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```
%Logan McBride
%September 8, 2023
%Project 1 Truncation vs Rounding Error
%EML 3034C Modeling Methods in MAE

% Clear command window, close all graphs, clear workspace
clc, close all, clear

% =====
fprintf("=====\\n")
fprintf("Project 1 - Round-off vs Truncation Error\\n")
fprintf("Logan McBride\\n")
display(datetime("today"))
fprintf("=====\\n\\n")
% =====

format short e

% Define the function f(x)
f = @(x) 2.5 - 1.25 * cos(3.8 * x); % Insert Function f(x)
df = @(x) 1.25 * 3.8 * sin(3.8 * x); % Insert Exact First Derivative df(x)
% Select x value to evaluate derivative
x = 5.25;

=====
Project 1 - Round-off vs Truncation Error
Logan McBride
    datetime

    01-Sep-2023

=====
```

## Double-Precision

loop through the exponents for each delta x value and evaluate the derivative

```
for i = 1:20
% calculate the delta x
del(i,1) = 10^(-i);
```

---

```

% Backward Finite Difference
backward(i,1) = (f(x) - f(x-del(i,1))) / (del(i,1));

% Forward Finite Difference
forward(i,1) = (f(x+del(i,1)) - f(x)) / (del(i,1));

% Central Finite Difference
central(i,1) = (f(x+del(i,1)) - f(x-del(i,1))) / (2*del(i,1));

% calculate the errors compared to exact (analytical) derivative
backward_error(i,1) = abs(backward(i,1) - df(x));
forward_error(i,1) = abs(forward(i,1) - df(x));
central_error(i,1) = abs(central(i,1) - df(x));
end

% Plot the error vs del x for each method on the log scale
figure
hold on
% Scale of log for axis
set(gca, 'XScale', 'log', 'YScale', 'log')
% the loglog() function is the same as plot() but on log scale for both axes
loglog(del, forward_error)
loglog(del, backward_error)
loglog(del, central_error)
xlabel('\Delta'), ylabel('Error')
title('Error on Forward, Backward, and Central')
legend('Forward','Backward','Central')
grid on, hold off
% assemble the results into a table (actually a matrix)
Table = [del, forward_error, backward_error, central_error];
% print to screen
fprintf("===== Double Precision Results =====\n")
fprintf("      Delta x      Forward      Backward      Central\n")
disp(Table)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

