

# Stat 346 Homework 2

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## 1 Problem 1, KNN #2.10

## 2 Problem 2

### 2.1 Part a

The confidence interval for  $\beta_1$  is defined as follows:

$$\beta_1 \pm t_{1-\alpha/2, n-1}(s(b_1)) \quad (1)$$

Substituting the provided values, we get:

$$3 \pm 2.09302 \quad (2)$$

### 2.2 Part b

$$H_0 : \beta_1 = 4 \quad (3)$$

$$H_a : \beta_1 \neq 4 \quad (4)$$

Based on this, we can set up a test statistic:

$$t^* = \frac{b_1 - 4}{s(b_1)} = -1 \quad (5)$$

How does one calculate the p value?

Ultimately, I think we fail to reject  $H_0$ .

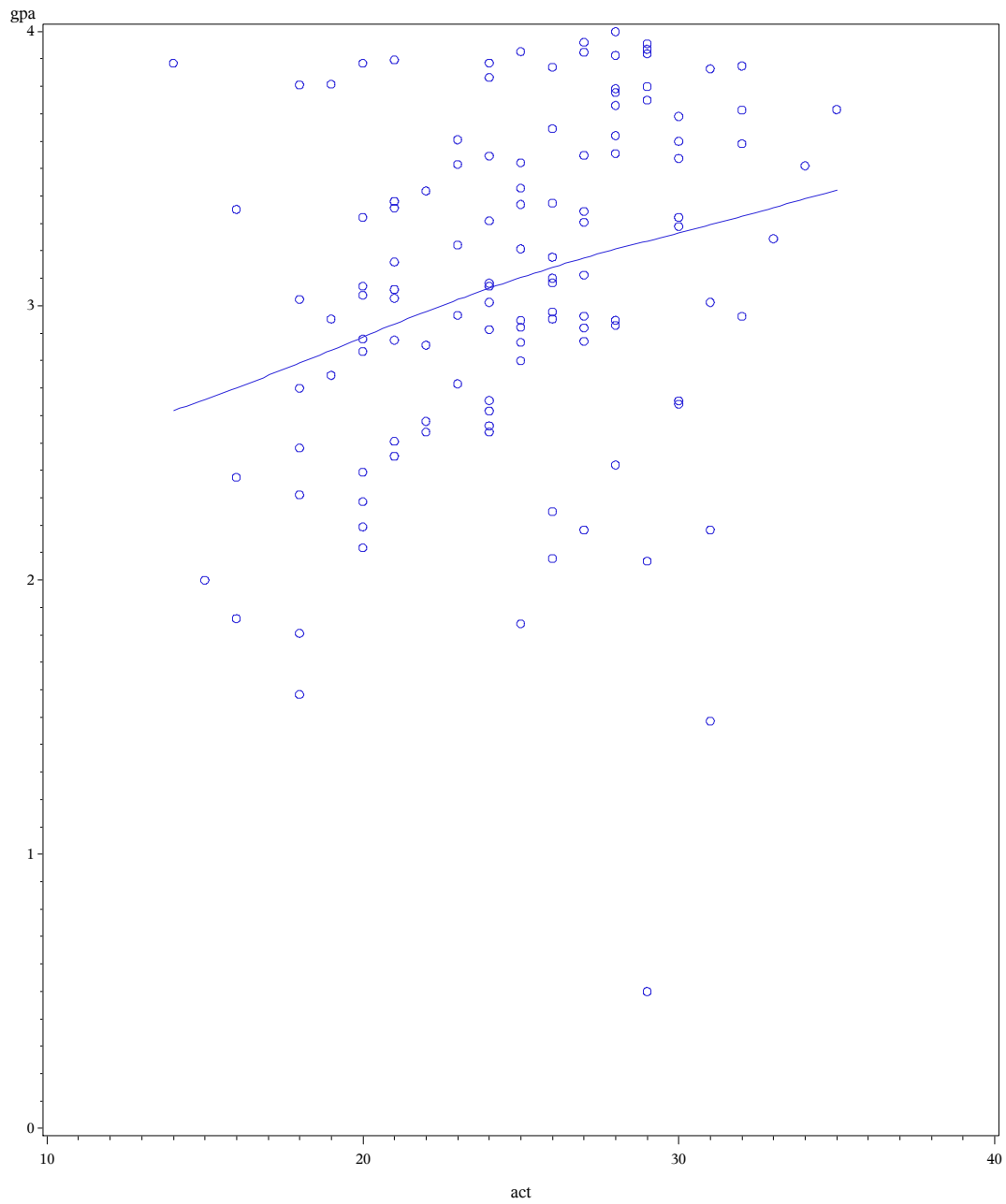
## 3 Problem 3

### 3.1 Part a

The scatterplot is shown in Figure 3.1 With sufficient smoothing, the relationship is vaguely linear.

