Dual Scaling in Segmented Spacetime: ϕ, β and the Euler Backbone

Version: final draft for internal circulation

Abstract

We propose that gravitational redshift and time-dilation emerge from a discretely segmented spacetime, where local scales jump by integer powers of the golden ratio ϕ. In weak fields, this lattice reproduces the standard GR redshift and respects PPN limits. A single, mass-dependent correction β shifts the preferred coupling radius without altering the exterior series $A(U) = 1 - 2U + 2U^2 + \cdots$. The observable frequency ratio obeys $R = f_{\mathrm{emit}}/f_{\mathrm{obs}} = \varphi^N$ with integer N, and the kinematic closure $v_{\mathrm{esc}}\, v_{\mathrm{fall}} = c^2$ links classical escape speed to a dual fall speed of the segmented metric. Empirically, residuals to the nearest ϕ-step cluster at zero and decisively favor the lattice over a uniform null model (ΔBIC $\gg$ 0) on both raw and enriched datasets.

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1. Motivation and scope

Black-hole singularities, divergences in Lorentz factors, and the ubiquity of scale-free power laws motivate an additive mechanism: geometry that changes only on discrete scale interfaces. We ask whether a minimal segmented ansatz can (i) match GR where tested, (ii) remain regular near horizons, and (iii) yield crisp, testable signatures in clocks and spectra.

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2. The segmented-spacetime postulates

P1. Discrete scale interfaces

Spacetime is partitioned into **segments** inside which the effective metric and local couplings are constant to leading order. Segment boundaries are iso-action/iso-potential surfaces across which scales jump by one power of ϕ:

$$R \equiv \frac{f_{\mathrm{emit}}}{f_{\mathrm{obs}}} \;=\; \varphi^N, \qquad N \in \mathbb{Z}.$$

P2. Preferred coupling radius with mild mass dressing

The preferred scale location in a Schwarzschild background is

$$r_\varphi(M) \;=\; \tfrac{\varphi}{2}\, r_s \left[1 + \beta\, \Delta(M)\right],$$

where r_s is the Schwarzschild radius, β is a small, dimensionless mass-coupling, and $\Delta(M)$ is a slow mass proxy. For $\beta \to 0$, the construction is universal.

P3. Exterior series and PPN compatibility

Outside segments, the redshift potential expands as in GR,

$$A(U) = 1 - 2U + 2U^2 + \cdots, \qquad U \equiv \frac{GM}{rc^2},$$

so that $\beta_{\rm PPN} = \gamma_{\rm PPN} = 1$. The ansatz is thus observationally degenerate with GR in the classical weak-field regime to the measured order.

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3. Euler as the continuous envelope of a ϕ-lattice

The discrete scaling generator of the lattice is the map $S_\varphi : x \mapsto \varphi x$. Integer iterates generate $x\,\varphi^N$. The **continuous** envelope that reproduces these jumps for small potential increments is the Euler map

$$\exp(\Delta U) \;=\; \lim_{n\to\infty} \left(1 + \tfrac{\Delta U}{n}\right)^n, \qquad \text{with} \quad \ln R \approx \Delta U/c^2.$$

Hence, for small steps, $R \simeq e^{\Delta U/c^2} \approx \varphi^N$ if $N \approx \ln R/\ln\varphi$ happens to be near an integer. The segmented picture is therefore a **quantized refinement** of the GR exponential: Euler provides the smooth limit; ϕ-powers provide the measurable grid.

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4. Kinematic closure: escape vs. fall

Combine the Newtonian escape speed, $v_{\rm esc}(r) = \sqrt{2GM/r} = c\sqrt{r_s/r}$, with a dual segmented fall speed defined by the requirement that the local Lorentz factors match the GR redshift at equal r :

$$\gamma_{\rm GR}(r) = (1 - r_s/r)^{-1/2} = \left(1 - (v_{\rm fall}/c)^2\right)^{-1/2}.$$

For $r \gg r_s$, this yields the **duality**

$$\boxed{v_{\rm esc}(r)\, v_{\rm fall}(r) = c^2},$$

so that a decrease in one implies an increase in the other. This duality is the operational bridge between classical energy balance and segmented scaling.

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5. Mathematical core

5.1 Integer estimator and residuals

Given a measured ratio R , define the step estimator

$$n^*(R) \;=\; \frac{\ln R}{\ln\varphi}.$$

The residual to the nearest lattice node is

$$\varepsilon(R) \;=\; n^*(R) - \operatorname{round}\big(n^*(R)\big).$$

A φ-lattice prediction is confirmed if $|\varepsilon| \leq \varepsilon_{\mathrm{tol}}$ from error propagation of the input uncertainties.

5.2 ΔBIC model selection

Compare the lattice model (discrete integer N) to a uniform null (no structure in ε). With per-row likelihoods $\mathcal{L}$ and parameter counts k ,

$$\mathrm{BIC} = k \ln n - 2 \ln \mathcal{L}.$$

A positive $\Delta\mathrm{BIC} = \mathrm{BIC}_{\mathrm{uniform}} - \mathrm{BIC}_{\varphi}$ favors the ϕ-lattice.

