Review:

message $(m_1 \cdots m_k) \longrightarrow \boxed{\text{modulation}} \xrightarrow{S_i(t)}$

$S_i(t)+n(t) \longrightarrow \boxed{\text{demodulation}} \longrightarrow (\hat{m}_1 \cdots \hat{m}_k)$

Error: $m_1 \cdots m_k \neq \hat{m}_1 \cdots \hat{m}_k$

$(m_1 m_2 \cdots m_k) \triangleq \vec{m} \longrightarrow \boxed{\text{Encoder}} \xrightarrow{\vec{C} = (C_1 \cdots C_n)} \boxed{\text{modulation}} \xrightarrow{S_i(t)}$

$S_i(t)+n(t) \longrightarrow \boxed{\text{demodulation}} \xrightarrow{(\hat{C}_1 \cdots \hat{C}_n) = \vec{r}} \boxed{\text{decoder}} \longrightarrow (\hat{m}_1 \cdots \hat{m}_k)$

$\{\vec{C}_1 \vec{C}_2 \cdots \vec{C}_{2^k}\} \Rightarrow \min_{i=1,\cdots,2^k} \Delta(\vec{r}, \vec{C}_i)$

Code rate $= k/n$.

hemming distance $\neq \Delta(\vec{C}_1, \vec{C}_2)$

A code $\{\vec{c}_1, \cdots \vec{c}_{2^k}\}$ has a ~~with~~ min hamming distance d.

if $d = \min\limits_{a,b} \Delta(\vec{c}_a, \vec{c}_b)$

$\Rightarrow$ Detect $d-1$ errors, correct $\lfloor \frac{d-1}{2} \rfloor$ errors

△. Linear block code

$0+0=0$, $0+1=1$, $1+1=0$,

$0\times0=0$ $0\times1=0$, $1\times1=1$

$$\underset{1\times k}{\vec{m}} \ \underset{k\times n}{G} = \underset{1\times n}{\vec{c}}$$

↑

Generator Matrix

Special Case: $G = [\underset{k\times k}{I}, \underset{k\times(n-k)}{P}]$

$$\vec{m}\, G = \underbrace{[\vec{m}, \vec{m} P]}_{\vec{c}}$$

Rx: parity check matrix. $H = [P^T \ I]$

$(n-k)\times k \quad \cancel{k\times k}\ (n-k)\times(n-k)$

$\Rightarrow \vec{m} G H^T = 0.$

$(n-k)\times \cancel{k}\ n$

$\Rightarrow$ Correct receiving will pass the check matrix

$\vec{c} =$ "1 1 0 0"

$\vec{e} =$ "0 0 1 0" error vector for the 3rd bit

$\Downarrow$

$\vec{r} = \vec{c} + \vec{e} =$ "1 1 1 0"

$\vec{r} H^T = \vec{c} H^T + \vec{e} H^T = \vec{e} H^T$

$(1\times n \ \ n\times \cancel{k}\ (n-k) \Rightarrow 1\times \cancel{k}\ (n-k))$

Pattern of $\vec{r} H^T$ ($\vec{e} H^T$) can be used to detect / correct error.

△ Convolution Code.

△ Interleaving

April 4

日	一	二	三	四	五	六
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

AWGN: $\vec{y} = \vec{x} + \vec{n}$

1 愚人节 Thursday / 星期四

Two-dimensional signal. — AWGN

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \cancel{h}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$ vector

$y_1 + j y_2 = x_1 + j x_2 + n_1 + j n_2$. baseband.

2 廿一 Friday / 星期五

$y = x + n$. complex.

~~Narrow b~~ transmit signal energy $|x|^2 = (x_1\ x_2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Narrowband (flat fading) — two-dimensional

3 廿二 Saturday / 星期六

$y = hx + n$.

↑

channel gain.

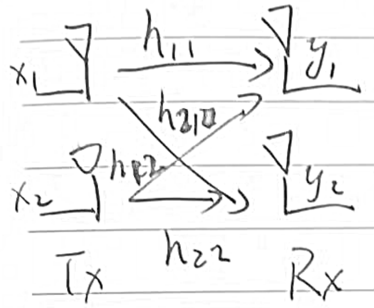
$x \to \boxed{AWGN} \to$ $y = x + n$

$x \to \boxed{Flat} \to$ $y = hx + n$.

4 清明 Sunday / 星期日

SISO: Single input Single output

Δ Narrowband MIMO Model



$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1$$
$$= (h_{11}\ h_{12})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + n_1$$
$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2$$
$$= (h_{21}\ h_{22})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + n_2$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

Suppose M_t transmit antennas and M_r receive antennas. channel gain for i-th transmit antenna to the j-th receive antenna is h_{ji}

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_r} \end{pmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1M_t} \\ \vdots & \ddots & \vdots \\ h_{M_r1} & \cdots & h_{M_rM_t} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{M_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{M_r} \end{bmatrix}$$

$$\vec{y} = H\vec{x} + \vec{n}$$