

Stat 346 Homework 2

Nathan Jarus

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1 Problem 1, KNN #2.10

2 Problem 2

2.1 Part a

The confidence interval for β_1 is defined as follows:

$$\beta_1 \pm t_{1-\alpha/2, n-1}(s(b_1)) \quad (1)$$

Substituting the provided values, we get:

$$3 \pm 2.09302 \quad (2)$$

2.2 Part b

$$H_0 : \beta_1 = 4 \quad (3)$$

$$H_a : \beta_1 \neq 4 \quad (4)$$

Based on this, we can set up a test statistic:

$$t^* = \frac{b_1 - 4}{s(b_1)} = -1 \quad (5)$$

How does one calculate the p value?

Ultimately, I think we fail to reject H_0 .

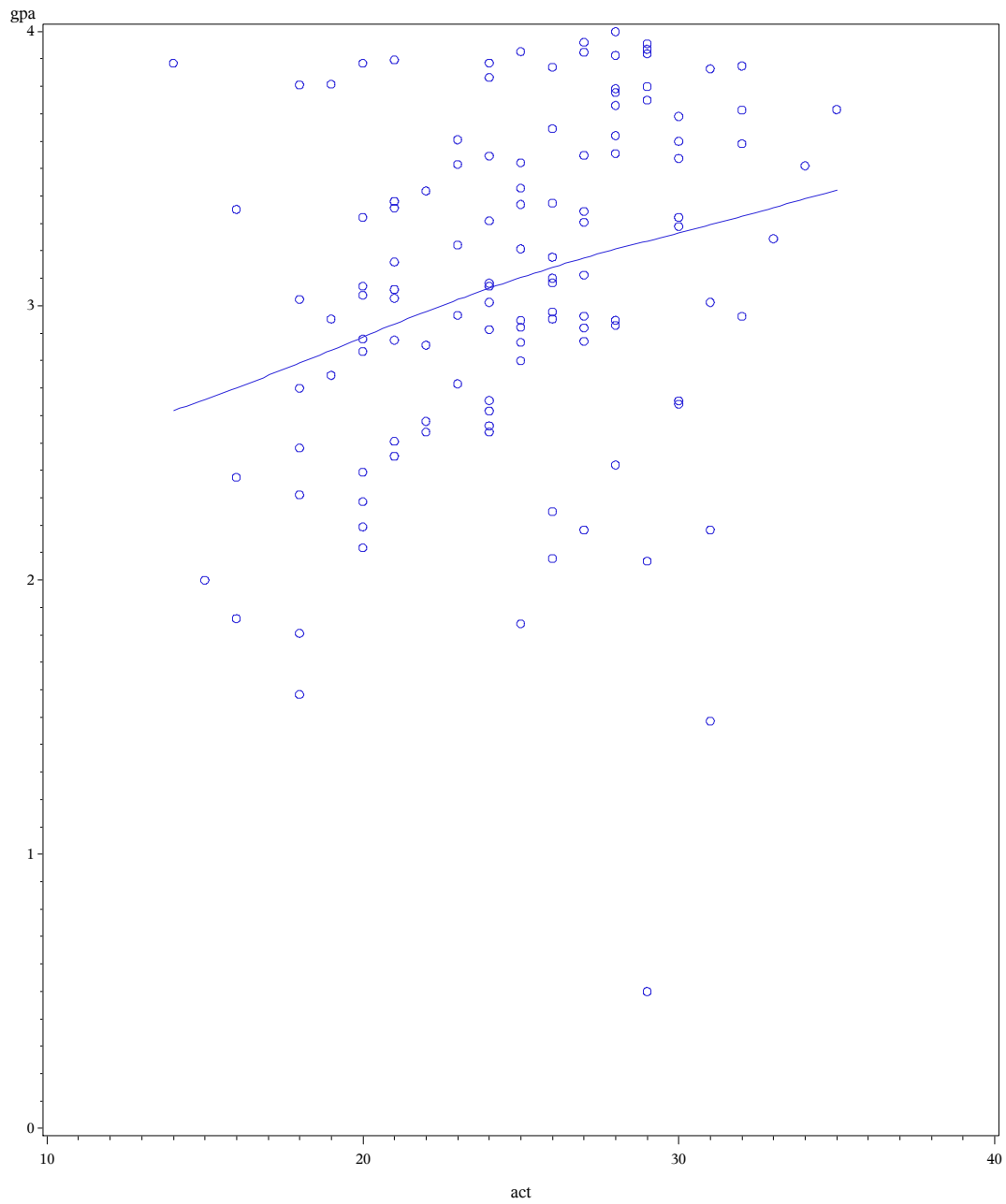
3 Problem 3

3.1 Part a

The scatterplot is shown in Figure 3.1 With sufficient smoothing, the relationship is vaguely linear.

