Discrete Mathematics Projects

The main goals of the course project are to

- allow students to choose a topic of personal interest for in-depth study and research
- provide experience in reading technical literature
- provide experience in self-directing a partnered or small-group project
- provide practice at presenting technical information (written and orally) at a level approaching that of professional journals/conferences
- provide a meaningful culminating experience for every student

Guidelines:

1. **Choose a topic and partner(s):** You must choose a topic that interests you and appropriately challenges you. You will work in groups of 2 or 3, and you will decide on a project topic in consultation with your partner(s) and professor. The end of this document contains a list of suggested topics, but you are encouraged to propose your own topic.
2. **Preliminary ideas:** A short statement (2-3 sentences) of your intended project topic is due at the end of class on April 25. You will be given time in class this day to work on your statement.
3. **Background research:** Each person in the group must research the chosen topic. I suggest starting with our textbook, then do web-searches, looking for scientific/technical articles, and using interlibrary-loan as necessary to procure books/journals (plan ahead!). Your bibliography should include professional technical material (available on- or off-line).
4. **Prepare project outline:** The project outline is due at the end of class on May 2. This should be typed but short. It will be a useful road-map for your final project, and it will ensure that you can perform any ILL requests before the break. It should include:
    a. A brief description (2-3 sentences) of the chosen area of study
    b. A list of the **specific goals** of the project. (What do you hope to accomplish? What exactly will you investigate?)
    c. A brief explanation of why the chosen topic is appropriate* given the backgrounds, interests, and learning goals of each student on the team. (*Do not propose to learn about algorithm complexity if you already know about algorithm complexity. Do propose to learn about algorithm complexity if you have no idea what this means but it sounds interesting.)
5. **Prepare Paper:** (Audience is your classmates in Math 4) Each team will write a 1400-2000 word paper (≈4-8 pages) on your chosen topic. You should work to fit high quality mathematics and exposition within the page limitation; be judicious in your use of space. The paper should be (nearly) professionally written and should (in most cases) include sections as follows:
    a. Abstract – Give a short, self-contained description of the project. This will be archived for future reference.
    b. Introduction - Give a short but informative introduction to your chosen topic, including the current "state of the field" as appropriate. Outline what you will do in the balance of the paper (ie. Explain the goals of the paper). Indicate why this topic is exciting/useful.

c. Background - Introduce any new definitions, notation, or other background information necessary for understanding the rest of the paper. (Your audience is your classmates in Math 4, not your professor.) Depending on your subject, your background and introduction sections might be combined.

d. Main Section(s) - Describe the heart of your project. What did you learn? What results did you discover? What examples did you find/invent? What theorems did you prove? Show off the new stuff that you now know.

   i. You can (and should) use references to guide your write-ups of your definitions, examples, theorems, proofs, etc., but you must explain the details in your own words and you must **cite your references**.

   ii. Somewhere in your paper, you must demonstrate that you learned some **hard/rigorous mathematics** – something that was hard to do or hard to learn.

   iii. Somewhere in your paper, you must demonstrate some **original thought**. For example, you might come up with a new algorithm that combines existing algorithms in a new way, an original comparison of different systems, a new application of some theory, a new class of examples, or a new proof of an old result. **Clearly indicate what is original and what was found in references.**

e. Conclusions - Wrap things up. This might involve drawing connections among the new things you learned or between the new things you learned and old things you learned in the class. Remind us what new knowledge your project produced and remind us why your project is exciting/useful. Deviating from the style of professional papers, include a short, final paragraph that reminds us of your "hard/rigorous mathematics" and your "original thought."

f. Bibliography. The items in your bibliography should be cited inline as they are used in your paper. Extraneous bibliographical items should not be included.

