Fall CO Courses Schedule Optimization

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Abstract

The purpose of this paper is to develop an optimization model to create a timetable that maximizes the instructors' overall satisfaction for courses and time slots they teach. This involves scheduling courses in available classrooms, assigning qualified teachers, and accommodating students' and teachers' availability, while also considering classroom capacity.

Introduction

Scheduling is a widely discussed topic that significantly impacts organizational efficiency. Various entities, including universities, companies, and governments across different cultures, focus on optimizing schedules to enhance productivity. Therefore, universities worldwide are actively seeking better scheduling solutions for the well-being of their staff and students. A disorganized schedule can lead to reduced productivity, confusion among workers and leaders, and negatively affect physical and mental health. Overworking can result in health issues among instructors and students such as sleep deprivation, a higher risk of stroke, and increased chances of depression, anxiety, and even suicide.

On the other hand, a well-structured schedule helps individuals manage their time effectively. For instructors, this means being able to adequately prepare for classes, provide timely feedback on assignments, and engage in research or other scholarly activities. A balanced schedule also allows instructors to fulfill their teaching commitments while attending meetings, participating in committees, and pursuing professional development opportunities.

Moreover, a thoughtfully planned schedule can create a more conducive teaching environment. Instructors can design their courses and assessments to enhance student learning and engagement. Overall, effective scheduling is crucial for instructors to perform their roles effectively and contribute positively to their institution's academic mission.

Description of the Problem

The problem our team aims to solve is to optimize university scheduling based on instructors' preferences for both time slots and courses. We are trying to create a schedule that maximizes the satisfaction level of instructors while adhering to constraints such as room availability, time slot overlapping situations, workload of instructors, etc.

In this case, we use linear programming as an approach to optimize the satisfaction level of instructors. Linear provides a mathematical framework for modeling the scheduling constraints and objectives as linear relationships. By formulating the problem as a linear program, we could use optimization techniques to find the best schedule that meets the specified criteria, taking into account the preferences of instructors and the university requirements to avoid overloading course tasks, conflicting time slots, and overpopulated classrooms. At the same time, we assign weights to the preference level of time slots and preference level of courses so that the model is suitable to adjust to different situations based on specific requirements.

A sample is delivered in the first place to explain our basic logic. Later in the report, we adapt the model to a real-life problem and analyze the solution and its implications afterward.

Hard Constraints

1) *Course:*
    a) Each course needs to be taught, and each course is taught by exactly one instructor.
    b) Each instructor can only teach up to two courses.
2) *Time Slot:*
    a) Each instructor can only teach up to one course at one time slot.
3) *Classroom Capacity:*
    a) The total number of classes being taught at the time slot is less or equal to the number of classrooms.

Sample Problem

This section includes a dataset where the elements {0, 1, 2} represent instructors' preference levels for teaching specific courses or at particular time slots. In this context, '0' indicates the instructor's least preferred option, '1' signifies that the instructor is willing to teach the course or at the time slot, and '2' denotes the instructor's most preferred choice for teaching the course or time slot.

Table 1: Sample Teaching Preferences Data

	Course 1	Course 2	Course 3
Instructor 1	0	1	1
Instructor 2	2	1	1
Instructor 3	1	0	2

In ***Table 1***, three instructors are assigned to teach three courses. Instructor 1 is able to teach course1, course 2 and course 3, but he is least willing to teach course 1. Instructor 2 can teach all three courses and has a preference to teach course 1. Instructor 3 prefers to teach course 3, followed by course 1 with a lower preference level, and has zero preference to teach course 2.

Table 2: Sample Time Preferences Data

	Time slot 1	Time slot 2	Time slot 3
Instructor 1	0	2	1
Instructor 2	1	1	1
Instructor 3	1	1	2

In ***Table 2***, three instructors are assigned to teach at three different time slots. Instructor 1 prefers to teach at time slot 2 the most, followed by time slot 3, he does not like to teach at time slot 1. Instructor 2 is open to three options, and all three options are equal to him/her. Instructor 3 prefers time slot 3 the most, and he can also teach at time slot 1 and time slot 2 at the same preference level.

