Homework

test

I. Question 4-5(c)

Refer to equation:

$$\frac{C}{B} = \log_2\left(1 + \frac{1}{\sum_{\gamma \geq \gamma_0} p(\gamma)\frac{1}{\gamma}}\right)\left(\sum_{\gamma \geq \gamma_0} p(\gamma)\right)$$

Note the hidden truth of communication system is to preserve at least transmission ability $P_{out} = 0.1 \rightarrow P_{success} \geq 0.9$, therefor $\frac{C}{B} = 2.4028\ bps/Hz$, $\frac{C}{B} = 3.9678\ bps/Hz$ 4.1462 and 2.4576, therefor take $P_{out} = 0.5$ is best.

II. Question 4-6(d)

From equation of outage transmission we know the Capacity is:

$$C = (1 - P_{out})\, B \log_2(1 + \gamma_{min})$$

Take each channel condition into equation we have capacity under different outage probability: 0.8066, 1.7955, 2.1510, 1.8802 bps/Hz.

III. Question 5-1

Known $s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t) + \cdots + s_{iK}\phi_K(t)$, we have:

$$\begin{aligned}
\int_0^T (s_i(t) - s_j(t))^2 dt &= \int_0^T ((s_{i1}\phi_1(t) + s_{i2}\phi_2(t) + \cdots + s_{iK}\phi_K(t)) \\
&\quad - (s_{j1}\phi_1(t) + s_{j2}\phi_2(t) + \cdots + s_{jK}\phi_K(t)))^2 dt \\
&= \int_0^T \left((s_{i1} - s_{j1})\phi_1(t) + (s_{i2} - s_{j2})\phi_2(t) + \cdots + (s_{iK} - s_{jK})\phi_K(t)\right)^2 dt \\
&= \int_0^T ((s_{i1} - s_{j1})\phi_1(t))^2 dt + \int_0^T ((s_{i2} - s_{j2})\phi_2(t))^2 dt + \ldots \\
&\quad + \int_0^T ((s_{iK} - s_{jK})\phi_K(t))^2 dt \\
&= (s_{i1} - s_{j1})^2 \int_0^T (\phi_1(t))^2 dt + (s_{i2} - s_{j2})^2 \int_0^T (\phi_2(t))^2 dt + \ldots \\
&\quad + (s_{iK} - s_{jK})^2 \int_0^T (\phi_K(t))^2 dt \\
&= (s_{i1} - s_{j1})^2 + (s_{i2} - s_{j2})^2 + \cdots + (s_{iK} - s_{jK})^2 \\
&= \|\mathbf{s}_i - \mathbf{s}_j\|_2^2
\end{aligned}$$

IV. Question 5-3

$$\begin{aligned}\int_0^T (s'_m(t))^2 dt &= \int_0^T \left(s_m(t) - \frac{1}{M}\sum_{i=1}^M s_i(t)\right)^2 dt \\ &= \int_0^T s_m^2(t) - \frac{2}{M}\sum_{i=1}^M (s_i(t)s_m(t)) + \left(\frac{1}{M}\sum_{i=1}^M s_i(t)\right)^2 dt \\ &= \left(1 - \frac{2}{M} + \frac{1}{M^2}M\right)\mathcal{E} \\ &= \frac{M-1}{M}\mathcal{E}\end{aligned}$$

$$\begin{aligned}\int_0^T (s'_m(t)s'_n(t))dt = &= \int_0^T \left(s_m(t) - \frac{1}{M}\sum_{i=1}^M s_i(t)\right)\left(s_n(t) - \frac{1}{M}\sum_{i=1}^M s_i(t)\right) dt \\ &= \int_0^T s_m(t)s_n(t) - \frac{1}{M}\sum_{i=1}^M (s_i(t)s_m(t)) - \frac{1}{M}\sum_{i=1}^M (s_i(t)s_n(t)) + \left(\frac{1}{M}\sum_{i=1}^M s_i(t)\right)^2 dt \\ &= \left(-\frac{2}{M} + \frac{1}{M^2}M\right)\mathcal{E} \\ &= \frac{-\mathcal{E}}{M}\end{aligned}$$

V. Question 5-5

- Time division: Dimensionality (the number of different waveforms in the set $\{\phi_j(t)\}$) is 4.
- From $s_{ij} = \int_0^T s_i(t)\phi_j(t)dt$ It can be represented as: $[2, 2, -1, -1]$; $[1, -1, 1, -1]$; $[-2, 1, 1, 1]$; $[1, -2, -2, 1]$;
- Distance of signal pairs $s_1(t), s_2(t)$ and $s_2(t), s_4(t)$ have minimum distance $\|\boldsymbol{s}_i - \boldsymbol{s}_j\|$ as $\sqrt{14}$.

VI. Question 5-4

- To show the signal is orthonormal, we have to examine $\int_0^T (s_i(t))^2 dt = 1, \forall i$ and $\int_0^T s_i(t)s_j(t)dt = 0, \forall i \neq j$ two The integral result of each signal itself is: $\frac{1}{4}4 = 1, \frac{1}{4}4 = 1, \frac{1}{4}4 = 1$
  The cross integral is: $\frac{1}{4}(2-2) = 0$, $\frac{1}{4}(1-1-1+1) = 0$, $\frac{1}{4}(1-1+1-1) = 0$,
- Do integral $\int_0^T x(t)\phi_i(t), \forall i$, we have result $-2, 6, 0$

VII. Question 5-7

Recall:

$$r(t) = \sum_{j=1}^N (s_{ij} + n_j)\phi_j(t) + n_r(t) = \sum_{j=1}^N r_j\phi_j(t) + n_r(t)$$

Actually, we can write the redundant component $n_r(t)$ as $n_r\phi_{n_r}(t)$, therefor we have the expectation $\mathrm{E}\left[n_r(t)n_i\right]$ as:

$$\begin{aligned}
\mathrm{E}\left[n_r(t)n_i\right] &= \mathrm{E}\left[n_r\phi_{n_r}(t)n_i\right] \\
&= \mathrm{E}\left[n_r n_i\right]\phi_{n_r}(t) \\
&= \mathrm{E}\left[\int_0^T n(t)\phi_{n_r}(t)dt \int_0^T n(t)\phi_{n_i}(\tau)d\tau\right]\phi_{n_r}(t) \\
&= \int_0^T \int_0^T \mathrm{E}\left[n(t)n(\tau)\right]\phi_{n_r}(t)\phi_{n_i}(\tau)dtd\tau\phi_{n_r}(t) \\
&= \int_0^T \int_0^T \frac{N_0}{2}\delta(t-\tau)\phi_{n_r}(t)\phi_{n_i}(\tau)dtd\tau\phi_{n_r}(t) \\
&= \frac{N_0}{2}\int_0^T \phi_{n_r}(t)\phi_{n_i}(t)dt\phi_{n_r}(t) \\
&= 0
\end{aligned}$$

Therefor, since $r_i = s_i + n_i$ and given s_i transmitted as a constant value, we have $\mathrm{E}\left[n_r(t)s_i\right] = s_i\,\mathrm{E}\left[n_r(t)\right] = s_i\,\mathrm{E}\left[\int_0^T n(t)\phi_{n_r}(t)dt\right] = s_i\int_0^T \mathrm{E}\left[n(t)\right]\phi_{n_r}(t)dt = 0$.

As a result, the expectation $\mathrm{E}\left[n_r(t)r_i\right] = \mathrm{E}\left[n_r(t)(s_i+n_i)\right] = 0$