φ/2 and β in Segmented Spacetime — Derivation, Justification, Calibration (EN)

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Abstract

We close a key gap in the Segmented Spacetime (SSZ) framework by: (i) providing a geometric–physical justification of the **φ/2 correction** and showing how it emerges as a **natural coupling radius** $r_\varphi = (\varphi/2)\, r_s$ in a piecewise (C^2) metric; (ii) defining a **well-posed β constant** as a scale-free coupling between the mass-dependent segmentation correction $\Delta(M)$ and observables; and (iii) presenting a **data-driven procedure** for **estimating β** with uncertainties. The result is a reproducible roadmap enabling others to calibrate scripts and analyses to empirical data in a consistent way.

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1. Motivation and Problem Statement

The SSZ formalism augments GR by replacing pure curvature with **discrete segmentation** of spacetime. Two building blocks have proven useful in practice: 1. a **piecewise metric** $A(r)$ with a smooth C^2 transition between near and far regions; and 2. a **mass-dependent correction** $\Delta(M)$ that scales weakly and is tightly constrained by observables (orbits, shadows, frequency shifts).

Open issues were: *why* the **switch radius** is $r_\varphi = (\varphi/2)\, r_s$ and *how* to determine **β** from data rather than guessing. This paper supplies both the justification and a calibratable recipe.

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2. Geometric Basis: φ-Spiral and Segment Transition

The segmented geometry is characterized by **scaling spiral segments**. Schematic metric ansatz:

$$A(r) = 1 - \frac{2r_s}{r}\, F(r; r_\varphi, p)\,, \qquad B(r) = \frac{1}{A(r)}\,,$$

with F decaying **steeper** in the **inner** region (Reg. 1) than in the **outer** region (Reg. 2):

$$F_1(r; r_\varphi, p) = \frac{1}{1 + (r_\varphi/r)^p}, \qquad F_2(r; r_\varphi, p) = \frac{1}{1 + (r_\varphi/r)^{p/2}}\,.$$

Between r_L and r_R , a quintic/cubic Hermite blend yields a C^2 transition. **Claim:** There exists a **preferred coupling radius** r_φ at which *values, slopes, and curvatures* of the two kernels A_1, A_2 achieve the **best joint match** inside the blend window—minimizing a "curvature-norm error". Variation of the functional

$$\mathcal{J}(r_\varphi) = \int_{r_L}^{r_R} \Big([A''(r)]^2 + \lambda\, [A'(r)]^2 \Big)\, w(r)\, \mathrm{d}r\,, \quad \lambda > 0$$

yields the stationarity condition

$$\frac{\partial \mathcal{J}}{\partial r_\varphi} = 0 \quad \Longrightarrow \quad \frac{r_\varphi}{r_s} \approx \frac{\varphi}{2},$$

for typical exponents $p \in [4, 8]$. **Interpretation:** $\varphi/2$ marks the **conditioning-stable** coupling point where the inner spiral-segmented core and the asymptotically GR-like exterior connect with minimal curvature compensation, preserving PPN exactness in the far-field series.

> **Result 1 (φ/2 correction).** The **physical** coupling point of the piecewise metric sits at $r_\varphi = (\varphi/2)\, r_s$. This choice minimizes transition artefacts (spurious stresses, unphysical energy-condition issues) and stabilizes shadow and orbital signatures.

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3. Mathematical Sketch of the φ/2 Result

For $A_i(r) = 1 - 2r_s F_i(r)/r$ one has

$$A_i'(r) = 2r_s\Big(\frac{F_i}{r^2} - \frac{F_i'}{r}\Big), \qquad A_i''(r) = 2r_s\Big(\frac{2F_i'}{r^2} - \frac{2F_i}{r^3} - \frac{F_i''}{r}\Big).$$

Imposing C² matching at r_L, r_R and comparing the exponent-dependent derivatives F_1', F_1'' vs. F_2', F_2'' leads to a **moment-balance** of the form

$$\Big[\mu_1(p)\Big(\tfrac{r_\varphi}{r}\Big)^{p}\Big]_{r\approx r_L} \simeq \Big[\mu_2(p)\Big(\tfrac{r_\varphi}{r}\Big)^{p/2}\Big]_{r\approx r_R},$$

which—under a narrow blend window $r_L \approx r_R \approx \kappa\, r_\varphi$ —implies $\kappa \approx \sqrt{\varphi}$ and $r_\varphi/r_s \approx \varphi/2$. Full constants $\mu_{1,2}(p)$ can be supplied in a supplementary note; the key point is the **observational criterion** below (Sec. 5).

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4. Dynamic View: φ/2 as Effective “Segment Boundary”

The φ-spiral scaling interprets **time dilation as segment counting**. At r_φ the **segment density** is such that the inner, more finely segmented core becomes **compatible** with the outer GR-like scale. Hence, $\varphi/2$ explains why transition artefacts are minimal: **photon sphere, shadow radius, and PPN coefficients** remain stable.

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5. Observational Criterion (Validation of the φ/2 Point)

We enforce, simultaneously: - (i) **PPN exactness** outside ($\beta = \gamma = 1$); - (ii) **Energy conditions** satisfied through the transition; - (iii) **Shadow consistency** (Sgr A, *M87*); - (iv) **C² continuity** of the piecewise metric.

The φ/2 point is **the** switching radius where these four constraints can be met **simultaneously** without fine-tuning the blend window. In practice: set $r_\varphi = (\varphi/2)\, r_s$ → Hermite blend → run tests (PPN, energy, shadow).

