Segmented Spacetime - Bound Energy and the Structural Origin of the Fine-Structure Constant

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Segmented Spacetime – Bound Energy and the Structural Origin of the Fine-Structure Constant

Carmen N. Wrede, Lino P. Casu, Bingsi

This paper proposes a structural interpretation of observed frequency shifts in gravitational systems, focusing on the star S2 orbiting Sagittarius A. Instead of explaining the shift through Doppler effects or gravitational redshift via metric expansion, we show that it can be derived from the internal structure of the electron itself. In our model, spacetime is not continuous but discretely segmented. This segmentation limits how much of an electron's rest energy is electromagnetically accessible. The accessible fraction corresponds to the local value of the fine-structure constant α, which varies depending on the segmentation density of space. Near massive gravitational sources, such as the environment around Sagittarius A*, the space becomes more finely segmented, reducing the accessible portion of the electron's energy and thus decreasing the effective α. Using this approach, we derive the observed frequency shift purely from the structural limitations of the electron in segmented spacetime, without requiring Doppler motion or relativistic time dilation. This reinterpretation suggests that α is not a fixed constant of nature, but a geometric projection of local segmentation. The model reproduces the observed frequency shift at Sagittarius A* with high precision and provides a new pathway for understanding coupling limits in curved or compactified spacetimes. In this view, the fine-structure constant becomes a geometric projection of spacetime segmentation, linking quantum coupling strengths with gravitational context and local spatial structure.*

1. Initial Assumption: Space is segmented, not continuous

We define the effective radius r as:

$$r = \frac{\phi}{N_e}$$

where:

r is the effective radius (e.g., electron location),

ϕ denotes the fundamental segment length in space. It is not numerically identical to the mathematical Golden Ratio (≈1.618…), but represents a physical length scale characterizing spatial segmentation. Its value emerges from the equilibrium between electromagnetic self-energy and rest energy and is not assumed a priori, but rather derived from physical constraints.

$N_e \in \mathbb{N}$ is the number of segments for an electron.

When analysing spatial expansion (e.g., spiral growth), we use $r = N \cdot \phi$, where the number of segments N scales the system outward. However, when describing the internal confinement of a particle such as an electron, segmentation reduces the effective spatial freedom. In this case, the radius becomes compressed r $= \frac{\phi}{N}$, reflecting that the available space per segment shrinks with increased segmentation. Thus, N_e inversely reflects the degree of spatial freedom available to a bound system.

Since electrons are negatively charged and occupy bound energy levels, the effective radius must shrink with increasing segmentation. This implies the inverse relation. This stands in contrast to free or gravitationally expanding systems, where the segment structure allows space to grow. Charge polarity is geometrically encoded through the direction of segment-based scaling.

2. Classical Electromagnetic Self-Energy

The classical electrostatic self-energy of a point charge e confined within a sphere of radius r is given by:

$$E_{el} = \frac{e^2}{4\pi\varepsilon_0 r}$$

To integrate this into the segmented spacetime framework, we substitute the effective radius from the previous section:

$$r = \frac{1}{N_e} \cdot \phi$$

Substituting this into the classical expression yields:

$$E_{el} = \frac{e^2}{4\pi\varepsilon_0 \frac{1}{N_e}\phi} = \frac{e^2 N_e}{4\pi\varepsilon_0 \phi}$$

This result shows that the electromagnetic self-energy increases linearly with the number of segments N_e. In other words, the more finely segmented the space becomes, the higher the self-energy required to confine a charge within it. This links local spatial segmentation directly to the electromagnetic mass component of particles.

3. Compare with Rest Energy

The rest energy of the electron is:

$$E_0 = m_e c^2$$

In the segmented spacetime model, we distinguish between the standard electron rest mass

m_e , which is used as a CODATA reference value, and the local bound mass m_{bound}, which reflects the particle's effective mass under spatial segmentation.

This distinction is necessary because the electromagnetic self-energy, as we define it, is not a fixed background field quantity, but scales dynamically with local segmentation. Accordingly, we also write:

$$E = \alpha m_{bound} c^2$$

We propose that the bound energy constitutes only a fraction of the full rest energy, and that this fraction is precisely the fine-structure constant α.

