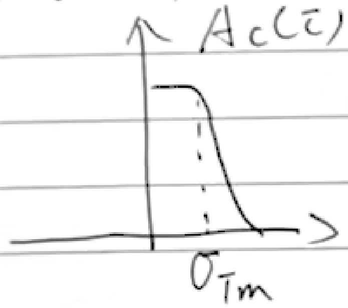
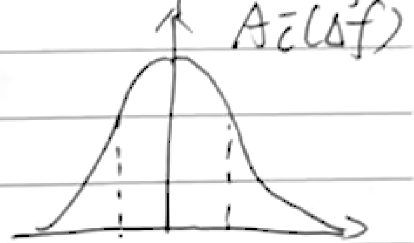
Let $A_{\tau}(\Delta f) = A_{\tau}(\Delta f, 0)$

$A_c(\tau) = A_c(\tau, 0)$.

$A_{\tau}(\Delta f)$ is the Fourier transform of $A_c(\tau)$





B_c : coherent bandwidth

$$B_c = 1/\sigma_{Tm}$$

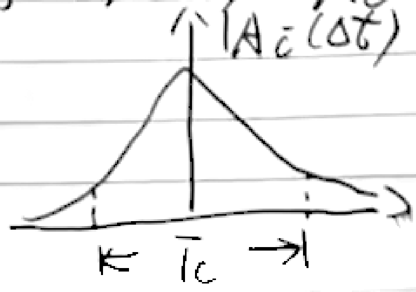
Narrowband : $B \ll B_c$

$\Rightarrow 1/T \ll 1/\sigma_{Tm}$

$\Rightarrow T \gg \sigma_{Tm}$.

$\triangle$ Doppler Power Spectrum

Channel coherent time can be observed from $A_{\tau}(\Delta f = 0, \Delta t) \triangleq A_{\tau}(\Delta t)$



Fourier transform of $A_{\bar{c}}(\Delta f, \Delta t)$. w.r.t. Δt

$$\int_{-\infty}^{+\infty} A_{\bar{c}}(\Delta f, \Delta t)\, e^{-j2\pi\rho\Delta t} \cdot d\Delta t$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_c(\tau, \Delta t)\, e^{-j2\pi\Delta f\cdot\tau}\, e^{-j2\pi\rho\Delta t}\, d\tau \cdot d\Delta t$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_c(\tau, \Delta t)\, e^{-j2\pi\rho\Delta t}\, d\Delta t\, e^{-j2\pi\Delta f\tau}\, d\tau.$$

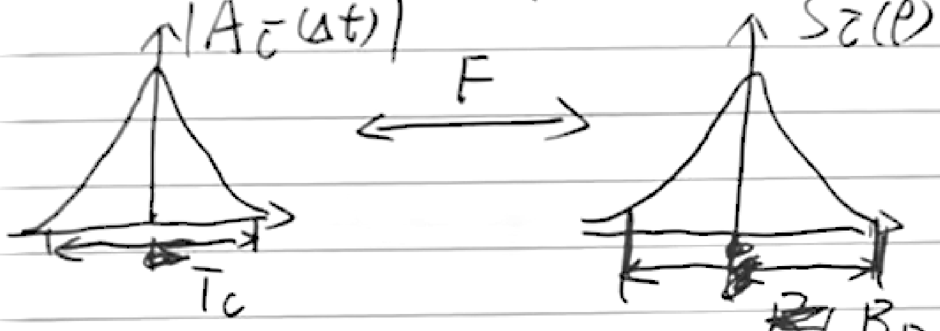
$$= \int_{-\infty}^{+\infty} \underbrace{S_c(\tau, \rho)}_{\text{scattering function.}} \cdot e^{-j2\pi\Delta f\tau}\, d\tau$$

$$\triangleq S_{\bar{c}}(\Delta f, \rho)$$

$$\begin{tikzcd}
S_{\bar{c}}(\Delta f, \rho) \arrow[d, leftrightarrow, "F"] & & \\
A_{\bar{c}}(\Delta f, \Delta t) & & S_c(\tau, \rho) \arrow[uull, bend right, leftrightarrow, "F"'] \\
& A_c(\tau, \Delta t) \arrow[ul, "F"] \arrow[ur, "F"'] & \\
& A_c(\tau_1, \tau_2; t_1, t_2) \arrow[u, Rightarrow, "\text{WSS. uncorrelated scattering}"'] &
\end{tikzcd}$$

