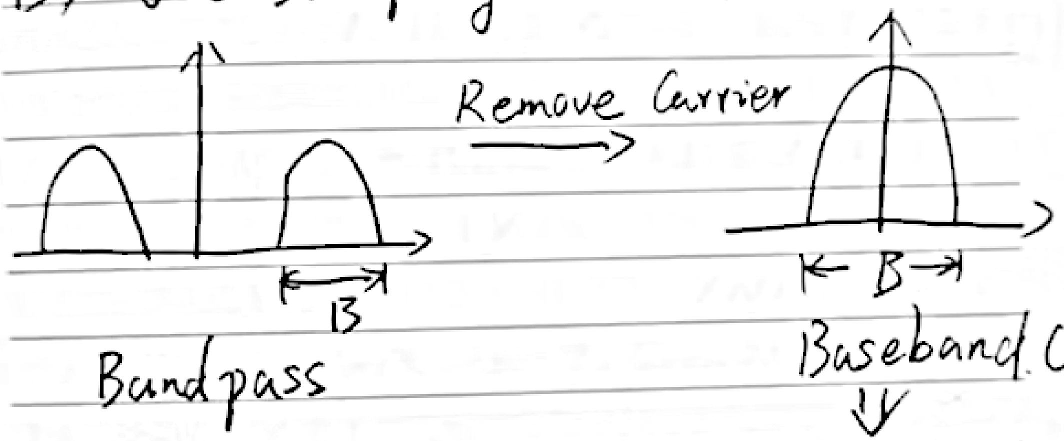
$$C = \max_{p(x)} I(X;Y) = \frac{1}{2} \log_2 (1+\gamma),$$

11 NOV

where γ is the SNR.

$$\gamma = \frac{P/2}{N_0 B/2} = \frac{P}{N_0 B} \left\{ P/2 = E[X^2] \right\}$$

In wireless communications, with bandwidth B, the sampling rate is B.



$\Downarrow$

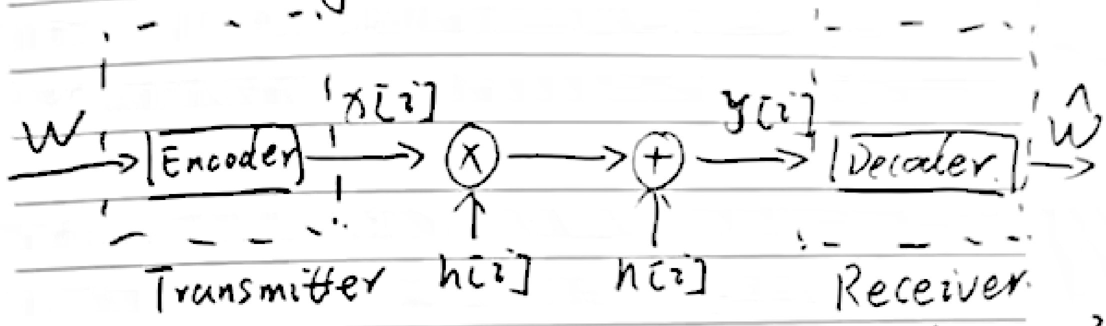
Nyquist Rate $= 2 \cdot \frac{B}{2} = B$

There are B samples per second.
Samples are complex

$\Rightarrow$ There are $2B$ AWGN transmissions per second.

$\Rightarrow$ Error-free bit rate $= 2B \cdot \frac{1}{2} \log_2(1+\gamma)$
(Capacity b/s) $= B \cdot \log_2(1+\gamma)$

Flat Fading Channel



Channel Power Gain $g[i] = |h[i]|^2$

$$y[i] = h[i]\,x[i] + n[i]$$

$\Downarrow$ CSI: Channel State/Side Information

$\Downarrow$ $CN(0, N_0B)$

△ If the CSI is available at the Rx (CSIR)

$$y'[i] = x[i] + n'[i]$$

$y'[i] = y[i]/h[i]$

$n'[i] = n[i]/h[i] \sim CN\left(0, \frac{N_0B}{g[i]}\right)$

⇒ Two AWGNs: Real + Imaginary

In each AWGN,

Signal Power: $E[|x[i]|^2/2] = P/2$

Noise Power: $N_0B/2g[i]$

SNR: $\gamma = P/N_0B/g[i] = g[i]P/N_0B$

Capacity: $\frac{1}{2}\log(1+\frac{g[i]P}{N_0B})$

Capacity with ~~Bandwidty~~ Bandwidth B:

$$C = B\log(1+\frac{g[i]P}{N_0B})$$

in the coherent time of $g[i]$

Δ If CSI is available at the Rx, but not available at the Tx.

