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Consensus of self-driven agents with avoidance of collisions

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In recent years, many efforts have been addressed on collision avoidance of collectively moving agents. In this paper, we propose a modified version of the Vicsek model with adaptive speed, which can guarantee the absence of collisions. However, this strategy leads to an aggregated state with slowly moving agents. We therefore further add a certain repulsion, which results in both faster consensus and longer safe distance among agents, and thus provides a powerful mechanism for collective motions in biological and technological multiagent systems.

I. INTRODUCTION

One of the most marvelous and ubiquitous phenomena in nature is *collective motion*, a kind of motion that can be observed at almost every scale: from bird flocks and fish schools at the macroscopic level to bacteria, individual cells, and even molecular motors at the microscopic level [1–9]. Although in most cases agents do not share global information and often travel in the absence of leaders or external forces, collective motion may still occur. Analogous behaviors are reported in engineering systems also, such as groups of autonomous mobile robots and air vehicles [10–16]. (See also a newly reported swarm model that may connect granular materials and agent-based models [17].) In order to uncover the underlying mechanism leading to the consensus of collective population, Vicsek *et al.* [18] proposed a model with self-driven agents to mimic the biological swarm, which displays a novel type of kinetic phase transition. From then on, the nature of the nonequilibrium phase transition of collective motion attracted greater and greater attention [19–24]. Due to simplicity and efficiency, many modified versions of the Vicsek model were proposed. For example, some new methods with effective leadership were introduced [15,25,26], and new moving protocols with adaptive speed to accelerate the consensus were designed [27,28]. Meanwhile some scholars have studied the consensus of collective motions via low-cost communication [29] and predictive mechanism [30–32], all of which can greatly enhance the global convergence.

Recently, much attention has been focused on how to keep distances among agents. A common way is to introduce attraction and/or repulsion [15–17,33–40]. However, any kind of repulsion alone cannot sufficiently avoid collision at all times because it is entirely possible that in a high-density area, two agents are compelled to collide for the purpose of avoidance of collision with other agents. In this paper, we propose a swarm model with adaptive speed to completely eliminate collisions. In a plane, each agent adjusts its direction as the average direction of its neighbors while it resets its speed according to the minimal distance from its neighbors. The farther an agent is away from its nearest neighbor, the higher speed it has. This strategy can completely avoid collisions; however, it results in an aggregated state where the agents move very slowly in average. Therefore, we further introduce a repulsion that can break down the aggregation of agents, and thus sharply speeds up the global convergence and enlarges the average distance among agents.

II. MODEL WITH ADAPTIVE SPEED

We consider each agent as an inelastic ball with radius a, limited in a square-shaped cell of linear size L with periodic boundary conditions. Initially, each agent is randomly distributed in the square, with moving direction randomly distributed in $[-\pi,\pi)$. At each time step, the position of the ith agent is updated as

$$\vec{x}_i(t+1)=\vec{x}_i(t)+\vec{v}_i(t), \quad (1)$$

and its direction is updated as

$$\theta_i(t+1)=\langle\theta_i(t)\rangle_r+\Delta\theta_i, \quad (2)$$

where $\Delta\theta_i$ denotes the thermal noise which is a random number uniformly distributed in the interval $[-\eta,\eta]$. (In the main context, we only consider the noise-free case, namely, $\eta=0$. A brief discussion about the effect of noise is presented in Sec. IV.) $\langle\theta_i(t)\rangle_r$ denotes the average direction of the agents within the horizon radius r of the ith agent (including the ith agent itself), which reads

$$\tan[\langle\theta_i(t)\rangle_r]=\langle v_i\sin\theta_i(t)\rangle_r/\langle v_i\cos\theta_i(t)\rangle_r. \quad (3)$$

