Exercise Sheet 03
Operator Algebras

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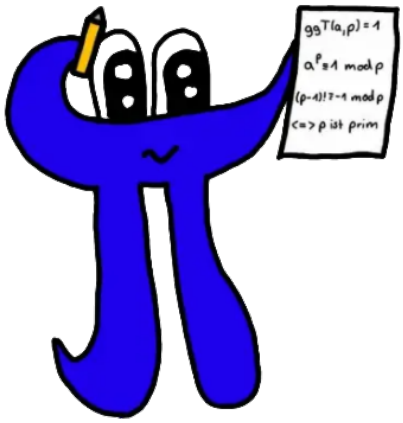

3.1

The first statement follows immediately from the fact that the canonical inclusion $\mathscr{B} \hookrightarrow \mathscr{A}$ is an injective *-homomorphism, so it is isometric as proven in the lecture.

If now $\mathscr{B}$ is a dense proper *-subalgebra of $\mathscr{A}$, assuming it could be turned into a C^*-algebra, the norm on that C^*-algebra would already have to be the norm on $\mathscr{A}$. But then the canonical inclusion is isometric and injective, so it has closed range and $\mathscr{B} \subseteq \mathscr{A}$ is closed and dense in $\mathscr{A}$. Now, however, we have $\mathscr{B} = \mathscr{A}$, a contradiction.