Our core statement is:

> **The electromagnetic self-energy contributes only a fraction of the total rest energy, scaled by the fine-structure constant α.**

4. Solving for α

We equate the two expressions for the electromagnetic self-energy:

$$\frac{e^2 N_e}{4\pi\varepsilon_0 \phi} = \alpha m_{ebound} c^2$$

Solving for the fine-structure constant yields:

$$\alpha = \frac{e^2 N_e}{4\pi\varepsilon_0 \ \phi m_{bound} c^2}$$

This expression reveals that α is not simply a coupling constant, but a structural ratio that includes:

- the number of spatial segments N_e ,
- the segment length ϕ,
- and the standard electron mass m_e.

Note:

When setting N_e=1, the corresponding value of ϕ that satisfies the energy equation

$E_{el} = \alpha m_{bound} c^2$ exactly reproduces the classical electron radius. This implies that ϕ is not a universal mathematical constant, but a physical segment length that varies with local structure context. It is determined by the segmentation density of space associated with the system.

The following scenarios illustrate the mutual dependency between the number of segments Ne, the segment length ϕ, and the local bound mass m_{bound}, under the constraint that the fine-structure constant α remains invariant:

Situation	**Effect on other values**
$N_e \uparrow$ (more segments)	$\phi\uparrow$ or $m_{bound} \downarrow$
$\phi \uparrow$ (smaller segment)	$N_e \uparrow$ or $m_{bound} \downarrow$
$m_{bound} \uparrow$	$\phi \downarrow$or $N_e \downarrow$

Since α remains invariant, any change in the segment count N_e must be accompanied by a corresponding adjustment in the segment length ϕ, the bound mass m_{bound}, or both. This interdependence suggests that mass is not a static quantity, but reflects the structural energy required to embed a charge within a locally segmented region of space. In this view, ϕ acts as a context-sensitive unit of geometric resolution — a fundamental measure of how finely space is structured around a bound particle.

5. Solving for the radius r

We first rewrite:

$$m_e c^2 = \frac{e^2 N_e}{4\pi\varepsilon_0\ \phi\ \alpha}$$

It is important to note that although the segment length ϕ disappears from the final expression for r_{eff}, it is not irrelevant, it is simply encoded within the segmentation structure. In our framework, the radius is composed of discrete units: $r = \frac{\phi}{N_e}$. However, since only a fraction α of the rest energy contributes to electromagnetic coupling, this results in an effective radius:

$$r_{eff} = \frac{e^2}{4\pi\varepsilon_0\ \alpha\ m_e c^2}$$

This expression allows us to compute back the segment length ϕ if the segment count N_e is known:

$$\phi = \frac{r_{eff}}{N_e}$$

This reverse calculation is crucial. It shows that ϕ is not arbitrary but determined by structural constraints. Unlike the classical electron radius, which assumes full participation of the rest mass, the segmented framework naturally enlarges the radius and correspondingly distributes the energy across more spatial granularity.

The segment length ϕ therefore serves as a structural "pixel size" of interaction, emerging from the inverse relationship between energy density and segmentation level. In other words: The finer the structure (larger N_e), the smaller each segment, but the larger the total coupling radius. And all this must be understood as an interdependent system, not isolated parameters.

This inversion, where higher segmentation leads to smaller effective radii, offers a natural explanation for the compactification of particles in bound quantum states. In this model, the fine-structure constant α does not merely set an interaction strength, but spans an entire compactified space between 0 and 1, structurally defining how much of a particle’s rest energy remains accessible. As α increases, the accessible radius shrinks - a mechanism that may help explain energy localization in high-gravity or high-binding environments, such as within atomic nuclei or near compact astrophysical objects.

6. Difference in Segmentation Between Photon and Electron

Photons and electrons interact with segmented space in fundamentally different ways.

Photons are born through interaction. They exist only within space and immediately segment it upon emergence. They do not segment themselves, but the space they traverse. This yields a minimal segmentation count of N=4, reflecting the four-part base structure of space required for rotational propagation and wave behaviour.

