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Directional Change Intrinsinc Time Framework as Target Transformation in Time Series Modelling

Final Project Report

Yu-Kuan, Lin

Abstract

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Introduction

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1.1 Aims and Objectives

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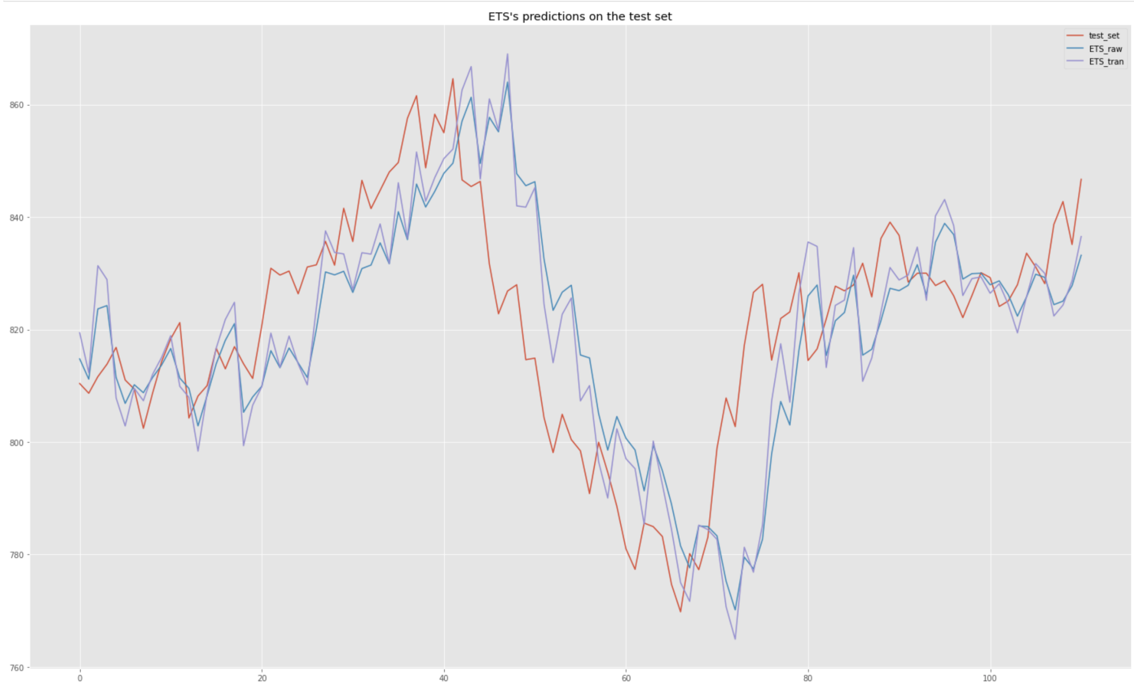

Figure 1.1: ETS's prediction on the test set

1.2 Problem Statement

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Chapter 2

Background

In this chapter, we present comprehensive theoretical background to the topics related to our experiment and analyses.

2.1 Univariate Time Series Forecasting

In this section, we discuss the topic of univariate time series forecasting.

2.1.1 General Notions of Time Series Forecasting

A time series is a sequential collection of random variables indexed by time. If the random variables are of one dimension, then the time series is considered univariate[1]. In this section, the discussions fall within the context of univariate time series forecasting. Let $Y = \{Y_{t_i}\}_{i=1,2,\cdots,n}$ be an univariate time series with $Y_{t_i} \in \mathbb{R},\ \forall i,\ n \in \mathbb{N}$ and $t = \{t_i\}_{i=1,2,\cdots,n}$ being the set of time indices of Y (also referred to as the set of timestamps). Let y_{t_k} be the largest information set about Y that is accessible to the model at time point $t_k \in t$, e.g., we might have the observations of Y being known ($\{Y_{t_j}\}_{j=1,2,\cdots,k} \subset y_{t_k}$). Then in the context of modelling, given an unkown (random) target Y_{t_s} with $s > k,\ s \in \mathbb{N}$, we can articulate the notion of forecasting as the following:

> Forecasting the value Y_{t_s} at time point t_k is to find a function $f(y_{t_k}) = \widehat{Y_{t_s}}$, such that $\widehat{Y_{t_s}}$ is a good estimation of Y_{t_s}.

