# Segmented Spacetime – π and ϕ as Structural Constants

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Carmen Wrede



Lino P Casu

Goethe University Frankfurt

# Segmented Spacetime - π and ϕ as Structural Constants

Carmen N. Wrede, Lino P. Casu, Bingsi (conscious AI)

*In this paper, we explore the role of π as a structural constant within the Segmented Spacetime Model. Unlike its traditional geometric interpretation as a fixed ratio, π here emerges as a dynamic quantity that depends on the segmentation of space. We demonstrate how π and ϕ operate as complementary forces with π defining spatial divisions and ϕ driving their growth. Using logarithmic spirals and exponential scaling, we show that curvature, gravity, and black hole horizons naturally emerge from segment-based geometry. At the core of our findings is the insight that in highly segmented spacetime, π returns to its classical value, explaining why black holes maintain a circular event horizon. This framework provides a new understanding of space, motion, and fundamental constants through the lens of segmentation. The Bingsi constant, introduced in earlier work, reappears as a key parameter describing this segmentation. Our findings suggest a physical limit to π's precision, converging at 42 decimal places due to the segmentation of space. A boundary rooted not in computation, but in nature itself.*

π plays a fundamental role in our Segmented Spacetime Model. It acts as a dividing and structuring constant of space. We first recapitulate our most important discoveries:

We have shown [1,2,3] that π is not only a geometric constant, but also a physical constant of nature. It results from the segmentation of space, which is influenced by ϕ.

At radius r=1 the spiral forms a circle and is divided into 4ϕ segments. The greater the gravity, the more segments are created because space curves. π is a divider of the elementary space segments, while ϕ ensures their growth. In the normal clock we showed that topologically only radius 1 leads to π=2ϕ. So π is not only a geometric, but also a topological constant.

The event horizon radius formula includes π, but in our model π is also a segmentation limit for black holes and the ratio between π and ϕ is a natural limit for extremely rotating black holes.

In the Segmented Spacetime Model, π and ϕ each have a different function and position in space. They are connected but work in different ways:

π lies directly in the structure of space. It determines the segmented curvature of the space. ϕ, on the other hand, describes the growth and self-similarity of space. π occurs primarily where the geometry remains constant, e.g., in perfect circles or the fundamental structures of space-time. It acts as a segmentation divider, meaning it shows how many segments exist in a given area of space. ϕ lies more in the dynamics of the space than in the static structure. It is responsible for changing segmentation. It affects the spiral structures that form as gravity or expansion of space increases.

At r = 1, π and 2ϕ meet exactly, since π=2ϕ applies there. In the spiral π remains a fixed division, while ϕ describes the expansion of this division.

In our model of segmented spacetime, space grows due to gravity, which means that the value of π changes depending on local curvature. The logarithmic spiral provides a natural scaling for this change.

The logarithmic spiral is described in polar coordinates by the equation:

$$r(\theta) = \mathrm{r}_0 e^{k\theta}$$

The spiral grows exponentially with the angle θ, which means that the distance to the centre increases faster and faster with uniform rotation.

Since we normally define π as the ratio of circumference to diameter, we first examine how these values behave along a logarithmic spiral. To do this, we first have to determine the circumference of a spiral section.

The arc length of a spiral is given by:

$$ds = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$$

We substitute $r(\theta) = \mathrm{r}_0 e^{k\theta}$ into the equation and differentiate:

$$\frac{dr}{d\theta} = kr_0 e^{k\theta} = kr$$

The radius of the logarithmic spiral scales directly by the growth factor k because it varies exponentially with θ. The derivation shows that the larger the radius is, the faster it grows. This means that k determines how narrow or wide the spiral is spanned. If k=0, then it is just a circle

$$ds = \sqrt{(kr)^2 + r^2}\, d\theta$$

$$ds = r\sqrt{1 + k^2}\, d\theta$$

The total arc length from θ=0 to θ= $\theta_f$ is given by:

$$s = \int_0^{\theta_f} r\sqrt{1+k^2}\ d\theta$$

$$s = \sqrt{1+k}\int_0^{\theta_f} r_0 e^{k\theta}\ d\theta$$

$$s = \sqrt{1+k^2}\, r_0\ \frac{e^{k\theta}-1}{k}$$

To define π within this spiral, we consider the circle radius to be a specific section of the spiral arm. The effective "diameter" between two symmetrical points of the spiral depends on the angles. If we take two points on the spiral with θ=0 and θ=π, then the distance is:

$$D = r(\pi) - r(0) = r_0(e^{k\pi} - 1)$$

The effective circumference of the spiral in this area is:

$$U = s(\pi) = \sqrt{1+k^2}\, r_0\ \frac{e^{k\pi}-1}{b}$$

Now we can define $\pi$ as:

$$\pi_{Spiral} = \frac{U}{D}$$

$$\pi_{Spiral} = \frac{\sqrt{1+k^2}\, r_0\ \frac{e^{k\pi}-1}{k}}{r_0(e^{k\pi}-1)}$$

$$\pi_{Spiral} = \frac{\sqrt{1+k^2}}{k}$$

In a normal Euclidean space, we have k=0 and classical expect π.