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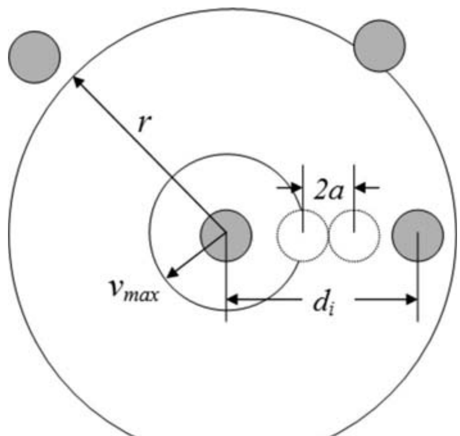

FIG. 1. Illustration of the current model with adaptive speed, where r denotes the horizon radius of an agent, d_i denotes the distance of the ith agent and its nearest neighbor, $v_{\max}$ denotes the possibly maximal velocity, and a denotes the size of an agent. Accordingly, $2a$ corresponds to the least distance of two agents.

In natural swarms, the speed of each agent is alterable; that is, agent may adjust not only its moving direction, but also its absolute velocity. In the common sense, to avoid collisions with other agents, an agent in a high-density group should adopt lower speed. Taking urban traffic as an example, the speed of an automobile is very low in the near-jammed situation, whereas it is generally of high speed when sparse automobiles take up the road. Accordingly, we set the speed of the ith agent to not more than $v_i=(d_i-2a)/2$, where d_i is the geographical distance between two centers of the ith agent and its nearest neighbor (see the illustration shown in Fig. 1). No matter how the ith agent and its nearest neighbor choose their directions in the next time step, the restriction can guarantee the distance between them no less than $2a$ and therefore avoid collision. In fact, this restriction is not only sufficient, but actually necessary. As the direction $\theta_i(t+1)$ of each agent in the next time step is determined by the average direction within its own horizon radius, it is impossible for an agent to know all the information of its neighbors, especially the moving directions of its neighbors in the next time step. Therefore, in order to avoid collision, it is obliged to take into account the worst circumstance, that is, two neighbors mutually approach. In this worst case, keeping the speed of each agent (labeled by i) no more than $(d_i-2a)/2$ is the only way to guarantee the absence of collisions.

Accordingly, the absolute velocity of each agent is updated with the following rule:

$$v_i(t+1) = \min\left(v_{\max}, \frac{d_i - 2a}{2}\right). \tag{4}$$

Clearly, when the distance between an agent and its nearest neighbor is longer than $2v_{\max}+2a$, its following speed can achieve the maximum; otherwise, its speed is limited as $(d_i-2a)/2$.

Moreover, in order to quantify the consensus of moving directions, an order parameter [18] is introduced as

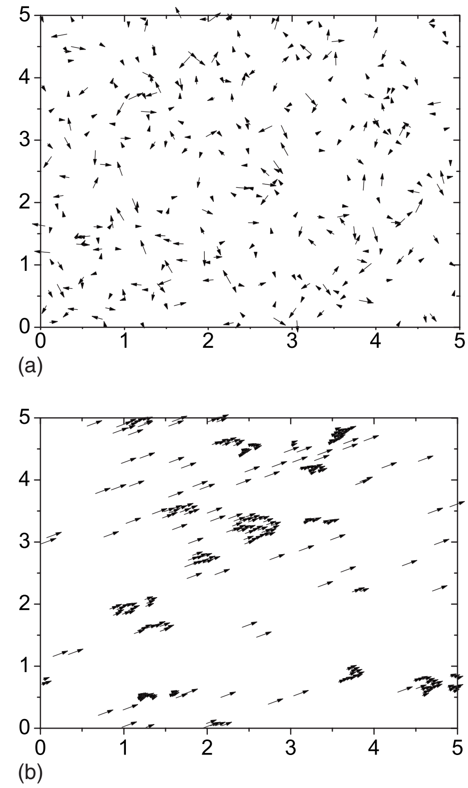

FIG. 2. Illustrations of locations and velocities (a) in the initial configuration, and (b) at the 500th time step. The parameters are set as $L=5$, $N=300$, $r=1$, $v_{\max}=0.03$, and $a=0.01$. The length and direction of an arrow represent the absolute value and direction of the corresponding agent's velocity.

