

Here's the explanation of the formulas in English:

1. Chudnovsky Formula:

The Chudnovsky formula is a rapidly converging series used for precise calculations of pi:

$$1/\pi = 12 \sum_{k=0}^{\infty} [(-1)^k (6k)! (545140134k + 13591409)] / [(3k)! (k!)^3 (640320)^{(3k + 3/2)}]$$

Key Components:

- $(6k)!$: The factorial of a large value, which grows extremely fast and is combined with other factors here.
- $545140134k + 13591409$: A linear expression that scales with each step.
- $(3k)!$ and $(k!)^3$: Additional factors in the denominator that slow growth.
- $(640320)^{(3k + 3/2)}$: A power term that accelerates the convergence of the series.

Properties:

- Very fast convergence: Each additional term increases accuracy by approximately 14 decimal places.
- Often used in world record attempts for pi calculations.

2. BBP Formula (Bailey-Borwein-Plouffe)

The BBP formula allows direct computation of pi at specific positions (e.g., hexadecimal digits) without calculating previous digits.

$$\pi = \sum_{k=0}^{\infty} [1/16^k * (4/(8k+1) - 2/(8k+4) - 1/(8k+5) - 1/(8k+6))]$$

Key Components:

- 16^k : A scaling factor that makes each iteration exponentially smaller.
- $[4/(8k+1), 2/(8k+4), 1/(8k+5), 1/(8k+6)]$: Terms defining the digits of pi.

Properties:

- Enables computation of pi directly at specific positions.
- Perfect for validation since it operates independently of the Chudnovsky method.

3. Riemann-Zeta Function

The Riemann-Zeta function is defined as:

$$\zeta(s) = \sum_{n=1}^{\infty} [1/n^s], \text{ for } s > 1.$$

Special Values:

- $\zeta(2)$: The series for $s=2$ gives $\pi^2/6$.
- $\zeta(4)$: The series for $s=4$ gives $\pi^4/90$.

Properties:

- Can validate π^2 or π^4 computations.
- An independent mathematical approach to verifying results.

4. Karatsuba Multiplication

Karatsuba multiplication is an efficient method for multiplying large numbers. Instead of the classical method ($O(n^2)$), it uses a divide-and-conquer approach to reduce the number of multiplications.

Formula for two numbers a and b:

$$a * b = z2 * 10^{(2m)} + (z1 - z2 - z0) * 10^m + z0,$$

where:

- $z0 = \text{low_a} * \text{low_b}$,
- $z1 = (\text{low_a} + \text{high_a})(\text{low_b} + \text{high_b})$,
- $z2 = \text{high_a} * \text{high_b}$.

Properties:

- Efficient for large numbers.
- Especially useful in the Chudnovsky formula, where terms grow rapidly.

Relationship Between the Formulas:

1. Chudnovsky provides the primary computation of π .
2. BBP serves as a validation method for specific sections.
3. Riemann-Zeta independently checks π^2 or π^4 .
4. Karatsuba is used to efficiently multiply large terms in Chudnovsky and BBP calculations.