Lecture 5 Digital Modulation and Detection

EE313/EE5028 Wireless Communications

Dr. WANG Rui @ SUSTech

Review of Last Lecture

- Capacity of AWGN channel
- Fundamental capacity of flat-fading channels depends on what is known at TX and RX.
    - Capacity with TX/RX CSI: variable-rate variable-power transmission (water-filling) optimal
    - Almost same capacity as with RX CSI only
    - Channel inversion practical, but should truncate
- Capacity of wideband channel obtained by breaking up channel into subbands
    - Similar to multicarrier modulation

Overview

- Signal Space Analysis
  - Signal and system model, Geometric representations of signals, Receiver structure and sufficient statistics, Decision regions and ML decision criterion, Error probability and the union bound
- Passband Modulation Principles
- Amplitude and Phase Modulation
  - Pulse amplitude modulation (PAM), Phase shift keying (PSK), Quadrature amplitude modulation (QAM), Differential modulation, Constellation shaping, Quadrature offset
- Frequency Modulation
  - Frequency shift keying (FSK) and minimum shift keying (MSK), Continuous-phase FSK (CPFSK), Noncoherent detection of FSK
- Pulse Shaping
- Symbol Synchronization and Carrier Phase Recovery

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What is digital modulation and detection?

Digital modulation consists of mapping the information bits into an analog signal for transmission over the channel.

Detection consists of determining the original bit sequence based on the signal received over the channel.

Advantages of digital modulation and detection over analog modulation:

- higher data rates
- high spectral efficiency (minimum bandwidth occupancy)
- high power efficiency (minimum required transmit power)
- robustness to channel impairments (minimum probability of bit error)
- better security and privacy
- low power/cost implementation
- etc.

Two main types of digital modulation: **constant envelope modulation** or **nonlinear modulation**, and **linear modulation**.

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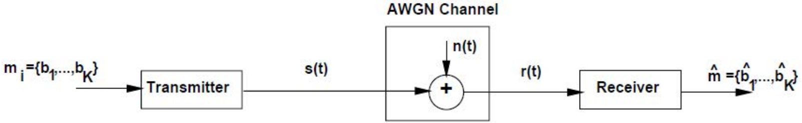
Signal and System Model

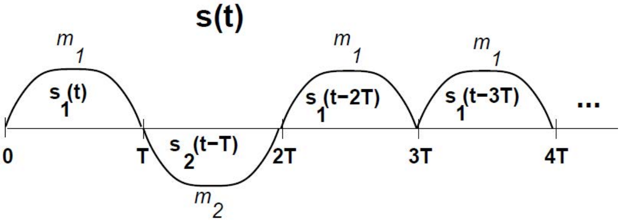
Consider a communication system sends $K = \log_2 M$ bits every T seconds, thus the data rate is $R = \frac{K}{T}$ bps. There are $M = 2^K$ possible sequences of K bits, $\mathcal{M} = \{m_1, \dots, m_M\}$ and $m_i = \{b_1, \dots, b_K\} \in \mathcal{M}$, and $\sum_{i=1}^{M} p_i = 1$.



Communication System Model.

m_i must be embedded into an analog signal for channel transmission, i.e., mapped to a unique analog signal $s_i(t) \in \mathcal{S} = \{s_1(t), \dots, s_M(t)\}$ in $[0, T)$, with energy $E_{s_i} = \int_0^T s_i^2(t)dt$. Detection of $s_i(t)$ at RX is equivalent to detection of m_i.

The transmitted signal is a sequence of analog signals over each time interval $[kT, (k+1)T)$, and $s(t) = \sum_k s_i(t - kT)$.



$$s(t) = s_1(t) + s_2(t - T) + s_1(t - 2T) + s_1(t - 3T).$$

Signal received by RX is $r(t) = s(t) + n(t)$ for AWGN, and RX determines the best estimate of which $s_i(t)$ was sent during $[kT, (k+1)T)$, and maps to a best estimate of $m_i(t)$. The goal of RX design in estimating $m_i(t)$ is to minimize the message probability error over each $[kT, (k+1)T)$:

$$P_e = \sum_{i=1}^{M} p(\hat{m} \neq m_i | m_i \text{ sent}) p(m_i \text{ sent}).$$

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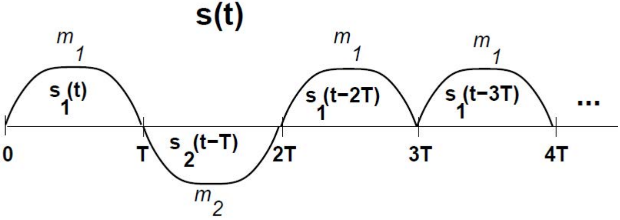

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Geometric Representation of Signals

M real energy signals $\mathcal{S} = \{s_1(t), ..., s_M(t)\}$ defined on $[0, T)$ can be represented as a linear combination of $N \leq M$ real **orthonormal** basis functions $\Phi = \{\phi_1(t), ..., \phi_N(t)\}$, i.e., Φ spans $\mathcal{S}$. We can use basis function representation to write each $s_i(t) \in \mathcal{S}$

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t),\ 0 \leq t < T$$

where $s_{ij} = \int_0^T s_i(t)\phi_j(t)dt$ is a real coefficient representing the projection of $s_i(t)$ onto the basis function $\phi_j(t)$ and

$$\int_0^T \phi_i(t)\phi_j(t)dt = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$$

$\{s_i(t)\}$ are linearly independent, $N = M$, otherwise $N < M$. The minimum N needed to represent any $s_i(t)$ of duration T and bandwidth B is roughly $2BT$ (**signal space dimension**).

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For linear passband modulation, Φ consists of the sine and cosine functions

$$\phi_1(t) = \sqrt{\tfrac{2}{T}}\cos(2\pi f_c t),\ \phi_2(t) = \sqrt{\tfrac{2}{T}}\sin(2\pi f_c t)$$

Thus, the complex passband representation of $s_i(t)$ in terms of its in-phase and quadrature components is given by

$$s_i(t) = s_{i1}\sqrt{\tfrac{2}{T}}\cos(2\pi f_c t) + s_{i2}\sqrt{\tfrac{2}{T}}\sin(2\pi f_c t)$$

Φ may have an initial phase offset ϕ_0, and may also include a baseband pulse-shaping filter $g(t)$ to improve the spectral characteristics, i.e.,

$$s_i(t) = s_{i1}g(t)\cos(2\pi f_c t) + s_{i2}g(t)\sin(2\pi f_c t)$$

$g(t)$ must maintain the orthonormal properties, i.e.,

$$\int_0^T g^2(t)\cos^2(2\pi f_c t)dt = 1,\ \int_0^T g^2(t)\cos(2\pi f_c t)\sin(2\pi f_c t)dt = 0$$

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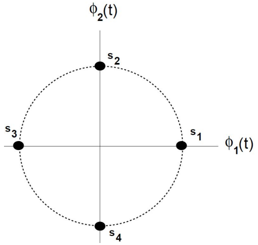
Example 1.

Binary phase shift keying (BPSK) transmits the signal $s_1(t) = \alpha\cos(2\pi f_c t)$ to send "1", and $s_2(t) = -\alpha\cos(2\pi f_c t)$ to send "0", $0 \le t < T$. Find the set of orthonormal basis functions and coefficients $\{s_{ij}\}$.

Solution: $\Phi = \phi(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$, $\therefore$ $s_1 = \alpha\sqrt{\frac{T}{2}}$ and $s_2 = -\alpha\sqrt{\frac{T}{2}}$.

Denote $\{s_{ij}\}$ as $\mathbf{s}_i = \{s_{i1}, ..., s_{iN}\} \in \mathcal{R}^N$, called signal constellation point corresponding to $s_i(t)$. The signal constellation consists of all constellation points $\{\mathbf{s}_1, ..., \mathbf{s}_M\}$, also called **signal space**. Given Φ, it is a one-to-one mapping between $s_i(t)$ and $\mathbf{s}_i$ (**signal space representation**).