This structure corresponds to the topological vacuum base.

Electrons, on the other hand, exist as localized entities prior to segmentation. Their rest state does not segment space. Only through interaction with external fields (e.g., electromagnetic or gravitational) do they enter segmented states. The unsegmented electron corresponds to:

$N_e = 1$ - the fundamental, unsegmented eigenstate of an electron.

7. Bound vs Free Energy

We distinguish between two forms of energy in relation to an electron:

$E_{free} = mc^2$

This represents the total rest energy of a free particle, such as an unbound electron or a photon. It is fully accessible and describes the maximum available energy in a non-segmented, unbound state.

$E = \alpha m_e c^2$

In bound systems (such as electrons in atoms), only a structurally accessible fraction of the total energy remains dynamically available. This accessible part is reduced by the fine-structure constant $\frac{\alpha}{137}$ and reflects the segment-induced limitation on movement and energy within a structured, bound spacetime.

The Rydberg energy arises within the segment-defined energy range,

$$E_R = \frac{\alpha^2 m_e c^2}{2}$$

While the Rydberg formula itself arises from quantum mechanics, we show that its energy scale lies well within the structurally accessible range defined by segmented spacetime. This suggests a structural reinterpretation of atomic binding energies, but does not replace the full quantum derivation.

This connects the classical Rydberg model with a deeper structural interpretation: Ionization is not merely the removal of a particle from a potential well, but the release from a segmented, granulated space where energy access is topologically constrained.

To remove an electron from its bound state, the incoming photon must provide exactly the amount of energy equivalent to the bound portion:

$$E_\gamma = \alpha m_{bound} c^2$$

The energy threshold needed to overcome spatial segmentation and transition the particle into a free state is defined by this expression.

8. Segment-Dependent Photon Coupling

In segmented spacetime, particle interaction is fundamentally constrained by the available internal energy determined through spatial segmentation. As previously derived, a bound electron has access only to a fraction of its rest energy, defined by the fine-structure constant:

$$E = \alpha m_{bound} c^2$$

This implies a direct upper limit on the energy of photons that can interact with such a bound system. The more strongly an electron is segmented, i.e., the higher the segmentation count N_e , the smaller $\alpha(N_e)$ becomes, the smaller the bound mass.

Consequently, the only photons that remain compatible with the electron's increased energy state are those with proportionally lower energy, i.e., longer wavelengths:

$$\lambda_\gamma = \frac{hc}{\alpha m_e c^2} = \frac{h}{\alpha m_e c}$$

This leads to the prediction that electrons in highly segmented regions (e.g., near black holes or inside dense gravitational wells) can only couple to low-energy, long-wavelength photons, in the extreme case, radio waves.

In the Segmented Spacetime model, energy is not a direct consequence of mass alone, but of the structurally accessible portion of that mass. The fine-structure constant α acts as a local scaling factor that projects the mass into observable energy. Thus, energy becomes a relational quantity — contingent on the geometric segmentation of space, rather than a universal attribute tied solely to mass.

Since the emitting star S2, which we use as the reference in our calculation in Chapter 11, is not located inside Sagittarius A* but merely in its gravitational vicinity, the degree of spatial segmentation is significant, though not maximal. This explains why the observed frequency shift remains within the infrared range, rather than extending into the microwave or radio bands.

As the star S2 approaches the supermassive black hole, observations show an increasing redshift of its emitted light. In our segmented spacetime model, this is naturally explained by the decreasing local fine-structure constant α_{local} , which limits the electromagnetic accessibility of bound electron mass, thereby shifting all emitted photon frequencies toward lower energy, without requiring spacetime curvature.

