HW 2: Kalman Filters

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Instructions

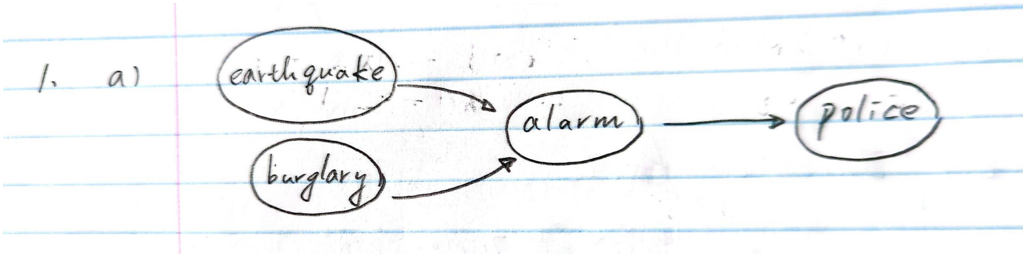
- Complete all the questions, *including* the "Resources Consulted" question. We expect that most students will use about a paragraph, or a few bullet points, to answer that question, but you can go longer or shorter.
- Submit the PDF *and* the code separately on Gradescope (look for "HW2: PDF" and "HW2: Code").
- For the PDF: To make things simpler for us to grade / inspect, please answer the questions in order (resources consulted first, then question one, then two, etc.). There is no page limit, but (as a high-level piece of advice) avoid writing excessively long-winded solutions. Use LaTeX for this; you can build upon this file. If you do that, you can remove the text that describes the actual questions if you want, please just make it *really easy* for us to see which question you are answering.
- For the code: some questions have code, which you will need to write. **Please use Python 3** for your code. Just submit everything as a single `.zip` file. Please include a brief README or brief comments in the code that make it *very easy* for us to run.

Resources Consulted

Question: Please describe which resources you used while working on the assignment. You do not need to cite anything directly part of the class (e.g., a lecture, the CSCI 545 course staff, or the readings from a particular lecture). Some examples of things that could be applicable to cite here are: (1) did you collaborate with a classmate; (2) did you use resources like Wikipedia, StackExchange, or Google Bard in any capacity; (3) did you use someone's code? When you write your answers, explain not only the resources you used but HOW you used them. If you believe you did not use anything worth citing, *you must still state that below in your answer* to get full credit.

Answer:
Reference for Kalman Filter:
https://reurl.cc/OG6oMy
Reference for coding:
https://github.com/rlabbe/Kalman-and-Bayesian-Filters-in-Python/blob/master/11-Extended-Kalman-Filters.ipynb
https://zhuanlan.zhihu.com/p/352443556

1. Answer each of the following questions.

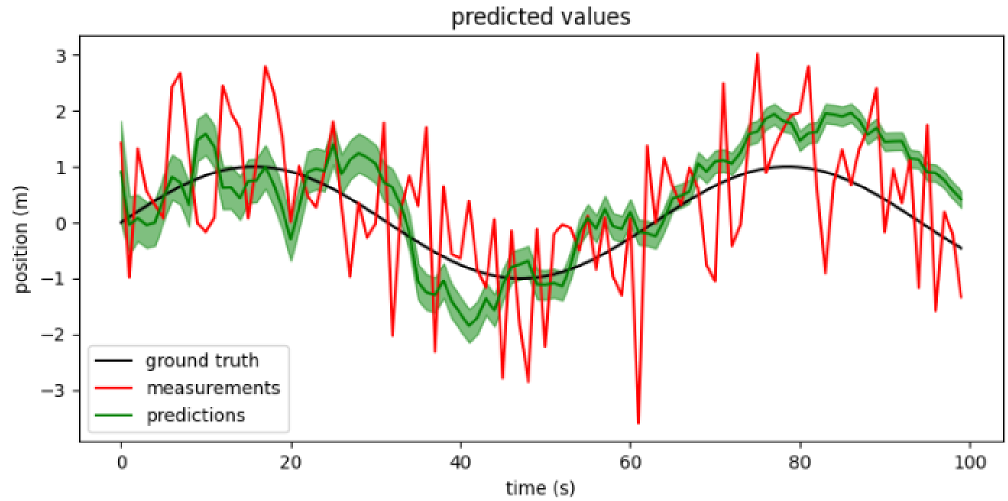
   (a) 

