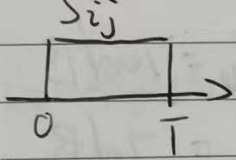
△ Pulse Shaping

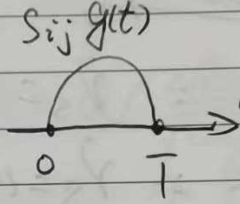
$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t) \qquad \forall i,\quad t \in [0, T)$$

$$\Downarrow$$

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,g(t)\cdot\phi_j(t)$$



$\times\,\phi_j(t) \Rightarrow$



$\times\,\phi_j(t)$

reduce sidelobes

△ Bit / Symbol Energy

$r(t) = s(t) + n(t)$ $\qquad s(t) = \mathrm{Re}\{u(t)\,e^{j2\pi f_c t}\}$

Symbol Energy $E_s = \int_0^{T_s} s^2(t)\,dt = \int_0^{T_s} |u(t)|^2\,dt$
$= \langle \vec{s}, \vec{s} \rangle$

Signal Energy per bit $E_b = E_s / \log_2 M$

SNR per Symbol $\gamma_s = \dfrac{P_r}{N_0 B} = \dfrac{E_s/T_s}{N_0 B} = \dfrac{E_s}{N_0 B}$

SNR per bit $\gamma_b = \frac{P_r/\log_2 M}{N_0 B} = \frac{E_b}{N_0} \triangleq \gamma_s/\log_2 M$

△ Error Prob. for BPSK and QPSK

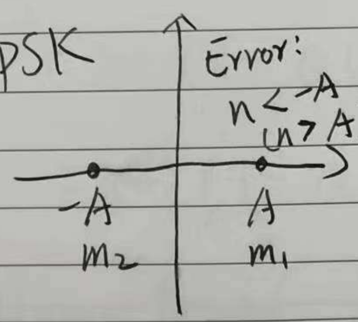
BPSK

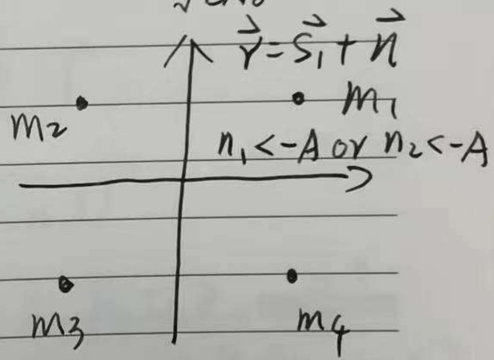
$s_1(t) = A g(t) \cos 2\pi f_c t \quad s_2(t) = -A g(t) \cos 2\pi f_c t$

$s_1 = A \quad s_2 = -A \quad E_s = E_b = \int_0^T s_i^2(t) = A^2$

$P_{s} = P_e[m_1] = Q\left(\frac{2A}{\sqrt{2N_0}}\right) = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_0}}\right) = Q(\sqrt{2\gamma_b})$

BPSK





QPSK: $\phi = \{ g(t) \cos 2\pi f_c t, \ -g(t) \sin 2\pi f_c t \}$

$s_1(t) = A g(t) \cos 2\pi f_c t - A g(t) \sin 2\pi f_c t$

~~$s_2(t) \ldots \quad s_3 \ldots \quad s_4$~~

$s_2(t), s_3(t), s_4(t) = \cdots$

$\vec{s}_1 = (A, A) \quad \cdots$

$E_s = \int_0^{T_s} s_1^2(t)\,dt = \langle \vec{s}_1, \vec{s}_1 \rangle = 2A^2$

$E_b = E_s / \log_2 4 = A^2$

$$P_s = P_e[m_1] = Pr[n_1 < -A \text{ or } n_2 < -A]$$

$$= 1 - Pr[n_1 > -A \text{ and } n_2 > -A]$$

$$= 1 - Pr[n_1 > -A]\, Pr[n_2 > -A]$$

$$= 1 - \left(1 - Q\left(\cancel{2A} \; 2A/\sqrt{2N_0}\right)\right)^2$$

$$\left[E_s = \langle \vec{S}_1, \vec{S}_1 \rangle = 2A^2 \quad E_b = E_s/2 = A^2. \right]$$

$$= 1 - \left(1 - Q\left(\sqrt{2}\sqrt{\frac{E_b}{N_0}}\right)\right)^2$$

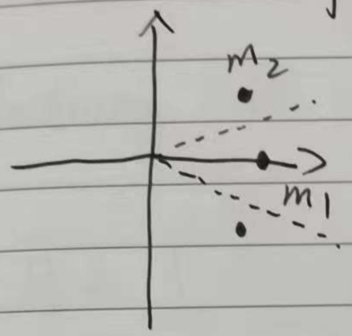
$$= 1 - \left(1 - Q\left(\sqrt{2\gamma_b}\right)\right)^2$$

