Models, Metamodels, and Model Transformation for Cyber-Physical Systems

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Outline

Cyber-Physical Systems

- Tight integration between a large-scale physical network and a cyber network that monitors and controls the physical network.
- Can be used to build sustainable infrastructure:
  - Make existing physical networks more dependable.
  - Reduce the physical resources needed to build new infrastructure.
- Examples:
  - Smart Power Grids
  - Intelligent Water Distribution Networks
  - Smart Transportation Systems

Modeling

- Designing cyber-physical systems requires constructing multiple models of each design
  - Performance models
  - Dependability models (reliability, resilience, survivability, etc.)
- Problems with modeling
  - How do you avoid re-making every model every time the system design changes?
  - How do you make sure each model is working off the same assumptions?

Model Transformation with Metamodeling

- Model transformation converts a model of one type (e.g. a performance model) to a model of another type (e.g. a reliability model)
- Can reduce workload and prevent mistakes when modeling complex systems
- Goals of a transformation technique:
  - *Correctness*: the generated model should model the same system as the source model
  - *Specificity*: the generated model should contain as much information from the source model as possible

Hierarchical Model Composition

- Build small models and link them together into complete system models
- Hierarchical models can contain heterogeneous submodels
- Not model transformation per se; no way to start with one model and derive a different one
- Projects: Ptolemy, Möbius

Graph Transformation

- Formulate models as graphs and model transformation as graph rewriting
- Each model type has a meta-model that describes how its graph can be transformed to graphs of other model types
- Projects: AToM^3, CHESS, CONCERTO
- (CHESS and CONCERTO are more focused on modeling multi-core computer systems)

Class Inheritance Transformation

- Each model type corresponds to a class in a class hierarchy
- Models are instances of their type's class
- Transformation occurs by using inheritance prinicples to cast models between different types
- Projects: OsMoSys, SIMTHESys

Coalgebraic Transformation

- Each modeling formalism is described as a coalgebra, a mathematical system useful for describing arbitrary transitions on an arbitrary state
- The coalgebras are placed in a lattice to provide a structure for determining how transformations are performed
- Can relate different types of models of the same system, such as a model of system functionality and a model of system power consumption
- Projects: Rosetta

Our Contribution

A model transformation technique based on Abstract Interpretation, a program analysis technique

- Models are viewed as syntactic representations of systems
- Properties can be *abstracted* from a model
- Semantically equivalent models can be *concretized* from a set of properties

Our Contribution

A model transformation technique based on Abstract Interpretation, a program analysis technique

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We can show that this technique is both correct and specific.

Order Relationships

Definition

A *partially ordered set* or *poset* $(L, \sqsubseteq)$ is a set L and an order relation $\sqsubseteq: L \times L \mapsto \{\textbf{true}, \textbf{false}\}$ (read 'less than or equal') that is

1. *Reflexive*: $l \sqsubseteq l, \forall l \in L$
2. *Transitive*: If $l_1 \sqsubseteq l_2$ and $l_2 \sqsubseteq l_3$, then $l_1 \sqsubseteq l_3, \forall l_1, l_2, l_3 \in L$
3. *Anti-symmetric*: If $l_1 \sqsubseteq l_2$ and $l_2 \sqsubseteq l_1$, then $l_1 = l_2, \forall l_1, l_2 \in L$

Meet and Join

Definition (Upper Bound)

A set $Y \subseteq L$ has $l \in L$ as an *upper bound* if $y \sqsubseteq l, \forall y \in Y$. l is a *least upper bound* if, for any upper bound l', $l \sqsubseteq l'$. We denote the least upper bound by $\bigsqcup Y$ (sometimes called the *meet* of Y). $\bigsqcup\{l_1, l_2\}$ can also be written as $l_1 \sqcup l_2$.

Definition (Lower Bound)

l is a lower bound of Y if $l \sqsubseteq y, \forall y \in Y$. The *greatest lower bound* is defined analogously to the least upper bound. It is denoted $\bigsqcap Y$ and sometimes called the *join* of Y.

Order-Preserving Functions

We can relate two posets P_1 and P_2 through functions with certain properties:

Definition

A function $f : P_1 \mapsto P_2$ between posets $(P_1, \sqsubseteq_1)$ and $(P_2, \sqsubseteq_2)$ is *monotone* (or *order-preserving*) if

$$p_1 \sqsubseteq_1 p_2 \implies f(p_1) \sqsubseteq_2 f(p_2), \forall p_1, p_2 \in P_1$$

Lattices

Definition

A *complete lattice* L is a partially ordered set where $\bigsqcup Y$ and $\bigsqcap Y$ is defined for all $Y \subseteq L$.