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**GPA and ACT**  
Scatter plot of ACT Score vs. GPA with Smoothing Line N=80



## 3.2 Part b

**GPA and ACT****Scatter plot of ACT Score vs. GPA with Smoothing Line N=80****The REG Procedure****Model: MODEL1****Dependent Variable: gpa**

<b>Number of Observations Read</b>	120
<b>Number of Observations Used</b>	120

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
<b>Model</b>	1	3.58785	3.58785	9.24	0.0029
<b>Error</b>	118	45.81761	0.38828		
<b>Corrected Total</b>	119	49.40545			

<b>Root MSE</b>	0.62313	<b>R-Square</b>	0.0726
<b>Dependent Mean</b>	3.07405	<b>Adj R-Sq</b>	0.0648
<b>Coeff Var</b>	20.27049		

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t	95% Confidence Limits	
<b>Intercept</b>	1	2.11405	0.32089	6.59	<.0001	1.47859	2.74951
<b>act</b>	1	0.03883	0.01277	3.04	0.0029	0.01353	0.06412

The estimated regression equation is

$$\hat{Y} = 2.11405 + 0.03883 * \hat{X} \quad (6)$$

For the slope, the 95% confidence limits are 0.01353 and 0.06412. Since they are both positive, we can conclude with confidence > 95% that there is a positive linear relationship between GPA and ACT scores.

### 3.3 Part c

A significance test for the slope,  $b_1$  uses the hypothesis

$$H_0 : b_1 = 0 \quad (7)$$

$$H_a : b_1 \neq 0 \quad (8)$$

Our test statistic is 3.04 with 118 DOF and the resulting p-value is 0.0029. Since  $p < 0.05$ , we reject  $H_0$ , meaning that there is a significant linear relationship between GPA and ACT scores.

### 3.4 Part d

The lowest X value in the data set is 14, so an ACT score of 0 (the intercept) would not be within the scope of the model.

### 3.5 Part e

Our mean response for an ACT score of 21 is a GPA of 2.92948. The 95% confidence interval is (2.7826, 3.0763).

### 3.6 Part f

The prediction is the same as for Part E, 2.92948. However, the confidence interval is (1.6868, 4.1721).

### 3.7 Part g

Percent of variance accounted for by ACT score:

$$\frac{3.58785}{49.40545} * 100 = 7.26205\% \quad (9)$$

Correlation: 0.26948

### 3.8 Part h

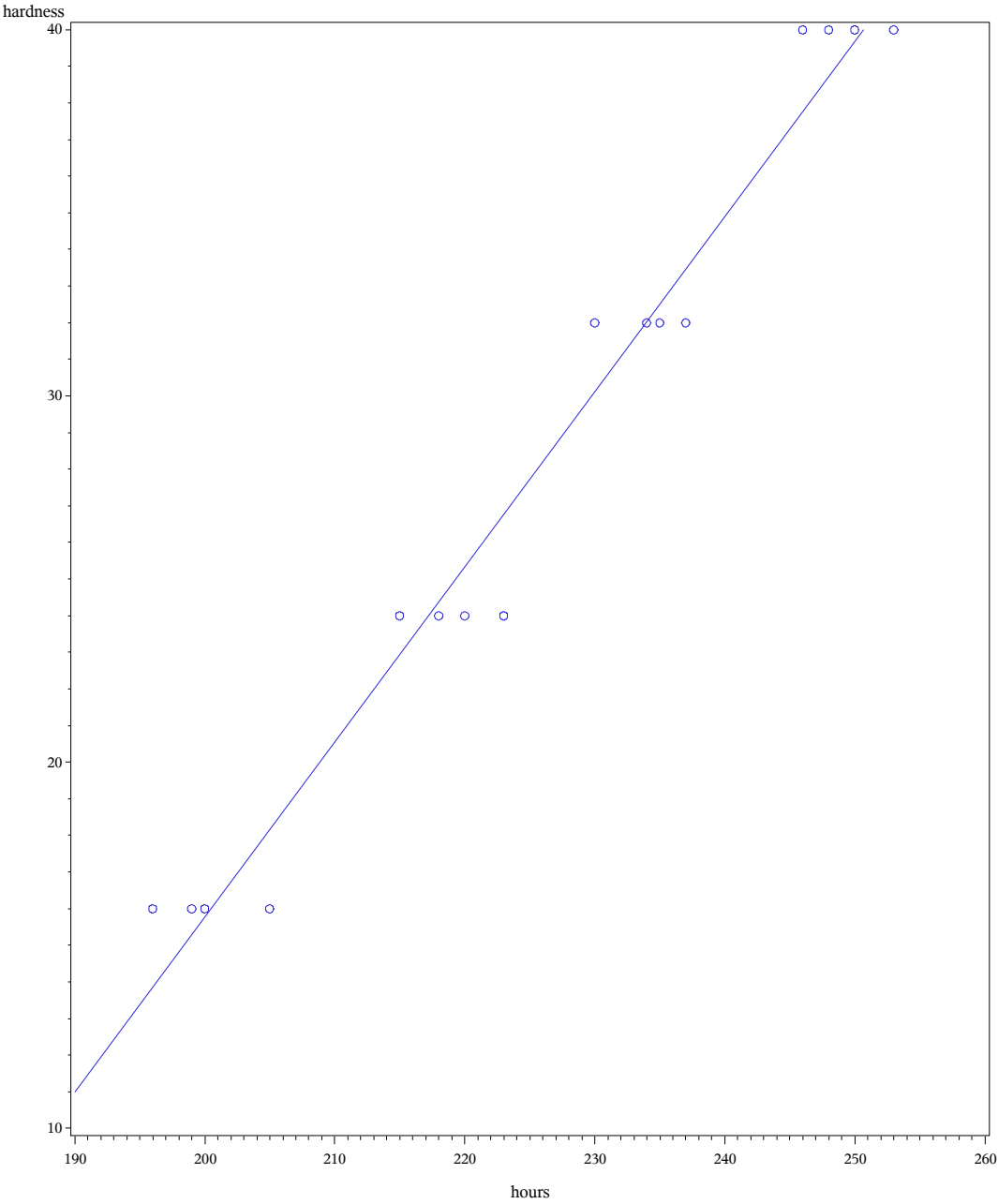
I do not think that ACT scores make a good predictor for GPA. The confidence intervals on predictions are very wide, and ACT score accounts for only 7% of the variance in GPA.

## 4 Problem 4

### 4.1 Part a

As shown in Figure 4.1, the data definitely follows a linear pattern, but seems to be sorted into buckets. This may demonstrate a nonlinear relationship between hardness and time, or it may be a limitation of the measurement method for hardness.

Plastic Hardness  
Scatter plot of hardness vs time with regression line



## 4.2 Part b

### *Plastic Hardness*

#### *Scatter plot of hardness vs time with regression line*

#### *The REG Procedure*

*Model: MODEL1*

*Dependent Variable: hardness*

<b>Number of Observations Read</b>	16
<b>Number of Observations Used</b>	16

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
<b>Model</b>	1	1245.57198	1245.57198	506.51	<.0001
<b>Error</b>	14	34.42802	2.45914		
<b>Corrected Total</b>	15	1280.00000			

<b>Root MSE</b>	1.56817	<b>R-Square</b>	0.9731
<b>Dependent Mean</b>	28.00000	<b>Adj R-Sq</b>	0.9712
<b>Coeff Var</b>	5.60059		

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t	95% Confidence Limits	
<b>Intercept</b>	1	-79.89337	4.81005	-16.61	<.0001	-90.20990	-69.57683
<b>hours</b>	1	0.47833	0.02125	22.51	<.0001	0.43275	0.52392

Estimated regression equation:

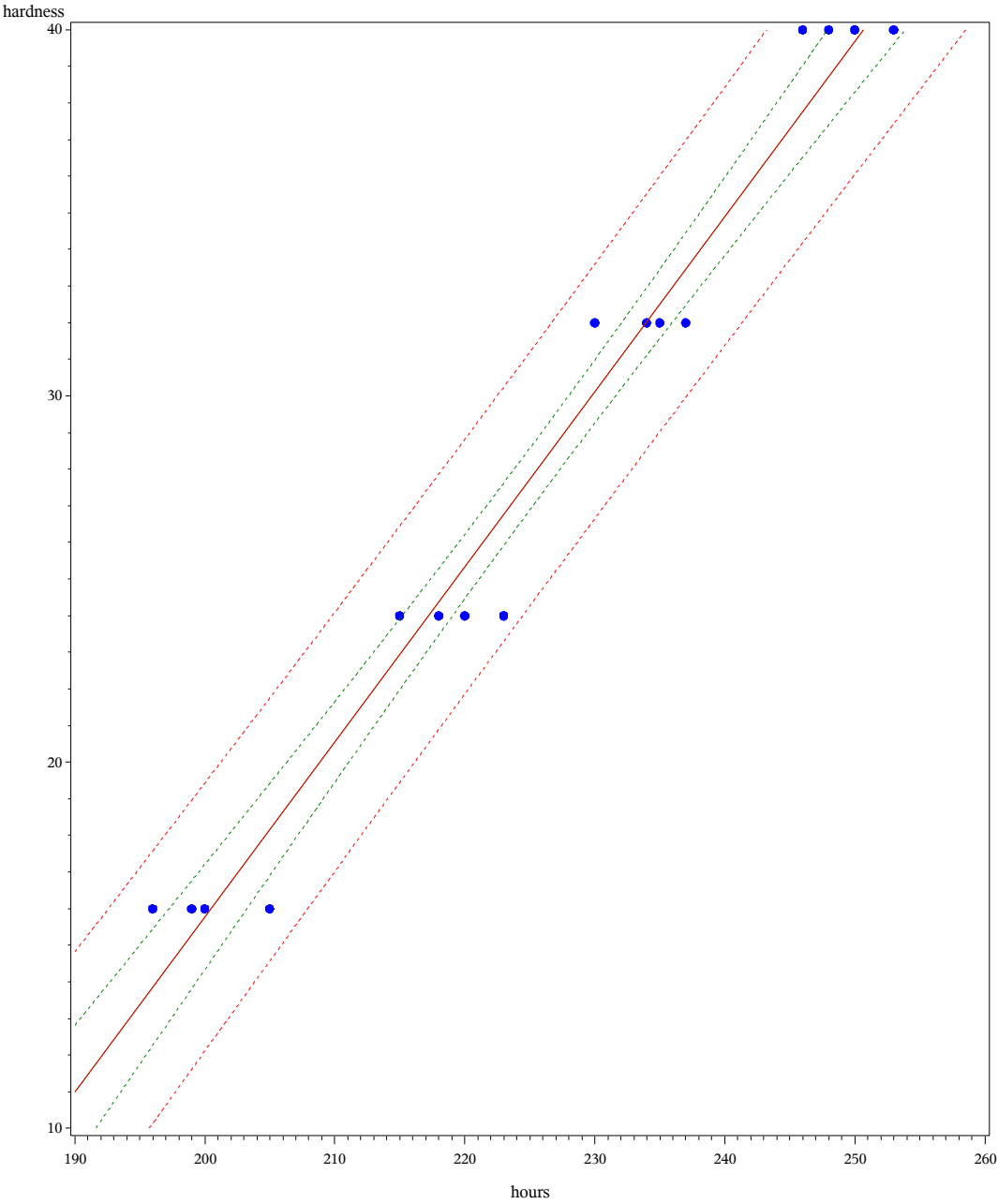
$$\hat{Y} = -79.89337 + 0.47833 * X \quad (10)$$

The slope 95% confidence interval is (0.43275, 0.52392).

## 4.3 Part c

As shown in Figure 4.3, the prediction band is wider to account for the extra variation due to the fact that new observations will probably not fall on the regression line.

Plastic Hardness  
Confidence and Prediction Bands



Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	45.20	45.20	43.88	<0.0001
Error	70	72.1	1.03		
Total	71	117.3			

Tab. 9: Problem 5 ANOVA Table

#### 4.4 Part d

97.31031% of the variance in hardness is due to time. The correlation coefficient between hardness and time is 0.98646.

#### 4.5 Part e

Time seems to be a very good predictor for plastic hardness.

### 5 Problem 5

$$H_0 : b_1 = 0 \quad (11)$$

$$H_a : b_1 \neq 0 \quad (12)$$

From the values in table 9, we can reject the null hypothesis and conclude that we have a reasonably good fit.