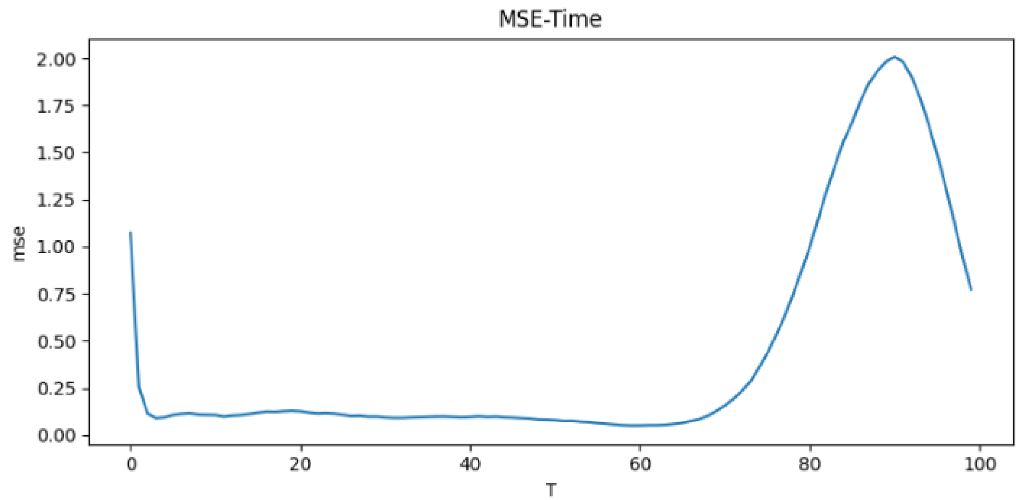
   (b) If at any point in Bayesian filtering the probability of a state assignment becomes 1, then that state will be considered certain, and the system will cease further update. This may cause future beliefs that contain the actual position not being taken into account. We can introduce more observation data or be aware of biases in measurement to avoid this situation.

   (c) Extended Kalman Filters (EKFs) work by linearizing nonlinear systems through Linear Taylor Expansion. However, this process of linearization performs poorly when dealing with multi-modal systems.

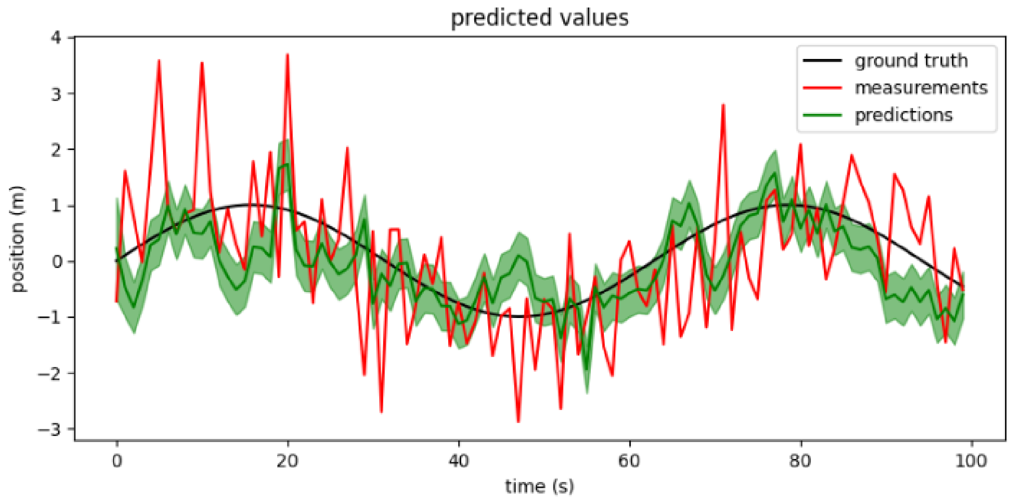
2. Kalman Filter:

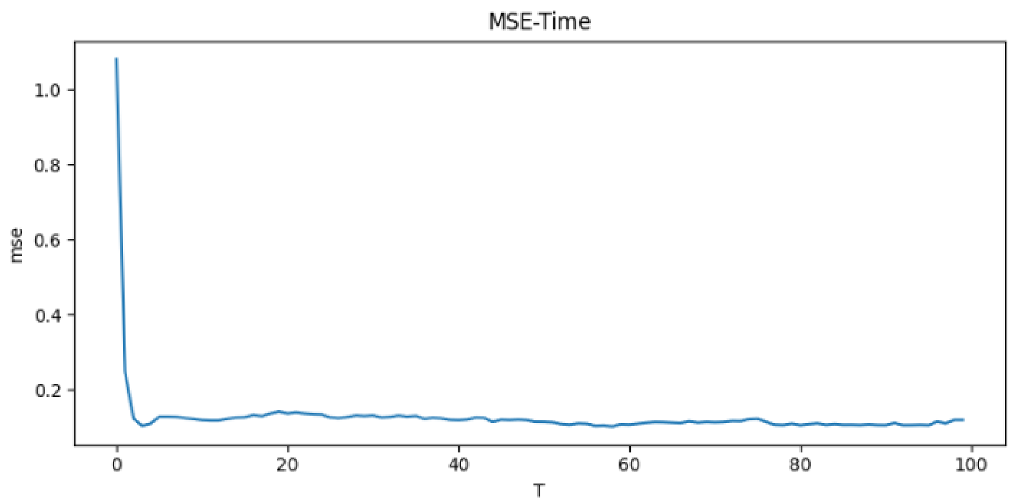
   (a) Without R:





(b) With R:


predicted values
ground truth
measurements
predictions
position (m)
time (s)
4
3
2
1
0
−1
−2
−3
0
20
40
60
80
100


