发射机端和接收机端都知道 H.

SVD分解

review: $\vec{y}=H\vec{x}+\vec{n}$ $\quad H=U\Lambda V^H$ $\quad \vec{x}=V\tilde{x}$ $\quad \tilde{y}=U^H\vec{y}$

$\tilde{y}=\Lambda\tilde{x}+\tilde{n}$ $\quad \tilde{n}=U^H\vec{n}$

$\tilde{y}_i=\sigma_i\tilde{x}_i+\tilde{n}_i$, $i=1,2,\cdots,R_H$, $R_H=\text{rank}(H)$.

$E[\vec{n}\vec{n}^H]=R_n=\sigma^2 I=E[\tilde{n}\tilde{n}^H]=R_{\tilde{n}}$, $E|\tilde{n}_i|^2=\sigma^2$

总发射功率: $Tr\, E[\vec{x}\vec{x}^H]\le P$

$\Rightarrow Tr\, E[\tilde{x}\tilde{x}^H]\le P$

$\sum_{i=1}^{R_H} E|\tilde{x}_i|^2\le P$ $\quad E|\tilde{x}_i|^2\triangleq P_i \Rightarrow \sum_{i=1}^{R_H} P_i\le P$

$C=\sum_{i=1}^{R_H} B\log_2\left(1+\sigma_i^2\frac{P_i}{\sigma^2}\right)$

$C^*=\max\sum_{i=1}^{R_H} B\log_2\left(1+\sigma_i^2\frac{P_i}{\sigma^2}\right)$

s.t $\sum_{i=1}^{R_H} P_i\le P$ $\qquad$ water-filling → 课本提取练习.

Lagrangian $J(\lambda,\{P_i\})=\sum_{i=1}^{R_H} B\log_2\left(1+\sigma_i^2\frac{P_i}{\sigma^2}\right)-\lambda\left(\sum_{i=1}^{R_H}P_i-P\right)$

$C^*=\min_{\lambda}\left(\max_{\{P_i\}} J(\lambda,\{P_i\})\right)$ $\qquad \frac{\partial J(\lambda,\{P_i\})}{\partial P_i}=0$

$\Rightarrow P_i=\left(\frac{B}{\lambda\ln 2}-\frac{\sigma^2}{\sigma_i^2}\right)^+=\max\left\{0,\frac{B}{\lambda\ln 2}-\frac{\sigma^2}{\sigma_i^2}\right\}$

$\Rightarrow \frac{\quad}{P}=\left(\frac{B}{P\lambda\ln 2}-\frac{\sigma^2}{P\sigma_i^2}\right)^+$, Let $\gamma_0=\frac{P\lambda\ln 2}{B}$ $\quad \gamma_i=\frac{P\sigma_i^2}{\sigma^2}$

$\Rightarrow \frac{P_i}{P}=\left(\frac{1}{\gamma_0}-\frac{1}{\gamma_i}\right)^+$, $\sum_{i=1}^{R_H}P_i\le P$, $\boxed{\sum_{i=1}^{R_H}\frac{P_i}{P}=1}$ ⇒ 求λ

$\sum_{i=1}^{R_H}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma_i}\right)^+=1 \Rightarrow \lambda=?$

γ_i 一般从大到小排列. $\gamma_1\ge\gamma_2\ge\cdots\ge\gamma_{R_H-1}\ge\gamma_{R_H}$

假设 ① $\gamma_1\ge\cdots\ge\gamma_{R_H}\ge\gamma_0$

$\sum_{i=1}^{R_H}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma_i}\right)=1 \Rightarrow \frac{R_H}{\gamma_0}=1+\sum_{i=1}^{R_H}\frac{1}{\gamma_i} \Rightarrow \gamma_0=\frac{R_H}{1+\sum_{i=1}^{R_H}\frac{1}{\gamma_i}}$

check if $\gamma_0\le\gamma_{R_H}$ $\quad$ yes $\gamma_0 \Rightarrow \lambda=?$

② if no, suppose $\gamma_1\ge\cdots\ge\gamma_{R_H-1}\ge\gamma_0>\gamma_{R_H} \Rightarrow \sum_{i=1}^{R_H-1}\frac{1}{\gamma_0}-\frac{1}{\gamma_i}=1$

$\Rightarrow \frac{R_H-1}{\gamma_0}=1+\sum_{i=1}^{R_H-1}\frac{1}{\gamma_i} \Rightarrow \gamma_0=\frac{R_H-1}{1+\sum_{i=1}^{R_H-1}\frac{1}{\gamma_i}}$, check if $\gamma_{R_H}<\gamma_0\le\gamma_{R_H-1}$

