

CSE 331 Final Exam Preparation

This is in no way a substitute for exam preparation, merely a compilation of all the key talking points.

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Counter Example

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Ex: Every day is a Wednesday, where a counter example would be Monday is not Wednesday.

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Assume what you want to prove is false, then show this leads to a contradiction.

Therefore, the original assumption has to be true.

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This is especially useful if the **scope** of F is smaller than the scope of E.

Direct Proof

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Remember though, that you must maintain *W.L.O.G*, that your proof can never be too specific and must be arbitrary.

Proof by Induction

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If proof needs to be correct for all numbers $\in \mathbb{N}$, and each step is dependant on the previous step, then *every* step can be reduced to a definitive base case that is easy to directly prove.

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Note: This isn't a runtime analysis, rather a proof that the algorithm terminates.

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HW4 “Attack on Alarms” and Interval Scheduling are examples of problems with greedy solutions.

Introduction

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Moreover, what is a **stable** matching?

Perfect Matchings

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With n members in each group, there are $n!$ perfect matchings.

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If m prefers n over their current matching **and** n prefers m over their current matching, the entire match is an instability.

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With the right data structures, the runtime can be reduced to $O(n^2)$.

Even though the runtime isn't linear, because the input size is $2n^2 \rightarrow \Theta(n^2)$ ¹, the runtime **with respect** to the input size is $O(N)$, or linear time.

¹This comes from n Group A members and n Group B members with their $2n$ preference lists

Stable Matching

Code:

```
Initially all  $m \in M$  and  $w \in W$  are free
While there is a man  $m$  who is free and hasn't proposed to every
woman
    Choose such a man  $m$ 
    Let  $w$  be the highest-ranked woman in  $m$ 's
        preference list to whom  $m$  has not yet proposed
    If  $w$  is free then
         $(m, w)$  become engaged
    Else  $w$  is currently engaged to  $m'$ 
        If  $w$  prefers  $m'$  to  $m$  then
             $m$  remains free
        If  $w$  prefers  $m$  to  $m'$ 
             $(m, w)$  become engaged
             $m'$  becomes free
        Endif
    Endif
EndWhile
Return the set  $S$  of engaged pairs.
```