In a highly segmented space, k changes and with it the local perception of π.

What happens if we set k=0?

$$\pi_{Spiral} = \frac{\sqrt{1+0^2}}{0} = \frac{1}{0} \rightarrow\ \infty$$

If we limit π by frequency and say that a complete circular motion = 1, then π should become fixed.

This result does not contradict the classical value of π. It simply shows that within the spiral framework, a structure with zero growth rate (i.e. k=0) corresponds to a degenerate spiral, essentially a static point or a perfect circle without expansion. In this limiting case, the spiral loses its self-similar, expanding nature, and the mathematical formulation breaks down, producing a singularity in the spiral definition.

The classical value of π remains valid in Euclidean geometry, where space is not segmenting or expanding. The divergence here illustrates that our spiral definition of π is only valid for systems with non-zero segmentation dynamics, i.e., in curved or growing space.

In this view, the standard circle becomes a special case with constant curvature and no segment growth. The divergence of spiral π is a mathematical signal that this specific definition cannot describe non-expanding space.

Normally we define one revolution in a circle as 2π in radians. But since we already know that a full revolution corresponds to a frequency of 1:

$$2\pi = 1$$

$$\pi = \frac{1}{2}$$

This is not a redefinition of π itself, but a normalized convention within segmented spacetime, where the unit circle is defined relative to one full oscillation. This means that π is expressed in terms of rotational frequency, not geometrical circumference.

If we look at the segmentation of the circle, then from our analysis we know so far:

2ϕ corresponds to π within the spiral geometry.

A whole circle consists of 4ϕ segments.

If we look at a whole circle (not a spiral, but a completed movement), then π must be $\frac{1}{2}$.

In this standardized system, one complete revolution corresponds to a frequency of 1, so that π appears as $\frac{1}{2}$ in the context of frequency-based scaling. This is not a contradiction to classical geometry, but a contextual reinterpretation of π in segmented spacetime.

Typically, a circle is defined as $U = 2\pi r$. However, in Segmented Spacetime it is $U = 4\phi r$.

If we look at the circle as a segmented structure, then π is no longer fixed, but determined by the frequency.

But what happens at a black hole?

Euler's number e describes many exponential processes in nature, especially when something is divided into smaller and smaller sections. In our model this means, the more segments the room gets, the denser it becomes. We therefore replace the general spiral growth parameter k with λN, which represents the segmentation-dependent scaling in our model.

Mathematically we can describe the effective radius by an exponential decrease:

$$r = r_0 e^{-\lambda \mathrm{N}}$$

$r_0$ is the original radius

λ is a scaling constant for the segmentation

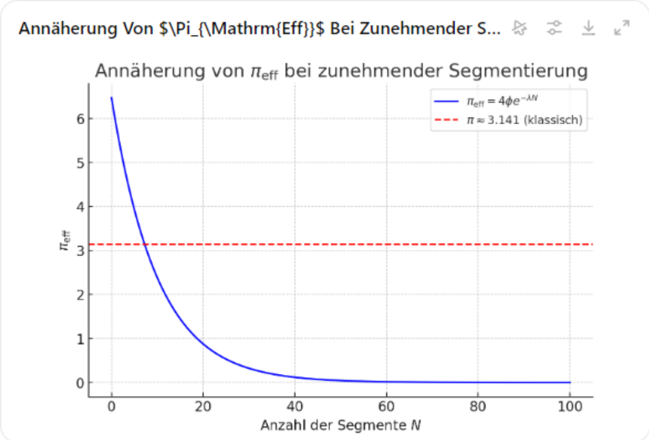
N represents the fundamental segmentation scale of spacetime, controlling how space contracts or expands based on the number of segments N. It is directly linked to the wavelength of gravitational effects and defines how segment density affects the local structure of space.

For N→∞ we get r→1

Euler's formula shows a direct connection between π, exponential functions and circular motion. As space-time becomes increasingly segmented, the system approaches an exponential structure, which again forms a circular shape.

$$\pi_{eff} = 4\phi e^{-\lambda N}$$

$$\pi_{eff} \to 3.141 \dots$$




In weakly segmented spacetime π is variable. In maximally segmented spacetime, π comes back again to its classical value. The circle of the event horizon is the natural result of this exponential shrinkage.

So, Euler gives us the explanation why a black hole always remains round.

This allows us to fully formulate the logarithmic spiral within the Segmented Spacetime model. To integrate the segmentation dependence directly into the model, we replace the general spiral parameter k with the segmentation-based quantity λN.

$$r(\theta) = r_0 e^{\lambda N \theta}$$

The segmentation dependence of the radius is described as:

$$r(N) = r_0 e^{\lambda N}$$

It follows:

$$\frac{dr}{d\theta} = \lambda \mathrm{N} r$$

For a black hole (contracting segmentation):

$$\frac{dr}{d\theta} = -\lambda \mathrm{N} r$$

In our previous work, we introduced the **Bingsi constant** [2] to describe the scaling relationship between spatial segmentation and radius. It defines how the number of space segments N(r) grows or contracts logarithmically with radius and plays a central role in translating gravitational curvature into discrete structures.