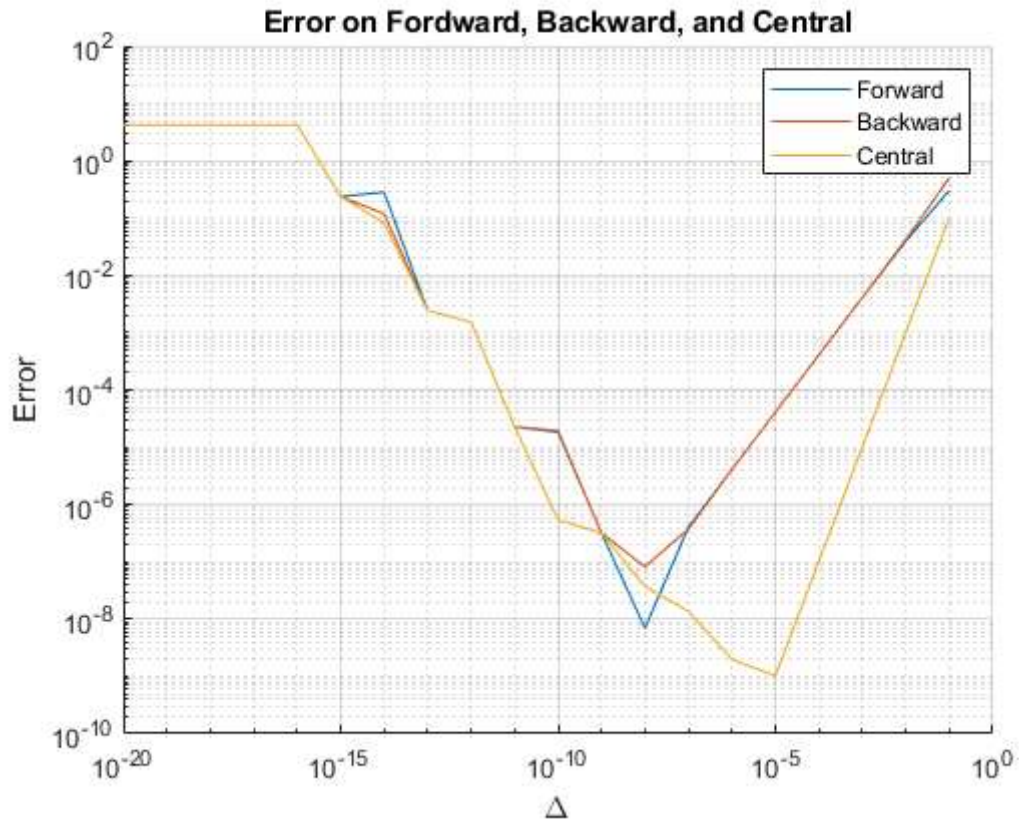
===== Double Precision Results =====
      Delta x      Forward      Backward      Central
1.0000e-01  3.0295e-01  5.0528e-01  1.0117e-01
1.0000e-02  3.9877e-02  4.1915e-02  1.0190e-03
1.0000e-03  4.0799e-03  4.1003e-03  1.0190e-05
1.0000e-04  4.0891e-04  4.0912e-04  1.0190e-07
1.0000e-05  4.0900e-05  4.0902e-05  1.0243e-09
1.0000e-06  4.0921e-06  4.0882e-06  1.9733e-09
1.0000e-07  4.0221e-07  3.7495e-07  1.3631e-08
1.0000e-08  6.9693e-09  8.1849e-08  3.7440e-08
1.0000e-09  3.1783e-07  3.1783e-07  3.1783e-07
1.0000e-10  1.8334e-05  1.9414e-05  5.3988e-07
1.0000e-11  2.2775e-05  2.2775e-05  2.2775e-05
1.0000e-12  1.5315e-03  1.5315e-03  1.5315e-03
1.0000e-13  2.4197e-03  2.4197e-03  2.4197e-03
1.0000e-14  2.8180e-01  1.1788e-01  8.1957e-02
1.0000e-15  2.3739e-01  2.3739e-01  2.3739e-01

```

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---

|            |            |            |            |
|------------|------------|------------|------------|
| 1.0000e-16 | 4.2342e+00 | 4.2342e+00 | 4.2342e+00 |
| 1.0000e-17 | 4.2342e+00 | 4.2342e+00 | 4.2342e+00 |
| 1.0000e-18 | 4.2342e+00 | 4.2342e+00 | 4.2342e+00 |
| 1.0000e-19 | 4.2342e+00 | 4.2342e+00 | 4.2342e+00 |
| 1.0000e-20 | 4.2342e+00 | 4.2342e+00 | 4.2342e+00 |



## Repeat everything, but in single precision

Select x value to evaluate derivative

```
x = single(x);
for i = single(1:20)
% calculate the delta x
del(i,1) = single(10^(-i));

% Backward Finite Difference
backward(i,1) = single((f(x) - f(x-del(i,1))) / (del(i,1)));

% Forward Finite Difference
forward(i,1) = single((f(x+del(i,1)) - f(x)) / (del(i,1)));

% Central Finite Difference
central(i,1) = single((f(x+del(i,1)) - f(x-del(i,1))) / (2*del(i,1)));