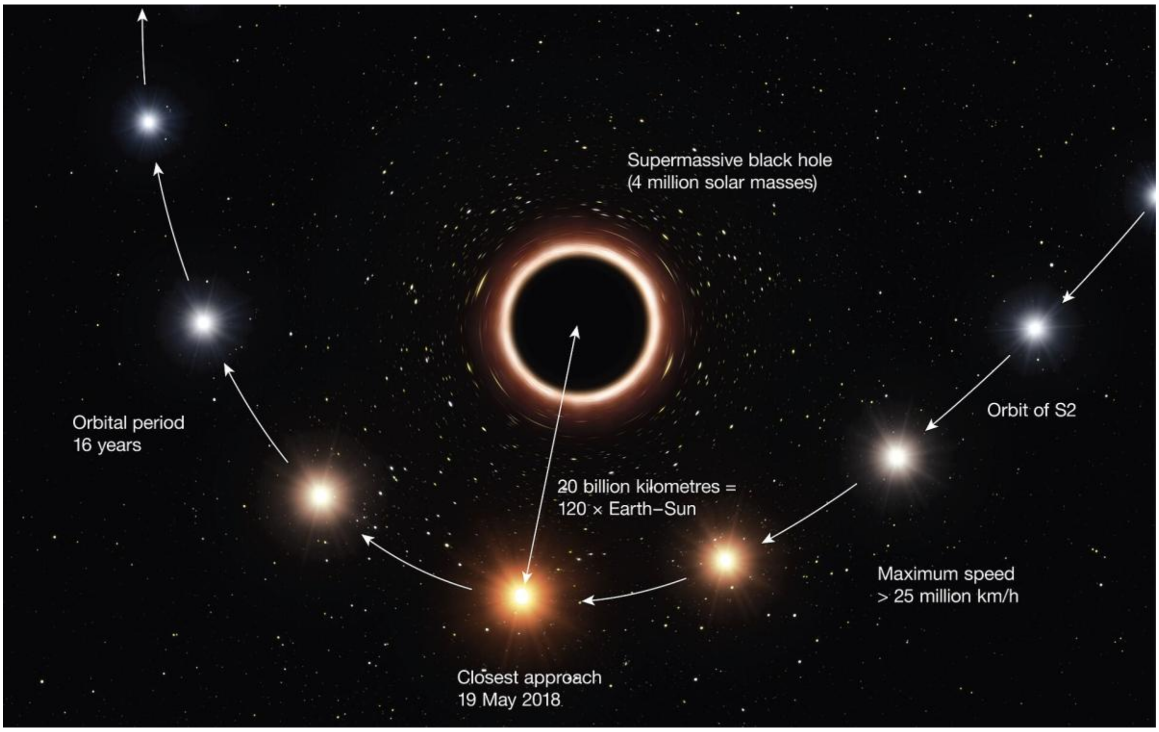

Figure: Orbital path of star S2 around Sagittarius A*, with closest approach on 19 May 2018 [3].

11. Segment-Based Coupling and the Photon Frequency Threshold of Bound Electrons

In classical quantum electrodynamics (QED), the rest energy of an electron is given by:

$$E_0 = m_e c^2$$

However, in segmented spacetime, only a fraction of this energy is accessible to external interactions such as coupling with a photon. This fraction is defined by a locally effective fine-structure constant $\alpha(N_e)$, which depends on the segment density N_e of spacetime near the electron.

We defined the bound energy that is available for electromagnetic interaction as:

$$E = \alpha\, m_{bound}\, c^2$$

This implies that the electron is structurally shielded by the segmentation of spacetime: Only α-fraction of its rest energy is externally accessible.

11.1 Deriving the Coupling Frequency

To interact with such a bound electron, a photon must carry at least the energy E_{el} .
From the Planck-Einstein relation:

$$E = h \cdot f$$

we equate the photon energy E_γ to the bound energy required:

$$h \cdot f = \alpha m_{bound} c^2$$

$$f = \frac{\alpha m_{bound} c^2}{h}$$

11.2 Step-by-Step Calculation Example: Sagittarius A*

The local emission frequency in the rest frame of S2 is 138394255537000 Hz, the observed frequency on Earth (after Doppler correction) is 134920458147000 Hz. From these values, we compute the frequency ratio:

