$(|\psi\rangle\langle\phi|) \in L(H)$

$|\psi\rangle\langle\phi|(|\eta\rangle) = \langle\phi|\eta\rangle|\psi\rangle$

vi vet denne er lineær

$\langle\phi_1 + c\phi_2 | \eta_1 + k\eta_2\rangle = \langle\phi_1|\eta_1\rangle + k\langle\phi_1|\eta_2\rangle$
$+ \bar{c}\langle\phi_2|\eta_1\rangle + c\langle\phi|\eta_2\rangle$

lineær betyr bare at $f(c_1v_1 + \dots c_nv_n)$
$= c_1f(v_1) + \dots + c_nf(v_n)$

$\mathcal{H}\otimes\bar{\mathcal{H}} \to L(H)$ ved $|\psi\rangle\otimes\langle\phi| \to |\psi\rangle\langle\phi|$

linearitet

Ser at $|\psi\rangle\otimes(\langle\phi_1| + \langle\phi_2|) \to |\psi\rangle(\langle\phi_1| + \langle\phi_2|)$
$= |\psi\rangle\langle\phi_1| + |\psi\rangle\langle\phi_2|$

Sjekke likt for $|\psi_1\rangle + |\psi_2\rangle$
eller $c|\psi_1\rangle + |\psi_2\rangle$

isomorfi sjekk at

$|\psi\rangle\langle\phi|(|\eta\rangle) = 0$, for alle $|\eta\rangle \in H \Leftarrow$

$\langle\phi| = 0$ eller $|\psi\rangle = 0 \Rightarrow |\psi\rangle\langle\phi| = 0$

4] linear functional

$\mathcal{H}\otimes\bar{\mathcal{H}} \to \mathbb{C}$ by $|\psi\rangle\otimes\langle\phi| \to \langle\phi|\psi\rangle$

linear functional $H\otimes\bar{H} \to \mathbb{C}$ means a function f taking values in $H\otimes\bar{H}$ returning values in $\mathbb{C}$. In this case

$f(|\psi\rangle\otimes\langle\phi|) = \langle\phi|\psi\rangle$, $|\psi\rangle \in H$, $\langle\phi| \in \bar{H}$

this means "in" or "belonging to"

clearly $\langle\phi|\psi\rangle \in \mathbb{C}$ since it is a complex number.

linearity:

$f(|\psi\rangle\otimes(\langle\phi_1| + c\langle\phi_2|)) = (\langle\phi_1| + c\langle\phi_2|)|\psi\rangle$
$= \langle\phi_1|\psi\rangle + c\langle\phi_2|\psi\rangle = f(|\psi\rangle\otimes\langle\phi_1|)$
$+ cf(|\psi\rangle\otimes\langle\phi_2|)$

so f is linear in $\bar{H}$. we need to check linearity in H as well, this is done in the same way but with $|\psi\rangle$ instead of $\langle\phi|$

Now we need to check that identifying $H \otimes \bar{H}$ by $L(H)$ gives us the trace operato

H is n-dimensional, pick an orthonormal basis: $\{|\psi_1\rangle, \dots, |\psi_n\rangle\}$

then $\{\langle\psi_1|, \dots, \langle\psi_n|\}$ is an orthonormal basis for $\bar{H}$ by the definition of $\bar{H}$

let $|\eta\rangle = \sum_{k=1}^{n} c_k |\psi_k\rangle$, $\langle\phi| = \sum_{k=1}^{n} a_k \langle\psi_k|$

then $|\eta\rangle\langle\phi| = \left(\sum_k c_k |\psi_k\rangle\right)\left(\sum_j a_j \langle\psi_j|\right)$

$$= c_1 a_1 |\psi_1\rangle\langle\psi_1| + c_1 a_2 |\psi_1\rangle\langle\psi_2| + \dots + c_1 a_n |\psi_1\rangle\langle\psi_n|$$
$$+ c_2 a_1 |\psi_2\rangle\langle\psi_1| + \dots + c_2 a_n |\psi_2\rangle\langle\psi_1|$$
$$+ \vdots$$
$$+ c_n a_1 |\psi_n\rangle\langle\psi_1| + \dots + c_n a_n |\psi_n\rangle\langle\psi_n|$$

this corresponds to the Matrix:

$$\begin{pmatrix} c_1 a_1 & c_1 a_2 & \cdots & c_1 a_n \\ c_2 a_1 & & \cdots & c_2 a_n \\ \vdots & & & \vdots \\ c_n a_1 & & \cdots & c_n a_n \end{pmatrix}$$

however $f(|\eta\rangle\langle\phi|)$

$$= \left(\sum_k a_k \langle\psi_k|\right)\left(\sum_j c_j |\psi_j\rangle\right) \underset{\substack{\uparrow \\ \text{orthonormality}}}{=} \sum_k a_k c_k = \mathrm{Tr}(|\eta\rangle\langle\phi|)$$