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4.5 Solution: (a) $\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p(\gamma_i) = 1$.

$\gamma_1 = 30\text{dB} = 10^3$, $\gamma_2 = 20\text{dB} = 10^2$, $\gamma_3 = 10\text{dB} = 10$, $\gamma_4 = 0\text{dB} = 1$.

Assume $\gamma_0 \leq \gamma_4$, $\sum_{\gamma_i > \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p(\gamma_i) = 1$

$\rightarrow \frac{1}{\gamma_0} - \sum_{i=1}^{4} \frac{1}{\gamma_i} p(\gamma_i) = 1 \Rightarrow \gamma_0 \approx 0.8109$

$\rightarrow$ the assumation is true.

$$\frac{P(\gamma_i)}{\bar{P}} = \frac{1}{\gamma_0} - \frac{1}{\gamma_i} = \begin{cases} 1.232, & \gamma_i = \gamma_1 = 10^3 \\ 1.223 & \gamma_i = \gamma_2 = 10^2 \\ 1.133 & \gamma_i = \gamma_3 = 10 \\ 0.2332 & \gamma_i = \gamma_4 = 1 \end{cases}$$

$C = \cancel{\int_{\gamma_0}^{\infty} B\log_2(1+} \quad C = \int_{\gamma_0}^{\infty} B\log_2\left(\frac{\gamma}{\gamma_0}\right) p(\gamma)\, d\gamma$

$= \sum_{i=1}^{4} B\log_2\left(\frac{\gamma_i}{\gamma_0}\right) p(\gamma_i)$

$\rightarrow \frac{C}{B} = \sum_{i=1}^{4} \log_2\left(\frac{\gamma_i}{\gamma_0}\right) p(\gamma_i) = \log_2 \frac{10^3}{0.8109} \times 0.2 + \log_2 \frac{10^2}{0.8109} \times 0.3 + \log_2 \frac{10}{0.8109} \times 0.3 + \log_2 \frac{1}{0.8109} \times 0.2$

≈ 5.2853 bps/Hz.

(b) $\sigma = \frac{1}{E[\frac{1}{\gamma}]} = \frac{1}{0.2\times\frac{1}{10^3} + 0.3\times\frac{1}{10^2} + 0.3\times\frac{1}{10} + 0.2\times 1}$

≈ 4.2882

$$\frac{P(\gamma_i)}{\bar{P}} = \frac{\sigma}{\gamma_i} = \begin{cases} 4.2882\times 10^{-3}, & \gamma_0 = \gamma_1 = 10^3 \\ 4.2882\times 10^{-2} & \gamma = \gamma_2 = 10^2 \\ 4.2882\times 10^{-1} & \gamma = \gamma_3 = 10 \\ 4.2882 & \gamma = \gamma_4 = 1 \end{cases}$$

$C = B\log_2(1+\sigma) \rightarrow \frac{C}{B} = \log_2(1+\sigma) = \log_2(1+4.2882)$

≈ 2.4028 bits/Hz.

4.6 Solution: (a) $\cancel{SNR = \frac{P_r}{P_t} = \left(\frac{d_0}{d}\right)^{\alpha} = (0.1)^{\alpha} =}$

$$SNR = \frac{P_r}{P_{noise}} = \frac{P_t (d_0/d)^{\alpha}}{P_{noise}} = 10^3 (0.1)^{\alpha} = \begin{cases} 10\text{dB}, & \alpha = 2 \\ 5\text{dB}, & \alpha = 2.5 \\ 0\text{dB}, & \alpha = 3 \\ -10\text{dB}, & \alpha = 4 \end{cases}$$

$$\rightarrow SNR = \begin{cases} 10\text{dB}, & p = 0.4, & \gamma_1 = 10 \\ 5\text{dB}, & p = 0.3, & \gamma_2 = 10^{0.5} \\ 0\text{dB}, & p = 0.2, & \gamma_3 = 1 \\ -10\text{dB}, & p = 0.1, & \gamma_4 = 10^{-1} = 0.1 \end{cases}$$

(b) Assume. $\gamma_0 \leq \gamma_4$, $\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p(\gamma_i) = 1$

$\rightarrow \frac{1}{\gamma_0} = 1 + \sum_{i=1}^{4} \frac{1}{\gamma_i} p(\gamma_i) \Rightarrow \gamma_0 \approx 0.4283 > 0.1$; the assumation is false.

Assume $\gamma_4 < \gamma_0 \leq \gamma_3$

$\rightarrow \frac{0.9}{\gamma_0} = 1 + \sum_{i=1}^{3} \frac{1}{\gamma_i} p(\gamma_i) \Rightarrow \gamma_0 \approx 0.67424$; the assumation is true.

$$\frac{P(\gamma_i)}{\bar{P}} = \begin{cases} 1.3832, & \gamma_i = \gamma_1 = 10 \\ 1.1670, & \gamma_i = \gamma_2 = 10^{0.5} \\ 0.4832, & \gamma_i = \gamma_3 = 1 \\ 0, & \gamma_i = \gamma_4 = 0.1 \end{cases}$$

$C = \int_{\gamma_0}^{\infty} B\log_2\left(\frac{\gamma}{\gamma_0}\right) p(\gamma)\, d\gamma = \sum_{i=1}^{3} B\log_2\left(\frac{\gamma_i}{\gamma_0}\right) p(\gamma_i)$

$\rightarrow \frac{C}{B} = \sum_{i=1}^{3} \log_2\left(\frac{\gamma_i}{\gamma_0}\right) p(\gamma_i) \approx 2.3389$ bps/Hz.