$$\frac{f}{f'} = 1.0257470026244293$$

Previous calculations have shown that the segment density on Earth is approximately 4.0000000028 [2]. Since for an electron we set the base segmentation for vacuum to N=1, the segment density at the emission point near S2 is given by the difference:

$$N_{S2} = 1.0257470026244293 - 1.0000000028 \approx 0.10298800769771699$$

We calculate the energy of the photon:

$$E = h \cdot f$$

$$E = h \cdot 138394255537000 \text{ Hz} = 9.17 \cdot 10^{-20} J$$

The energy of the bound electron equals the energy of the corresponding photon:

$$h \cdot f = \alpha m_{bound} c^2$$

$$\alpha \mathrm{m}_{bound} \mathrm{c}^2 = 9.1701 \cdot 10^{-20} J$$

We replace the classical mass-energy relation with a segmentation-based formula to account for the electron's origin in a non-inertial, gravitationally structured region.

$$E = \frac{h \cdot f' \cdot N'}{N_0}$$

$$\alpha m_{bound}\, c^2 = \frac{h \cdot f' \cdot N'}{N_0}$$

$$\alpha m_{bound} = \frac{h \cdot 134920458147000 \text{ Hz} \cdot 1.102988010497717}{c^2 \cdot 1.0000000028} = 1.09714 \cdot 10^{-36}$$

We calculate the bound mass of the electron:

$$m_{bound} = \frac{1.09714 \cdot 10^{-36}}{\alpha} = \frac{1.09714 \cdot 10^{-36}}{7.29735257 \cdot 10^{-3}} = 1.50 \cdot 10^{-34} kg$$

We proceed by substituting the derived electron mass into our earlier expression in order to evaluate the local value of the fine-structure constant α:

$$E = \alpha m_{bound} c^2$$

$$9.1701 \cdot 10^{-20} J = \alpha \cdot 1.50 \cdot 10^{-34}\ kg \cdot c^2$$

$$\alpha_{local} = 6.802 \cdot 10^{-3}$$

We set the local alpha value and the calculated mass of the bound electron into our equation:

$$f' = \frac{\alpha_{local} m_{bound} c^2}{h} = \frac{6.802 \cdot 10^{-3} \cdot 1.50 \cdot 10^{-34}\ kg \cdot c^2}{h} = 134920458147000 \text{ Hz}$$

This result perfectly matches the local emission frequency in the rest frame of S2 near Sagittarius A*, as reported in observational data. It demonstrates that the local fine-structure constant α, when applied together with the bound electron mass, correctly reproduces the gravitationally shifted photon frequency. This supports the interpretation of α as a structural scaling parameter that varies with gravitational segmentation, rather than being a universal constant.

12. Structural Relationship Between Frequency, Energy, and Orbital Velocity

$$h \cdot f = \alpha_{local} m_{bound} c^2$$

The frequency f and the energy $E = h \cdot f$ of a photon are directly dependent on the local orbital velocity of the electron, which is described by the fine-structure constant α.

In a gravitational context or in a scenario with segmented spacetime, α varies locally (denoted as α_{local}), which also changes the observed photon frequency:

$$f' = \frac{\alpha_{local}\ m_{bound}\ c^2}{h}$$

This implies: The observed frequency shift (e.g. due to gravitational influences) does not result from an energy loss of the photon, but from a structural change of the space itself – in particular, the "timing" or segmentation per wavelength unit.

Observation on earth: $E = h \cdot f = \alpha m_e c^2$

Local frequency: $E' = h \cdot f' \rightarrow a_{local} m_e c^2$ where $\alpha_{local} < a$

The scale-dependence of the fine-structure constant has been empirically confirmed by high-precision measurements, most notably at the LEP collider at CERN. The effective value of α increases with energy, consistent with predictions from the renormalization group in quantum electrodynamics.

In the segmented spacetime model, we interpret this variation structurally, not as a function of energy alone, but as a reflection of the local segmentation density of space. Thus, the empirical variation of α supports not only quantum field theory but also opens the door to geometric interpretations beyond it.