③ if no, suppose $\gamma_1\ge\cdots\ge\gamma_{R_H-2}\ge\gamma_0>\gamma_{R_H-1}\ge\gamma_{R_H}$

△ Space-Time coding. (CSIR only)

$\vec{y}=H\vec{x}+\vec{n}$

$\underbrace{\vec{y}_1\vec{y}_2\cdots\vec{y}_T}_{Y\sim M_r\times T} \Leftarrow \underbrace{\vec{x}_1\vec{x}_2\cdots\vec{x}_T}_{X\sim M_t\times T}$ $\qquad \underbrace{\vec{n}_1,\vec{n}_2\cdots\vec{n}_T}_{N\sim M_r\times T}$

$Y=HX+N$

QPsk: $\mathcal{X}=\{\pm1\pm i\}$ $\quad |\mathcal{X}|=4 \Rightarrow X\sim 4^{M_t\times T}$

Receiver: $H, Y \Rightarrow X=?$ $\qquad \|X\|_F^2$

ML Detector: $\underset{X}{\arg\min}\|Y-HX\|_F^2=\underset{X}{\arg\min}\sum_{i=1}^{T}\|\vec{y}_i-H\vec{x}_i\|^2$

Alamouti Code — 2 Tx antennas

$S_0\ S_1$ (发射机有两根天线,要发两个信号).

$\underset{\vec{x}_1}{\begin{pmatrix}S_0\\S_1\end{pmatrix}}\underset{\vec{x}_2}{\begin{pmatrix}-S_1^*\\S_0^*\end{pmatrix}}$ → 两次传输.

Example: 1 Rx antenna.

$y_1=(h_1\ h_2)\begin{pmatrix}S_0\\S_1\end{pmatrix}+n_1$

$y_2=(h_1\ h_2)\begin{pmatrix}-S_1^*\\S_0^*\end{pmatrix}+n_2 \Rightarrow y_2^*=-h_1^*S_1+h_2^*S_0+n_2^*$

$\Rightarrow y_2^*=(h_2^*\ -h_1^*)\begin{pmatrix}S_0\\S_1\end{pmatrix}+n_2^* \Rightarrow \begin{pmatrix}y_1\\y_2^*\end{pmatrix}=\underbrace{\begin{pmatrix}h_1&h_2\\h_2^*&-h_1^*\end{pmatrix}}_{H}\begin{pmatrix}S_0\\S_1\end{pmatrix}+\begin{pmatrix}n_1\\n_2^*\end{pmatrix}$

$HH^H=(|h_1|^2+|h_2|^2)I$

$\vec{y}=H\begin{pmatrix}S_0\\S_1\end{pmatrix}+\vec{n}$ $\qquad$ 10.1 10.2 10.3.1 10.6.1

$H^H\vec{y}=(|h_1|^2+|h_2|^2)\begin{pmatrix}S_0\\S_1\end{pmatrix}+H^H\vec{n}$ ⟶ 估计 S_0,S_1, $\frac{H^H\vec{y}}{|h_1|^2+|h_2|^2}$

复习:

△ wireless channel.

$\begin{cases}\text{large-scale fading}\begin{cases}\text{path loss}\\\text{shadowing}\end{cases}\\\text{small-scale fading — multi-path}\begin{cases}\text{narrowband} \to \text{Rayleigh}\\\text{wideband}\end{cases}\end{cases}$

$$\begin{tikzcd}
h(t,\tau) \Rightarrow A_c(\tau_1,\tau_2,t_1,t_2) \arrow[r, Rightarrow, "WSS"'] & A_c(\tau,\Delta t) \arrow[dl, "对\Delta t傅立叶"'] \arrow[dr, "对\tau傅立叶"] & \\
S_c(\tau,\rho) \arrow[dr, "对\tau傅立叶"'] & & A_c(\Delta f,\Delta t) \arrow[dl, "对\Delta t傅立叶"] \\
& S_c(\Delta f,\rho) &
\end{tikzcd}$$

△ Channel capacity

AWGN

flat fading $\begin{cases}\text{CSIR}\\\text{CSIR+CSIT}\end{cases}$

△ Digital Modulation

Signal Space

constellation.

AWGN Receiver

ML detection ⇒ minimum distance ⇒ decision region.

Error Probability ⇒ union bound.

△ MIMO.

narrowband MIMO model $\quad \vec{y}=H\vec{x}+\vec{n}$

$\begin{cases}\text{CSIT + CSIR} \Rightarrow \text{SVD}\\\text{CSIR} \Rightarrow \text{space time coding.}\end{cases}$

ML detection.