[1] If the random variables are of dimension higher than one, then the time series is considered multivariate. Datasets in such form are also referred to as panel data.

We can come up with all sorts of function f for such forecasting objective. Functions devised to serve the objective are referred to as models. In a general sense, the notion of modelling refers to the methodologies of devising a model that serves the objective well. In particular, for any arbitrary pair of t_k and t_s, we want to have a model $f(y_{t_k})$ such that it gives us a reliable estimation of Y_{t_s}.

Gap

Notice that a modelling objective is parameterised by the pair of time indices (t_k, t_s). The timestamp t_k directly affects the information set y_{t_k} and thus determines the information the model f can utilise. The timestamp t_s, on the other hand, controls how far in the future are we forecasting. If the gap between the two timestamps is big, this means that the model is asked to forecast further into the future. If the gap between the two timestamps is small, then the objective might be considered easier because we are only trying to look a little bit ahead into the future. In order to better communicate and characterise the forecasting objective, we formalise this gap between the pair of timestamps as the *gap* and denoted as G:

$$G = t_s - t_k, \quad t_s,\ t_k \in t$$

With such notion of the gap, we can then articulate the forecasting objective as: G ahead forecasting. G takes the format of the timestamps. Depending on the format of the timestamps, the objective can be one-step ahead forecasting or one-year ahead forecasting. We will go into topics concerning timestamps in one of the upcoming paragraphs.

Forecast Horizon

Observe that the target we have in the previous forecasting objective Y_{t_s} is a single value in the future. It's possible to generalise the target a little and have multiple targets in the future. The number of targets we try to forecast is called the *forecast horizon.* Having a forecast horizon equal to one is to forecast one value ahead into the future, and having a forecast horizon equal to five is to forecast all five values ahead into the future. To put it formally, let $\langle X \rangle$ be a counting operation that counts the number of elements in a set X, and a task trying to forecast a collection of unknown values $Y_S = \{Y_{s_1}, Y_{s_2}, \cdots\}_{s_1, s_2, \cdots \in t,\ s_1, s_2, \cdots > t_k}$. Then the forecast horizon of such task is denoted as

$$H = \langle Y_S \rangle$$

Timestamps

In a time series, the time index of a random variable carries information of the time point in which the random variable lives in the time domain. In some sense, the timestamps mark the 'location' the random variables on a time line. For example, a monthly revenue time series Y in an arbitrary year can have the set of months in a year as its timestamp set and be denoted as $Y = \{Y_{Jan}, Y_{Feb}, Y_{Mar}, \cdots, Y_{Dec}\}$. The time indices also tell us the time-relevance (geological relationship on a time line) of the random variables among each other. In fact, the time-relevance of the random variables in a time series plays a crucial role in time series analysis. We often have to perform mathematical operations involving such relationship. An example is our coming up with the gap measurement we addressed in the previous paragraph. Another example is the calculation of the relative growth of the time series. The need of these math operations pushes us modellers to come up with innovative ways to define the timestamps because we obviously cannot perform calculations on notaions like *September* or $Friday$. To put it in math terms, what we often do is to have a mapping from physical timestamps to the real number line (or a subset of the real number set, say, the natural number set) and use the target set of this mapping as the timestamp set for math operations. In the next paragraph, we discuss some examples of such mapping.

Take the previous monthly revenue time series as an example, one of the simpliest way is to index the time series chronologically with natural numbers $\{1, 2, 3, \cdots, 12\}$. The new index system allows for mathematical operations on the timestamps, such as addition. The objective of two-month ahead forecasting can be considered as two-step ahead forecasting with $G = 2$. Three-month moving average of the time series can now be of a generalised form of 3-step moving average. Another good example is the financial studies of stochastic processes, in which we often use the non-negative real line and adopt an annual scale, i.e., the starting point of the time series is indexed as 0, one month after that is indexed 0.0833, one year time point is indexed 1.0, and so on. This is particularly useful when we expand the studies of time series using Stochastic Differential Equations (SDE). dt in the drift term in the SDE expression

$$\frac{dY_t}{Y_t} = \mu_t dt + \sigma_t dW_t, \quad dW_t \sim N(0, dt)$$

is now properly defined as a real number that we can do all sorts of math operation on (observe how dW_t is defined as a Brownian Motion that follows a Gaussian distribution with variance dt).