Formulation for The Sample Problem

Decision Variables and Parameters

Binary Variables:

$x_{i,j}$ indicates whether instructor ***i*** teaches course ***j***.

$x_{i,j} = 1$ if the instructor teaches the course ***j.***

$x_{i,j} = 0$ otherwise.

$y_{i,t}$ indicates whether the instructor ***i*** teaches at time slot ***t***.

$y_{i,j} = 1$ if the instructor teaches at the time slot ***t.***

$y_{i,j} = 0$ otherwise.

Constants:

$J = \{1, 2, 3\}$ denotes the set of courses.

$I = \{1, 2, 3\}$ denotes the set of instructors.

T denotes the set of time slots.

K denotes the number of classrooms available.

$P_{i,j}$ denotes the preference level for instructor ***i*** to teach course ***j***.

$Q_{i,t}$ denotes the preference level for instructor ***i*** to teach in time slot ***t***.

w_1, w_2 denotes the weight of preference level of courses and time slots respectively, where $w_1 + w_2 = 1$

*Note: Larger preference level number indicates instructors' **stronger** tendency to the options*

Sample Formulation

The objective function is the weighted sum of satisfaction level of courses and time slots for all instructors, if we assume table 1 and table 2 are equally weighted, w_1 = 0.5, w_2 = 0.5, so it can be formed as

$$\textbf{Maximize } \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} p_{i,j}x_{i,j} + \frac{1}{2}\sum_{i=1}^{3}\sum_{t=1}^{3} q_{i,j}y_{i,t} =$$

$$\begin{aligned} 0.5\,((0)(x_{1,1}) + (1)(x_{1,2}) + (2)(x_{1,3}) + \\ (2)(x_{2,1}) + (1)(x_{2,2}) + (1)(x_{2,3}) + \\ (2)(x_{3,1}) + (0)(x_{3,2}) + (1)(x_{3,3})) + \end{aligned}$$

(From Table 1)

$$\begin{aligned} 0.5\,((0)(y_{1,1}) + (2)(y_{1,1}) + (1)(y_{1,1}) + \\ (1)(y_{2,1}) + (1)(y_{2,2}) + (1)(y_{2,3}) + \\ (1)(y_{3,1}) + (1)(y_{3,2}) + (2)(y_{3,3}) \end{aligned}$$

(From Table 2)

Constraint 1 Each course can only be taught by exactly one instructor:

$$\text{s.t.} \quad \begin{aligned} (x_{1,1}) + (x_{2,1}) + (x_{3,1}) = 1 \\ (x_{1,2}) + (x_{2,2}) + (x_{3,2}) = 1 \\ (x_{1,3}) + (x_{2,3}) + (x_{3,3}) = 1 \end{aligned}$$

(From Table 1)

Constraint 2 Each instructor can only teach up to two courses:

$$\begin{aligned} (x_{1,1}) + (x_{1,2}) + (x_{1,3}) \le 2 \\ (x_{2,1}) + (x_{2,2}) + (x_{2,3}) \le 2 \\ (x_{3,1}) + (x_{3,2}) + (x_{3,3}) \le 2 \end{aligned}$$

(From Table 1)

Constraint 3 The professor can't take a time slot without teaching any course, and one professor can't teach two courses in one time slot:

$$\begin{aligned} (y_{1,1}) + (y_{1,2}) + (y_{1,3}) = (x_{1,1}) + (x_{1,2}) + (x_{1,3}) \\ (y_{2,1}) + (y_{2,2}) + (y_{2,3}) = (x_{2,1}) + (x_{2,2}) + (x_{2,3}) \\ (y_{3,1}) + (y_{3,2}) + (y_{3,3}) = (x_{3,1}) + (x_{3,2}) + (x_{3,3}) \end{aligned}$$

(From Table 2)

Constraint 4 Binary Constraints

$$x_{i,j}, y_{i,t} \in \{0,\ 1\},\ \ \forall\, i \in I,\ j \in J,\ t \in T$$

The solutions are summarized in the following tables, where 1 means the instructor teaches the course or in the time slot, and 0 means not.