Let $A_{\tau}(\Delta t) \triangleq A_{\tau}(\Delta f = 0, \Delta t)$

$S_{\tau}(\rho) \triangleq S_{\tau}(\Delta f = 0, \rho)$ Doppler Power Spectrum



Coherent time   Doppler ~~Delay~~ Spread

$$T_c = 1/\cancel{B_d}\, B_D$$

$$S_{\tau}(\rho) = F_{\tau}\{S_c(\tau, \rho)\}\Big|_{\Delta f = 0}$$

$$= \int_{-\infty}^{+\infty} S_c(\tau, \rho)\, e^{-j2\pi\tau\cdot\Delta f}\, d\tau \Big|_{\Delta f = 0}$$

$$= \int_{-\infty}^{+\infty} S_c(\tau, \rho)\, d\tau.$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A_c(\tau, \Delta t)\, d\tau \cdot e^{-j2\pi\rho\Delta t}\, d\Delta t$$

$$= \int_{-\infty}^{+\infty} \underbrace{\sum_{n=1}^{N(t)} E[d_n^2(t)]\, e^{j2\pi f_{Dn}\Delta t}}_{\text{with all Doppler frequencies.}} \cdot e^{-j2\pi\rho\Delta t}\, d\Delta t.$$

AWGN Channel.

$$X \longrightarrow \boxed{\text{AWGN}} \longrightarrow Y$$

$\uparrow$
Noise

$$Y = X + N$$

(X, Y) are correlated R.V.s $\sim p(x,y) = p(x) \cdot p(y|x)$

N: Gaussian Noise. $N(0, \frac{N_0 B}{2})$

Information from X to Y without error.

$$I(X;Y) \triangleq \sum_{x,y} p(x,y) \cdot \log \frac{p(x,y)}{p(x)p(y)}$$

$$= H(Y) - H(Y|X)$$

$$H(Y) \triangleq -\sum_{y} p(y) \log p(y)$$

$$H(Y|X) \triangleq -\sum_{x,y} p(x,y) \cdot \log p(y|x)$$

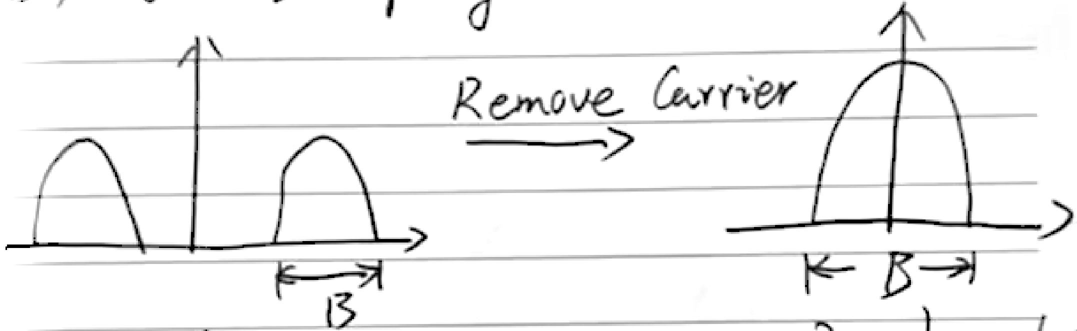
Capacity : maximum amount of information from X to Y ~~w/o~~ without error.

$$C = \max_{p(x)} I(X;Y) = \frac{1}{2} \log_2 (1+\gamma),$$

where γ is the SNR.

$$\gamma = \frac{P/2}{N_0 B/2} = \frac{P}{N_0 B} \quad \left\{ \frac{P}{2} = E[X^2] \right\}$$

In wireless communications, with bandwidth B, the sampling rate is B.



Bandpass

Baseband ($B/2$)

$\Downarrow$

Nyquist Rate $= 2 \cdot \frac{B}{2} = B$

There are B samples per second. }
Samples are complex

$\Rightarrow$ There are $2B$ AWGN transmissions per second.

$\Rightarrow$ Error-free bit rate $= 2B \cdot \frac{1}{2} \log_2(1+\gamma)$
(Capacity b/s) $= B \cdot \log_2(1+\gamma)$