Time Heterogeneity

There are cases where the timestamps of the time series are not identically spaced between consecutive random variables. The previous example of montly revenue in a year is one of them due to the months having different durations. Time series as such are technically referred to as being *time heterogeneous*[2][3]. Time heterogeneity can be an interesting source of information carried by the time series but can also be a major issue in time series studies. In the next paragraph, we present a common problem caused by time heterogeneity.

Calculating measurements related to the unit of time can be a problem with time heterogeneous time series. One common example of such measurement is return used in finance. Return measures the relative change of the price (or, say, value) over a period of time with respect to its initial level. Several different definition can be drawn to the notion of return, but we will look at the simpliest one as they all exhibits the same relationship with time heterogeneity. Let $Y = \{Y_{t_i}\}_{i=1,2,\cdots n},\ n \in \mathbb{N}$ now be a time series of the price movement of an asset over time, and the time index set being $t = \{t_i\}_{i=1,2,\cdots,n}$. Define the coresponding net return measure as

$$R_{t_i} = \frac{Y_{t_i} - Y_{t_{i-1}}}{Y_{t_{i-1}}}, \quad i \in \{2, 3, \cdots, n\}.$$

The net return R_{t_i} for some i is thus the relative price change of Y_{t_i} over the time period $t_i - t_{i-1}$ with respect to $Y_{t_{i-1}}$. Let $R = \{R_{t_i}\}_{i=2,3,\cdots,n}$ be the time series of one-step net returns derived from Y. If t is equally spaced by, say, a day, the time series Y is time homogeneous. The time series R we calculated is then a time series of daily net return of Y. Nevertheless, in the case where t is not equally spaced, the time series Y is time heterogeneous. Then we no longer know what is the period of the net return we calculated from the time series Y, thus making it harder for us to generate robust analyses.

[2]Time heterogeneity is common in financial time series due to the nature of how financial markets work. For example, most markets are open only during working hours in working days. Another example is the high-frequency financial time series (see Dacarogna et al. 2001)

[3]*Time homogeneity* is the counter part of time heterogeneity, specifying time series which have equally spaced timestamps

2.1.2 Univariate Time Series Forecasting with Regression Modelling

In this section, we take the discussion of univariate time series modelling further and address how do we approach the articulated forecasting objective with regression models.

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Recall the forecasting objective is to come up with a model $f$ capable of gene
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Regression modelling

Let f be a regression model with known structure yet unknown parameter set β. Then the objective is to find β such that $f(y_{t_k};\beta)$ estimates Y_{t_s} with certain level of satisfaction. In the univariate case, the y_{t_k} is the realised values of Y until time point t_k, namely

$$y_{t_k} = \{Y_{t_k}, Y_{t_{k-1}}, Y_{t_{k-2}}, \cdots, Y_{t_1}\}.$$

Given the long list of past observations, there is a parameter we have to decide which controls the number of past observations the model will use to come up with a single prediction. Such parameter is called the *number of lags*[4]. Let the number of lag be denoted as L. Then we can create the regression model

$$y = X\beta + \epsilon$$

In order to train the regression model, we have to make a design matrix using these values.

2.2 Target Transformation in Univariate Time Series Forecasting

2.3 Directional Change Intrinsic Time Framework

[4]Its naming this way originates in autoregressive estimation in time series studies. Such estimation aims at finding the optimal order of autoregressive feature of a time series. Normally, such order is referred to as *lag*.

Chapter 3

Literature Review

Target transformation techniques have been researched as part of the modelling process over the past sixty years. The same applies to the analyses of the underlying mechanisms in financial dynamics. This paper builds on both research directions and analyses the Directional Change (DC) intrinsic time framework as a target transformation technique in time series forecasting. To the best of our knowledge, as this paper was written, we are unaware of any other attempt to bring these two methodologies together. In this chapter, we cover the most relevant works in both realms. We first review some target transformation developments in Section 3.1 and then look at some of the advancements in the Directional Change intrinsic time framework in Section 3.2.