% calculate the errors compared to exact (analytical) derivative
```

---

```

backward_error(i,1) = single(abs(backward(i,1) - df(x)));
forward_error(i,1) = single(abs(forward(i,1) - df(x)));
central_error(i,1) = single(abs(central(i,1) - df(x)));
end
% Plot the error vs del x for each method on the log scale
figure
hold on
% Scale of log for axis
set(gca, 'XScale', 'log', 'YScale', 'log')
% the loglog() function is the same as plot() but on log scale for both axes
loglog(del, forward_error)
loglog(del, backward_error)
loglog(del, central_error)
xlabel('\Delta'), ylabel('Error')
title('Error on Forward, Backward, and Central')
legend('Forward','Backward','Central')
grid on
% assemble the results into a table (actually a matrix)
fprintf("\n\n\n")
Table = [del, forward_error, backward_error, central_error];
% print to screen
fprintf("===== Single Precision Results =====\n")
fprintf("      Delta x      Forward      Backward      Central\n")
disp(Table);

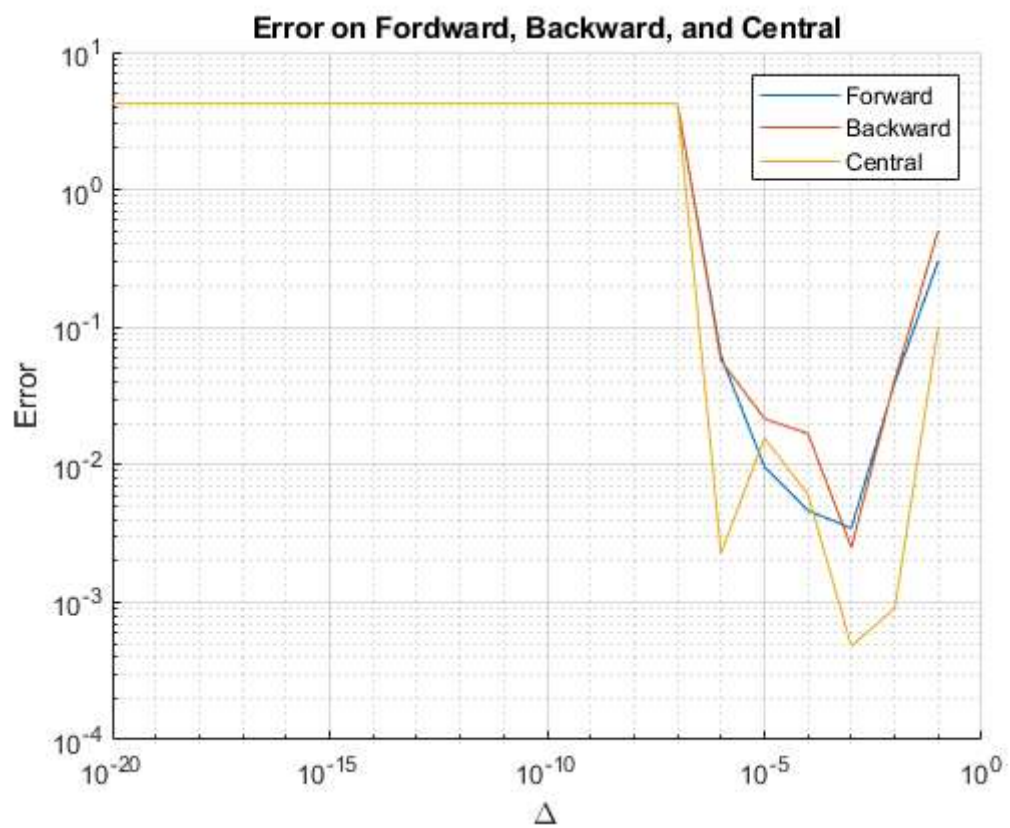
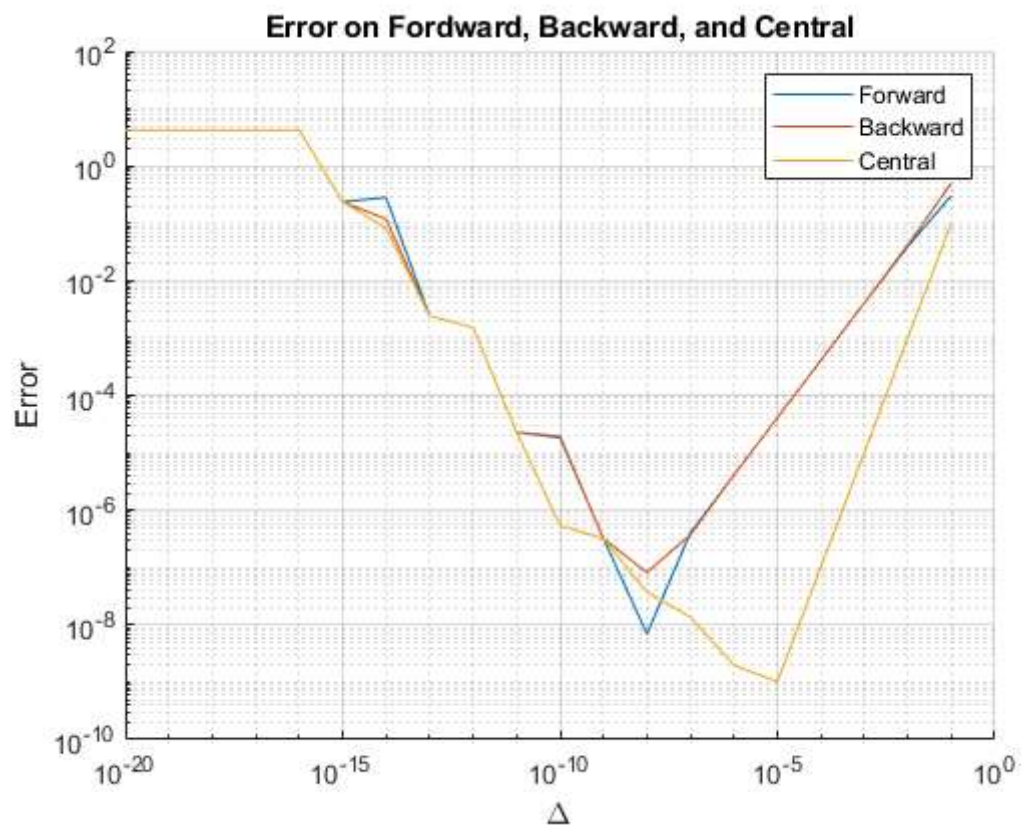
```