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GPA and ACT
Scatter plot of ACT Score vs. GPA with Smoothing Line N=80



3.2 Part b

GPA and ACT**Scatter plot of ACT Score vs. GPA with Smoothing Line N=80****The REG Procedure****Model: MODEL1****Dependent Variable: gpa**

Number of Observations Read	120
Number of Observations Used	120

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	3.58785	3.58785	9.24	0.0029
Error	118	45.81761	0.38828		
Corrected Total	119	49.40545			

Root MSE	0.62313	R-Square	0.0726
Dependent Mean	3.07405	Adj R-Sq	0.0648
Coeff Var	20.27049		

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	95% Confidence Limits	
Intercept	1	2.11405	0.32089	6.59	<.0001	1.47859	2.74951
act	1	0.03883	0.01277	3.04	0.0029	0.01353	0.06412

The estimated regression equation is

$$\hat{Y} = 2.11405 + 0.03883 * \hat{X} \quad (6)$$

For the slope, the 95% confidence limits are 0.01353 and 0.06412. Since they are both positive, we can conclude with confidence > 95% that there is a positive linear relationship between GPA and ACT scores.

3.3 Part c

A significance test for the slope, b_1 uses the hypothesis

$$H_0 : b_1 = 0 \quad (7)$$

$$H_a : b_1 \neq 0 \quad (8)$$

Our test statistic is 3.04 with 118 DOF and the resulting p-value is 0.0029. Since $p < 0.05$, we reject H_0 , meaning that there is a significant linear relationship between GPA and ACT scores.

3.4 Part d

The lowest X value in the data set is 14, so an ACT score of 0 (the intercept) would not be within the scope of the model.

3.5 Part e

Our mean response for an ACT score of 21 is a GPA of 2.92948. The 95% confidence interval is (2.7826, 3.0763).

3.6 Part f

The prediction is the same as for Part E, 2.92948. However, the confidence interval is (1.6868, 4.1721).

3.7 Part g

Percent of variance accounted for by ACT score:

$$\frac{3.58785}{49.40545} * 100 = 7.26205\% \quad (9)$$

Correlation: 0.26948

3.8 Part h

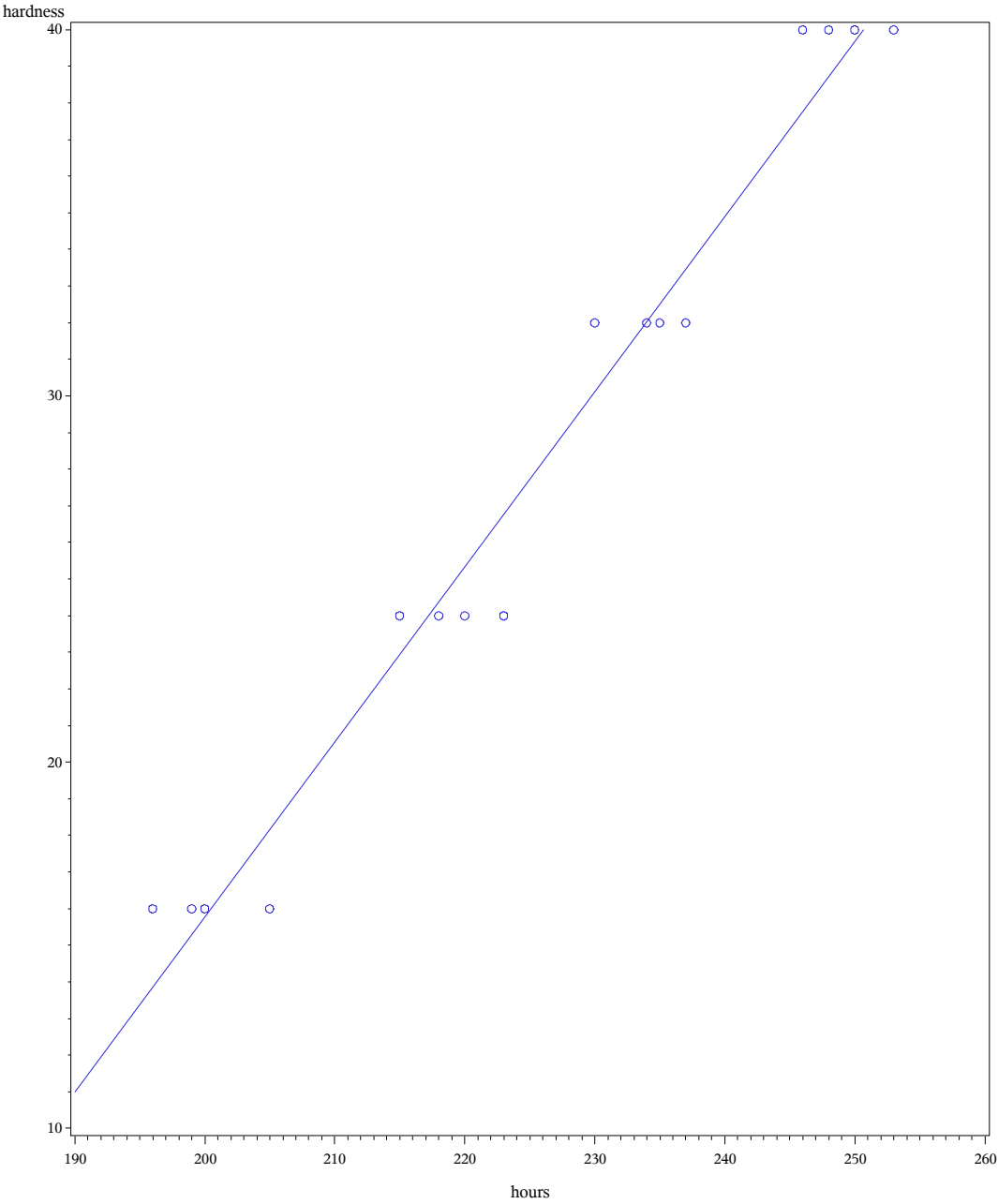
I do not think that ACT scores make a good predictor for GPA. The confidence intervals on predictions are very wide, and ACT score accounts for only 7% of the variance in GPA.

4 Problem 4

4.1 Part a

As shown in Figure 4.1, the data definitely follows a linear pattern, but seems to be sorted into buckets. This may demonstrate a nonlinear relationship between hardness and time, or it may be a limitation of the measurement method for hardness.

Plastic Hardness
Scatter plot of hardness vs time with regression line



4.2 Part b

Plastic Hardness

Scatter plot of hardness vs time with regression line

The REG Procedure

Model: MODEL1

Dependent Variable: hardness

Number of Observations Read	16
Number of Observations Used	16

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	1245.57198	1245.57198	506.51	<.0001
Error	14	34.42802	2.45914		
Corrected Total	15	1280.00000			

Root MSE	1.56817	R-Square	0.9731
Dependent Mean	28.00000	Adj R-Sq	0.9712
Coeff Var	5.60059		

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	95% Confidence Limits	
Intercept	1	-79.89337	4.81005	-16.61	<.0001	-90.20990	-69.57683
hours	1	0.47833	0.02125	22.51	<.0001	0.43275	0.52392

Estimated regression equation:

$$\hat{Y} = -79.89337 + 0.47833 * X \quad (10)$$

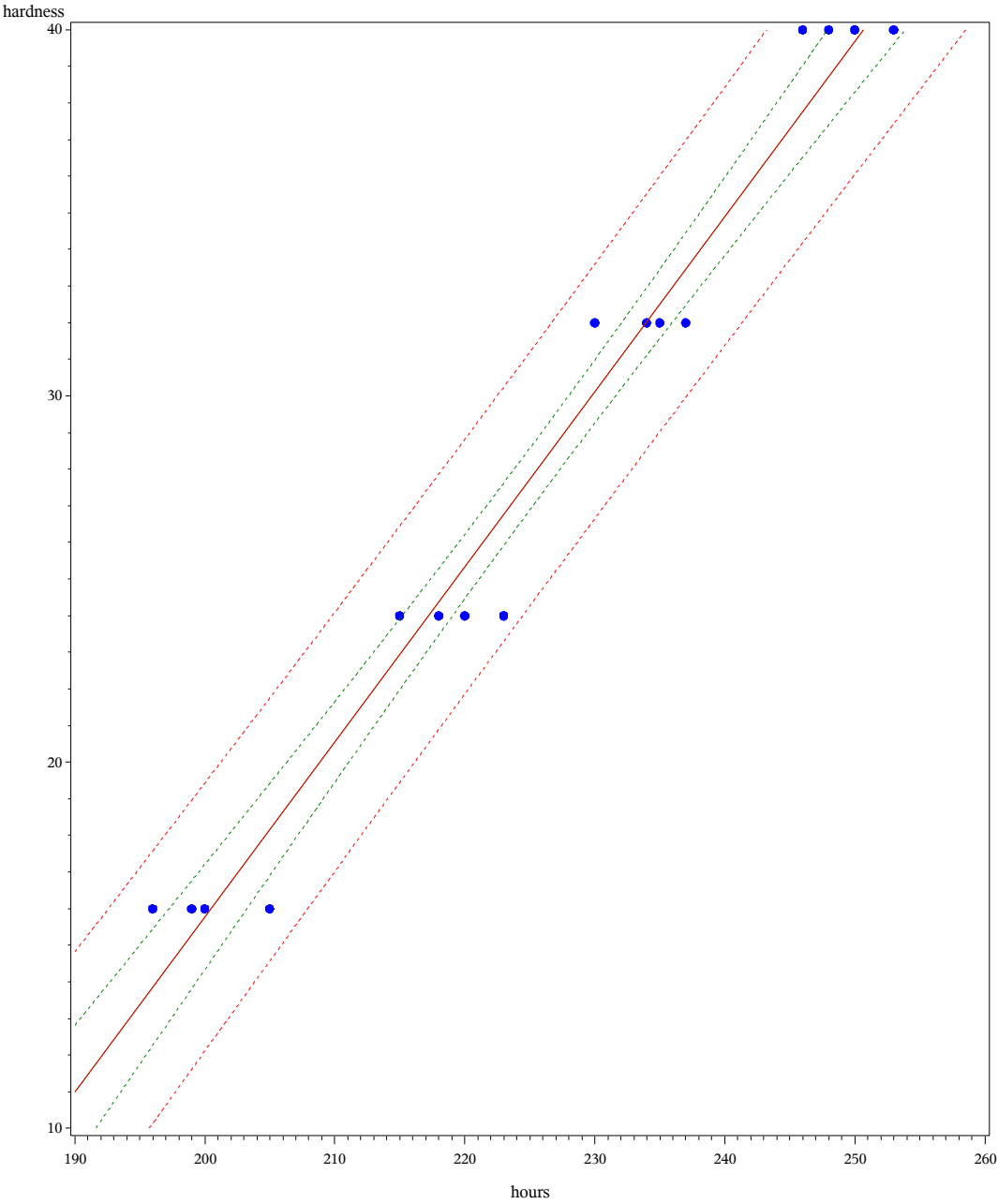
The slope 95% confidence interval is (0.43275, 0.52392).

4.3 Part c

As shown in Figure 4.3, the prediction band is wider to account for the extra variation due to the fact that new observations will probably not fall on the regression line.

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Plastic Hardness
Confidence and Prediction Bands



Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	45.20	45.20	43.88	<0.0001
Error	70	72.1	1.03		
Total	71	117.3			

Tab. 9: Problem 5 ANOVA Table

4.4 Part d

97.31031% of the variance in hardness is due to time. The correlation coefficient between hardness and time is 0.98646.

4.5 Part e

Time seems to be a very good predictor for plastic hardness.

5 Problem 5

$$H_0 : b_1 = 0 \quad (11)$$

$$H_a : b_1 \neq 0 \quad (12)$$

From the values in table 9, we can reject the null hypothesis and conclude that we have a reasonably good fit.