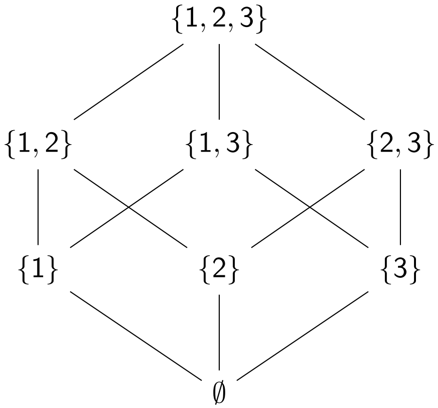
A consequence of 3.5 is that every complete lattice has a least element, $\bot = \bigsqcap L$, and a greatest element, $\top = \bigsqcup L$.

Powerset Lattices

A common complete lattice is the set of all subsets of a set S, called the powerset and denoted $\mathcal{P}(S)$. $(\mathcal{P}(S), \subseteq)$ forms a complete lattice with $\bigsqcup = \bigcup$ and $\bigsqcap = \bigcap$.

$\top = S$ and $\bot = \emptyset$

$\mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$



Galois Connections

Definition

A *Galois connection* (P, α, γ, Q) between two posets $(P, \sqsubseteq_P)$ and $(Q, \sqsubseteq_Q)$, is a pair of monotone functions $\alpha : P \mapsto Q$ and $\gamma : Q \mapsto P$ such that

$$p \sqsubseteq_P (\gamma \circ \alpha)(p), \forall p \in P$$

$$(\alpha \circ \gamma)(q) \sqsubseteq_Q q, \forall q \in Q$$

- P is referred to as the *concrete domain* and Q as the *abstract domain*.
- α is called the *abstraction operator* and γ the *concretization operator*.

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- P is referred to as the *concrete domain* and Q as the *abstract domain*.
- α is called the *abstraction operator* and γ the *concretization operator*.
- We will use Galois Connections to abstract properties from models and concretize models from properties.

Properties of Galois Connections

- If you have one side of a Galois Connection, you can construct the other:
  - α uniquely determines γ by $\gamma(q) = \bigsqcup\{p : \alpha(p) \sqsubseteq_Q q\}$
  - γ uniquely determines α by $\alpha(p) = \bigsqcap\{q : p \sqsubseteq_P \gamma(q)\}$

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  - γ uniquely determines α by $\alpha(p) = \bigsqcap\{q : p \sqsubseteq_P \gamma(q)\}$
- Repeated abstraction and concretization is 'free'; i.e. it does not lose precision: $\alpha \circ \gamma \circ \alpha = \alpha$ and $\gamma \circ \alpha \circ \gamma = \alpha$

Systems, models, and properties

Let's consider a system **S**. We write

- $\mathbf{S} \vdash m$ if the model $m \in M$ describes **S**
- $\mathbf{S} \vdash m_1 \rightsquigarrow m_2$ if m_1 can be transformed to m_2 (while still describing **S**)
- We don't require $\rightsquigarrow$ to be easily calculable or even a function in the mathematical sense

Systems, models, and properties

Let's consider a system $\mathbf{S}$. We write

- $\mathbf{S} \vdash m$ if the model $m \in M$ describes $\mathbf{S}$
- $\mathbf{S} \vdash m_1 \rightsquigarrow m_2$ if m_1 can be transformed to m_2 (while still describing $\mathbf{S}$)
- We don't require $\rightsquigarrow$ to be easily calculable or even a function in the mathematical sense
- $\mathbf{S} \vdash p$ if the set of properties $p \in P$ hold for $\mathbf{S}$
- $\mathbf{S} \vdash p_1 \rhd p_2$ if the properties p_1 can be transformed into properties p_2 while still describing $\mathbf{S}$
- We **do** require $\rhd$ to be deterministic!

Idea: properties are easier to reason about than models.