6. **Prepare Oral Report:** Each team will deliver a 10-15 minute (depending on team size) presentation on their project. While the paper will include all technical details, the oral will generally omit certain details in favor of providing an overview of new material to your peers in a way that is digestible. You will have had a lot of practice in your topic by the end of the semester, but most of your peers will be seeing the material for the first time. Structure your presentation as you would hope others would structure theirs! Make it interesting and easy to understand. It is generally helpful to include many examples. It should have an easy-to-follow structure (introduction through conclusions). You may include brief interactive activities with the class, but be mindful of time because these activities often take longer than anticipated.

Final preparations: Proof-read, proof-read, and proof-read your paper and presentation slides multiple times. The goal is to have your writing/exposition at a level approaching that of a professional publication. Practice the oral presentation, and then practice, practice, and practice again. The timing should be precise (10-14 minutes depending on your team size), the content should be precise, any activities should be well rehearsed, and extraneous chatter should be eliminated in order to achieve a high-density talk. The talk should be rehearsed enough to present at a conference with an undergraduate audience.

Evaluation: Please see the "Presenting Mathematics" document (posted on Moodle) for criteria for effective papers and oral presentations. Keep these criteria in mind as you proceed with your project. These criteria will be used in the evaluation of your work. Each student will also evaluate every other presentation (peer report form is posted on Moodle).

Project Ideas

You may choose your own project idea in the broad area of discrete mathematics, subject to the constraints mentioned above (appropriately challenging, allows for the learning of hard/rigorous mathematics, and allows for creative thought). For basic definitions of different areas of discrete math, and many ideas for topics nicely organized under umbrella topics, see http://mathworld.wolfram.com/topics/DiscreteMathematics.html Below, I give a few ideas, including enough words that you can Google to get a sense of whether you are interested. I am also more than happy to chat about your interests and personal goals to help you choose a project that will be a good match. Within each topic defined below, I am happy to help define specific goals – these are just topics to get you started.

- Strong Mathematical induction: how does it compare to weak mathematical induction? What type of results can be used as a proof tool where weak mathematical induction fails? Connect mathematical induction to the well-ordering principle with proof and rigor.

- Second-order linear homogeneous recurrence relations: describe them in their general form. How can they be solved? What applications use first and second order linear homogeneous recurrence sequences?

- Infinity and Beyond: What does it mean for there to be different "levels" of infinity? How are these infinities defined? How did Cantor (and others) prove this complexity around set cardinality?

- Fuzzy logic and set theory: What is fuzzy logic and/or set theory and how does it compare to standard propositional logic and set theory? What applications can it be used in where standard set theory/logic fails?

- Cryptosystems/hash functions: Learn the mathematics (usually some number theory) behind and the implement of cryptosystems or hash functions (Ceasar cypher, RSA, elliptical curve techniques, etc). El Gamal is a good place to start.

- Graph algorithms: There are many problems that people need to solve on graphs. These projects might include researching the related theory and implementing the related algorithms. Some ideas: Traveling salesman, network flows, network survivability, breadth first vs depth first searches, etc. Some of these are covered (briefly) in your book.

- Russell and Whitehead's *Principia Mathematica* and Kurt Godel's Incompleteness Theorems: What do they say about number theory and formal mathematical systems?

- Subgraph enumeration and/or spanning trees. In these topics, which are useful in a variety of applications, you are looking to identify various subgraphs that satisfy various conditions. A related topic involves Cayley's formula, which counts the number of labelled trees. Projects in these areas might involve graph algorithms, combinatorics (theory of counting), and possibly statistics.

- Group theory – these are discrete mathematical objects with lots, tons!, of applications, as examples:
    - Group theory involved with Rubik's cube and other games
    - Group theory (or actually field theory) involved with error-control codes

- How matrices and linear algebra are used in graph theory.

- Algorithm efficiency: Big O, Big Ω, Big Θ! Polynomial time versus Non-polynomial time algorithms (P Vs. NP!). See Ch 11 for a list of subtopics in this area.

- Regular expressions and finite-state automata – see Ch 12 for various aspects of Modeling Computation.

- See sections/chapters of our book that we skipped