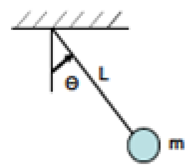
EML 3034 – Fall 2023: COMPUTER PROJECT #9
Matlab Project

Grading:

1) [80%] Complete assignment, input results in webcourses project assignment Quiz as instructed.
2) [20%] and uploaded Matlab code files and output.

You must upload your codes and output to receive credit for this part of the assignment. <u>Failure to upload your Matlab code will result in a loss of 50 points for the assignment.</u>

Solution of 2nd order Non-linear ODE



You are to find the non-linear response of a simple pendulum of length *L=4.5m* and mass *m* released with zero initial velocity $\dot{\theta}_o = 0$ and at an initial angle $\theta_o = \dfrac{\pi}{6.25} = 28.8^0$ at is given by the following expression

$$T(k) = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2} \frac{1}{\sqrt{1-k^2\sin^2(\theta)}}\,d\theta$$

where, the parameter *k* is related to the intial angle θ_o as

$$k = \sin\left(\frac{1}{2}\theta_o\right)$$

Using a value of g =9.81 m/s^2, you should compute using a quadrature a period of T= 4.3237s. Use Gauss-Kronrod quadrature https://www.mathworks.com/help/matlab/ref/quadgk.html in the Matlab toolbox).

> *quadgk(fun,a,b) integrates the function handle fun from a to b using high-order global adaptive quadrature and default error tolerances.*

In this assignment, you are to develop a Matlab code utilizing ODE45 to find the response of the simple pendulum whose angular position $\theta(t)$ is governed by the following non-linear ODE

$$G.E.: \quad \frac{d^2\theta(t)}{dt^2} + \left(\frac{g}{L}\right)\sin\theta(t) = -C_d\,\dot{\theta}(t)\,|\,\dot{\theta}(t)\,|$$

$$I.C.'s: \quad \theta(0) = \theta_o$$

$$\dot{\theta}(0) = \dot{\theta}_o$$

where C_d is an effective drag coefficient. You are to solve the problem for time interval $t \in (0s, 200s)$ for two cases:

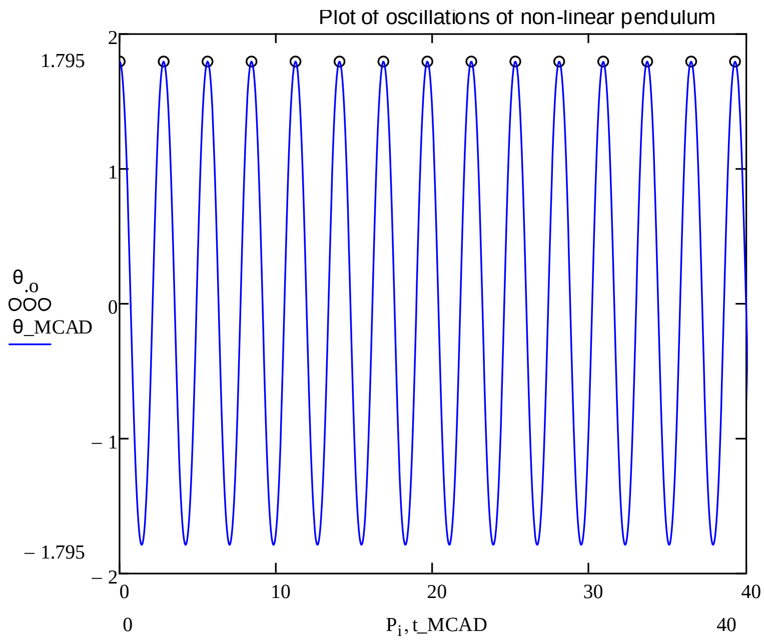
<u>Case 1- no drag acting on the pendulum mass:</u> $C_d = 0$

1) Report the solution for $\theta(t)$ and $\dot{\theta}(t)$ at t =132.65s using a tolerance of 10^{-3} and 10^{-6}. Since ODE45 is an adaptive time stepping routine you may likely have to interpolate your results (tabulated in the output for all time steps used by ODE 45) to obtain the solution the requested time: ***<u>use linear interpolation report to 2 decimal places and chop.</u>***

   You can write your own Lagrange interpolation routine or use the Matlab interp1 routine

   > ***interp1(x , v , xq)*** *returns interpolated values of a 1-D function at specific query points using linear interpolation. Vector x contains the sample points, and v contains the corresponding values, v(x). Vector xq contains the coordinates of the query points.*

2) Report the Poincare' map (also called the phase plot of $\theta(t)$ vs $\dot{\theta}(t)$) for the non-linear pendulum using the TOL=10^{-6}.
3) Verify that you computed the correct solution using the analytical prediction for the period matches the period from your computation using the Runge-Kutta solver ode45. As a check of your results, report the plots of $\theta(t)$ and superimpose the computed period from your previous project to check that you have computed correct solutions. That is produce such a plot: (*note that is an example that I pulled from a class presentation with different initial conditions, so that your numbers will be different but the general plot should look the same). You can use the Mathcad pseudocode that I provided in class and on the website to figure out how to carry out such a plot.*



You may use Matlab's plotting routines or export data to EXCEL, MATHCAD or any such a program to produce the required plot.

4) Report the phase plot of $\theta(\mathrm{t})$ vs $\dot{\theta}(\mathrm{t})$) for the pendulum using the TOL=10^{-6}.

<u>Case 2- drag acting on the pendulum mass:</u> $C_d = 0.2$

5) Report the solution for $\theta(\mathrm{t})$ and $\dot{\theta}(\mathrm{t})$ at t = 132.65s using a tolerance of TOL=10^{-3} and TOL=10^{-6}. Since ODE45 is an adaptive time stepping routine you may likely have to interpolate your results (tabulated in the output for all time steps used by ODE 45) to obtain the solution the requested time: ***<u>use linear interpolation report to 2 decimal places and chop.</u>***

6) Report the phase plot of $\theta(\mathrm{t})$ vs $\dot{\theta}(\mathrm{t})$) for the pendulum using the TOL=10^{-6}.

This is the last project for the course.