Relating Models and Properties

- If properties p hold for a model m, we write $m \mathrel{R} p$
- $\rhd$ preserves R: if $\mathbf{S} \vdash m_1 \rightsquigarrow m_2$, $\mathbf{S} \vdash c_1 \rhd p_2$, and $m_1 \mathrel{R} p_1$, then $m_2 \mathrel{R} p_2$

$$\begin{array}{ccccc}
\mathbf{S} & \vdash & m_1 & \rightsquigarrow & m_2 \\
& & \| & & \| \\
& & R & \Longrightarrow & R \\
& & \| & & \| \\
\mathbf{S} & \vdash & p_1 & \rhd & p_2
\end{array}$$

Relating Models and Properties

- Even if we know that there exists an m_2 such that $m_2 \, R \, p_2$, it may be hard to find such a m_2
- We can use lattices to provide structure that helps us approximate m_2
- Let's view $(P, \sqsubseteq)$ is a lattice and R is constrained by two conditions:
  - If $m \, R \, p_1$ and $p_1 \sqsubseteq p_2$, then $m \, R \, p_2$
  - If $m \, R \, p$ for all $p \in P' \sqsubseteq P$, then $m \, R \, \bigsqcap P'$
- In other words, the more specific p is, the better; and there exists a best p to describe any model

Representation Functions

- We define a representation function $\beta : M \mapsto P$ that maps $m \in M$ to the most specific $p \in P$ describing it
- β is also preserved by $\rhd$: if $\mathbf{S} \vdash m_1 \leadsto m_2$, $\mathbf{S} \vdash p_1 \rhd p_2$, and $\beta(m_1) \sqsubseteq p_1$, then $\beta(m_2) \sqsubseteq p_2$

$$\begin{array}{ccccc} \mathbf{S} & \vdash & m_1 & \leadsto & m_2 \\ & & \beta\big\downarrow & \Longrightarrow & \beta\big\downarrow \\ & & \sqsubseteq & & \sqsubseteq \\ \mathbf{S} & \vdash & p_1 & \rhd & p_2 \end{array}$$

Introducing Correctness

- We can show β is correct by constructing a Galois Connection between $\mathcal{P}(M)$ and P
- $\gamma(p) = \{m \in M : \beta(m) \sqsubseteq p\}$
- $\alpha(M') = \bigsqcup\{\beta(m) : m \in M'\}$
- Intuitively, concretizing properties gives you the models those properties hold for
- Abstracting a set of models gives the most specific set of properties that hold for all those models

$$
\begin{tikzcd}
\mathcal{P}(M) \arrow[rr, shift left, "\alpha"] & & P \arrow[ll, shift left, "\gamma"] \\
M \arrow[u, "\{\cdot\}"] \arrow[urr, "\beta"] & &
\end{tikzcd}
$$

Applying Abstract Interpretation to Model Transformation

- Let's suppose we have two model types, M_1 and M_2, and associated Galois connections $(\mathcal{P}(M_1), \alpha_1, \gamma_1, P)$ and $(\mathcal{P}(M_2), \alpha_2, \gamma_2, P)$
- If we have $\mathbf{S} \vdash m_1$ for $m_1 \in M_1$, we can abstract properties from m_1 by taking $p_1 = \beta_1(m_1)$

Applying Abstract Interpretation to Model Transformation

- Let's suppose we have two model types, M_1 and M_2, and associated Galois connections $(\mathcal{P}(M_1), \alpha_1, \gamma_1, P)$ and $(\mathcal{P}(M_2), \alpha_2, \gamma_2, P)$
- If we have $\mathbf{S} \vdash m_1$ for $m_1 \in M_1$, we can abstract properties from m_1 by taking $p_1 = \beta_1(m_1)$
- We can concretize p_1 into models in M_2 by letting $M_2' = \gamma_2(p_1)$
- This gives us a set of M_2 models where the properties p_1 hold

Specificity

- Ideally, concretizing any set of properties will result in a single model
- This is not usually the case, since a destination model may require an assumption about the system not specified in the source model
- The function $\sigma : \mathcal{P}(M) \mapsto M$ allows the user to select one model from a set of transformed models
- E.g. $\sigma(M_2') = m_2$ introduces additional assumptions needed to get a unique model as the result of a transformation

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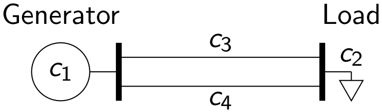
- The function $\sigma : \mathcal{P}(M) \mapsto M$ allows the user to select one model from a set of transformed models
- E.g. $\sigma(M_2') = m_2$ introduces additional assumptions needed to get a unique model as the result of a transformation
- We can capture the properties of these additional assumptions by taking $p_2 = \beta_2(m_2)$
- These properties can be combined with the properties from m_1 by taking $p = p_1 \sqcup p_2$