Table 1.1: Sample Teaching Preferences Solutions

	Course 1	Course 2	Course 3
Instructor 1	0	1	0
Instructor 2	1	0	0
Instructor 3	0	0	1

Table 2.1: Sample Time Preferences Solutions

	Time slot 1	Time slot 2	Time slot 3
Instructor 1	0	1	0
Instructor 2	0	0	1
Instructor 3	0	0	1

From the ***Table 1.1*** and ***Table 2.1***, we can see that
$x_{1,2} = 1$, $y_{1,2} = 1$ indicates that instructor 1 teaches course 2 in time slot 2.
$x_{2,1} = 1$, $y_{2,3} = 1$ indicates that instructor 2 teaches course 1 in time slot 3.
$x_{3,3} = 1$, $y_{3,3} = 1$ indicates that instructor 3 teaches course 1 in time slot 3.

General Formulation

From sample problem, now we can formulate this scheduling problem in general:

$$\max\ \ w_1 \sum_{i\in I}\sum_{j\in J} p_{i,j}x_{i,j} + w_2 \sum_{i\in I}\sum_{t\in T} p_{i,j}y_{i,t}$$

Constraints:
Course Constraints

$$\sum_{i\in I} x_{i,j} = 1\ , \forall\, j \in J$$

$\sum_{j \in J} x_{i,j} \leq 2, \forall i \in I$

Time slot constraints

$\sum_{j \in J} x_{i,j} = \sum_{t \in T} y_{i,t} \forall i \in I$

Classroom Usage

$\sum_{i \in I} y_{i,t} \leq K \forall t \in T$

Others

$w_1 + w_2 = 1$

$x_{i,j} \in \{0, 1\} \forall i \in I, j \in J$

$y_{i,t} \in \{0, 1\} \forall i \in I, t \in T$

$P_{i,j} \in \{0, 1, 2, 3, 4, 5\}$

$Q_{i,t} \in \{0, 1, 2, 3, 4, 5\}$

Data and Computation

Now consider a real-life example. ***Table 3*** shows the majority of CO courses that will be offered in the Fall term along with instructor preferences. And ***Table 4*** lists instructors' time preferences accordingly.

Table 3: CO Courses Teaching Preferences (Fall)

	CO 330	**CO 342**	**CO 351**	**CO 353**	**CO 367**	**CO 370**	**CO 372**	**CO 430**	**CO 431**	**CO 434**
Instructor 1	2	2	1	0	3	3	4	3	5	5
Instructor 2	5	5	1	2	1	5	4	3	2	2
Instructor 3	1	1	5	1	1	1	0	1	1	2
Instructor 4	4	3	2	0	1	1	4	4	4	4
Instructor 5	2	2	2	1	1	2	3	3	4	5
Instructor 6	2	2	3	4	1	0	1	1	2	3

Instructor 7	2	1	1	1	5	4	4	5	4	4

Table 4: CO Courses Time Preferences (Fall)

	8:30 - 9:50	**10:00 - 11:20**	**11:30-12:50**	**13:00-14:20**	**14:30-15:50**	**16:00-17:20**
Instructor 1	3	2	4	1	5	3
Instructor 2	5	4	5	3	1	2
Instructor 3	3	5	1	1	2	2
Instructor 4	2	0	0	1	2	1
Instructor 5	1	2	3	2	1	1
Instructor 6	1	2	0	0	0	0
Instructor 7	1	2	2	2	1	1

Assume $w_1 = 0.6$ and $w_2 = 0.4$. Assume there are 3 classrooms in total for those CO courses, so K = 3. Note I ∈ {1,2,...,10}, J ∈ {1,2,...,7}, T ∈ {1,2,...,6}.