MSE-Time
mse
T
1.0
0.8
0.6
0.4
0.2
0
20
40
60
80
100

3. Consider the scalar system:

(a) Write the equations:

3. (a)

$$x(t+1) = \alpha x(t) + w(t) \qquad w(t) \sim G(0, R)$$

$$z(t) = \sqrt{x(t)^2 + 1} + v(t) \qquad v(t) \sim G(0, Q).$$

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ \alpha \end{bmatrix} = \begin{bmatrix} \alpha_{t-1} x(t) + w(t) \\ \alpha_{t-1} \end{bmatrix} = \begin{bmatrix} g_1(x, \alpha) \\ g_2(x, \alpha) \end{bmatrix}$$

$$\bar{z}(t) = z(t)$$

$$\Rightarrow g_1(x, \alpha) = \alpha_{t-1} x(t-1) + w(t)$$

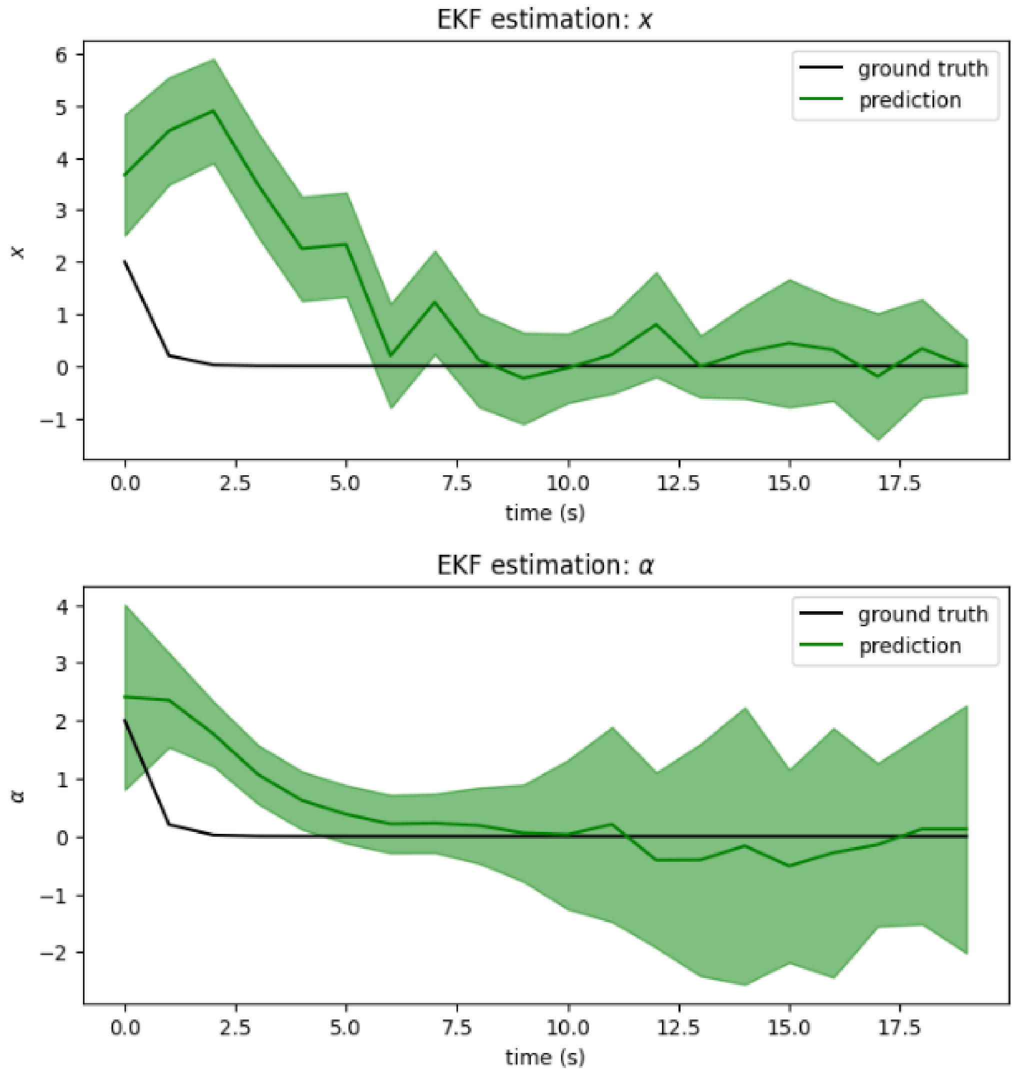
$$g_2(x, \alpha) = \alpha_{t-1}$$

$$\Rightarrow G_t = \begin{bmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial \alpha} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} \alpha_{t-1} & x(t-1) \\ 0 & 1 \end{bmatrix}$$

$$h = \sqrt{x(t)^2 + 1}$$

$$H = \frac{\partial h}{\partial x} = \begin{bmatrix} \frac{x(t)}{\sqrt{x(t)^2+1}} & 0 \end{bmatrix}$$

(b) Figure:


EKF estimation: x
ground truth
prediction
x
time (s)
EKF estimation: α
ground truth
prediction
α
time (s)