EML 3034 – Fall 2023: COMPUTER PROJECT #8
MATLAB Project: 1st order ODE – thermocouple

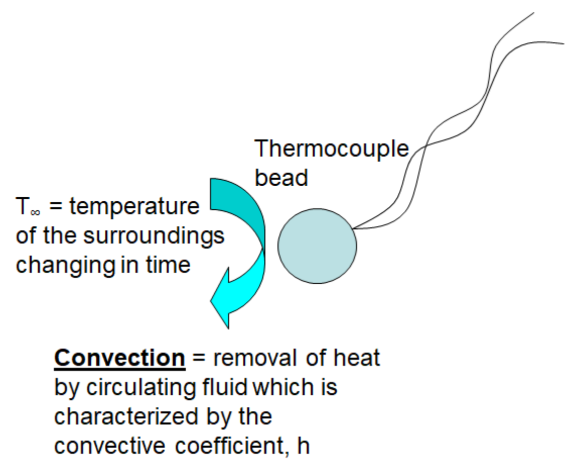
Grading:

1. [80%] Complete assignment, input results in webcourses project assignment Quiz as instructed.
2. [20%] and uploaded Matlab code files and output.

You must upload your codes and output to receive credit for this part of the assignment. Failure to upload your Matlab code will result in a loss of 50 points for the assignment.

Numerical Solution of first order ODEs

We will consider the transient temperature response of a small spherical thermocouple bead T(t) that is being cooled from an initial temperature T_o by a fluid at temperature $T_\infty(t)$ which is changing linearly in time.



Take the following properties of the thermocouple bead and convective heat transfer coefficient in SI units:

$T_o := 298$	**initial temperature [K]**
$\rho := 16600$	**density [kg/m^3]**
$c_p := 160$	**specific heat [J/kgm^3]**
$h := 125$	**convection coefficient [W/m^{2}K]**
$D := 1.5 \cdot 10^{-3}$	**diameter [m]**

The ambient fluid at temperature $T_\infty(t)$ is growing according to: $T_\infty(t) = T_o + Bt$.
where, B = 0.35 [K/s].

Since the initial temperature is elevated, radiation heat transfer plays a role and the governing equation for the temperature of the thermocouple is highly non-linear and it is given as the following first order ODE:

$$GE: \quad \frac{dT(t)}{dt} = \left[\frac{A[h + h_r(T,t)]}{\rho V c_p} \right] T_\infty(t) - \left[\frac{A[h + h_r(T,t)]}{\rho V c_p} \right] T(t)$$

$$IC: \quad T(0) = T_o$$

Where the properties ρ, c_p, D, and h are given above and,

$A = 4\pi R^2$ is the bead surface area

$V = \frac{4}{3}\pi R^3$ is the bead volume

$h_r(T,t) = \varepsilon\sigma\left[T(t)^2 + T_\infty(t)^2\right] \times \left[T(t) + T_\infty(t)\right]$ is the radiative heat transfer coefficient

$\varepsilon = 0.925$ is the emissivity of the thermocouple bead (a radiative property of the bead material)

$\sigma = 5.68 \times 10^{-8}$ is the Stefan-Boltzmann radiation constant [$\mathrm{W/m^2}K^4$]

So that we have a standard first order ODE problem

$$GE: \quad \frac{dT(t)}{dt} = f(T,t)$$

$$IC: \quad T(0) = T_o$$

With a known non-linear right-hand side:

$$f(T,t) = \left[\frac{A[h + h_r(T,t)]}{\rho V c_p} \right] T_\infty(t) - \left[\frac{A[h + h_r(T,t)]}{\rho V c_p} \right] T(t)$$

Part A: Solve the governing equation for the temperature using the Euler method for the time interval $t \in [0, 200s]$, using a time step of

1. $\Delta t = 10[s]$ or using 20 time steps.
2. $\Delta t = 5[s]$ or using 40 times steps.
3. $\Delta t = 1[s]$ or using 200 time steps.

Plot the numerical solution for the temperature as a function of time for all three listed time steps. What do you observe, what do you expect happened in the numerical solution of the problem?

If you obtained a solution with a given time step of 5s, should the time step should be reduced further below 5s? how would you determine this?

Provide the solution at time *t=50s, 150s, and 200 s* computed with time steps above. There are values at specific requested time points where you might not evaluate at directly because it is not divisible by the time step (Δt) you are using. For all of the values to be reported on the answer sheet, use linear interpolation. This is the *interp1()* function in Matlab. See below for an example.

EXAMPLE: Suppose we want to find the solution value at t = 2.5 for this set of data:

```
t = [1, 2, 3];  % time points
y = [2, 4, 6];  % solution
interpolated_solution = interp1(t, y, 2.5)  % this evaluates to
                                            % 5 decimal places
```

Report results to 2 decimal places and chop.

Part B: Solve the governing equation using **ODE45** with (1) the default tolerance value 10^{-3} and (2) by setting the tolerance to 10^{-6} using **odeset**.

Provide the solution at time *t=50s, 150s and 200s* computed with the tolerance set 10^{-3} and then again by setting the tolerance to 10^{-6} using ODE set. **Report results to 2 decimal places and chop.**

- **Complete Webcourses Project Assignment Quiz questions and uploading project code (Matlab and PDF format).**