We can formulate the above problem in general:

$$\textbf{Maximize} \quad w_1 \sum_{i \in I} \sum_{j \in J} p_{i,j} x_{i,j} + w_2 \sum_{i \in I} \sum_{t \in T} q_{i,j} y_{i,t}$$

$$\text{s.t.} \quad \sum_{i=1}^{10} x_{i,j} = 1, \forall j \in J$$

$$\sum_{j=1}^{7} x_{i,j} \leq 2, \forall i \in I$$

$$\sum_{j=1}^{7} x_{i,j} = \sum_{t=1}^{7} y_{i,t}, \forall i \in I$$

$$\sum_{i=1}^{10} y_{i,t} \leq K,\ \forall\, t \in T$$

$$x_{i,j} \in \{0,\ 1\},\ \forall\, i \in I,\ j \in J$$

$$y_{i,t} \in \{0,\ 1\},\ \forall\, i \in I,\ t \in T$$

$$P_{i,j} \in \{0,\ 1,\ 2,\ 3,\ 4,\ 5\}$$

$$Q_{i,t} \in \{0,\ 1,\ 2,\ 3,\ 4,\ 5\}$$

Analysis of the Solution

With the assistance of computer program, we obtain the following solutions with the optimal value 42.2:

Instructor 1 teaches Course 7 in time slot 3, 11:30- 12:50.
Instructor 1 teaches Course 9 in time slot 5, 14:30- 15:50.
Instructor 2 teaches Course 2 in time slot 1, 8:30 - 9:50.
Instructor 2 teaches Course 6 in time slot 3, 11:30- 12:50.
Instructor 3 teaches Course 3 in time slot 2, 10:00 - 11:20.
Instructor 4 teaches Course 1 in time slot 5, 14:30- 15:50.
Instructor 5 teaches Course 10 in time slot 3, 11:30- 12:50.
Instructor 6 teaches Course 4 in time slot 2, 10:00 - 11:20.
Instructor 7 teaches Course 5 in time slot 2, 10:00 - 11:20.
Instructor 7 teaches Course 8 in time slot 4, 13:00- 14:20.

Overall, each course is assigned to an instructor and a time slot, ensuring that all courses are covered within the scheduling framework. However, we can see that the time slot 16:00 - 17:20 is not in use. So the solution utilizes five out of the six available time slots. Besides, not all instructors are fully utilized, as some instructors are assigned to teach only one course. These points indicate that there is some room for optimization in the scheduling process, and we can take further investigation into them and reveal opportunities for improvement.

Conclusion and Future Work

In summary, we derived the general formulation from a sample problem and subsequently applied it to a real-life scenario. We also noticed that some real-life scenarios often entail additional constraints that must be taken into account. For instance, considerations such as student enrollment figures and instructor retention rates may introduce new constraints to be factored into the analysis.

Besides, we identified both the strengths and weaknesses inherent in this Linear Program. Strengths include its efficiency and the requirement to satisfy all constraints.

However, a notable weakness lies in its provision of only one optimal solution, despite the potential benefit of having multiple options to choose from. Additionally, Linear Programs cannot guarantee a feasible solution, and when an infeasible solution arises, it can be challenging to pinpoint which conditions cannot be fulfilled.

So we analyzed the situation when the program generates infeasible solution:

Analysis of Infeasibility

First thing we know about infeasibility is that there has to be constraints to be violated so that the solution is not feasible. Also, there might be more than one constraints that are violated when trying to solve the model. There could be several reasons cause the infeasibility of the model, below are some examples:

1. Lack of instructor to some specific course: There is one course that no instructor is able to lecture on.
2. Overload course arrangement to instructors: One instructor needs to teach more than two courses.
3. Time slot conflicts: Instructor needs to teach two courses in the same time slot.
4. Lack of teaching space: In some time slots, there aren't enough classrooms for instructors to teach the course.

One additional point to note is our ability to easily incorporate new constraints in the future. For instance, if we wish to enforce a rule that instructors cannot have two consecutive time slots, we can modify our code to flag any violations of such constraints. By iterating over each constraint, we can sequentially test the feasibility of the solution with each new rule.

For example, if our model initially includes four standard constraints and we want to add constraints A and B, our program can automatically test the solution with constraint set Standard + A, then with Standard + A + B. If the solution becomes infeasible at any step, the program will pinpoint which constraint led to the infeasibility. This iterative approach streamlines the process of adding and testing new constraints, thereby enhancing the efficiency of constraint design.

References:

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