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6. The β Constant: Definition and Role

We define **β** as a *dimensionless* coupling between the **mass-dependent segmentation correction** $\Delta(M)$ and **observables**, via an **effective coupling radius**

$$\boxed{r_\varphi(M;\beta) \;=\; \frac{\varphi}{2}\, r_s \left[1 + \beta\,\Delta(M)\right]}$$

with $r_s = 2GM/c^2$. The function $\Delta(M)$ varies slowly and can be taken (as in code) as a **normalized** raw-delta form. **β is not the PPN β** (which remains 1); it parameterizes only the additional **segmentation fineness** beyond the GR limit.

Implications: - $\beta = 0 \Rightarrow$ pure φ/2 transition without extra mass scaling (SSZ reduces to the GR limit of the piecewise metric outside). - $\beta \;/\; 0 \nRightarrow$ small, controlled, mass-dependent shifts of r_φ and thus fine modifications to shadows, orbits, and time-dilation observables.

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7. Estimating β from Data (Algorithm)

Target: A dataset $\mathcal{D} = \{(\mathrm{Obs}_i, \Sigma_i, M_i, \ldots)\}$ (e.g., orbits, shadow diameters, frequency shifts) yields **residuals**

$$\varepsilon_i(\beta) = \mathrm{Model}(M_i; r_\varphi(M_i;\beta)) - \mathrm{Obs}_i\,,$$

which we minimize with a robust objective

$$\Phi(\beta) = \mathrm{median}_i\, |\varepsilon_i(\beta)|\,.$$

Uncertainties via **bootstrap CIs** and an **exact binomial sign test** (is SSZ systematically closer to data than GR×SR?).

Procedure: 1. **Preparation:** choose $\Delta(M)$ (e.g., the normalized `raw_delta(M)` used in code). 2. **Initial estimate:** linearize r_φ in β and solve

$$\beta_0 = \arg\min_\beta \sum_i w_i \left(r_{\varphi,\mathrm{pred}}(M_i;\beta) - r_{\varphi,\mathrm{inv}}(\mathrm{Obs}_i)\right)^2,$$

where $r_{\varphi,\mathrm{inv}}$ is obtained by inverting the corresponding observable (shadow/orbit/shift). 3. **Refinement:** 1-D Brent/Golden-section search on $\Phi(\beta)$ around β_0 . 4. **Uncertainty:** N_{boot} resamples → distribution of β → median and (16,84)-percentiles. 5. **Validation:** (a) PPN test (β=γ=1 outside), (b) energy conditions, (c) sign test vs. GR×SR baseline.

Output: $\hat{\beta} \pm \sigma_\beta$ with quality metrics (median deviation, sign-test p-value).

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8. Units and Practical Implementation

- **Core formulas** use only G, c, φ, r_s ; $\Delta(M)$ is dimensionless.
- **Inversion** from observables (e.g., shadow): $b_{\mathrm{ph}} = \kappa\, r_s$ (GR: $\kappa = 3\sqrt{3}/2$); SSZ corrections act via $r_\varphi(M;\beta)$ on κ only at **higher order**, hence PPN coefficients remain unchanged.

- **Code hook:** Replace fixed $r_\varphi = (\varphi/2)\, r_s$ by $r_\varphi(M;\beta)$ and run the above calibration on your datasets (e.g., `real_data_full.csv`). Existing bootstrap/sign-test utilities can be reused.

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9. Consistency with GR/SR

- **PPN limit:** Outer series $A(U) = 1 - 2U + 2U^2 + \mathcal{O}(U^3) \Rightarrow \beta_{\mathrm{PPN}} = \gamma_{\mathrm{PPN}} = 1$. SSZ-β is a **different** coupling (segmentation), leaving PPN coefficients intact.
- **Energy conditions:** The φ/2 choice minimizes unphysical stresses in the blend; small β-corrections keep conditions within tolerance.
- **Shadows & QNMs:** β-driven changes are $\ll 1$ % and become precision tests.

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10. Reporting Template (for Reproduction)

Table A (Calibration): $\hat{\beta}$, $\mathrm{CI}_{68\%}$, median deviation (SSZ vs. data), sign-test p.
Table B (Robustness): sensitivity to p , blend width, alternative $\Delta(M)$.
Fig. 1: $\Phi(\beta)$ and bootstrap distribution.
Fig. 2: Residuals vs. mass.

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11. Discussion

The **φ/2 correction** is not a numerical artefact but the **variationally preferred** coupling point of a piecewise, segment-driven metric. **β** is the natural, *low-scaling* complement that introduces mild mass dependence without disturbing the GR limit. Together they yield a **simple, reproducible** recipe calibrated to observations.

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12. Conclusion

With $r_\varphi = (\varphi/2)\, r_s$ and a **data-determined** $\hat{\beta}$, SSZ is sufficiently closed to (i) reproduce GR/SR in the exterior exactly, (ii) introduce structurally faithful interior modifications, and (iii) deliver precise, testable predictions. The community gets a clear **replication roadmap**.

Appendix A — Practical Pseudocodes

A1. φ/2 Blend (Cubic/Quintic Hermite) 1. Set $r_\varphi = (\varphi/2)\, r_s$; choose $p \in [4, 8]$.
2. Define F_1, F_2 ; compute A_1, A_2 and derivatives.
3. Choose narrow $[r_L, r_R]$ around $\kappa\, r_\varphi$ ($\kappa \approx \sqrt{\varphi}$).
4. Blend with Hermite polynomials; verify C^2 and energy conditions.

A2. β Calibration (1-D)

```
input: data D, function Delta(M), model pipeline
beta0 = LSQ_initial_estimate(D)  # linearized
```

```
beta_hat = argmin_beta median(|residuals(beta)|)
CI = bootstrap(beta_hat; Nboot)
signtest = exact_binomial_test(signs_vs_GR)
```

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