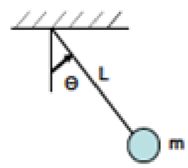
EML 3034H – Fall 2023: COMPUTER PROJECT #7
MATLAB Project: Gauss Quadratures

Grading:

1) [80%] Complete assignment, input results in webcourses project assignment Quiz as instructed.
2) [20%] and uploaded Matlab code files and output.

You must upload your codes and output to receive credit for this part of the assignment. Failure to upload your Matlab code will result in a loss of 50 points for the assignment.

Gauss Quadrature



You are to write an use a 5 point Gauss-Legendre quadrature to evaluate the period *T(k)* of the non-linear response of the simple pendulum of length $L=14.5\ m$ and mass *m* released with zero initial velocity at an initial angle $\theta_o = 28.8^\circ$ that is given by the following expression

$$T(k) = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2} \frac{1}{\sqrt{1-k^2\sin^2(\theta)}}\,d\theta$$

where, the parameter *k* is related to the intial angle θ_o as

$$k = \sin\left(\frac{1}{2}\theta_o\right)$$

Take $g = 9.81\ m/s^2$. The 5-point Gauss quadrature rule weights and abscissae are provided in the table below:

	Gauss-Legendre weights (w_i)	Gauss-Legendre abscissae (ξ_i)
1	0.236926885056189	+0.906179845938664
2	0.478628670499366	+0.538469310105683
3	0.568888888888889	0.000000000000000
4	0.478628670499366	- 0.538469310105683
5	0.236926885056189	- 0.906179845938664

Report results to 8 decimal places and round-up.

1. Verify your MATLAB code by using it to integrate,

$$\int_{1.2}^{2.5} (12.5x^9 + 3.2x^4)dx$$

and **report the computed value compared to the analytical (exact) value of this integral.**

2. **Report the computed period of the pendulum for the parameters:**

> **(a) using a single interval for the quadrature [0,π/2].**
> **(b) using two intervals [0,π/4] + [π/4,π/2].**

You have been provided a working set of Matlab files for Gauss-Legendre quadrature that you can find on the CANVAS site for this project, and you may use these as a guide and/or modify them as you see fit for this project. This code integrates using a 2 point Gauss-Legendre rule and uses a test function $f(x) = x^2$ as an example to verify the code.

Historical Note:

The Foucault pendulum is a simple device (https://en.wikipedia.org/wiki/Foucault_pendulum) named after French physicist Léon Foucault and conceived as an experiment to demonstrate the Earth's rotation. The pendulum was introduced in 1851 and was the first experiment to give simple, direct evidence of the Earth's rotation. Foucault pendulums today are popular displays in science museums and universities. At UCF, there is a Foucault pendulum located at the entrance of the math and physics building.

Léon Foucault

Jean Bernard Léon Foucault (1819 – 11 February 1868) was a French physicist best known for his demonstration of the Foucault pendulum, a device demonstrating the effect of the Earth's rotation. He also made an early measurement of the speed of light, discovered eddy currents, and is credited with naming the gyroscope. https://en.wikipedia.org/wiki/L%C3%A9on_Foucault

The first public exhibition of a Foucault pendulum took place in February 1851 in the Meridian of the Paris Observatory. A few weeks later, Foucault made his most famous pendulum when he suspended a 28-kilogram brass-coated lead bob with a 67-meter-long wire from the dome of the Panthéon in Paris (pictured on the next page). The proper period of the pendulum was approximately

$$2\pi\sqrt{\frac{L}{g}} \sim 16.5s\,.$$

At either the Geographic North Pole or Geographic South Pole, the plane of oscillation of a pendulum remains fixed relative to the distant masses of the universe while Earth rotates underneath it, taking one sidereal day to complete a rotation. So, relative to Earth, the plane of oscillation of a pendulum at the North Pole, viewed from above, undergoes a full clockwise rotation during one day; a pendulum at the South Pole rotates counterclockwise.

When a Foucault pendulum is suspended at the equator, the plane of oscillation remains fixed relative to Earth. At other latitudes, the plane of oscillation precesses relative to Earth, but more slowly than at the pole; the angular speed, ω (measured in clockwise degrees per sidereal day), is proportional to the sine of the latitude, φ:

$$\omega = 360^{o} \sin \varphi / day$$

where latitudes north and south of the equator are defined as positive and negative, respectively. A "pendulum day" is the time needed for the plane of a freely suspended Foucault pendulum to complete an apparent rotation about the local vertical. This is one sidereal day divided by the sine of the latitude. For example, a Foucault pendulum at Orlando’s 28.5° North latitude, viewed from above by an earthbound observer, rotates clockwise 171.8° in a day.

Foucault's pendulum in the Panthéon, Paris

UCF's pendulum was crafted by hand at the California Academy of Sciences in San Francisco. It fell into disrepair, and it was restored back into operation in 2016 thanks to an endowment from a UCF Alumna https://www.ucf.edu/news/ucfs-pendulum-swings-again-thanks-to-alumna/.