A 2-D signal space $\mathbf{s}_i \in \mathcal{R}^2$, with the ith axis corresponding to $\phi_i(t)$, $i = 1, 2$. We can analyze the infinite-dimensional functions of $s_i(t)$ as vectors $\mathbf{s}_i$ in finite-dimensional vector space $\mathcal{R}^2$. MPSK and MQAM are two-dimensional.

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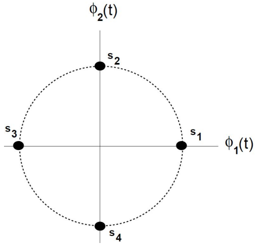

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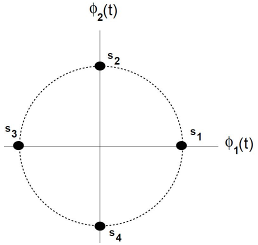

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The length of a vector in $\mathcal{R}^N$ is defined as

$$\| \mathbf{s}_i \| = \sqrt{\sum_{j=1}^{N} s_{ij}^2}.$$

The distance between $\mathbf{s}_i$ and $\mathbf{s}_k$ is thus

$$\| \mathbf{s}_i - \mathbf{s}_k \| = \sqrt{\sum_{j=1}^{N} (s_{ij} - s_{kj})^2} = \sqrt{\int_0^T (s_i(t) - s_j(t))^2 dt}.$$

The inner product $\langle s_i(t), s_k(t) \rangle$ of $s_i(t)$ and $s_k(t)$ on $[0, T)$ is

$$\langle s_i(t), s_j(t) \rangle = \int_0^T s_i(t) s_k(t) dt.$$

The inner product between real vectors $\langle \mathbf{s}_i, \mathbf{s}_k \rangle = \mathbf{s}_i \mathbf{s}_k^T = \langle s_i(t), s_j(t) \rangle$.
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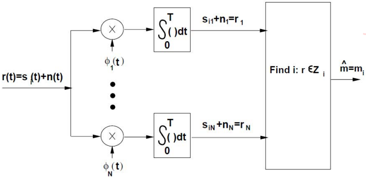
Receiver Structure and Sufficient Statistics

RX determines which $\mathbf{s}_i$, or m_i was sent on $[0, T)$ (a similar procedure is done for $[kT, (k+1)T)$), via converting $r(t)$ over each T into a vector.



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$$\sum_{j=1}^{N}(s_{ij}+n_j)\phi_j(t)+n_r(t) = \sum_{j=1}^{N} r_j\phi_j(t)+n_r(t),$$

$n_r(t) = n(t) - \sum_{j=1}^{N} n_j\phi_j(t)$ denotes the "remainder" noise ($\perp$ **signal space**).

If the optimal detection of $\mathbf{s}_i$ given $r(t)$ does not make use of $n_r(t)$, then RX can make $\hat{m}$ of m_i as a function of $\mathbf{r} = (r_1, ..., r_N)$, called a **sufficient statistic** for $r(t)$ in the optimal detection of m_i.

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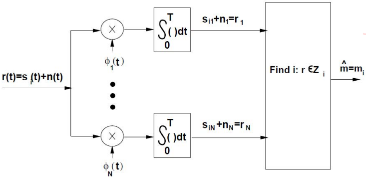

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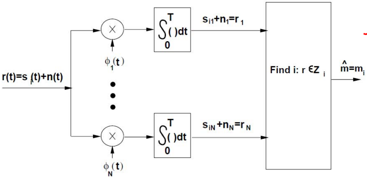

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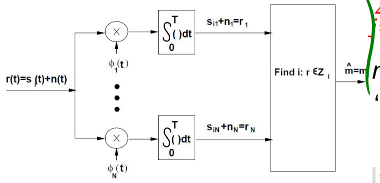

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$\triangle$ $\phi = \{\phi_1(t), \phi_2(t), \cdots, \phi_N(t)\}$

$t \in [0, T)$

$<\phi_i(t), \phi_j(t)> = \int_0^T \phi_i(t)\phi_j(t)dt$

$= \begin{cases} 1, & \text{if } i=j \\ 0, & \text{if } i \neq j \end{cases}$

$\{s_1(t), s_2(t), \cdots S_m(t)\}$

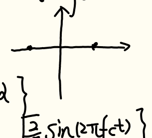
$\forall i$, $S_i(t) = \sum_{j=1}^{N} S_{ij}\phi_j(t)$

$S_i(t) \Rightarrow (S_{i1}, S_{i2}, \cdots, S_{iN}) \triangleq \vec{S_i}$

$\|\vec{S_i}\|$ $\|\vec{S_i} - \vec{S_j}\|$

Example. BPSK. $\pm \alpha\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$

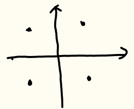
$\phi : \{\sqrt{\frac{2}{T}}\cos(2\pi f_c t)\}$



$\{\vec{S_1} = S_1 = \alpha.$ $\vec{S_2} = S_2 = -\alpha\}$

QPSK: $\phi = \{\sqrt{\frac{2}{T}}\cos(2\pi f_c t), \sqrt{\frac{2}{T}}\sin(2\pi f_c t)\}$

$\begin{cases} \vec{S_1} = (\alpha, \alpha) & \vec{S_3} = (-\alpha, \alpha) \\ \vec{S_2} = (\alpha, -\alpha) & \vec{S_4} = (-\alpha, -\alpha) \end{cases}$



$\triangle$ Rx Structure

AWGN.

if m_i is sent $r(t) = S_i(t) + n(t)$ → Gaussian Noise.

$E[n(t)] = 0$, $E[n(t)n(\tau)] = \frac{N_0}{2}\delta(t-\tau)$

$r_j = <r(t), \phi_j(t)> = \int_0^T r(t)\phi_j(t)dt = \int_0^T S_i(t)\phi_j(t)dt + \int_0^T n(t)\phi_j(t)dt$

$n(t) = \sum_{j=1}^{N} n_j\phi_j(t) + n_r(t)$, $n_j = <n(t), \phi_j(t)>$

$n_j = \int_0^T n(t)\phi_j(t)dt = \int_0^T [\sum_{k=1}^{N} n_k\phi_k(t)]\phi_j(t)dt + \int_0^T n_r(t)\phi_j(t)dt$

$= n_j + <n_r(t), \phi_j(t)>.$

$\Rightarrow <n_r(t), \phi_j(t)> = 0$, $\forall j$ $n_r(t) \perp \phi_j(t)$

$r_j = S_{ij} + n_j.$

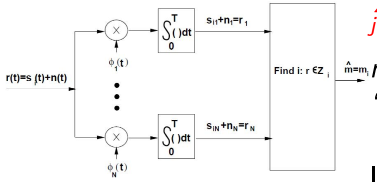
$r(t) = \sum_{j=1}^{N} S_{ij}\phi_j(t) + \sum_{j=1}^{N} n_j\phi_j(t) + n_r(t)$

$= \sum_{j=1}^{N}(S_{ij}+n_j)\phi_j(t) + n_r(t) = \sum_{j=1}^{N} r_j(t)\phi_j(t) + n_r(t)$

→ $\vec{r} = (r_1, r_2, \cdots r_N)$, $\vec{r}$ is a sufficient statistic for $r(t)$.

Receiver Structure and Sufficient Statistics

RX determines which $\mathbf{s}_i$, or m_i was sent on $[0, T)$ (a similar procedure is done for $[kT, (k+1)T)$), via converting $r(t)$ over each T into a vector.



