Mat3420 Oblig Jakob Lange s.4

3] b) Let $W \in G$. then W is unitary and $\det(W) = 1$ so $W \in SU(H)$

now let $U \in SU(H)$. Then $\exists\ |\psi\rangle \in H$ s.t $U|\psi\rangle = |\phi_1\rangle$ we know that there is W in G s.t $W|\psi\rangle = |\phi_1\rangle$

then $UW^{-1}|\phi_1\rangle = |\phi_1\rangle$ and $\det(UW^{-1}) = \det(U)\det(W^{-1}) = 1$

so $UW^{-1} \in G_1$ and hence in G

then $U = (UW^{-1})W$ is also in G ■

4] Let again H be a finite dimensional hilbert space of dimension at least 2.
G is the set of unitary operator with determinant 1 s.t for $W \in G$: $W|\phi\rangle = |\phi\rangle$. Let V be a unitary operator s.t $|\phi\rangle$ is not an eigenvector of V.

for any $W \in G$: $\det(VWV^*) = \det(V)\det(W)\det(V^*)$

$$= \det(W)\det(V)\det(V^*)$$
$$= \det(VV^*) = 1$$

let $|\psi\rangle = V|\phi\rangle$, since $|\phi\rangle$ is not an eigenvector of V then $|\psi\rangle \neq |\phi\rangle$ but for $W \in G$:

$VWV^*|\psi\rangle = VW|\phi\rangle = V|\phi\rangle = |\psi\rangle$

let G' be the set of unitary operators on H s.t for $W' \in G'$ $W'|\psi\rangle = |\psi\rangle$, $\det(W') = 1$

then $V^*W'V = W \in G$ so VGV^* generates G'

hence by problem 3 we let $|\phi\rangle = |\phi_1\rangle$, $|\psi\rangle = |\phi_2\rangle$ and see that $SU(H)$ is generated by G and VGV^*