$$V_a = \frac{\left|\sum_{i=1}^{N} \vec{v}_i\right|}{\sum_{i=1}^{N} v_i}, \quad 0 \le V_a \le 1, \tag{5}$$

where $v_i=|\vec{v}_i|$. A larger value of V_a indicates better consensus. Since the speed in this model is no longer constant, it is necessary to define another order parameter V_b to evaluate the consensus of the absolute velocity as

$$V_b = \frac{\sqrt{\Delta v^2}}{v}, \quad V_b \ge 0, \tag{6}$$

where $v=\langle v_i\rangle$ is the average absolute velocity of all the agents, and Δv^2 is the variance of the absolute velocity. Apparently, a smaller V_b corresponds to better consensus. Especially when $V_b=0$, all agents share the same speed.

Numerical results reveal that after the direction consensus, speed still varies. Figures 2(a) and 2(b) respectively illustrate the locations and velocities of all the agents in the initial configuration and at the 500th time step. After a certain time period from the beginning, the positions of agents are not uniformly distributed and an aggregation phenomenon appears [see Fig. 2(b)]. Therein the average speed in a high-density area is much slower than that in a low-density area. This aggregated state can be understood as follows:

agents in a high-density region agglomerate together and mutually move in a low speed; thus they can seldom disperse apart. Meanwhile they take up the way of their subsequent peers whose speed is higher, making the high-density area congregate more agents, and in turn achieving even higher density and slower speed. (Of course, on the other hand, the density is limited by the size of agents, a.) Moreover, the motions of agents are similar to the laminar flow in hydromechanics: when V_a gets close to 1, each agent is moving along a line with the same direction and will never diverge from its final track. Thus, different layers present various flowing speeds.

In the current model, the nearest distance among agents in high-density areas is very close to $2a$, making the involved agents move in a very low speed (close to 0). In the meantime the nearest distances among agents in low-density areas are usually more than $2a+2v_{\max}$; accordingly the involved agents can achieve a high speed (close to $v_{\max}$). Consequently, the absolute velocities of all agents in the whole system can be in a high diversity. Only in a low-density layer can the agents maintain high-speed movement in a comparatively long term. As a matter of fact, the swarm never get speed consensus even with identical directions, as shown later in Fig. 5(b). For the purpose of making all the agents achieve the consensus with higher speed, it is necessary to define a certain repulsion to avoid agglomeration. In addition, denoting $r_{ij}-2a$ as the safe distance between the ith and the jth agents, where r_{ij} denotes the geographical distance between the ith and the jth agents. In real applications of unmanned air vehicles and autorobots, the longer safe distance is favorable. Therefore we hope a properly designed moving protocol with repulsion could make the safe distance longer.

III. SCATTERING MODEL

Based on the strategy with adaptive speed mentioned above, in this section, we define a repulsion to enlarge the safe distances among agents. We assume: (1) the direction of the repulsion should be along the line of two agents, and (2) the magnitude of the repulsion should decrease with the increase in distance between two agents. Moreover, as long as the distance between two agents is over $2v_{\max}+2a$, no matter how they choose their directions and speeds, collision will not occur in the following steps. Considering this, the repulsion in our model should be a short-distance force and work only when the distance is shorter than r_0 ($r_0=2v_{\max}+2a$). Accordingly, we set the form of repulsion force as

$$\vec{f}_{ij}=\begin{cases} u\,\exp\left(-\dfrac{1}{1-r_{ij}/r_0}\right)\dfrac{\vec{r}_{ij}}{r_{ij}}, & r_{ij}<r_0 \\ 0, & r_{ij}\geq r_0, \end{cases} \qquad (7)$$

where u is a free parameter. Since the mass of an agent plays no role in the present model, we suppose the repulsive effect (caused by the repulsion) can directly affect the velocity vector in the next time step (see Fig. 3 for an illustration).