$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt,\ n_j = \int_0^T n(t)\phi_j(t)dt,$$

$$\sum_{j=1}^{N}(s_{ij}+n_j)\phi_j(t)+n_r(t) = \sum_{j=1}^{N} r_j\phi_j(t)+n_r(t),$$

$n_r(t) = n(t) - \sum_{j=1}^{N} n_j\phi_j(t)$ denotes the "remainder" noise ($\perp$ **signal space**).

If the optimal detection of $\mathbf{s}_i$ given $r(t)$ does not make use of $n_r(t)$, then RX can make $\hat{m}$ of m_i as a function of $\mathbf{r} = (r_1, ..., r_N)$, called a **sufficient statistic** for $r(t)$ in the optimal detection of m_i.

$\vec{r} = (r_1, r_2, \cdots r_N)$ $r_j = s_{ij} + n_j$.

assume m_i is sent.

$n_j = \int_0^T n(t)\phi_j(t)dt$, $E[n_j] = 0$.

$E[n_j^2] = E[\int_0^T\int_0^T n(t)\phi_j(t)n(\tau)\phi_j(\tau)dt d\tau]$

$= \int_0^T\int_0^T \frac{N_0}{2}\delta(t-\tau)\phi_j(t)\phi_j(\tau)d\tau dt = \frac{N_0}{2}$

$n_j \sim N(0, \frac{N_0}{2})$

$Conv[n_j, n_k] = E[n_j, n_k]$.

$= \int_0^T\int_0^T \frac{N_0}{2}\delta(t-\tau)\phi_j(t)\phi_k(\tau)dt d\tau$.

$= \int_0^T \frac{N_0}{2}\phi_j(\tau)\phi_k(\tau)d\tau = 0$, for $j \neq k$.

$r_j | m_i \sim N(s_{ij}, \frac{N_0}{2})$

$Cov[r_j, r_k] = E[(r_j - s_{ij})(r_k - s_{ik})]$

$$= \begin{cases} \frac{N_0}{2}, & j = k \\ 0, & j \neq k. \end{cases}$$

$p(\vec{r}|m_i) = \prod_{j=1}^{N} p(r_j|m_i)$

$= \prod_{j=1}^{N} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(r_j - s_{ij})^2}$

$= \frac{1}{(\pi N_0)^{\frac{N}{2}}} e^{-\frac{1}{N_0}\|\vec{r} - \vec{s}_i\|^2}$

$p(m_i \text{ is sent}|\vec{r}) = \frac{P(m_i, \vec{r})}{P(\vec{r})} \cdot \frac{P(m_i)}{P(m_i)} = p(\vec{r}|m_i) \cdot \frac{P(m_i)}{p(\vec{r})}$

optimal Rx. $= \arg\max_i p(m_i|\vec{r})$, $i = 1, \cdots, m$.

$= \arg\max_i p(\vec{r}|m_i)\frac{P(m_i)}{P(\vec{r})} = \arg\max_i p(\vec{r}|m_i)\frac{1}{m}$.

发射该符号的概率

$= \arg\max_i p(\vec{r}|m_i) \rightarrow$ Maximum likehood (ML).

likihood function $L(m_i) = L(\vec{s}_i) \triangleq$

Since $n(t)$ is Gaussian, $r(t) = s_i(t) + n(t)$ and $\mathbf{r} = (r_1, ..., r_N)$ are Gaussian. Recall $r_j = s_{ij} + n_j$, conditioned on a transmitted constellation $\mathbf{s}_i$, we have

$$\mu_{r_j|\mathbf{s}_i} = E[r_j|\mathbf{s}_i] = E[s_{ij} + n_j|s_{ij}] = s_{ij},$$
$$\sigma^2_{r_j|\mathbf{s}_i} = E[r_j - \mu_{r_j|\mathbf{s}_i}]^2 = E[s_{ij} + n_j - s_{ij}|s_{ij}]^2 = E[n_j^2] = \frac{N_0}{2},$$

and

$$Cov[r_j r_k|\mathbf{s}_i] = E[(r_j - \mu_{r_j})(r_k - \mu_{r_k})|\mathbf{s}_i] = \begin{cases} \frac{N_0}{2}, j = k \\ 0, j \neq k \end{cases}$$

Thus, $r_j \sim \mathcal{N}(s_{ij}, \frac{N_0}{2})$, and the conditional distribution of $\mathbf{r}$ is given by

$$p(\mathbf{r}|\mathbf{s}_i \text{ sent}) = \prod_{j=1}^{N} p(r_j|m_j) = \frac{1}{(\pi N_0)^{\frac{N}{2}}} \exp[-\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2]$$

It can also be shown that r_j conditioned on $\mathbf{s}_i$ (so as s_{ij}) and $n_r(t)$ are Gaussian and uncorrelated, and therefore independent.

$\{m_1, m_2, \cdots, m_M\}$

$\Downarrow$

$\{s_1(t), s_2(t), \cdots, s_M(t)\}$

$\vec{s}_1, \vec{s}_2, \cdots, \vec{s}_M$

Assume the channel is AWGN.

$r(t) = s(t) + n(t)$, $r(t) = h(t)s(t) + n(t)$

$\Rightarrow \vec{r} = \vec{s}_i + \vec{n}$

optimal: $\hat{i} = \arg\max_i p(m_i|\vec{r})$

if transmitter $p(m_i) = \frac{1}{M}$ (重要前提).

then $\arg\max_i p(m_i|\vec{r})$ 等价于 $\arg\max_i p(\vec{r}|m_i)$

(likelyhood function 似然)

ML: $L(\vec{s}_i) = p(\vec{r}|m_i) = \frac{1}{(\pi N_0)^{\frac{N}{2}}} e^{\wedge}\{-\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2\}$

$\Rightarrow L(\vec{s}_1)\ L(\vec{s}_2) \cdots L(\vec{s}_M)$

log-likelyhood function.

$l(\vec{s}_i) \triangleq -\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2 = -\frac{1}{N_0}\|\vec{r} - \vec{s}_i\|^2 \longrightarrow$ ML: $\arg\max_i L(\vec{s}_i) = \arg\max_i l(\vec{s}_i) = \arg\min_i \|\vec{r} - \vec{s}_i\|^2$

the distance between $\vec{r}$ and $\vec{s}_i$.

Since $n(t)$ is Gaussian, $r(t) = s_i(t) + n(t)$ and $\mathbf{r} = (r_1, ..., r_N)$ are Gaussian. Recall $r_j = s_{ij} + n_j$, conditioned on a transmitted constellation $\mathbf{s}_i$, we have

$$\mu_{r_j|\mathbf{s}_i} = E[r_j|\mathbf{s}_i] = E[s_{ij} + n_j|s_{ij}] = s_{ij},$$

$$\sigma^2_{r_j|\mathbf{s}_i} = E[r_j - \mu_{r_j|\mathbf{s}_i}]^2 = E[s_{ij} + n_j - s_{ij}|s_{ij}]^2 = E[n_j^2] = \frac{N_0}{2},$$

and

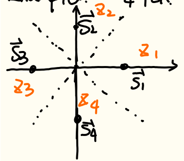
$$Cov[r_j r_k|\mathbf{s}_i] = E[(r_j - \mu_{r_j})(r_k - \mu_{r_k})|\mathbf{s}_i] = \begin{cases} \frac{N_0}{2}, j = k \\ 0, j \neq k \end{cases}$$

Thus, $r_j \sim \mathcal{N}(s_{ij, \frac{N_0}{2}})$, and the conditional distribution of $\mathbf{r}$ is given by

$$p(\mathbf{r}|\mathbf{s}_i \text{ sent}) = \prod_{j=1}^{N} p(r_j|m_j) = \frac{1}{(\pi N_0)^{\frac{N}{2}}} \exp[-\frac{1}{N_0} \sum_{j=1}^{N} (r_j - s_{ij})^2]$$

It can also be shown that r_j conditioned on $\mathbf{s}_i$ (so as s_{ij}) and $n_r(t)$ are Gaussian and uncorrelated, and therefore independent.