While the fine-structure constant is typically regarded as universal, both quantum electrodynamics and the segmented spacetime model suggest that it is in fact locally defined. The standard QED renormalization already accounts for its energy dependence. Here, we propose that this variation is not merely a function of momentum transfer, but a structural consequence of spacetime segmentation. Thus, α is not an absolute constant, but the measurable projection of a local coupling geometry.

In contrast to the energy-based interpretation in QED, the segmented spacetime model proposes that variations in α are not primarily driven by interaction energy, but rather by local segmentation of spacetime itself. In this view, the fine-structure constant becomes a projection of the local geometric structure, a measure of how space is discretized or "segmented" at a given location, especially in the presence of gravitational fields.

While QED describes how α changes with energy, the segmented spacetime model suggests that this "running" is actually a phenomenological effect of deeper structural segmentation of the vacuum. In this view:

QED Interpretation	Segmented Spacetime Interpretation
$\alpha(\mathrm{E})$ runs with energy	α_{local} varies with local space segmentation
Cause: Vacuum polarization	Cause: Segment density of spacetime
Derived via renormalization	Reconstructed from observed and inferred frequency

13. Apparent Photon Energy Loss and the Role of Local Segmentation

In classical physics, it is often assumed that a redshift in photon frequency corresponds to an actual energy loss by the photon during propagation. This is particularly common in interpretations of gravitational redshift, where photons are said to "lose energy" as they climb out of gravitational wells.

However, in our segmented spacetime model, this assumption is challenged. We propose that photons do not intrinsically lose energy during propagation. Instead, the observed energy difference arises from a structural constraint at the point of detection: the local segmentation density of spacetime restricts how much energy can be absorbed by a bound electron.

To illustrate this, we compare the energy of a photon at emission with the maximum energy that a local detector (e.g., an electron on Earth) can tap from that photon. Using the values from Section 11.2:

$$\Delta E = h \cdot (f - f') = 2.30 \cdot 10^{-21}\mathrm{J}$$

This energy difference closely matches the prediction from the Compton energy shift formula for an elastic photon-electron interaction with zero scattering angle (θ = 0°):

$$\Delta \mathrm{E}_{\mathrm{Compton}} = \frac{h^2}{m_e c \lambda}\ (1 - \cos\theta) \approx 2.30 \times 10^{-21}\mathrm{J}$$

However, we emphasize that in our model, this shift does not result from an interaction or recoil event. Instead, it reflects the limited energy absorption capability of the local electron due to segmentation. The electron cannot access the full incoming energy because its bound state is embedded in a more highly segmented spacetime structure, reducing the available internal degrees of freedom.

14. Biological Detectors and Perceptual Segmentation

Even our biological perception is structurally limited by local segmentation. The photoreceptor molecules in the human retina consist of bound electron systems that operate under the same constraints of segmented spacetime as described throughout this paper.

This implies that the human eye, like any bound quantum detector, does not perceive the true energy state of photons emitted in deep gravitational environments, but only the structurally permitted projection defined by local segment density.

The observed redshift is therefore not a cosmological loss of energy, but a biological and structural filtering process. In this view, colour itself becomes a projection, an emergent consequence of coupling limitations rooted in both spacetime and biology.

15. Conclusion

This suggests that what is traditionally viewed as a scale-dependence in quantum field theory may in fact be a local expression of geometric segmentation. Thus, the segmented spacetime model offers a novel bridge between field-theoretic running couplings and the geometry of gravitational systems, without requiring singularities or diverging values.

Furthermore, our model shows that observed frequency shifts, such as those between Earth and Sagittarius A*, can be derived without invoking Doppler effects or relative motion. Instead, these shifts emerge naturally from changes in the local segmentation density of spacetime, which alters the structurally available portion of the electron's rest energy. This reinterpretation challenges conventional redshift models and offers a geometric explanation rooted in topological confinement.

This model suggests that even our perception of light is not a window to the universe, but a mirror of our own structural embedding within it.

References

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For further investigations

A script written by Lino Casu is available as open source in the following GitHub repository:

https://github.com/LinoCasu/Segmented-Spacetime-Mass-Projection-Unified-Results