

EE 503 : Homework 1

Due : 09/05/2023, Tuesday before class.

1. Show that

$$P(\cup_{i=1}^n E_i) \leq \sum_{i=1}^n P(E_i)$$

This is known as Boole's inequality. **Hint:** Either use $P(E \cup F) = P(E) + P(F) - P(EF)$ and mathematical induction, or else show that $\cup_{i=1}^n E_i = \cup_{i=1}^n F_i$, where $F_1 = E_1$, $F_i = E_i \cap_{j=1}^{i-1} E_j^c$, and use the axioms of probability.

2. The dice game craps is played as follows. The player throws two dice and if the sum is seven or eleven, then she wins. If the sum is two, three, or twelve, then she loses. If the sum is anything else, then she continues throwing until she either throws that number again (in which case she wins) or she throws a seven (in which case she loses). Calculate the probability that the player wins.
3. Suppose that 5 percent of men and 0.25 percent of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.
4. A deck of 52 playing cards, containing all 4 aces, is randomly divided into 4 piles of 13 cards each. Define E_i to be the event "the i^{th} pile has exactly one ace, ($i = 1, 2, 3, 4$)". Determine $P(E_1 E_2 E_3 E_4)$, the probability that each pile has an ace.
5. If the occurrence of B makes A more likely, does the occurrence of A make B more likely?
6. Stores A , B and C have 50, 75 and 100 employees, and, respectively, 50, 60, 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store C ?
7. Three prisoners are informed by their jailer that one of them has been chosen at random to be executed, and the other two are to be freed. Prisoner A asks the jailer to tell him privately which of his fellow prisoners will be set free, claiming that there would be no harm in divulging this information since he already knows that at least one will go free. The jailer refuses to answer this question, pointing out that if A knew which of his fellows were to be set free, then his own probability of being executed would rise from $\frac{1}{3}$ to $\frac{1}{2}$, since he would then be one of two prisoners. What do you think of the jailer's reasoning?
8. Ordering a 'deluxe' pizza means you have four choices from 15 available toppings. How many combinations are possible if:

- a) Toppings cannot be repeated?
 - b) Toppings can be repeated?
 - c) Suppose toppings can be repeated and all combinations are equally likely, what is the probability that a customer chooses 4 different toppings?
9. A deck of cards contains 10 red cards numbered 1 to 10 and 10 black cards numbered 1 to 10. How many ways are there of arranging the 20 cards in a row? Suppose we draw the cards at random and lay them in a row. What is the probability that red and black cards alternate?
10. A lot of 100 items contains k defective items. M items are chosen at random and tested. What is the probability that m are found defective?