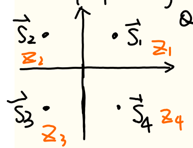
Example: 4-PSK



decision region

$z_i \triangleq \{\vec{r} \,|\, \|\vec{r}-\vec{s}_j\| \geq \|\vec{r}-\vec{s}_i\|, \forall j \neq i\}$

QPSK



△Error Prob & Union Bound.

$P_e = \sum_{i=1}^{M} P_r(m_i) P_r(\vec{r} \notin z_i | m_i)$

if $P_r(m_i) = \frac{1}{M}$ (重要前提)

$= \frac{1}{M} \sum_{i=1}^{M} P_r(\vec{r} \notin z_i | m_i)$

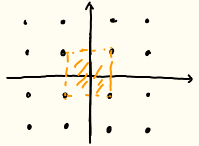
$= 1 - \frac{1}{M} \sum_{i=1}^{M} P_r(\vec{r} \in z_i | m_i)$

$= 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{z_i} p(\vec{r}|m_i) d\vec{r}$

$\because \vec{r} - \vec{s}_i \sim N(0$

$\therefore$ let $\vec{n} = \vec{r} - \vec{s}_i \Rightarrow = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{z_i - \vec{s}_i} p(\vec{n}) dn.$

16-QAM



$\int_{z_i - \vec{s}_i} p(\vec{n}) dn$

To minimize $P_e = p(\hat{m} \neq m_i | r(t)) = 1 - p(\hat{m} = m_i | r(t))$, we maximize $p(\hat{m} = m_i | r(t))$. Since

?

$$\begin{aligned} p(\mathbf{s}_i \text{ sent} | r(t)) &= p((s_{i1}, \dots, s_{iN}) \text{ sent} | (r_1, \dots, r_N, n_r(t))) \\ &= \frac{p((s_{i1}, \dots, s_{iN} \text{ sent}), r_1, \dots, r_N, n_r(t))}{p(r_1, \dots, r_N, n_r(t))} \\ &= \frac{p((s_{i1}, \dots, s_{iN}) \text{ sent}, r_1, \dots, r_N) p(n_r(t))}{p(r_1, \dots, r_N) p(n_r(t))} \\ &= p((s_{i1}, \dots, s_{iN}) \text{ sent} | (r_1, \dots, r_N)) \end{aligned}$$

i.e., $\mathbf{r}$ is a sufficient statistic for $r(t)$ detecting m_i, also called **received vector** associated with $r(t)$.

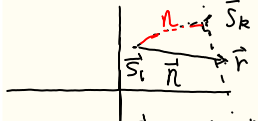
Union Bound: upper bound.

A_{ik}: Given $\vec{s}_i$, exist $\vec{s}_k$, make $\|\vec{s}_i - \vec{r}\| > \|\vec{s}_k - \vec{r}\|$

$P_e(m_i) = P_r[\bigcup_{k \neq i} A_{ik}] \leq \sum_{k \neq i} P_r[A_{ik}]$

交集

$P_r[A \cup B] \leq P_r(A) + P_r(B)$.



$\vec{n} = (n_1, n_2, \cdots n_i)$

$N(0, \frac{N_0}{2})$

Noise along $\vec{s}_k - \vec{s}_i$

$\sim N(0, \frac{N_0}{2})$

let $d_{ik} = \|\vec{s}_i - \vec{s}_k\|$

$P_r[A_{ik}] = P_r[n > \frac{d_{ik}}{2}] = Q(\frac{d_{ik}}{\sqrt{2N_0}})$

$\Rightarrow P_e(m_i) \leq \sum_{k \neq i} Q(\frac{d_{ik}}{\sqrt{2N_0}})$

$P_e(m_i) \leq \sum_{k \neq i} Q(\frac{d_{ik}}{\sqrt{2N_0}}) \leq \frac{1}{M} \underbrace{\sum_{i=1}^{M} \sum_{i \neq k} Q(\frac{d_{ik}}{\sqrt{2N_0}})}_{M(M-1)}$

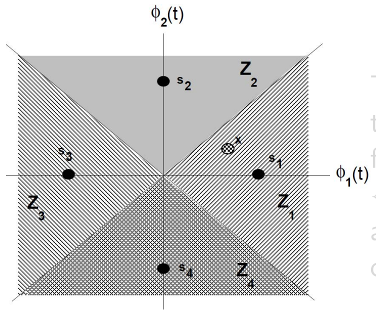
再放缩 $d_{min} = \min_{i,k} d_{ik}$

$\leq (M-1) Q(\frac{d_{min}}{\sqrt{2N_0}})$

Decision Regions and the Maximum Likelihood Criterion

RX selects $\hat{m} = m_i$ (or $\mathbf{s}_i$), so that $p(\mathbf{s}_i \text{ sent}|\mathbf{r}) > p(\mathbf{s}_j \text{ sent}|\mathbf{r})$, $\forall j \neq i$.
Define a set of decisions regions $\{Z_1, ..., Z_M\} \subset \mathcal{R}^N$ by

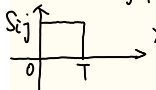
$$Z_i = (\mathbf{r} : p(\mathbf{s}_i \text{ sent}|\mathbf{r}) > p(\mathbf{s}_j \text{ sent}|\mathbf{r}) \forall j \neq i)$$



These regions do not overlap, they partition the signal space assuming there is no $\mathbf{r} \in \mathcal{R}^N$ for which $p(\mathbf{s}_i \text{ sent}|\mathbf{r}) = p(\mathbf{s}_j \text{ sent}|\mathbf{r})$. If so, $\{\mathbf{s}_i\}$ is partitioned with decision regions by arbitrarily assigning such points to either Z_i or Z_j.

△ Pulse Shaping

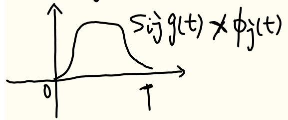
$S_i(t) = \sum_{j=1}^{N} S_{ij}\phi_j(t), t \in (0,T)$



$\times\ \phi_j(t)$

pulse shaping.

$S_i(t) = \sum_{j=1}^{N} S_{ij}\, \underline{g(t)\phi_j(t)}$



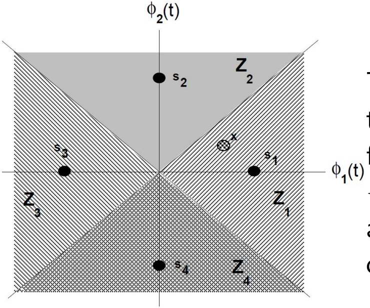
△ 5.2 bandpass modulation.

Fig 5.7 match filter.

Decision Regions and the Maximum Likelihood Criterion

RX selects $\hat{m} = m_i$ (or $\mathbf{s}_i$), so that $p(\mathbf{s}_i \text{ sent}|\mathbf{r}) > p(\mathbf{s}_j \text{ sent}|\mathbf{r})$, $\forall j \neq i$.
Define a set of decisions regions $\{Z_1, ..., Z_M\} \subset \mathcal{R}^N$ by

$$Z_i = (\mathbf{r} : p(\mathbf{s}_i \text{ sent}|\mathbf{r}) > p(\mathbf{s}_j \text{ sent}|\mathbf{r}) \forall j \neq i)$$



These regions do not overlap, they partition the signal space assuming there is no $\mathbf{r} \in \mathcal{R}^N$ for which $p(\mathbf{s}_i \text{ sent}|\mathbf{r}) = p(\mathbf{s}_j \text{ sent}|\mathbf{r})$. If so, $\{\mathbf{s}_i\}$ is partitioned with decision regions by arbitrarily assigning such points to either Z_i or Z_j.

