A COMPLETE LIST OF g-VECTORS FOR \mathcal{R}_4

We recall that $\mathcal{R}_4 = \mathbb{F}Q/I$ is given by

$$Q: 1 \xrightarrow[\beta_1]{\alpha_1} 2 \xrightarrow[\beta_2]{\alpha_2} 3 \xrightarrow[\beta_3]{\alpha_3} 4$$

with

$$I: \left\langle \begin{array}{c} \alpha_1 \beta_1, \alpha_1 \alpha_2, \beta_2 \beta_1, \\ \alpha_2 \beta_2 - \beta_1 \alpha_1, \alpha_3 \beta_3 - \beta_2 \alpha_2 \end{array} \right\rangle.$$

Then, the complete list of g-vectors for \mathcal{R}_4 is as follows.

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & -1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 & -1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$(7) \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & -1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & -1
\end{pmatrix}$$

$$(10) \begin{pmatrix} 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & -1 & 1 \\
0 & 1 & -1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
0 & 1 & -1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & 0 & 1 \\
1 & -1 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$(14) \begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 1 & -1 \\
0 & 1 & 0 & -1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$(17) \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & -1
\end{pmatrix}$$

$$(19) \begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
1 & -1 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$(22) \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
0 & 1 & -1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 1 & -1 & 0
\end{pmatrix}$$

$$(24) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & -1 & 1 \\
0 & 0 & -1 & 1 \\
0 & 1 & -1 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & -1 & 1 \\
-1 & 1 & 0 & 0 \\
0 & 1 & -1 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$(27) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
1 & -1 & 0 & 1 \\
1 & -1 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & -1 & 1 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & -1 & 1 \\
0 & 0 & 0 & 1 \\
-1 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & 1 & -1 \\
0 & 0 & 1 & -1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 & 0 & -1 \\
0 & 1 & -1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & -1 \\
0 & 0 & 1 & -1 \\
0 & 1 & 0 & -1 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & -1 \\
0 & 1 & -1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
-1 & 1 & 0 & -1 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & 1 & 0 \\
1 & -1 & 1 & -1 \\
1 & 0 & 0 & 0 \\
1 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
1 & -1 & 1 & 0 \\
1 & -1 & 1 & -1 \\
0 & 0 & 1 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & 1 & -1 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
-1 & 1 & 0 & -1 \\
0 & 0 & 1 & -1 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$(40) \begin{pmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & -1 & 1 \\
0 & 0 & -1 & 1 \\
0 & 1 & -1 & 1 \\
0 & 1 & -1 & 0
\end{pmatrix}$$

$$(42) \begin{pmatrix} -1 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & 0 & 1 \\
0 & 0 & -1 & 1 \\
1 & 0 & 0 & 0 \\
1 & -1 & 0 & 0
\end{pmatrix}$$

$$(44) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 1 \\
-1 & 1 & -1 & 1 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$(46) \begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$(47) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & -1 & 1 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$(49) \begin{pmatrix} -1 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(50) \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & -1 \\
0 & 1 & 0 & -1 \\
0 & 1 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 1 & -1 \\
-1 & 1 & 0 & -1 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
-1 & 1 & 0 & -1 \\
0 & 1 & 0 & -1 \\
0 & 1 & -1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
1 & -1 & 1 & 0 \\
1 & -1 & 1 & -1 \\
1 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
0 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 \\
0 & 0 & 1 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
0 & -1 & 1 & -1 \\
0 & 0 & 1 & -1 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & 0 & 0 \\
-1 & 1 & 0 & -1 \\
-1 & 1 & -1 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & -1 & 1 \\
1 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 \\
0 & 0 & -1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & -1 & 1 \\
-1 & 1 & -1 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & -1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & -1 & 1 \\
0 & 0 & -1 & 1 \\
-1 & 1 & -1 & 0 \\
0 & 1 & -1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & -1 & 1 \\
-1 & 1 & 0 & 0 \\
-1 & 1 & -1 & 0 \\
0 & 1 & -1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 0 & 1 \\
1 & -1 & 0 & 1 \\
0 & 0 & -1 & 1 \\
1 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 1 \\
0 & -1 & 0 & 1 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & -1 & 1 \\
-1 & 1 & -1 & 1 \\
-1 & 1 & -1 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 0 & 1 \\
0 & -1 & 1 & 0 \\
1 & -1 & 0 & 1 \\
1 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
0 & -1 & 0 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$(70) \begin{pmatrix} 0 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(71) \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(72) \begin{pmatrix} -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(73) \begin{pmatrix} -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(74) \begin{pmatrix} -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(75) \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$(76) \begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$(77) \begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$(78) \begin{pmatrix} 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(79) \begin{pmatrix} 0 & 0 & -1 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & -1 & 1 \\
0 & -1 & 0 & 1 \\
1 & -1 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 0 & 1 \\
0 & 0 & -1 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
0 & -1 & 0 & 1 \\
1 & -1 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & -1 \\
1 & -1 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & -1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
-1 & 1 & -1 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & -1 & 1 & 0 \\
0 & -1 & 1 & -1 \\
1 & -1 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & -1 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0
\end{pmatrix}$$