After defining such a repulsion, the moving direction of each agent is determined not only by the average direction within its horizon radius, but also by the repulsive effect. The synthesis of repulsive effect $\vec{f}_i$ [$\vec{f}_i=\Sigma_{j=1}^{N}\vec{f}_{ji}$, determined by Eq. (7)] and the average velocity $\vec{v}_i$ [whose direction and magnitude are respectively determined by Eqs. (3) and (4)] is set as the following moving direction of the agent (see Fig. 3). On the other hand, the absolute velocity should also follow Eq. (4). Numerical simulations, as shown in Fig. 4, indicate that this protocol can effectively scatter the aggregated agents [take Fig. 2(b) as an example for comparison]. Actually, under this protocol, each agent can hold a certain distance (much longer than the system mentioned in Sec. II) with its neighbors, and therefore achieves its maximal speed $v_{\max}$.

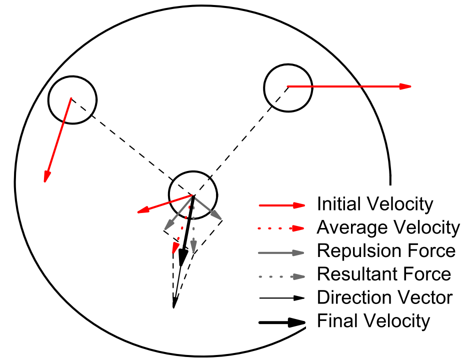

FIG. 3. (Color online) Illustration of the motion protocol with repulsive effect.

We also investigate the effects of repulsion strength by adjusting the parameter u. Figure 5(a) shows that the convergence of moving direction is not sensitive to the repulsion strength. However, a larger value of u corresponds to a shorter time for the system to achieve the consensus of speed, as well as a higher average speed in the steady state [see Figs. 5(b) and 5(c)]. Considering Eq. (4), the larger average speed actually implies that the average distance between agents is longer. Note that when $u=0$, V_b cannot approach 0, and the average speed is very low.

IV. CONCLUSION AND DISCUSSION

As long as we consider the sizes of agents, it is not only possible but actually necessary to propose a protocol to avoid

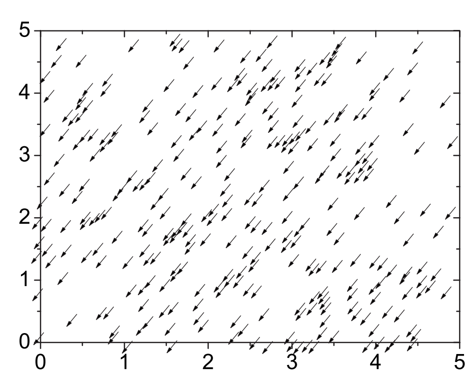

FIG. 4. The distribution of positions and velocities at 500th time step in the scattering model. The parameters are set as $L=5$, $N=300$, $r=1$, $v_{\max}=0.03$, and $a=0.01$. The length and direction of an arrow represent the absolute value and direction of the corresponding agent's velocity.