```

===== Single Precision Results =====

```

| <i>Delta x</i> | <i>Forward</i> | <i>Backward</i> | <i>Central</i> |
|----------------|----------------|-----------------|----------------|
| 1.0000e-01     | 3.0293e-01     | 5.0528e-01      | 1.0117e-01     |
| 1.0000e-02     | 3.9889e-02     | 4.1698e-02      | 9.0408e-04     |
| 1.0000e-03     | 3.4590e-03     | 2.5015e-03      | 4.7874e-04     |
| 1.0000e-04     | 4.6473e-03     | 1.6810e-02      | 6.0816e-03     |
| 1.0000e-05     | 9.6579e-03     | 2.1579e-02      | 1.5618e-02     |
| 1.0000e-06     | 6.1868e-02     | 5.7342e-02      | 2.2631e-03     |
| 1.0000e-07     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-08     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-09     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-10     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-11     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-12     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-13     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-14     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-15     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-16     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-17     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-18     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-19     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |
| 1.0000e-20     | 4.2342e+00     | 4.2342e+00      | 4.2342e+00     |



---

# Machine epsilon

```
E_dbl = eps  
E_sgl = eps("single")
```

```
E_dbl =  
2.2204e-16
```

```
E_sgl =  
single  
1.1921e-07
```

*Published with MATLAB® R2022b*