Project 2
Root Solving Using Newton-Rapshon and Secant Method – finding the friction factor in a pipe

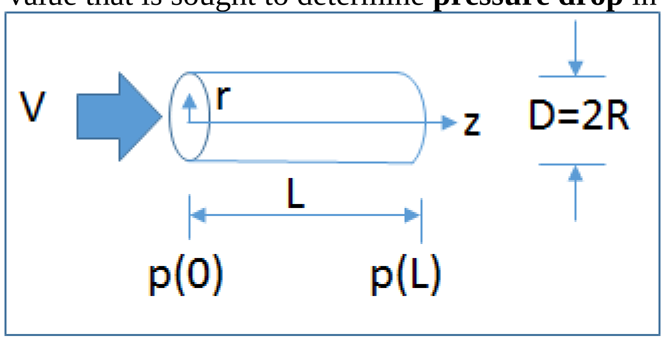
---

Grading:

1. [80%] Complete assignment, input results in webcourses project assignment Quiz as instructed in the results in webcourses project assignment Quiz.
2. [20%] and uploaded Matlab code files and output.

You must upload your codes and output to receive credit for this part of the assignment. Failure to upload your Matlab code will result in a loss of 50 points for the assignment.

Instructions: You will write a program in Matlab to solve a non-linear equations using the Newton-Raphson method and the Secant method. The application is from fluid mechanics. The **friction factor**, *f*, is a value that is sought to determine **pressure drop** in pipes and channels.



According to extensive experiments, Colebrook arrived at the following expression for the friction factor, *f*, which is referred to as the Colebrook equation in his honor

$$F\_factor(f) := f^{-\frac{1}{2}} + 2.0 \cdot \log\left(\frac{r}{3.7} + \frac{2.51}{Re \cdot f^{\frac{1}{2}}}\right)$$

Where,

f	is the unknown friction factor
r	is the ratio of surface roughness of the pipe wall to diameter of the pipe. It is a given value for a type of pipe material.
Re	is called the Reynolds number and it is a non-dimensional number characterizing the ratio of linear momentum to viscous effects in the pipe flow. It is a know value for a given fluid, pipe diameter and flow rate.

The plot of this relation for a range of roughness ratios and Reynolds numbers is the famous *Moody Chart* which is difficult at times to read, while we can readily compute the friction factor by using methods from EML 3034C. Once the friction factor is known, the pressure drop due to friction for a fluid of density ρ is given by

$$\Delta p = p(0) - p(L) = \rho g h_f$$

Where

g	is the acceleration due to gravity (in appropriate units).
ρ	is the density of the fluid.
h_f	is the head loss due to friction for a pipe of length L and diameter. D in which fluid flows at a mean velocity V.

The head loss is related to the friction factor by the relation:

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

So that if the English system is used, the head loss, h_f, is given in feet (of water) and it is the equivalent pressure due to a column of water of height h_f. Assuming flow in a pipe such that:

L= 2500 ft	length of pipe.
D = 12 in	diameter of pipe.
r = 3 x 10^{-4}	pipe wall roughness ratio.
V= 0.35 ft/s	mean velocity in the pipe.
Re = 3.1818 x10^4	Reynolds number of the flow.
g = 32.2 ft/s^2	acceleration due to gravity.

(1) Use the **<u>Newton Raphson</u>** method to solve the Colebrook equation for the friction factor in the pipe using an initial guess of *f = 0.001*. You can determine the derivative of the Colburn equation by using the symbolic manipulator in Mathcad (on UCF apps) or in Mathematica (free use of certain functions online), or carry this out by hand (see appendix).

(2) From the result computed in (1), find the head loss h_f in feet.

This code should output: the iteration number, the guess for the root at the current iteration, the iterative convergence at each iteration, and the residual at each iteration in one matrix, so that you can view the results in a single table. **<u>Set the stopping criteria to when both the residual and iterative convergence are less than 10^{-5}.</u>**

It is also worthwhile to vary the initial guess and see how the results change. You should note that the root is still found correctly for various initial guesses, but the number of iterations (which corresponds directly to run-time) changes quite a bit. Try comparing the number of iterations required for an initial guess of f=0.001, f=0.01 and that for an initial guess of f=0.1, do you obtain convergence with all initial guesses?

Note that it was necessary to compute the derivative of the Colburn relation to apply the Newton-Raphson method. An alternative for this problem would be the Secant method that does not require derivative evaluation.

(3) Repeat the exercise using the **Secant method**. You will need to write a new routine to call to solve your problem. Use initial guesses of *f=0.01* and *f=0.1*.
(4) Report the friction factor and head loss as before which you now have obtained using the Secant method. How do the answers compare?

It would be a good practice to write a main code that has the logic of the problem solution and two routines, one for Newton Raphson and one for Secant method, that can be called from the main routine to solve the equation of interest.

Once your program(s) is(are) completed, you should add a plot of the residual vs. iteration number using the given initial guesses(s) for both Newton-Raphson and Secant method. Refer to last week's assignment if you need help creating a plot.

This completes Project 2 assignment. **Now report the required values on the Webcourses on-line quiz for project 2, specifically report:**

a) **The friction factor that you found using the initial guess *f=0.001*.**
b) **The head loss in feet that you compute for the length of pipe.**

for Newton Raphson and for the Secant method (with the initial guesses of *f=0.001 and f=0.01)*.

This completes your Project: **Upload your Matlab code and a PDF of your output results.**

It is good practice to print your output to a file using the fprintf command. You can then make a PDF of that file or you can use the *diary* command and then make a PDF of the text output.

You will be quizzed regarding the Newton-Rapshon and Secant methods. Make sure you have a secant code that will work for any f(x). You will be asked questions in a webcourses quiz on this subject.

Appendix:

Colebrook equation: Hint: use $\frac{d}{dx}\log(u) = \frac{\log(e)}{u}\frac{du}{dx}$

$$F(f) = f^{-1/2} + 2.0\log\left(\frac{r}{3.7} + \frac{2.51}{Re\, f^{1/2}}\right)$$

. then

$$\frac{dF}{df} = -\frac{1}{2}f^{-3/2} + \cancel{2}.0\log(e)\left[\frac{1}{\left(\frac{r}{3.7} + \frac{2.51}{Re\, f^{1/2}}\right)}\right]\left(\frac{2.51}{Re}\right)\left(-\frac{1}{\cancel{2}}f^{-3/2}\right)$$

$$\boxed{\frac{dF}{df} = -\frac{1}{2}f^{-3/2} - \left[\frac{\frac{2.51}{Re}\log(e)}{\frac{r}{3.7} + \frac{2.51}{Re\, f^{1/2}}}\right]f^{-3/2}}$$