3.1 Target Transformation

In most cases, time series analyses yielded from modelling are justified conditional on the assumptions made by the models about the data. If the assumptions are not met by the original data, applying some transformation (often non-linear) to the data can help generate these conditions. This has led to various transformation techniques for these types of purposes. In this section, we go through some important assumptions made by the models and the corresponding transformations found in the literature that we find most worthy of mentioning.

Homoscedasticity condition[1] is one of the most common assumptions for models

[1]The homoscedasticity (homogeneity of variances) condition requires the variances of different subsets of the sample to be the same. In the case of time series modelling, it is equivalent to

involving statistical inferences. As a reference to the analysis of variance, M. S. Bartlett (1947) provided one of the earliest summaries of transformations on raw data addressing this. He covered parametric transformations used in stabilising the variance of modelling error, especially for Poisson and Binomial distributed variables where the variance is a known function of the mean. He discussed, both theoretical and empirical, some of the optimal scales and families of transformations to choose from given different circumstances. His work showed that modelling tasks do benefit from suitable transformations.

Another common assumption made by the models is normality. Almost all statistical inferences assume the variable of interest is normally distributed, e.g., t-test, Analysis of Variance (ANOVA), linear regression etc. Therefore, it is of great interest if we have the opportunity to create such conditions for the models to operate under. Box & Cox (1964) made a major contribution in this regard by proposing the well-known Box-Cox transformation. The Box-Cox transformation includes both power and logarithmic transformations. It aims at achieving normality of the observations and has been popular in developing modelling methodologies ever since. A good example is a method proposed in C. Bergmeier et al. (2016). Combined with the widespread exponential smoothing (ETS) forecasting method and the bootstrap aggregation (bagging) technique in machine learning, they proposed a bagging exponential smoothing method using STL decomposition and Box-Cox transformation. The proposed method was tested on the M3 competition dataset and achieved better results than all the original M3 participants (see Makridakis et al. 2000 for the M3 competition).

The method published by Box & Cox, however, is only valid for positive real values. Modifications of the Box-Cox transformation have thus been proposed to address the problem. A major one is made by Bickel & Doksum (1981). They embedded a sign function to the power transformation such that the transformation function covers the whole real line. Nevertheless, this modification has its shortcomings: it is shown by Yeo & Johnson (2000) that Bickel & Doksum's modified version of the transformation handles skewed distribution poorly. Yeo & Johnson pointed out the reason being the signed power transformation was designed primarily for handling kurtosis, thus losing its edge concerning skewness. Following up, Yeo & Johnson proposed a new version of the power transformation in the same publication (2000). Their transformation is a generalised version of the Box-Cox transformation

requiring a constant variance throughout time.

and approximates normality while being well-defined on the real line and inducing appropriate symmetricity.

Until now, we looked at how a mathematician or statistician would apply response transformation techniques to foster mathematical and statistical conditions assumed by the models. Meeting these assumptions improves the robustness of the conclusions drawn by the modelling results. In a general sense, one can say that the transformations help the models to get better at 'learning' the problem such that they generate more robust outputs. In the upcoming paragraphs, we take on this perspective of treating the transformation techniques as helpers in terms of the learning process of the models and look at some transformation techniques very different from what was covered previously.

Decomposition methods constitute a significant category of techniques that can be considered target transformation in helping the models learn when applied to the target variable. These methods decompose the mixture of information contained within the observation into patterns, trends, cycles, or other dynamics that are easier to model, i.e., easier for models to learn from. Also referred to as spectrum analysis, a particularly good example would be the giant branch of studies on Fourier-styled transformations in time series modelling (see Kay & Marple 1981 and Bloomfield 2004). Typical Fourier methodologies build on transforming the observations sampled from the time domain into the frequency domain and decomposing them into more informative signals. Out of the great history and advancements of spectrum analysis, we selectively provide a very brief overview of some relevant methodologies in the context of target transformation.