Denote $p(\mathbf{s}_i \ \text{sent}|\mathbf{r})$ as $p(\mathbf{s}_i|\mathbf{r})$, and $p(\mathbf{s}_i|\mathbf{r}) = \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})}$. To minimize P_e, we can

已给 $\vec{r}$

$$\arg\max_{\mathbf{s}_i} \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})} = \arg\max_{\mathbf{s}_i} p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i), \ \ i = 1, ..., M$$

Assuming equally likely messages $p(\mathbf{s}_i) = \frac{1}{M}$, so

$$\arg\max_{\mathbf{s}_i} p(\mathbf{r}|\mathbf{s}_i), \ \ i = 1, ..., M$$

Likelihood function $L(\mathbf{s}_i) = p(\mathbf{r}|\mathbf{s}_i)$. Given $\mathbf{r}$, a **maximum likelihood (ML) receiver** outputs $\hat{m} = m_i$ corresponding to $\mathbf{s}_i$ that maximizes $L(\mathbf{s}_i)$. Further define the **log likelihood function** $l(\mathbf{s}_i) = \log L(\mathbf{s}_i)$, given by

$$l(\mathbf{s}_i) = -\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2 = -\frac{1}{N_0} \parallel \mathbf{r} - \mathbf{s}_i \parallel^2$$

Thus, $l(\mathbf{s}_i)$ depends only on the distance between the received vector $\mathbf{r}$ and the constellation point $\mathbf{s}_i$.

Denote $p(\mathbf{s}_i \text{ sent}|\mathbf{r})$ as $p(\mathbf{s}_i|\mathbf{r})$, and $p(\mathbf{s}_i|\mathbf{r}) = \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})}$. To minimize P_e, we can

$$\arg\max_{\mathbf{s}_i} \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})} = \arg\max_{\mathbf{s}_i} p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i), \;\; i = 1, ..., M$$

Assuming equally likely messages $p(\mathbf{s}_i) = \frac{1}{M}$, so

$$\arg\max_{\mathbf{s}_i} p(\mathbf{r}|\mathbf{s}_i), \;\; i = 1, ..., M$$

Likelihood function $L(\mathbf{s}_i) = p(\mathbf{r}|\mathbf{s}_i)$. Given $\mathbf{r}$, a **maximum likelihood (ML) receiver** outputs $\hat{m} = m_i$ corresponding to $\mathbf{s}_i$ that maximizes $L(\mathbf{s}_i)$. Further define the **log likelihood function** $l(\mathbf{s}_i) = \log L(\mathbf{s}_i)$, given by

$$l(\mathbf{s}_i) = -\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2 = -\frac{1}{N_0} \parallel \mathbf{r} - \mathbf{s}_i \parallel^2$$

Thus, $l(\mathbf{s}_i)$ depends only on the distance between the received vector $\mathbf{r}$ and the constellation point $\mathbf{s}_i$.

Denote $p(\mathbf{s}_i \text{ sent}|\mathbf{r})$ as $p(\mathbf{s}_i|\mathbf{r})$, and $p(\mathbf{s}_i|\mathbf{r}) = \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})}$. To minimize P_e, we can

$$\arg\max_{\mathbf{s}_i} \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})} = \arg\max_{\mathbf{s}_i} p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i), \ i = 1, ..., M$$

Assuming equally likely messages $p(\mathbf{s}_i) = \frac{1}{M}$, so

$$\arg\max_{\mathbf{s}_i} p(\mathbf{r}|\mathbf{s}_i), \ i = 1, ..., M$$

Likelihood function $L(\mathbf{s}_i) = p(\mathbf{r}|\mathbf{s}_i)$. Given $\mathbf{r}$, a **maximum likelihood (ML) receiver** outputs $\hat{m} = m_i$ corresponding to $\mathbf{s}_i$ that maximizes $L(\mathbf{s}_i)$. Further define the **log likelihood function** $l(\mathbf{s}_i) = \log L(\mathbf{s}_i)$, given by

$$l(\mathbf{s}_i) = -\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{ij})^2 = -\frac{1}{N_0} \parallel \mathbf{r} - \mathbf{s}_i \parallel^2 \checkmark$$

Thus, $l(\mathbf{s}_i)$ depends only on the distance between the received vector $\mathbf{r}$ and the constellation point $\mathbf{s}_i$.

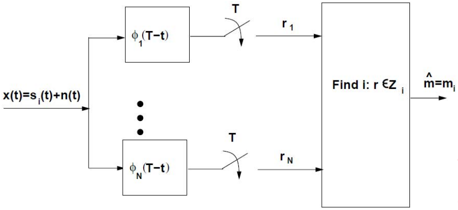
$\mathbf{r}$ is computed from $r(t)$, and then the signal constellation closest to $\mathbf{r}$ is determined as the constellation point $\mathbf{s}_i$ satisfying

$$\arg\min_{\mathbf{s}_i} \sum_{j=1}^{N} (r_j - s_{ij})^2 = \arg\min_{\mathbf{s}_i} \parallel \mathbf{r} - \mathbf{s}_i \parallel^2$$

This $\mathbf{s}_i$ is determined from the decision region Z_i that contains $\mathbf{r}$,

$$Z_i = (\mathbf{r} : \parallel \mathbf{r} - \mathbf{s}_i \parallel < \parallel \mathbf{r} - \mathbf{s}_j \parallel \; \forall j \neq i), \; i = 1, ..., M.$$

For BPSK, $s_1 = A$ and $s_2 = -A$, then $Z_1 = (\mathbf{r} : \mathbf{r} > 0)$, $Z_2 = (\mathbf{r} : \mathbf{r} < 0)$.



An alternate receiver structure is the **matched filter**, $\psi(t) = \phi(T - t)$, $0 \leq t < T$. A signal is passed through a filter matched to itself, the output SNR is maximized.

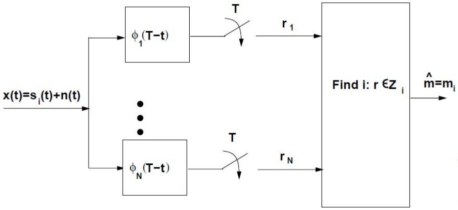
$\mathbf{r}$ is computed from $r(t)$, and then the signal constellation closest to $\mathbf{r}$ is determined as the constellation point $\mathbf{s}_i$ satisfying

$$\arg\min_{\mathbf{s}_i} \sum_{j=1}^{N} (r_j - s_{ij})^2 = \arg\min_{\mathbf{s}_i} \parallel \mathbf{r} - \mathbf{s}_i \parallel^2$$

This $\mathbf{s}_i$ is determined from the decision region Z_i that contains $\mathbf{r}$,

$$Z_i = (\mathbf{r} : \parallel \mathbf{r} - \mathbf{s}_i \parallel < \parallel \mathbf{r} - \mathbf{s}_j \parallel \; \forall j \neq i), \; i = 1, ..., M.$$

For BPSK, $s_1 = A$ and $s_2 = -A$, then $Z_1 = (\mathbf{r} : \mathbf{r} > 0)$, $Z_2 = (\mathbf{r} : \mathbf{r} < 0)$.



