תרגיל בית מס' 2

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BCE loss function:

$$L(y, \hat{y}) = -[y \cdot \log(\hat{y}) + (1-y) \cdot \log(1-\hat{y})]$$

y: true lable ($y \in \{0, 1\}$)

$\hat{y}$: Predicted Probability which is the output of the sigmoid activation function

$$z = W^T x + b$$

Sigmoid activation:

$$\hat{y} = \sigma(z) = \frac{1}{1+e^{-z}}$$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial z}{\partial w}$$

$$\frac{\partial L}{\partial \hat{y}} = -\left[\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}\right]$$

$$\frac{\partial \hat{y}}{\partial z} = \hat{y}(1-\hat{y})$$

the gradient is:

$$\frac{\partial z}{\partial w} = x$$

$$\frac{\partial L}{\partial w} = -\left(\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}\right) \cdot \hat{y}(1-\hat{y}) \cdot x$$

④ $$\frac{\partial L}{\partial w} = -\left(\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}\right) \cdot \hat{y}(1-\hat{y}) \cdot x$$

אם $y=1$

$$\frac{\partial L}{\partial w} = \left(-\frac{1}{\hat{y}}\right) \cdot \hat{y}(1-\hat{y}) \cdot x$$

$$\frac{\partial L}{\partial w} = \left(\frac{-1}{\hat{y}}\right) \cdot \hat{y}(1-\hat{y}) \cdot x \Rightarrow \frac{\partial L}{\partial w} = -(1-\hat{y}) \cdot x$$

אם $y=0$

$$\frac{\partial L}{\partial w} = -\left(\frac{1}{1-\hat{y}}\right) \cdot \hat{y}(1-\hat{y}) \cdot x$$

$$\frac{\partial L}{\partial w} = \left(-\frac{1}{1-\hat{y}}\right) \cdot \hat{y}(1-\hat{y}) x = -\hat{y} x$$

$$\frac{\partial L}{\partial w} = -x \quad \Leftarrow \quad \hat{y} = 0 \quad \text{and} \quad y = 1$$

$$\frac{\partial L}{\partial w} = 0 \quad \Leftarrow \quad \hat{y} = 1 \quad \text{and} \quad y = 1$$

$$\frac{\partial L}{\partial w} = -x \quad \Leftarrow \quad \hat{y} = 1 \quad \text{and} \quad y = 0$$

$$\frac{\partial L}{\partial w} = 0 \quad \Leftarrow \quad \hat{y} = 0 \quad \text{and} \quad y = 0$$

2. Given the following confusion matrix for a multi-class classification problem with three classes (A, B, and C):

	Predicted: A	**Predicted: B**	**Predicted: C**
Actual: A	50	5	10
Actual: B	7	60	8
Actual: C	4	6	80

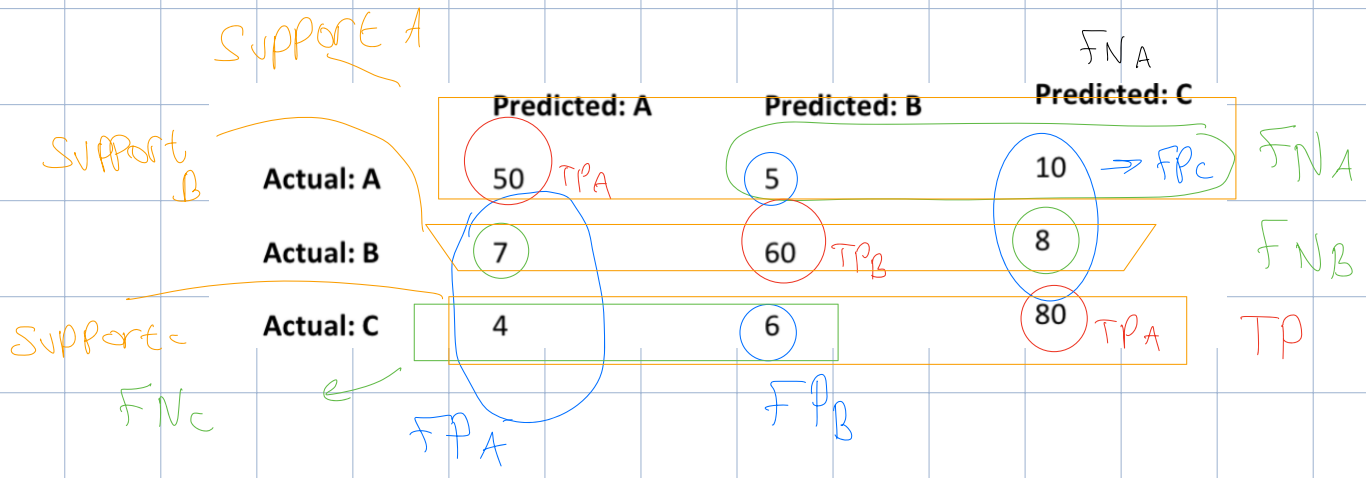
- Calculate the accuracy, precision, recall, and F1-score for each class.
- Provide the overall accuracy and the weighted average F1-score for the model.