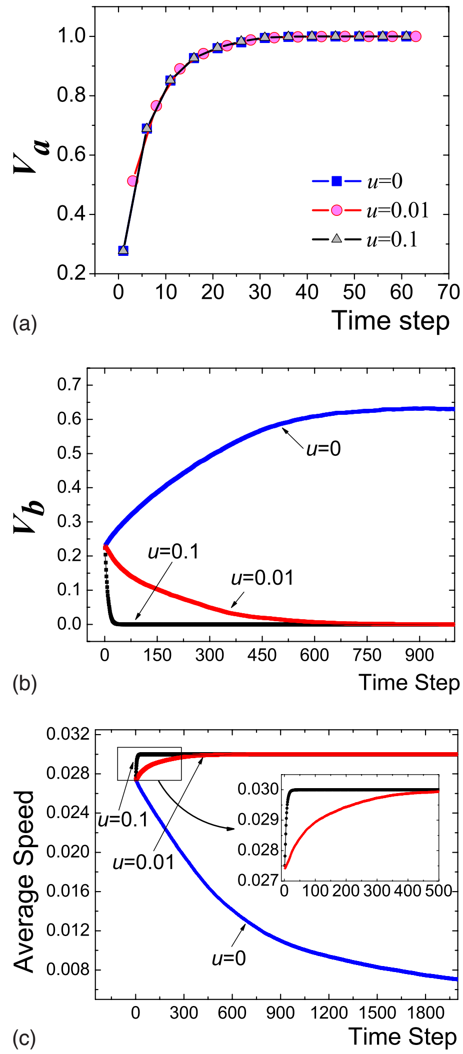

FIG. 5. (Color online) V_a, V_b, and the average speed versus time steps under different repulsion strengths. The parameters are set as $L=5$, $N=300$, $r=1$, $v_{\max}=0.03$, and $a=0.01$. All the data come from the average results of 500 independent runs.

collisions among them. Although the Vicsek model [18] has achieved a great success in mimicking the self-driven swarm, it cannot guarantee the absence of collisions. We report in Fig. 6 a simple simulation of the noise-free Vicsek model neglecting the sizes of agents. As the population grows, in the stable state, the minimal geographical distance between pairs of agents decreases quickly. If the size of agent is set as $a=0.01$, then the minimal distance to avoid collisions must be larger than $2a=0.02$. That is to say, the standard Vicsek model can only hold less than 200 agents with size of 0.01 in a 5×5 square. In comparison, the current model with adaptive speed can hold thousands of such agents.

However, numerical simulations showed an aggregation phenomenon in the current model, which impedes the convergence of speed. To overcome this blemish, we define a repulsion to scatter the aggregated agents. The simulation results are exciting: each agent can hole a certain personal space. What is more, they can quickly achieve speed consensus and move in a very high speed. Numerical results also indicate that the stronger the repulsive effect is, the less con-

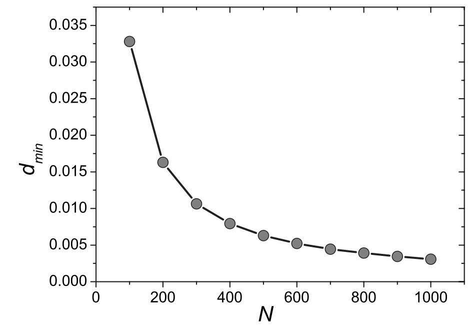

FIG. 6. Minimal geographical distance between pairs of agent in the stable state of the standard Vicsek model, $d_{\min}$, versus the number of agents, N. Each data point is the average of 1000 independent runs. The restriction to avoid collisions with agent size $a=0.01$ corresponds to $d_{\min}>0.02$.

vergence time it takes to achieve the consensus. In Sec. II, we have already proved that even when two neighbors mutually approach, the adaptive strategy can still help to avoid possible collision. Therefore, in any event, collision will never occur in the scattering model.

Furthermore, it is well known that the thermal noise can also play a significant role in determining the moving directions of agents. Thus, we need to check whether our rule is robust in the presence of noise. The numerical result indicates that even in the noisy environment, in the stable state, the average distance and average speed are both larger than those without the repulsion. The order parameter for direction consensus of course decreases with the increase in noise strength η, and it exhibits almost the same trend as the standard Vicsek model (actually, it is a little bit larger than the Vicsek model; see please the simulation result shown in Fig. 7).