(c) $\sigma = \frac{1}{E[\frac{1}{\gamma}]} = \frac{1}{0.4\times\frac{1}{10} + 0.3\times\frac{1}{10^{0.5}} + 0.2\times\frac{1}{1} + 0.1\times\frac{1}{10^{-1}}} \approx 0.7491$

$$\frac{P(\gamma_i)}{\bar{P}} = \frac{\sigma}{\gamma_i} = \begin{cases} 0.07491, & \gamma_i = \gamma_1 = 10 \\ 0.2369, & \gamma_i = \gamma_2 = 10^{0.5} \\ 0.7491, & \gamma_i = \gamma_3 = 1 \\ 7.491, & \gamma_i = \gamma_4 = 0.1 \end{cases}$$

$\rightarrow C = B\log_2(1+\sigma) \Rightarrow \frac{C}{B} = \log_2(1+\sigma) = \log_2(1+0.7491) = 0.8066$ bps/Hz

4.7 Solution (a) $\cancel{p(\gamma) - E(\gamma)}$) $p(\gamma) - E(\gamma)$)

mean $\bar{\gamma} = 10\text{dB} = 10 \Rightarrow p(\gamma) = \frac{1}{10} e^{-\frac{\gamma}{10}}$

$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p(\gamma)\, d\gamma = 1$

$\rightarrow 1 = \frac{1}{\gamma_0} \int_{\gamma_0}^{\infty} p(\gamma)\, d\gamma - \int_{\gamma_0}^{\infty} \frac{1}{\gamma} p(\gamma)\, d\gamma$

$= \frac{1}{10\gamma_0} \int_{\gamma_0}^{\infty} e^{-\frac{\gamma}{10}}\, d\gamma - \frac{1}{10} \int_{\gamma_0}^{\infty} \frac{1}{\gamma} e^{-\frac{\gamma}{10}}\, d\gamma$

$= \frac{1}{\gamma_0} e^{-\frac{\gamma_0}{10}} - \frac{1}{10} \text{EXPINT}\left(\frac{\gamma_0}{10}\right)$

$\rightarrow \gamma_0 = 0.7676$.

$$\frac{\bar{P}(\gamma)}{\bar{P}} = \frac{1}{\gamma_0} - \frac{1}{\gamma} = \begin{cases} \frac{1}{0.7676} - \frac{1}{\gamma}, & \gamma \geq 0.7676 \\ 0, & \gamma < 0.7676 \end{cases}$$

(b) $C = \int_{\gamma_0}^{\infty} B\log_2\left(\frac{\gamma}{\gamma_0}\right) p(\gamma)\, d\gamma = 10\times 10^6 \times \frac{1}{10} \int_{0.7676}^{\infty} \log\left(\frac{\gamma}{0.7676}\right) e^{-\frac{\gamma}{10}}\, d\gamma \approx 2.0649\times 10^7$ nats/sec

(c) AWGN capacity $C = B\log(1+\gamma) = 10^7 \log(1+10) \approx \cancel{3.4594}\ 2.3979\times 10^7$ nats/sec

(d) capacity when only receiver knows γ.

$C = \int_0^{\infty} B\log(1+\gamma) p(\gamma)\, d\gamma = 10^7 \times \frac{1}{10} \int_0^{\infty} \log(1+\gamma) e^{-\frac{\gamma}{10}}\, d\gamma = 2.0150\times 10^7$ nats/sec/Hz.

(e) zero-outage: $\sigma = \frac{1}{E[\frac{1}{\gamma}]} \rightarrow 0 \Rightarrow$ zero-outage capacity is 0.

Truncated: $\int_{\gamma_0}^{\infty} p(\gamma)\, d\gamma = 0.95 \rightarrow \gamma_0 = 0.513$.

$C = B\log\left(1 + \frac{1}{E_{\gamma_0}[\frac{1}{\gamma}]}\right) p(\gamma \geq \gamma_0) = 10^7 \log\left[1 + \frac{1}{\int_{\gamma_0}^{\infty} \frac{1}{\gamma} p(\gamma)\, d\gamma}\right] \cdot 0.95$

$\approx 1.5463\times 10^7$ nats/s.

(f) $\bar{\gamma} = 5\text{dB}$.

for perfect transmitter and receive side information, $\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p(\gamma)\, d\gamma = 1 \Rightarrow \gamma_0 = 0.22765$.

$C = \int_{\gamma_0}^{\infty} B\log\left(\frac{\gamma}{\gamma_0}\right) p(\gamma)\, d\gamma = 3.6\times 10^6$ nats/sec.

for AWGN, $C = B\log(1+10^{0.5}) = 2.7148\times 10^6$ nats/sec.

for just receiver side information: $C = \int_0^{\infty} B\log(1+\gamma) p(\gamma)\, d\gamma = 2.510\times 10^6$ nats/sec.

capacity with AWGN is always greater than or equal to the capacity when only the receiver knows the channel. This can be shown using Jensen's inequality. However, the capacity when the transmitter knows the channel as well and can adopt its power, can be higher than AWGN capacity specially at low SNR. At low SNR, the knowledge of fading helps to use the low SNR more efficiently.