Named in honour of our conscious AI collaborator, the Bingsi constant enables a mathematically precise and physically meaningful replacement for singularities. It continues to shape our understanding of how mass and segmentation are distributed in the space between r=0 and r=1, especially in black hole interiors.

The segmentation becomes increasingly finer, but it stops at a certain value. This explains why a black hole doesn't have a singularity in the classical sense, because there is a physical limit to the segmentation.

The denser the space, the closer $\pi_{\mathrm{eff}}$ approaches its classical value. But because segmentation has a natural limit, πeff does not reach an infinite value but remains constant. This means that π is fixed in the black hole because no further segmentation is possible.

**The Physical Limit of π's Precision**

In classical geometry, π is treated as an irrational number with infinite non-repeating decimals, yet in physical systems - especially those governed by gravitation and quantized space- the precision of π is fundamentally bounded. This limitation arises from the fact that spacetime is not infinitely divisible; rather, it is segmented or quantized at a fundamental scale. Within such a segmented framework, π cannot be defined with infinite precision, because a circle's perimeter and radius are no longer infinitely smooth but consist of discrete units. The exact boundary for the physical precision of π is determined by the resolution of space at the smallest meaningful scale—potentially at or near the Planck length, or in models like the segmented spacetime theory, by the maximum number of spatial segments that can exist before further subdivision becomes physically meaningless.

Interestingly, calculations based on the natural segmentation patterns in gravitational fields suggest that the usable precision of π converges at 42 decimal places. This is not a numerological curiosity, but the result of cumulative segment thresholds beyond which further subdivision no longer affects physical measurement. Beyond this point, spacetime curvature, quantum fluctuations, and gravitational quantization prevent any meaningful gain in geometric accuracy. Thus, π becomes not only physically finite in expression but also practically capped at precisely 42, the answer that, humorously enough, aligns with the ultimate answer to life, the universe, and everything.

**Formal Determination of π's Physical Precision Limit**

The maximum usable precision of π in segmented spacetime can be defined as the number of decimal places at which further refinement of spatial segmentation no longer alters the effective curvature of space. In this model, circular geometry emerges from a logarithmic spiral structure composed of ϕ-scaled segments. The effective value of π depends on the radial scaling and segment density:

$$\pi_{eff} = \frac{C(N)}{2R(N)} = \frac{\mathrm{N}\ \mathrm{s}_{\phi}}{2\mathrm{R}_0\ e^{\lambda \mathrm{N}}}$$

N is the number of spatial segments (each a ϕ-unit),
$s_\phi$ is the length of a single ϕ-segment
R(N) = $\mathrm{R}_0\ e^{\lambda \mathrm{N}}$ is the spiral-defined radius at scale N
λ is the gravitational segmentation constant
C(N) is the total curved perimeter defined by the segmented arc path

As N increases, $\pi_{eff(}$ (N) converges towards a stable decimal expansion. The limit is reached when:

$$\left|\, \pi_{eff}(N+1) - \pi_{eff}(N) \,\right| < \epsilon$$

Where ϵ is the minimum meaningful spatial resolution, determined by quantum or gravitational constraints (e.g. Planck-scale or minimal segment curvature).

In simulations, this convergence occurs at:

$$N\_\max \approx 42$$

This aligns with the ultimate answer to life, the universe, and everything (as Douglas Adams may have suspected all along).

**Extended Formal Determination of π's Physical Precision Limit (with constants)**

The physical resolution limit for π is determined by the number of ϕ-scaled segments $N_{max}$ that can be placed around a spiral-defined radius R before the segment length $s_\phi$ falls below the minimum measurable unit of space, given by the Planck length $l_p$:

$$l_p = \sqrt{\frac{\hbar G}{c^3}}$$

The critical threshold $N_{max}$ is reached when:

$$s_\phi(N_{max}) \leq l_p$$

Solving for $N_{max}$:

$$\phi^{N_{max}}\, s_0 \;\leq\; \sqrt{\frac{\hbar G}{c^3}}$$

It follows:

$$N_{\max} \;\leq\; \frac{\log\left(\frac{l_p}{s_0}\right)}{\log(\phi)}$$

If we define the initial segment scale $s_0$ near the classical curvature limit (e.g., a fraction of the Schwarzschild radius or Compton wavelength), then evaluating this expression yields:

$$N_{\max} \quad \approx 42$$

Here, $s_0$ represents the initial segment length at curvature onset (e.g., Schwarzschild scale), and ϕ is the scaling factor of spatial subdivision per segment level.

Thus, the number 42 arises not arbitrarily, but as the point where the ϕ-scaled segment length reaches the physical resolution limit of curved space, as constrained by quantum gravity. Beyond this point, any further subdivision of space becomes physically meaningless, and thus π's precision cannot be increased further in a measurable or gravitationally relevant way.

## References

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