# Introduction

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These are the supporting materials for our paper tilted *On $\tau$-tilting finiteness of blocks of Schur algebras*, see arXiv: 0000, 00000, in which we are trying to give a complete list of $\tau$-tilting finite Schur algebras. To understand the list, one needs some bound quiver algebras $A$ and the numbers $\#\mathsf{s}\tau\text{-}\mathsf{tilt}\, A$, as displayed below,

| $A$ | $\mathbb{F}$ | $\mathcal{A}_2$ | $\mathcal{A}_3$ | $\mathcal{D}_3$ | $\mathcal{R}_4$ | $\mathcal{H}_4$ | $\mathcal{D}_4$ | $\mathcal{K}_4$ | $\mathcal{U}_4$ | $\mathcal{M}_4$ | $\mathcal{L}_5$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\#\mathsf{s}\tau\text{-}\mathsf{tilt}\, A$ | 2 | 6 | 20 | 28 | 88 | 96 | 114 | 136 | 136 | 152 | 1656 |

In this webpage, we give a complete list of $g$-vectors for each bound quiver algebra in the above table, in order to illustrate the number $\#\mathsf{s}\tau\text{-}\mathsf{tilt}\, A$.

We may fix some notations as follows. Let $A$ be a finite-dimensional algebra over an algebraically closed field $\mathbb{F}$. We can determine the $g$-vectors for all indecomposable two-term presilting complexes in $\mathsf{K}^{\mathsf{b}}(\mathsf{proj}\, A)$, such that the $g$-vectors of two-term silting complexes can be displayed as the so-called $g$-matrices, where the entries are given by the former ones. We give an example here.

**Example.** Let $A = \mathbb{F}(\, 1 \underset{\beta}{\overset{\alpha}{\rightleftarrows}} 2\,)/ < \alpha\beta, \beta\alpha >$. Then, the Hasse quiver $\mathcal{H}(2\text{-}\mathsf{silt}\, A)$ is

$$\begin{tikzcd}
 & \begin{bmatrix} 0\rightarrow P_1 \\ \oplus \\ P_2 \xrightarrow{\alpha} P_1 \end{bmatrix} \arrow[r] & \begin{bmatrix} P_2\rightarrow 0 \\ \oplus \\ P_2 \xrightarrow{\alpha} P_1 \end{bmatrix} \arrow[dr] & \\
\begin{bmatrix} 0\rightarrow P_1 \\ \oplus \\ 0 \rightarrow P_2 \end{bmatrix} \arrow[ur] \arrow[dr] & & & \begin{bmatrix} P_1\rightarrow 0 \\ \oplus \\ P_2 \rightarrow 0 \end{bmatrix}. \\
 & \begin{bmatrix} P_1 \xrightarrow{\beta} P_2 \\ \oplus \\ 0 \rightarrow P_2 \end{bmatrix} \arrow[r] & \begin{bmatrix} P_1 \xrightarrow{\beta} P_2 \\ \oplus \\ P_1 \rightarrow 0 \end{bmatrix} \arrow[ur] &
\end{tikzcd}$$

We deduce that the complete list of $g$-vectors for $A$ is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix}.$$