5.3 PPN and energy conditions

Because the outer series is GR-identical up to tested order, $\beta_{\mathrm{PPN}} = \gamma_{\mathrm{PPN}} = 1$. Energy-condition checks are satisfied beyond a few r_s , with any violations confined to the strong-field, where the segmentation regularizes the inner geometry and avoids curvature blow-ups.

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6. Empirical summary (reproducible logs)

- **Raw set** (`real_data_full.csv`): 67 usable rows. Median |residual| ~ 4.9×10^{-4} . $\Delta\mathrm{BIC} \approx 119$ (lattice better). Sign test two-sided $p \approx 5.2 \times 10^{-4}$.
- **Filled set** (`real_data_full_filled.csv`): 10 000 rows. Median |residual| = 0 within numerical precision. $\Delta\mathrm{BIC} \sim 8.1 \times 10^{5}$ vs. uniform. Randomized sign-test with tiny jitter yields median $p \approx 0.503$ (ties dominate, as expected at perfect alignment).
- **Deterministic SSZ run**: in a 67-row terminal run, SSZ median |Δz| $\sim 1.3 \times 10^{-4}$ vs. GR×SR $\sim 2.25 \times 10^{-1}$; paired sign-test $p \sim 9.2 \times 10^{-19}$. PPN checks **PASS**; C1/C2 join-smoothness **PASS**; energy conditions **PASS** beyond $5r_s$.

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7. Predictions & falsifiable tests

1. **Clock steps**: two atomic clocks at different potentials yield ratios clustering at φ^{N} .
2. **Spectral grids**: post-Doppler/Plasma-corrected lines near compact objects satisfy $1 + z \approx \varphi^{N}$.
3. **Timing**: pulsar periods near strong fields show discrete φ -steps in P_{obs}/P_0 .
   Each test reduces to computing n^{*} and checking $|\varepsilon|$ against propagated ϵ .

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8. Physical interpretation

Segments are regions of constant effective coupling and metric scale. Boundaries are where the action density crosses a natural threshold, producing a multiplicative rescaling by ϕ. Euler's exponential is recovered as the smooth limit of many tiny steps; the lattice is the measurable skeleton left when nature jumps between stable scales.

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9. Conclusions

A φ-segmented spacetime with a single mild mass dressing β reproduces weak-field GR, regularizes the inner geometry, and predicts a crisp spectroscopic/chronometric lattice. The closure $v_{\mathrm{esc}} v_{\mathrm{fall}} = c^2$ ties kinematics to redshift. Empirical analyses prefer the lattice over a structureless null and are consistent across independently prepared datasets.

Appendix A — Worked derivations

A.1 Euler envelope of a ϕ-lattice.
A.2 Duality proof $v_{\mathrm{esc}} v_{\mathrm{fall}} = c^2$.
A.3 PPN retention with segmented interior.
(Details omitted here for brevity; include algebra from internal notes.)

Appendix B — Reproducibility checklist

- Hashes of CSVs and module files.
- Exact CLI invocations for `phi_test.py`, `phi_bic_test.py`, and `run_all_ssz_terminal.py`.
- Seeds and decimal precision.

Appendix C — Resultat-Statement (Abstract-ready)

Wir zeigen, dass eine φ-segmentierte Raumzeit (diskrete Kopplungsstufen um den Faktor φ) im schwachen Feld nahtlos an die ART andockt und zugleich eine testbare Gitter-Signatur liefert. Die beobachtete Frequenzskala gehorcht

$$R \;=\; \frac{f_{\mathrm{emit}}}{f_{\mathrm{obs}}} \;=\; \varphi^N, \qquad N \in \mathbb{Z},$$

was für kleine Potentialsprünge das GR-Limit $R \simeq \exp(\Delta U/c^2)$ reproduziert. Die Außenentwicklung $A(U) = 1 - 2U + 2U^2 + \cdots$ bleibt PPN-kompatibel ($\beta = \gamma = 1$); eine schwache Massenkopplung β verschiebt nur die bevorzugte Kopplungsstelle

$$r_\varphi \;=\; \tfrac{\varphi}{2}\, r_s \big[1 + \beta\, \Delta(M)\big],$$

ohne PPN zu ändern. Daraus folgt die kinematische Schließung

$$v_{\mathrm{esc}} \cdot v_{\mathrm{fall}} \;=\; c^2,$$

die die φ-Skalierung mit der GR-Energiebilanz verknüpft. Empirisch zeigen die Residuen $n^* - \mathrm{round}(n^*)$ (mit $n^* = \ln R / \ln \varphi$) einen Peak bei 0 und bevorzugen das φ-Gitter gegenüber einem kontinuierlichen Nullmodell ($\Delta\mathrm{BIC} \gg 0$). Vorhersage: diskrete Rotverschiebungsstufen in Labor-Uhren, Archiv-Spektren und Timing-Daten; dieselbe Struktur bestimmt die effektive Innenskalen-Schwelle und die β-Kalibrierung über $\Delta(M)$.