Project 5 – Fall 2023

Solving Non-Linear Equations Using Newton-Raphson

Grading:

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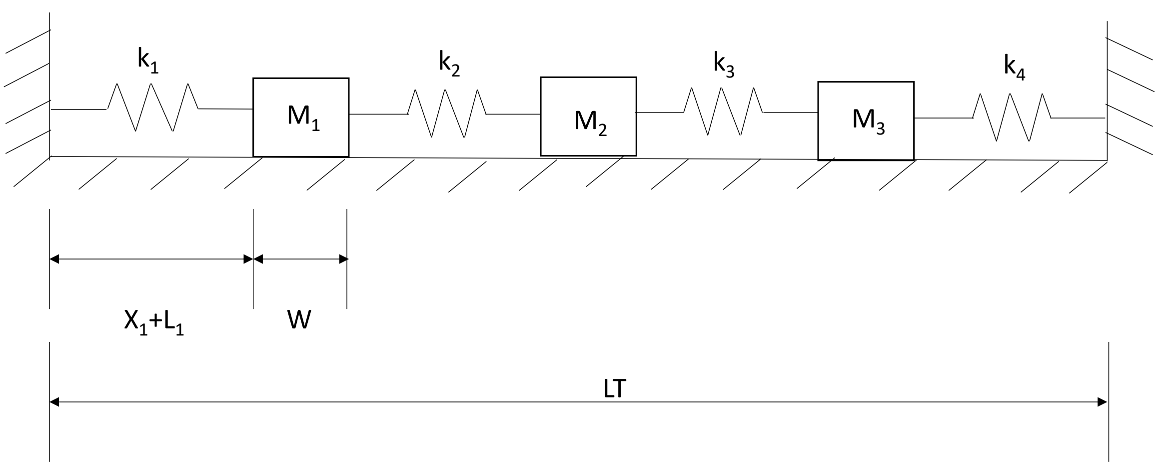
1. **[80%]** Complete assignment, input results in webcourses project assignment Quiz.
2. **[20%]** and uploaded Matlab code files and output.

You must upload your codes and output to receive credit for this part of the assignment. Failure to upload your Matlab code will result in a loss of 50 points for the assignment.

Systems of blocks and springs are used in models to study structural vibrations as we have seen in the eigenvalue problem shown in class. Consider as system of 3 blocks of masses (M_1, M_2 and M_3) each of width, W, that are connected by four springs attached between two walls separated by a distance LT, as shown in the figure below.



The total length of each spring is $x_i + L_i$, where L_i is the un-stretched length of the ith spring. This is displayed for the first spring in the figure. The blocks slide on the well-lubricated frictionless supporting surface.

Each spring has a non-linear force/displacement relation that is given by a cubic relation: $F_{spring,i} = k_i x_i + k_{ii} x_i^3$. Considering the static equilibrium case, the equations governing the system are:

1. The constraint on the total length:

$$(x_1+L_1)+(x_2+L_2)+(x_3+L_3)+(x_4+L_4)+3W = LT$$

2. The force balance on each mass:

$$k_1 x_1 + k_{11} x_1^3 = k_2 x_2 + k_{22} x_2^3$$
$$k_2 x_2 + k_{22} x_2^3 = k_3 x_3 + k_{33} x_3^3$$
$$k_3 x_3 + k_{33} x_3^3 = k_4 x_4 + k_{44} x_4^3$$

This leads to system of four (4) simultaneous nonlinear equations:

$$(x_1 + L_1) + (x_2 + L_2) + (x_3 + L_3) + (x_4 + L_4) + 3W - LT = 0$$
$$k_1 x_1 + k_{11} x_1^3 - (k_2 x_2 + k_{22} x_2^3) = 0$$
$$k_2 x_2 + k_{22} x_2^3 - (k_3 x_3 + k_{33} x_3^3) = 0$$
$$k_3 x_3 + k_{33} x_3^3 - (k_4 x_4 + k_{44} x_4^3) = 0$$

Take the following values for the parameters of the problem:

$$L_1 = L_2 = L_3 = L_4 = 1m$$
$$W = 0.435m$$
$$LT = 15m$$
$$k_1 = 1N/m \qquad k_{11} = 0.125N/m$$
$$k_2 = 2N/m \qquad k_{22} = 0.255N/m$$
$$k_3 = 3N/M \qquad k_{33} = 0.325N/m$$
$$k_4 = 4N/m \qquad k_{44} = 0.425N/m$$

Use an initial guess for the spring stretches of:

$$x_1 = 1m$$
$$x_2 = 1m$$
$$x_3 = 1m$$
$$x_4 = 1m$$

A. Solve the non-linear problem for spring stretches, x_i , using the Newton-Raphson method. The Jacobian matrix must be evaluated at each iteration step of the Newton-Raphson algorithm, and the resulting linear system for the update vector may be solved by using the intrinsic MATLAB linear solver (such as the *linsolve* routine). You are to evaluate the elements of the Jacobian by using a *first order backward finite differencing* approximation with a step size of $\Delta x = 10^{-5}$ to evaluate the partial derivatives.

Set the convergence factor *10^{-5}* for both the iterative norm and the residual norm. Set the maximum number of iterations to *K=100*, and report:

1. The initial elements of the Jacobian **(use 3 decimal places and round off).**
2. The spring stretches, x_i **(use 3 decimal places and round off).**
3. The initial residual norm **(use 3 decimal places and round off).**

B. Using the same values for the linear spring coefficients k_i but setting the non-linear coefficients $k_{ii} = 0$ to zero, solve the linear problem and report:

4. The spring stretches, x_i **(use 3 decimal places and round off).**

This completes this project. Report your results on the Webcourses Project 5 assignment quiz, and upload your MATLAB code, and the PDF of the MATLAB code.