$\Rightarrow$ with CSIR, but no CSIT

$\Rightarrow$ Capacity $C = B\log(1+gP/N_0B)$ is feasible.

But Tx doesn't know g and $C = B\log(1+\frac{gP}{N_0B})$

$\Rightarrow$ Tx fixes a data rate

$R = B\cdot\log(1+\gamma_{min})$

$$\begin{cases} \text{If } \gamma = gP/N_0B \geq \gamma_{min} \Rightarrow \text{Error free} \\ \text{If } \gamma < \gamma_{min} \Rightarrow \text{packet loss} \Rightarrow \text{outage} \end{cases}$$

$\gamma = \frac{gP}{N_0 B}$ is random due to random g at Tx.

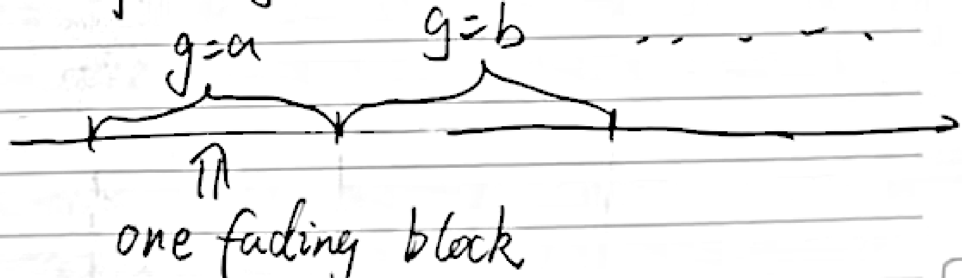
$\Rightarrow \Pr[\gamma_{min} > \gamma]$ outage probability (P_{out}).

$\Rightarrow$ Average data rate $= R \cdot (1 - P_{out})$

$\cancel{R \cdot \Pr[\gamma_{min} > \gamma] = B \log_2(1 + \gamma_{min} \Pr[\gamma_{min} > \gamma])}$

△ CSIR, no CSIT — Ergodic Capacity

Block fading



Transmission Scheme 1:

[packet] [packet] . . . .

$R \le C$: successful $R > C$: outage

Transmission Scheme 2:

[large packet . . .

Average Capacity / Ergodic Capacity

$$\bar{C} = \int_0^\infty B\log(1+\gamma)\cdot p(\gamma)\,d\gamma$$

$$= E[B\log(1+\gamma)]$$

$$\begin{cases} \text{If } R \le \bar{C} \Rightarrow \text{successful transmission} \\ \text{If } R > \bar{C} \Rightarrow \text{packet loss} \end{cases}$$

Although CSIT is not available, the distribution of γ (Channel Distribution Information, CDI) is usually known to the Tx.

$$\Downarrow$$

Tx knows $\bar{C}$

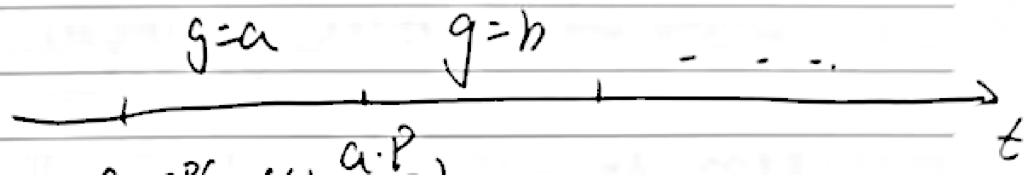
$$\Downarrow$$

$R = \bar{C}$ is alway true.

or $\bar{C}$ is achievable

or data rate is $\bar{C}$

Δ CSIT & CSIR



$C_a = B\log(1+\frac{a\cdot P}{BN_0})$

Because of CSIT: $R = C_a$ $\quad R = C_b$ . . . . .

Average Capacity: $C = \sum_{s\in S} C_s \cdot \underbrace{Pr[g=s]}_{p(s)}$

For continuous R.V. g.

$$C = \int_0^{+\infty} B\log(1+\underbrace{\frac{gP}{BN_0}}_{\gamma})\cdot P(g)\cdot dg$$

$$= \int_0^{\infty} B\log(1+\gamma)\cdot p(\gamma)\cdot d\gamma$$

$$= \int_0^{+\infty} C_\gamma \cdot p(\gamma)\, d\gamma$$

$\Rightarrow$ ~~B~~ With both CSIR & CSIT, ~~erd~~

ergodic capacity is achievable with shorter