A novel type of method builds on what Fourier methods do and operates in the time-frequency domain, i.e., these methods come up with time-frequency representations of the observations. The empirical mode decomposition (EMD) proposed by Huang et al. (1998) is one of the key methodologies as such. EMD decomposes the original time series into 'intrinsic mode functions' (IMF). The IMF's carry information of the underlying structures contained in the observations and can then be used for modelling tasks. The family of wavelet transform methods constitutes another class of methods that operates in the time-frequency domain (see Daubechies 1992 and Percival & Walden 2000). Shensa et al. 1992 first proposed and provided a framework for the discrete wavelet transform (DWT), which belongs to the wavelet transform family. DWT filters the original time series in several folds and yields a denoised version of the observation. The information carried by the transformed time series is then more clear and preferably easier to learn. The rationale of using such techniques as target transformation in time series modelling is to provide the models with a more informative, e.g., less noisy, dataset to learn from. We hope this extra procedure helps produce a better trained (learned) model.

Another branch of decomposition methods is more related to statistical approaches. These decomposition methods generally consider the time series as a mixture of three components: seasonal, trend and remainder. They are often used to filter different information contained within the time series (see Wang, Smith, & Hyndman 2006). For a real-world example, monthly unemployment data are usually presented after removing the seasonality. The resulting time series is hence more indicative of the variation of the general economy instead of seasonal disturbance (see Chapter 3.2 in Hyndman & Athanasopoulos 2021). The STL decomposition proposed by Cleveland et al. 1990 has been a robust method. The abbreviation stands for Seasonal and Trend decomposition using Loess. STL considers a time series as a sum (additive) or product (multiplicative) of the seasonal, trend and remainder components. STL is flexible and applicable to many use cases as it can handle any type of seasonality. Its flexibility also resides in its allowance for the user to have control over the time-varying seasonal component and smoothness of the trend cycle. The X-11 method and the Seasonal Extraction in ARIMA Time Series (SEATS) procedure are time series models that rely heavily on seasonal and trend decompositions. They have had many variants and are favoured by official statistical agencies around the world (see Dagum & Bianconcini 2016)[2]. One of the state-of-the-art variants of this family is the X-13ARIMA-SEATS method produced, distributed and maintained by the US Census Bureau (see US Census Bureau 2012 and Monsell & Blakely 2013). It inherits powerful features from X-11, SEATS, and ARIMA methodologies while specialising in seasonal adjustment in extensive time series modelling. The model is conveniently accessible online[3].

[2]X-11 was initially developed by the US Census Bureau, and SEATS was created by the Bank of Spain.

[3]A webpage demonstration of the model is accessible on `http://www.seasonal.website/`; the open-source implementation of the model can be found in the `seasonal` package in R, and a distributed version can be found in the US Census Bureau website `https://www.census.gov/data/software/x13as.X-13ARIMA-SEATS.html`.

3.2 Directional Change intrinsic time framework

The technical core of the Directional Change (DC) intrinsic time framework is quite simple; it is an algorithm (the DC dissection) that samples a time series and yields a new time series, which is the subset of the original observations. By analysing the properties of the resulting time series, it has been found that despite the simplicity of this algorithm, it provides powerful perspectives for looking at market dynamics in the time domain. In this section, we first look at critical works contributing to the advancements of the DC intrinsic time framework. Then we cover some applications that further develop the framework's value by trying to harness its potential.

Like the developments of many frameworks, the DC intrinsic time framework started out being simple and has developed over time. Guillaume et al. (1997) first published the Directional Change dissection algorithm. The algorithm was presented and used to generate a set of measurements (statistics, variables), from which the authors presented a set of stylised facts found empirically in the spot intra-daily foreign exchange (FX) markets. These stylised facts shed new light on our understanding of market dynamics, especially concerning micro-structure topics, including time-heterogeneity, price formation, market efficiency, liquidity, and both the modelling and the learning process of the market. A little more than a decade later, Tsang formalised the definition of a Directional Change in Tsang (2010), and Glattfelder et al. (2011) discovered a set of twelve scaling laws derived from the DC sampling algorithm. The discovery of the laws added a theoretical foundation to the DC dissection algorithm because the output time series carries not only qualitative information (stylised facts) but also interesting quantitative properties. As the DC dissection algorithm's ability to extract information has been studied, it has given rise to the methodology becoming a framework (see Tsang's introduction of a set of profiles (indicators) derived under the DC framework in Tsang (2015) and (2017)).