① accuracy: $\frac{TP}{Total}$

$Precision = \frac{TP}{TP + FP}$

$Recall = \frac{TP}{TP + FN}$

$F1\ Score = \frac{Precision \cdot Recall}{Precision + Recall} \cdot 2$



$Total = 4 + 6 + 80 + 7 + 60 + 8 + 50 + 5 + 10 = 230$

$Accuracy\ A = \frac{TP_A + TN}{total} = \frac{50 + 154}{230} = 0.886$

עבור הקטגוריה A:

$TP_A = 50$

$FP_A = 7 + 4$

$FN_A = 10 + 5$

$TN_A = Total - (TP_A + FP_A + FN_A) = 154$

$$Precision_A = \frac{TP_A}{TP_A + FP_A} = \frac{50}{50 + 11} = 0.819$$

$$Recall = \frac{TP_A}{TP_A + FN_A} = \frac{50}{50 + 15} = 0.769$$

$$F1\text{-}Score_A = \frac{Precision_A \cdot Recall}{Precision_A + Recall} \cdot 2 = 0.793$$

עבור הקטגוריה B:

$TP_B = 60$

$FP_B = 8 + 10 = 18$

$FN_B = 6 + 4 = 10$

$TN_B = 230 - 88 = 142$

$$Accuracy_B = \frac{TP_B + TN_B}{Total} = \frac{60 + 142}{230} = 0.878$$

$$Precision_A = \frac{TP_B}{TP_B + FP_B} = \frac{60}{60 + 11} = 0.845$$

$$\text{Recall} = \frac{TP_B}{TP_B + FN_B} = \frac{60}{60+15} = 0.8$$

$$F1\text{-}Score_B = \frac{Precision_B \cdot Recall}{Precision_B + Recall} \cdot 2 = 0.821$$

עבור C מחלקה:

$$TP_c = 80$$

$$FP_c = 8+10 = 18$$

$$FN_c = 6+4 = 10$$

$$TN_c = 230 - 108 = 122$$

$$Accuracy_C = \frac{TP_c + TN_c}{Total} = \frac{80+122}{230} = 0.878$$

$$Precision_A = \frac{TP_c}{TP_c + FP_c} = \frac{80}{80+18} = 0.816$$

$$Recall = \frac{TP_c}{TP_c + FN_c} = \frac{80}{80+10} = 0.888$$

$$F1\text{-}Score_C = \frac{Precision_c \cdot Recall}{Precision_c + Recall} \cdot 2 = 0.850$$

(A) accuracy: $\frac{TP\,all}{Total} = \frac{50+60+80}{230} = 0.896$

f1 score weight everage

$$\text{Weight } f1 = \frac{\sum_{i=1}^{K} f_i \cdot \text{support}_i}{\text{Total support}}$$

Support_A: $50+5+10 = 65$

Support_B: $7+60+8 = 75$

Support_C: $4+6+80 = 90$

Total support = $\text{support}_A + \text{support}_B + \text{support}_C$

$= 65+75+90 = 230$

$f1_A = 0.793$

$f1_B = 0.821$

$f1_C = 0.850$

$$f1\,\text{weight} = \frac{0.793 \cdot 65 + 0.821 \cdot 75 + 0.850 \cdot 90}{230}$$

$f1\,\text{weight} = 0.824$