Topological Model



$$
\begin{aligned}
\text{components} &\subseteq \textbf{component} \\
\text{attributes} &\subseteq \textbf{component} \times \{\texttt{generator}, \texttt{load}, \texttt{line}\} \\
\text{links} &\subseteq \textbf{component} \times \textbf{component} \times \textbf{component} \\
\text{neighbors} &\subseteq \textbf{component} \times \mathcal{P}(\textbf{component})
\end{aligned}
$$

$$
\begin{aligned}
\text{components}(T) &= \{c_1, c_2, c_3, c_4\} \\
\text{attributes}(T) &= \{(c_1, \texttt{generator}), (c_2, \texttt{load}), \\
&\qquad (c_3, \texttt{line}), (c_4, \texttt{line})\} \\
\text{links}(T) &= \{(c_1, c_3, c_2), (c_1, c_4, c_2)\} \\
\text{neighbors}(T) &= \{(c_1, \{c_3, c_4\}), (c_2, \{c_3, c_4\})\}
\end{aligned}
$$

MIS model

	Components	
States	c_3	c_4
S_1	1	1
S_2	1	0
S_3	0	1
S_4	0	0

$$\Pi_0 = [1, 0, 0, 0]$$

$$u = [1, 1, 1, 0]$$

$$P_{c_3} = \begin{bmatrix} p_L & 0 & q_L & 0 \\ 0 & p_L & 0 & q_L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{c_4} = \begin{bmatrix} p_L & q_L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & p_L & q_L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \Pi_0^T * P_{c_3} * P_{c_4} * u = p_L^2 + 2p_L q_L$$

Properties of MIS model

$$
\begin{aligned}
\text{components} &\subseteq \textbf{component} \\
\text{attributes} &\subseteq \textbf{component} \times [0,1] \\
\text{functional\_states} &\subseteq \mathcal{P}(\textbf{component}) \\
\text{initial\_probability} &\subseteq \mathcal{P}(\textbf{component}) \times [0,1]
\end{aligned}
$$

$$
\begin{aligned}
\text{components}(M) &= \{c_3, c_4\} \\
\text{attributes}(M) &= \{(c_3, p_L), (c_4, p_L)\} \\
\text{functional\_states} &= \{\{c_3, c_4\}, \{c_3\}, \{c_4\}\} \\
\text{initial\_probability} &= \{(\{c_3, c_4\}, 1)\}
\end{aligned}
$$

MIS model generated from Topology model properties

$$\Pi_0 = [\mathfrak{s}_1, \cdots, \mathfrak{s}_{15}]$$
$$u = [\mathfrak{u}_1, \cdots, \mathfrak{u}_{15}]$$
$$P_{c_1} = \begin{bmatrix} \mathfrak{p}_{\mathfrak{c}_1} & 0 & \cdots & \mathfrak{q}_{\mathfrak{c}_1} & 0 & \cdots \\ \vdots & \ddots & & \vdots & \ddots & \\ 0 & \cdots & \mathfrak{p}_{\mathfrak{c}_1} & 0 & \cdots & \mathfrak{q}_{\mathfrak{c}_1} \\ 0 & \cdots & 0 & 1 & 0 & \cdots \\ \vdots & & \vdots & \vdots & \ddots & \\ 0 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix}$$
$$R = \Pi_0^T * P_{c_1} * P_{c_2} * P_{c_3} * P_{c_4} * u$$

- If σ_{MIS} sets $\mathfrak{p}_{\mathfrak{c}_1} = \mathfrak{p}_{\mathfrak{c}_2} = 1$, then $\mathfrak{u}_5 = \cdots = \mathfrak{u}_{16} = 0$ and $\mathfrak{s}_5 = \cdots = \mathfrak{s}_{16} = 0$
- Binding $\mathfrak{p}_{\mathfrak{c}_3} = \mathfrak{p}_{\mathfrak{c}_4} = p_L$, $\mathfrak{s}_1 = 1$, $\mathfrak{u}_1 = \mathfrak{u}_2 = \mathfrak{u}_3 = 1$, and $\mathfrak{u}_4 = 0$ suffice to generate an MIS model equivalent to our first MIS model.

Conclusions

- We present a system for transforming models based on Abstract Interpretation
- This approach is both correct and specific
- It can be used to generate cross-domain models and nonfunctional models from functional models or vice-versa
- This power makes correctly modeling sustainable infrastructure easier