An alternate receiver structure is the **matched filter**, $\psi(t) = \phi(T - t)$, $0 \leq t < T$. A signal is passed through a filter matched to itself, the output SNR is maximized.

r is computed from $r(t)$, and then the signal constellation closest to **r** is determined as the constellation point $\mathbf{s}_i$ satisfying

$$\arg\min_{\mathbf{s}_i} \sum_{j=1}^{N} (r_j - s_{ij})^2 = \arg\min_{\mathbf{s}_i} \| \mathbf{r} - \mathbf{s}_i \|^2$$

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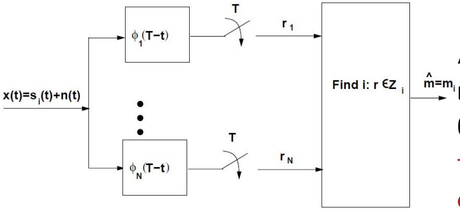

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Error Probability and the Union Bound

For ML with M equally likely messages, we have

$$\begin{aligned}
P_e &= \sum_{i=1}^{M} p(\mathbf{r} \notin Z_i | m_i \text{ sent}) p(m_i \text{ sent}) = \frac{1}{M}\sum_{i=1}^{M} p(\mathbf{r} \notin Z_i | m_i \text{ sent}) \\
&= 1 - \frac{1}{M}\sum_{i=1}^{M} p(\mathbf{r} \in Z_i | m_i \text{ sent}) = 1 - \frac{1}{M}\sum_{i=1}^{M}\int_{Z_i} p(\mathbf{r}|m_i) d\mathbf{r} \\
&= 1 - \frac{1}{M}\sum_{i=1}^{M}\int_{Z_i} p(\mathbf{r} = \mathbf{s}_i + \mathbf{n}|\mathbf{s}_i) d\mathbf{r} = 1 - \frac{1}{M}\sum_{i=1}^{M}\int_{Z_i - \mathbf{s}_i} p(\mathbf{n}) d\mathbf{n}
\end{aligned}$$

Union bound: $P_e(m_i \text{ sent}) = p\left(\bigcup_{k=1,k\neq i}^{M} A_{ik}\right) \leq \sum_{k=1,k\neq i} p(A_{ik})$, where $p(A_{ik}) = p(\| \mathbf{s}_k - \mathbf{r} \| < \| \mathbf{s}_i - \mathbf{r} \| \; |\mathbf{s}_i \text{ sent}) = p(\| \mathbf{n} + \mathbf{s}_i - \mathbf{s}_k \| < \| \mathbf{n} \|)$.

Error Probability and the Union Bound

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&= 1 - \frac{1}{M} \sum_{i=1}^{M} p(\mathbf{r} \in Z_i | m_i \text{ sent}) = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i} p(\mathbf{r} | m_i) d\mathbf{r} \\
&= 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i} p(\mathbf{r} = \mathbf{s}_i + \mathbf{n} | \mathbf{s}_i) d\mathbf{r} = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i - \mathbf{s}_i} p(\mathbf{n}) d\mathbf{n}
\end{aligned}
$$

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$p(A_{ik}) = p(\| \mathbf{s}_k - \mathbf{r} \| < \| \mathbf{s}_i - \mathbf{r} \| \, | \mathbf{s}_i \text{ sent}) = p(\| \mathbf{n} + \mathbf{s}_i - \mathbf{s}_k \| < \| \mathbf{n} \|)$.

$P(A_{ik}) = P(\|\vec{s}_k - \vec{r}\| < \|\vec{s}_i - \vec{r}\| \, | \vec{s}_i \text{ sent}) = \vec{P}(\|\vec{s}_k - \vec{s}_i - \vec{n}\| < \|\vec{s}_i - \vec{s}_i - \vec{n}\|)$

$= \vec{P}(\|\vec{n} + \vec{s}_i - \vec{s}_k\| < \|\vec{n}\|)$.

A_{ik} occurs if $\mathbf{n}$ is closer to $\mathbf{s}_i - \mathbf{s}_k$ than to 0. Let $d_{ik} = \| \mathbf{s}_i - \mathbf{s}_k \|$, then

$$p(A_{ik}) = p(n > \frac{d_{ik}}{2}) = \int_{\frac{d_{ik}}{2}}^{\infty} \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-v^2}{N_0}\right] dv = Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right)$$

which yields

$$P_e(m_i \text{ sent}) \leq \sum_{k=1, k\neq i}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right),$$

$Q(z) = p(x > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{1}{2}\text{erfc}(\frac{z}{\sqrt{2}}) \left(\leq \frac{e^{-z^2/2}}{z\sqrt{2\pi}},\ z \gg 0\right)$.
Hence the union bound can be obtained as

$$P_e = \sum_{i=1}^{M} p(m_i) P_e(m_i \text{ sent}) \leq \frac{1}{M} \sum_{i=1}^{M} \sum_{k=1, k\neq i}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right)$$

Define $d_{min} = \min_{i,k} d_{ik}$, we have a looser one $P_e \leq (M-1) Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$.

Based on nearest neighbor approximation $P_e \approx M_{d_{min}} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right)$.

For binary modulation, $M = 2$, then $P_b = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$.

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Chapter 6.

△ Bit/Symbol Energy

$r(t)=s(t)+n(t) \quad t\in[0,T_s)$

$s(t)=\mathrm{Re}\{u(t)e^{-j2\pi f_c t}\}$.

Symbol Energy:

$E_s=\int_0^{T_s} s^2(t)\,dt=\int_0^{T_s}|u(t)|^2dt=\langle\vec{s},\vec{s}\rangle$

Bit Energy:

$E_b=E_s/\log_2 M$ (S中有M个symbol，一个S_i中有$\log_2 M$ bit)

SNR per symbol:

$\gamma_s=\dfrac{E_s/T_s}{N_0B}$ $\dfrac{\rightarrow \text{信号平均功率}}{\rightarrow \text{噪声平均功率}}$

Generally, $B \approx \frac{1}{T_s}$

$\rightarrow \gamma_s=\dfrac{E_s}{N_0}$

SNR per bit

$\gamma_b \triangleq \dfrac{E_b}{B}=\dfrac{E_s/\log_2 M}{B}=\dfrac{\gamma_s}{\log_2 M}$

△ Error Prob for BPSK, QPSK.

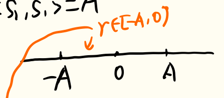
$s_1(t)=A\,g(t)\cos(2\pi f_c t)$ g(t) pulse shaping.

$s_2(t)=-A\,g(t)\cos(2\pi f_c t)$.

$s_1=A. \quad s_2=-A$

$E_s=E_b=\langle s_1,s_1\rangle=A^2$

BPSK.



$r=s_i+n, \quad N(0,\frac{N_0}{2})$.

因为BPSK对称，所以Error probability $=P_e(m_i)$

$P_s=P_e(m_1)=\Pr[n<-A]=Q\left(\dfrac{2A}{\sqrt{2N_0}}\right)$

$=Q\left(\sqrt{2}\sqrt{\dfrac{E_b}{N_0}}\right)=Q(\sqrt{2\gamma_b})=Q(\sqrt{2\gamma_s})$

QPSK: $\phi=\{g(t)\cos(2\pi f_c t), -g(t)\sin(2\pi f_c t)\}$.

$s_1(t)=A\,g(t)\cos(2\pi f_c t)-A\,g(t)\sin(2\pi f_c t)$

$\vec{s}_1=(A,A)$

$s_2(t)=\cdots \quad s_3(t)=\cdots \quad s_4(t)\cdots$

$\vec{s}_2=(-A,A) \quad \vec{s}_3=(-A,-A) \quad \vec{s}_4=(A,-A)$.

Passband Modulation Principles

Generally, modulated carrier signals encode information in the amplitude $\alpha(t)$, frequency $f(t)$, or phase $\theta(t)$ of a carrier signal,

$$s(t) = \alpha(t)\cos[2\pi(f_c + f(t))t + \theta(t) + \phi_0] = \alpha(t)\cos[2\pi f_c t + \phi(t) + \phi_0].$$

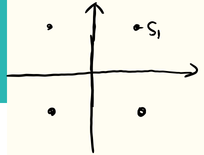
We can use in-phase and quadrature components to write $s(t)$,

$$\begin{aligned} s(t) &= \alpha(t)\cos\phi(t)\cos(2\pi f_c t) - \alpha(t)\sin\phi(t)\sin(2\pi f_c t) \\ &= s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t). \end{aligned}$$

We can also write $s(t)$ in its complex baseband representation as

$$s(t) = \mathcal{R}\left\{u(t)e^{j2\pi f_c t}\right\}$$

where $u(t) = s_I(t) + js_Q(t)$.



