chapter-3 习题课

（part1，HW2）

3-1. Consider a two-ray channel consisting of a direct ray plus a ground-reflected ray, where the transmitter is a fixed base station at height h and the receiver is mounted on a truck (also at height h). The truck starts next to the base station and moves away at velocity v. Assume that signal attenuation on each path follows a free-space path-loss model. Find the time-varying channel impulse at the receiver for transmitter–receiver separation $d = vt$ sufficiently large for the length of the reflected ray to be approximated by $r + r' \approx d + 2h^2/d$.

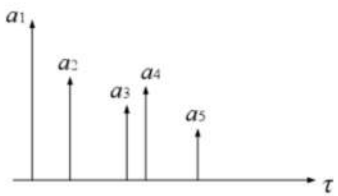
$$c(\tau,t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t))$$

$c(\tau,t)$: impulse response of the channel at time t to the impulse input at time t-τ

$\alpha_n(t)$: random amplitude

$\varphi_n(t)$: random phase

$\tau_n(t)$: random delay



$d = vt$

$r + r' = d + \frac{2h^2}{d}$

Equivalent low-pass channel impulse response is given by

$$c(\tau,t) = \alpha_0(t)e^{-j\phi_0(t)}\delta(\tau - \tau_0(t)) + \alpha_1(t)e^{-j\phi_1(t)}\delta(\tau - \tau_1(t))$$

3-1. Consider a two-ray channel consisting of a direct ray plus a ground-reflected ray, where the transmitter is a fixed base station at height h and the receiver is mounted on a truck (also at height h). The truck starts next to the base station and moves away at velocity v. Assume that signal attenuation on each path follows a free-space path-loss model. Find the time-varying channel impulse at the receiver for transmitter–receiver separation $d = vt$ sufficiently large for the length of the reflected ray to be approximated by $r + r' \approx d + 2h^2/d$.

$$\alpha_0(t) = \frac{\lambda\sqrt{G_l}}{4\pi d} \text{ with } d = vt$$
$$\phi_0(t) = 2\pi f_c \tau_0(t) - \phi_{D_0}$$
$$\tau_0(t) = d/c$$
$$\phi_{D_0} = \int_t 2\pi f_{D_0}(t)dt$$
$$f_{D_0}(t) = \frac{v}{\lambda}\cos\theta_0(t)$$
$$\theta_0(t) = 0 \ \forall t$$

同理，

$$\alpha_1(t) = \frac{\lambda R\sqrt{G_l}}{4\pi(r+r')} = \frac{\lambda R\sqrt{G_l}}{4\pi(d+\frac{2h^2}{d})} \text{ with } d = vt$$
$$\phi_1(t) = 2\pi f_c \tau_1(t) - \phi_{D_1}$$
$$\tau_1(t) = (r + r')/c = (d + \frac{2h^2}{d})/c$$
$$\phi_{D_1} = \int_t 2\pi f_{D_1}(t)dt$$
$$f_{D_1}(t) = \frac{v}{\lambda}\cos\theta_1(t)$$
$$\theta_1(t) = \pi - \arctan\frac{h}{d/2} \ \forall t$$

3-2. Find a formula for the multipath delay spread T_m for a two-ray channel model. Find a simplified formula when the transmitter–receiver separation is relatively large. Compute T_m for $h_t = 10$ m, $h_r = 4$ m, and $d = 100$ m.

For the 2 ray model:

$$\tau_0 = \frac{l}{c}$$

$$\tau_1 = \frac{x + x'}{c}$$

$$\therefore \text{delay spread}(T_m) = \frac{x + x' - l}{c} = \frac{\sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}}{c}$$

when $d \gg (h_t + h_r)$

$$T_m = \frac{1}{c}\frac{2h_t h_r}{d}$$

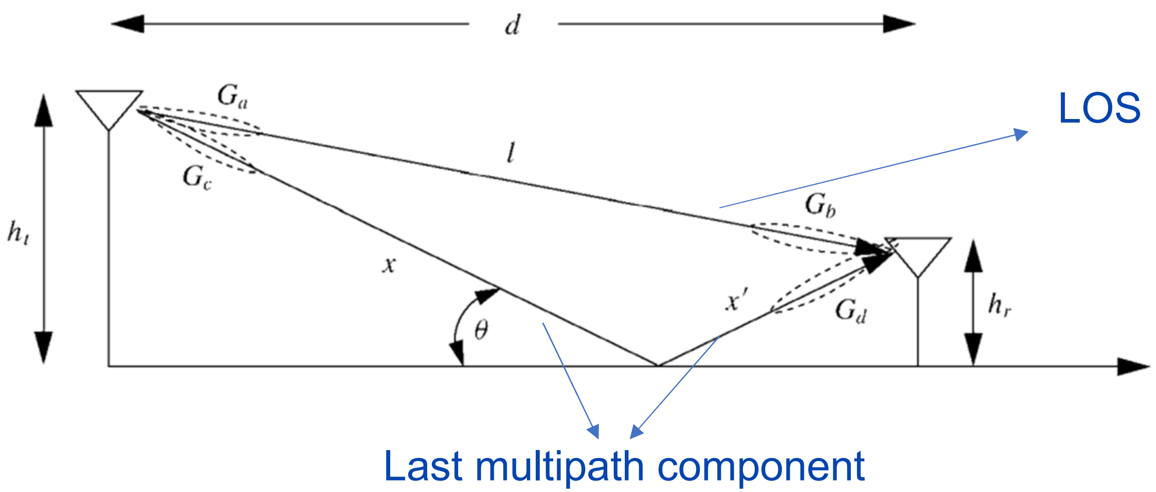
$$h_t = 10m,\ \ h_r = 4m,\ \ d = 100m$$

$$\therefore T_m = 2.67 \times 10^{-9} s$$

3-3. Consider a time-invariant indoor wireless channel with LOS component at delay 23 ns, a multipath component at delay 48 ns, and another multipath component at delay 67 ns. Find the delay spread assuming that the demodulator synchronizes to the LOS component. Repeat assuming that the demodulator synchronizes to the first multipath component.

- Delay spread:
  - Def: difference between the time of arrival of the **earliest** significant multipath component (typically the LOS) and the time of arrival of the **last** multipath components.

---- (from Wiki)



3-3. Consider a time-invariant indoor wireless channel with LOS component at delay 23 ns, a multipath component at delay 48 ns, and another multipath component at delay 67 ns. Find the delay spread assuming that the demodulator synchronizes to the LOS component. Repeat assuming that the demodulator synchronizes to the first multipath component.

Delay for LOS component $= \tau_0 = 23$ ns
Delay for First Multipath component $= \tau_1 = 48$ ns
Delay for Second Multipath component $= \tau_2 = 67$ ns

$\tau_c =$ Delay for the multipath component to which the demodulator synchronizes.

$$T_m = \max_m \tau_m - \tau_c$$

So, when $\tau_c = \tau_0$, $T_m = 44$ ns. When $\tau_c = \tau_1$, $T_m = 19$ ns.