EE 503: Problems from Past Finals

1. You enter the DMV to get your brand new customized license plates EXPDIST for your car.

   There are three clerks. It takes clerk i an exponentially distributed amount of time of rate μ_i to perform this task, $i = 1, 2, 3$. Suppose $\mu_1 < \mu_2 < \mu_3$. Find how much time you spend at DMV in each of the following cases:

   (i) There is nobody at DMV when you enter, and you pick clerk i with probability $p_i, i = 1, 2, 3$.

   (ii) As soon as you enter, all three clerks start serving you but they dont colloborate.

   (iii) When you enter, each clerk is working on one customer. You join the first clerk that becomes free and complete service with him/her.

2. In P2P systems, user cooperation is a key element to achieve good system performance. In this problem, we will study how incentive schemes improve system performance. Lets consider a P2P file sharing system. Assume there is one seed, who has the complete file of interest, in the system at the beginning with service rate μ_s. Leechers, who want to download the file, arrive the system in accordance with a Poisson process of rate λ, and denote by μ_l the service rate of leechers. Further, assume resource is fair shared by all leechers. That is if there are K leechers in the system, the seed will provide each leecher a service rate $\frac{\mu_s}{K}$. Finally, assume the file download time of a leecher is exponential distributed with mean $\frac{1}{\mu_a}$ if the leecher receives an aggregate service rate μ_a.

   (i) If the system doesn't adapt any incentive scheme, leechers will not provide service to system (to other leechers) and they will leave the system as soon as they finish their downloads. Model this system as a CTMC by defining states and sketch the corresponding state transition diagram.

   (ii) Now we enforce an incentive scheme so that leechers will provide a portion, say α, of their service rates to the system (fair shared by all other leechers) but they still leave the system after they download the file. Model this system as a CTMC by defining states and sketch the corresponding state transition diagram.

   (iii) Find the stability condition of the two systems (with and without incentive scheme).

   (iv) Let $\mu_s = 15, \mu_l = 10, \alpha = 0.2$. Assume now there are 5 leechers in the system. Compute the expected time until all leechers finish their downloads for both systems.

   (v) Comment how incentive schemes improve system performance in terms of system stability and file download delay.

   (iv) Now we adapt a good incentive scheme so that leechers not only provide a portion of their service rates to the system but also stay in the system (become seeds) for an exponential time with mean $\frac{1}{\beta}$ after they download the file. Model this system as a CTMC by defining states and sketch the corresponding state transition diagram.

   (Hint: Now the number of seeds in the system is not fixed. You need to define states that incorporate seed dynamics.)

3. Consider the following Markov Chain (MC), whose transition diagram is shown in Figure 1. For state $3 \ldots 8$, each state will jump to any of its neighbors with equal probability. (Note that state 3's neighbors are 4 and 8. You cannot go from state 3 to state 1.)

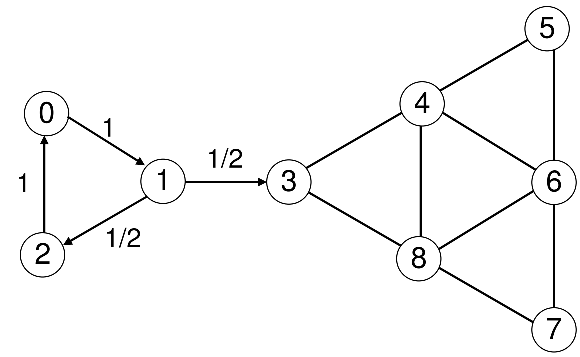

Figure 1: The Markov chain.

(i) Characterize the states of the MC (e.g. transient, recurrent, absorting, etc.).

(ii) If the chain starts from state 0, what is the probability that the chain will be in the same state after 4 transitions? If the chain starts from state 3, what is the probability it will be at state 6 after 2 transitions?

(iii) Find the expected number of transitions until the MC, starting in state 0, hits state 1. Find also the expected number of transitions until the MC, starting in state 0, hits state 3.

(iv) Does the MC have a stationary distribution? (Justify your answer.)

(v) Assume we start from state 3. Find the stationary distribution of the MC consisting of state $3 \ldots 8$.