$E_s = \langle \vec{s}_1, \vec{s}_1 \rangle = 2A^2$

$E_b = \dfrac{E_s}{\log_2 M} = A^2$

$\vec{r} = \begin{pmatrix} A \\ A \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}.$

$\vec{s}_1 \quad \vec{n}$

因为对称 所以Error Probility $= P_e(m_i)$

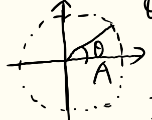
$P_s = P_e(m_1) = P_r[n_1 < -A \text{ or } n_2 < -A]$

union $\leq 2P_r[n_1 < -A] = 2Q(\sqrt{2\gamma_b})$

$= 1 - P_r[n_1 > -A \text{ and } n_2 > -A] = 1 - P_r^2[n_1 > -A]$

$= 1 - \left[1 - Q\left(\dfrac{2A}{\sqrt{2N_0}}\right)\right]^2 = 1 - \left[1 - Q(\sqrt{2\gamma_b})\right]^2$

Δ MPSK



$\theta = \frac{2\pi}{M}.$ $\vec{s}_i = \begin{pmatrix} s_{i1} \\ s_{i2} \end{pmatrix} = \begin{pmatrix} A\cos[2\pi(i-1)/M] \\ A\sin[2\pi(i-1)/M] \end{pmatrix}.$

$i = 1, 2, \ldots M$

$E_s = \langle \vec{s}_i, \vec{s}_i \rangle = A^2$

$P_s = P_e[m_1] = 1 - P_r[\vec{r} \in Z_1 | m_1]$

$= 1 - \int f(\vec{r}|m_1)d\vec{r}$

Q function → 标准正态分布的右尾函数

$Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = 1 - \Phi(x).$

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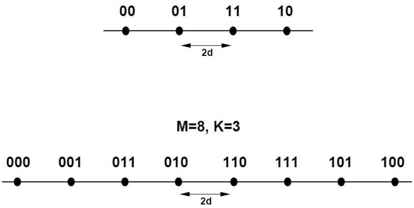
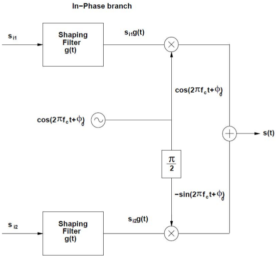
There are 3 main types of amplitude/phase modulation:

- Pulse Amplitude Modulation (MPAM): information encoded in amplitude only.
- Phase Shift Keying (MPSK): information encoded in phase only.
- Quadrature Amplitude Modulation (MQAM): information encoded in both amplitude and phase.

Pulse Amplitude Modulation (MPAM)

Transmitted signal: $s_i(t) = \mathcal{R}\{A_i g(t) e^{j2\pi f_c t}\} = A_i g(t)\cos(2\pi f_c t)$, $0 \le t \le T \gg \frac{1}{f_c}$. $A_i = (2i - 1 - M)d$, $i = 1, \ldots, M$ and d is typically a function of the signal energy. $\overline{E_s} = \frac{1}{M}\sum_{i=1}^{M} A_i^2$, $d_{min} = 2d$.





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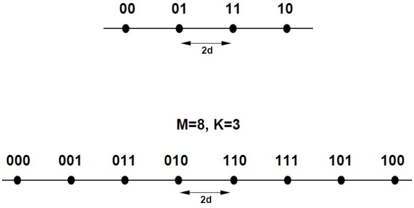
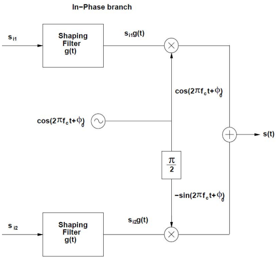
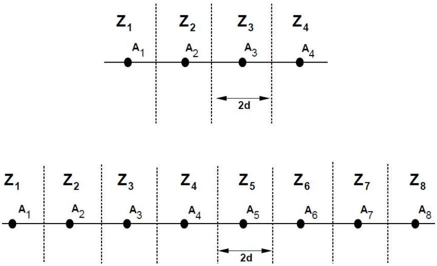




Decision Regions of MPAM

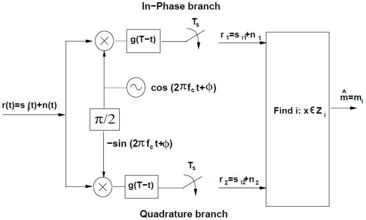
Mathematically, for any M, these decision regions are defined by

$$Z_i = \begin{cases} (-\infty, A_1 + d), i = 1 \\ [A_i - d, A_i + d), 2 \le i \le M - 1 \\ [A_M - d, \infty), i = M \end{cases}$$

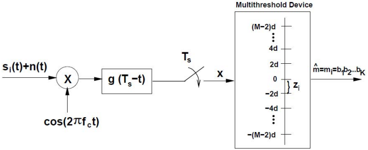


Decision Regions for MPAM

Demodulation of MPAM



Amplitude/Phase Demodulator

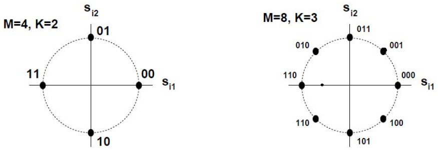


Coherent Demodulator for MPAM

Phase Shift Keying (MPSK)

Transmitted signal: $s_i(t) = Ag(t)\cos\left[2\pi f_c t + \frac{2\pi(i-1)}{M}\right]$,

$d_{min} = 2A\sin(\frac{\pi}{M})$. $E_{s_i} = A^2$.

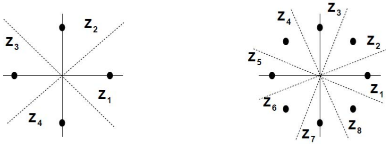


Gray Encoding for MPSK

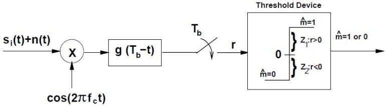
2PSK is often referred to as binary PSK or BPSK, while 4PSK is often called quadrature phase shift keying (QPSK), and is the same as MQAM with $M = 4$.

Decision Regions of MPSK

$$Z_i = \left\{re^{j\theta} : \frac{2\pi(i-1.5)}{M} \leq \theta < \frac{2\pi(i-0.5)}{M}\right\}$$



Decision Regions for 4PSK and 8PSK



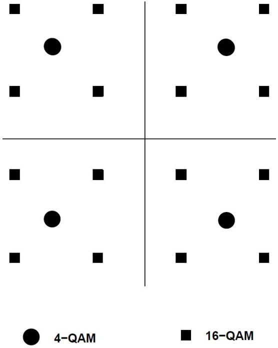
Coherent Demodulator for BPSK

Decision Regions of MQAM

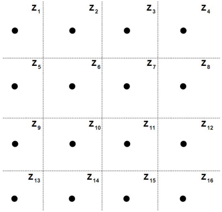
Quadrature Amplitude Modulation (MQAM)

$s_i(t) = \mathcal{R}\{A_i e^{j\theta_i} g(t) e^{j2\pi f_c t}\}$,

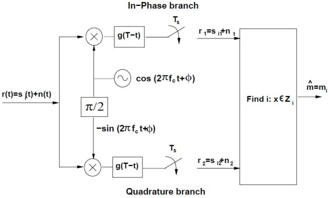
$d_{ij} = \| \mathbf{s}_i - \mathbf{s}_j \|$. $E_{s_i} = A_i^2$.