In the noise-free Vicsek model, given r and L, the convergence is faster with more agents (i.e., larger N) since they will have more frequent communications in a denser circumstance. Actually, a recent numerical study [28] indicates that the convergence time scales approximately as $(\ln N)^{-1.3}$; that is, the larger the population is, the shorter the convergence time is. In Fig. 8, we report the simulation result on the convergence time in the noise-free Vicsek model (see the

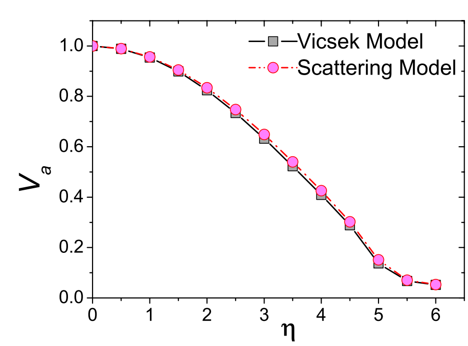

FIG. 7. (Color online) Comparison of order parameters V_a in the Vicsek model and the scattering model under noisy environment. The parameters are set as $L=5$, $N=300$, $r=1$, and $v_{\max}=0.03$. In scattering model, $u=0.01$ and $a=0.01$. All the data come from the average over 500 independent runs.

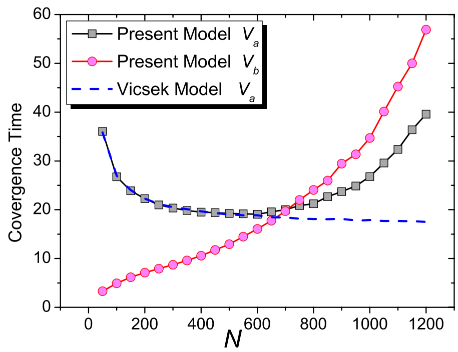

FIG. 8. (Color online) Comparison of convergence times between the standard Vicsek model and the present model (i.e., the scattering model) in the absence of noise. In the Vicsek model and the present model, the convergence time for V_a is defined as the required time steps making V_a larger than 0.99, while the convergence time for V_b is defined as the required time steps making V_b smaller than 10^{-3}. Blue dashed curve represents the simulation result for the Vicsek model, while the black squares and red circles represent the results for V_a and V_b, respectively. The parameters are set as $L=5$, $r=1$, and $v_{\max}=0.03$. In the present model, $u=0.1$ and $a=0.01$. All the data come from the average over 5×10^3 independent runs.

blue dashed curve), where the threshold quantile is set as $V_a=0.99$. It decreases monotonously with the increase in N, in accordance with Ref. [28]. In contrast, in the present scattering model, more effort should be taken to avoid collisions in the denser circumstance. Figure 8 compares the convergence times between the standard Vicsek model and the scattering model in the absence of noise. One can find that in the sparse circumstance, $N\leq600$, the convergence times of the Vicsek model and the scattering model are almost the same, while in the denser range, the convergence time in the scattering model quickly increases versus the slow decrease in that in the Vicsek model. The convergence time for absolute velocity increases even most quickly than that for moving direction. This result indicates a limitation of the present model, namely, it cannot efficiently deal with the systems with huge population. Accordingly, how to design an efficient method to simultaneously guarantee the absence of collisions and the quick convergence is still an open problem for us. Anyway, in the case of $a=0.01$, the standard Vicsek model can avoid the collisions only if the number of agents is less than or about 100 (see Fig. 6), while the scattering model can hold about 600 agents with the same speed of convergence. We therefore believe that the scattering model can find applications in the design of motion protocol for self-driven agents.

Some difficult yet important problems about the conservative model remain to be further explored. For example, if the ahead ones of a group of agents need not pay attention to the following ones (that is, each agent only receives information in a sector ahead in its moving direction rather than all the neighbors within its sight radius [41]), collisions may automatically disappear. If the swarm needs shorter time to get convergence while avoiding the collisions, it may indicate that the complete communication is not always the most efficient manner while partial communication may be better in some cases [29,41]. In addition, the properties of the phase transition induced by the noise (see, for example, in Ref. [34], Grégoire and Chaté showed that a swarm model with repulsion as well as the minimal Vicsek model suffers a discontinuous phase transition) remains an open issue. Though not the focus in this article, it is worth a detailed investigation in the future.

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