Union Bound. $P_s \le Pr[n_1 < -A] + Pr[n_2 < -A]$

$= \cdots$

(textbook)

△. Error Prob. of MPSK.



$$\vec{s}_i = \begin{pmatrix} s_{i1} \\ s_{i2} \end{pmatrix} = \begin{pmatrix} A\cdot\cos 2\pi(i-1)/M \\ A\sin 2\pi(i-1)/M \end{pmatrix}$$

$i = 1, 2, \cdots, M.$

$$\cancel{E_b}\ E_s = \langle \vec{s}_i, \vec{s}_i \rangle = A^2$$

$$P_s = P_e[m_1]$$

$$= 1 - P_r[\vec{r} \text{ in } Z_1 | m_1]$$

$$= 1 - \int_0^{+\infty}\int_{-\pi/M}^{\pi/M} \frac{r}{\pi N_0} e^{-\frac{1}{N_0}(r^2 - 2\sqrt{E_s}\, r\cdot\cos\theta + E_s)} \, d\theta \, dr$$

$$\approx ?$$

△ Error Prob. of MPAM. MQAM

△ Error Prob. Approximation.

4. Flat Fading Channel.

Receiving SNR is γ random $\Rightarrow$ Average Error Prob.

$$\bar{P}_s = \int_0^{+\infty} \underbrace{P_s(\gamma)}_{\text{modulation}} \underbrace{p_{\gamma_s}(\gamma)}_{\text{channel fading}} \, d\gamma$$

Rayleigh fading

$$\cancel{p(h) = \frac{h}{\sigma^2} e^{-h^2/2\sigma^2}} \qquad \vec{r} = h\vec{s}_i + \vec{n}$$

$$|h| \sim p(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}$$

$$\cancel{r(t) = h\,s_i(t) + n(t) \quad t \in [0, T_s)}$$

Suppose average transmission power is α:

$$\alpha = \frac{1}{T_s}\int_0^{T_s} s_i^2(t)\,dt = \frac{1}{T_s}\langle \vec{s}_i, \vec{s}_i \rangle$$

Average receiving signal power

$$= \frac{1}{T_s}\int_0^{T_s} |h|^2 s_i^2(t)\,dt = \alpha |h|^2$$

Noise power $= N_0 \cdot B = N_0 / T_s$

SNR $\gamma_s = \alpha|h|^2/N_0 = T_s\alpha|h|^2/2\sigma_n^2$ 五

$$\sigma_n^2 = N_0/2$$

PDF of γ_s: $p_{\gamma_s}(\gamma) = \frac{\sigma_n^2}{\sigma^2\alpha T_s} e^{-\gamma\sigma_n^2/\sigma^2 T_s\alpha}$.

with expectation $\sigma^2 T_s\alpha/\sigma_n^2 \triangleq \bar{\gamma}_s$

$$\Rightarrow p_{\gamma_s}(\gamma) = \frac{1}{\bar{\gamma}_s} e^{-\gamma/\bar{\gamma}_s}$$

$$\Rightarrow \bar{P}_s = \int_0^{+\infty} P_s(\gamma)\cdot\frac{1}{\bar{\gamma}_s} e^{-\gamma/\bar{\gamma}_s}\, d\gamma$$

for Rayleigh fading channel.

channel

△ ~~Block~~ code

$$\xrightarrow{\vec{m}} \boxed{En} \xrightarrow{\vec{c}} \boxed{Mod} \xrightarrow{S(t)}$$

$(m_1 \cdots m_k)$ $(c_1 \cdots c_n)$ ~~code word~~

message $n \geq k$ codeword

Code rate = k/n.

2^k messages $\Rightarrow 2^k$ codewords $\Rightarrow 2^n$ Rx CW

Hamming distance $\Delta(\vec{c}_1, \vec{c}_2)$ = # of different bits

Let $j = 2^k$. $\{\vec{c}_1, \vec{c}_2, \cdots \vec{c}_j\}$ be the set of right codewords (codebook)

Let $\vec{r}$ be the received codeword.

Estimated message = $\arg\min_i \Delta(\vec{r}, \vec{c}_i)$

"0" → 000 → 001 → "0"

"1" → 111 channel.