4QAM and 16QAM Constellations



Decision Regions for 16-QAM



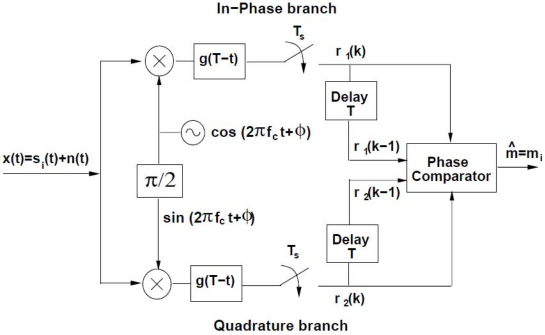
Coherent demodulator for QAM

Differential PSK

- Recovery of phase ϕ_0 is required in coherent MPSK and MQAM, which brings more complexity and cost in RX.
- Differential encoding:

Original data:	**A**	**A**	**-A**	**A**	**-A**	**-A**	$(s_{k-1} = -A)$
Encoding:	**1**	**0**	**1**	**1**	**1**	**0**	

- Differential PSK demodulator



Differential PSK Demodulator.

Constellation Shaping

- Rectangular and hexagonal constellations have a better power efficiency than the square or circular constellations associated with MQAM and MPSK, respectively.

- These irregular constellations can save up to 1.3 dB of power at the expense of increased complexity in the constellation map.

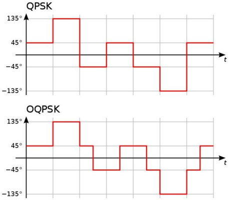
- The optimal constellation shape is a sphere in N-dimensional space, which must be mapped to a sequence of constellations in 2-dimensional space in order to be generated by the modulator.

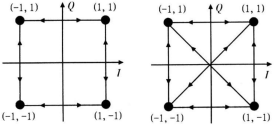
- For uncoded modulation, the increased complexity of spherical constellations is not worth their energy gains, since coding can provide much better performance at less complexity cost.

Quadrature Offset: OQPSK and $\frac{\pi}{4}$-QPSK

OQPSK: offset-QPSK

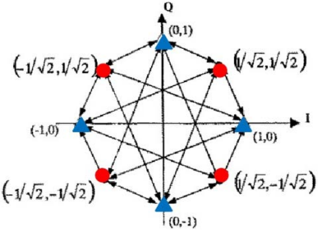
- prevent π phase transition
- maximum phase transition $\frac{\pi}{2}$





$\frac{\pi}{4}$-QPSK

- maximum phase transition $\frac{3\pi}{4}$



Overview

- Signal Space Analysis
  - Signal and system model, Geometric representations of signals, Receiver structure and sufficient statistics, Decision regions and ML decision criterion, Error probability and the union bound
- Passband Modulation Principles
- Amplitude and Phase Modulation
  - Pulse amplitude modulation (PAM), Phase shift keying (PSK), Quadrature amplitude modulation (QAM), Differential modulation, Constellation shaping, Quadrature offset
- Frequency Modulation
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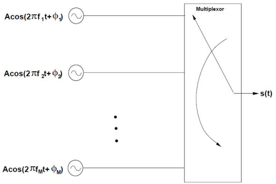
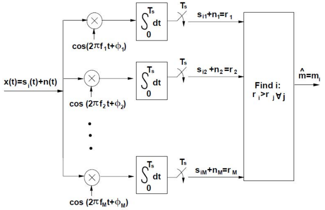
Frequency Modulation

$s_i(t) = A\cos(2\pi f_i t + \phi_i)$, i is the index of the ith message corresponding to the $\log_2 M$ bits and ϕ_i is the phase associated with the ith carrier.
The signal space representation is $s_i(t) = \sum_j s_{ij}\phi_j(t)$, $s_{ij} = A\delta(i-j)$ and $\phi_j(t) = \cos(2\pi f_j t + \phi_j)$.
The orthogonality of the basis functions requires a minimum separation between different carrier frequencies of $\Delta f = \min_{ij}|f_i - f_j| = \frac{1}{2T_S}$.
Frequency Shift Keying (FSK) and Minimum Shift Keying (MSK)
In MFSK, $s_i(t) = A\cos[2\pi f_c t + 2\pi\alpha_i\Delta f_c t + \phi_i]$, $\alpha_i = (2i-1-M)$, and $\Delta f = 2\Delta f_c$. **MSK**: $2\Delta f_c = \frac{1}{2T_S}$, minimum frequency separation.

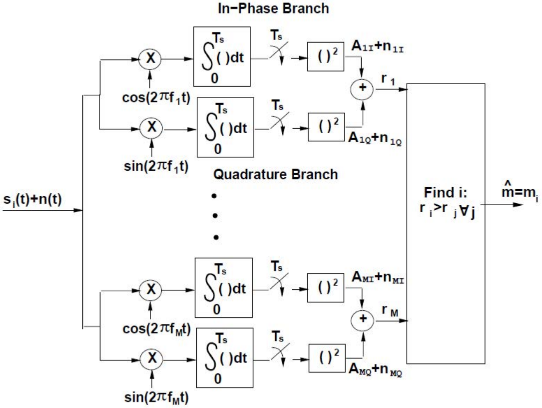




Continuous-Phase FSK (CPFSK)

$s_i(t) = A\cos[2\pi f_c t + 2\pi\Delta f_c \int_{-\infty}^{t} u(\tau)d\tau] = A\cos[2\pi f_c t + \theta(t)]$,
bandwidth: $B_s \approx 2M\Delta f_c + 2B_g$, where B_g is the BW of $g(t)$.

Noncoherent Detection of FSK



Noncoherent FSK Demodulator

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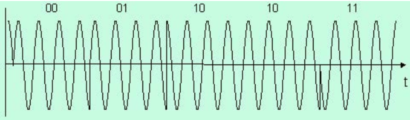
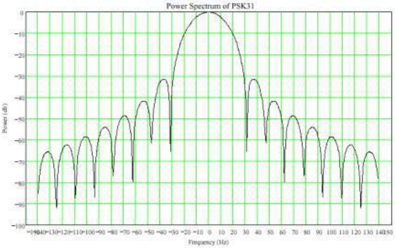
Pulse Shaping

- The bandwidth of modulated signal depends on the pulse shape $g(t)$.
- If $g(t)$ is a rectangular pulse, the spectral sidelobe is high, which causes adjacent channel interference.
- Pulse shaping can reduce the energy of sidelobes, while prevent intersymbol interference.





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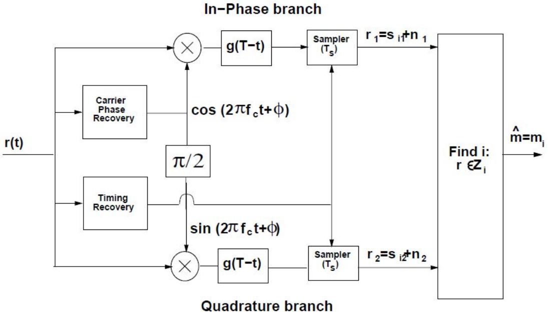
Symbol Synchronization and Carrier Phase Recovery

- Receiver Structure with Phase and Timing Recovery



- Maximum Likelihood Phase Estimation: Phase Lock Loop
- Maximum Likelihood Timing Estimation: Decision-Directed Timing Estimation, Early-Late Gate Synchronizer

Main Points

- Signal space, decision regions, ML decision criterion, error probability, and the union bound.
- Amplitude/Phase Modulation (MPSK,MQAM)
  - Information encoded in amplitude/phase
  - More spectrally efficient than frequency modulation
  - Issues: differential encoding, pulse shaping
- Frequency Modulation (FSK)
  - Information encoded in frequency
  - Continuous phase (CPFSK) special case of FM
  - More robust to channel and amplifier nonlinearities