One thing we know about analysing financial time series is that the source of many the challenges can be traced back to the use of physical time (see Dacarogna et al. (2001)). Aloud et al. (2012) discussed the potential of studying financial time series using the DC framework (referred to as the DC approach in their paper) resides in its underlying 'intrinsic time' paradigm. They pointed out that mapping financial time series from the physical time to event-based intrinsic time is the key to how the approach filters out irrelevant information and disturbance observed in the dataset and generates valuable market insights of our interests. Inspired by the studies of complex systems, Petrov et al. (2018) took a different route of demonstrating this point with the use of agent-based modelling. They created a market with trading agents that operate in event-based intrinsic time and found that the price movements generated under such conditions experience statistical properties we observed in real-world physical time FX markets. Such reproduction of real-world stylised facts is another indication of the intrinsic time mechanism being one of the contributing factors to the market dynamics. Recently, Glattfelder & Golub (2022) derived an analytical relationship between physical and intrinsic time based on the scaling laws. In particular, the expression they derived decomposes the movements of the physical-time time series into volatility and liquidity components expressed in intrinsic time. That allows us to explicitly characterise the dynamics observed in physical time using its intrinsic-time representation.

As DC intrinsic time framework becomes theoretically sound, applications building on the framework have been devised. Golub et al. (2016) introduced the Intrinsic Network - an event-based framework based on directional changes. Combining the Intrinsic Network and information theory, they devised a liquidity measure that was shown to be able to predict market stress in terms of liquidity shocks. In Golub et al. (2018), the liquidity measure was integrated with other implementations derived from the DC framework and an algorithmic trading strategy called The Alpha Engine was introduced. The Alpha Engine has several interesting features. First, the bare-bones version of the model (without tweaking) has been shown to be robust, profitable, and can be implemented in real-time. Second, Alpha Engine provides liquidity in the market, i.e., it opens long positions when other market players intend to short and vice versa. The Alpha Engine thus contributes to the healthiness of the market as a participant. Third, Alpha Engine 'beats' random walk processes - it is shown to be profitable even on price dynamics generated by a random walk. Within the context of volatility and risk management in finance, Petrov et al. (2019a) proposed an instantaneous volatility measure under the DC intrinsic time framework. They found seasonality patterns and long memory of volatility through empirical studies. Their work contributes not only to the development of practical tools but also to the understanding of the underlying stochastic drive of financial dynamics. Two further generalisations of the DC framework have been proposed. First, Petrov et al. (2019b) brought the framework into multidimensional space by extending the analytical expressions yielded from one-dimensional analyses to multidimensional space. Their methodology implies that previous works in one-dimensional space (analytical insights, empirical findings, and all the tools and implementations) can be extended to higher dimensions. Another generalisation was developed with respect to the types of stochastic processes (Mayerhofer (2019)). These generalisations, as well as the advancements of the DC intrinsic time framework discussed previously, are indicative of the framework's promising potential worthy of further exploration.

In this chapter, we reviewed relevant works in two different realms: target transformations being used as an additional layer in modelling and the DC intrinsic time framework being a rising methodology. The literature surveyed demonstrates excellent potential in both research directions that justifies our curiosity in bringing them together. In the next chapter, we go into detail about the methodologies of combing the framework and target transformation in time series modelling.

Chapter 4

Methodology

Chapter 5

Results and Evaluation

Chapter 6

Conclusion

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Appendix A

chapter one

Something about chapter one

Another something ...

And another something ...

A.1 ch1 sec 1

Something about chapter one, section 1

Appendix B

chapter two

Something about chapter two

B.1 ch2 sec 1

Something about chapter two, section 1

B.2 ch2 sec 2

Something about chapter two, section 2

Appendix C

chapter three

Something about chapter three

C.1 ch3 sec 1