Stat 346 Homework 3

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1 Problem 1

1.1 Part a

The residuals are not normally distributed, indicating a nonlinear relationship. Possible fixes include non-linear regression or a transformation on X.

1.2 Part b

These residuals seem to be normally distributed with constant variance. This indicates that the model is a good fit.

1.3 Part c

The errors are not normally distributed. This might be fixed with a transformation on Y.

1.4 Part d

The variance on the residuals is not constant. This might be fixed with a transformation on Y.

2 Problem 2

2.1 Part a

Plots 2.1 and 2.1. The residuals seem normally distributed and support a linear trend.

2.2 Part b

$s\{b_1\} = 2.125 \times 10^{-2}$
$t(1 - 0.05/2, 16 - 2) = t(0.975, 14) = 2.145$
$b_1 = 0.47833 \pm 2.145 * 2.125 \times 10^{-2}$
$(0.43275, 0.52392)$

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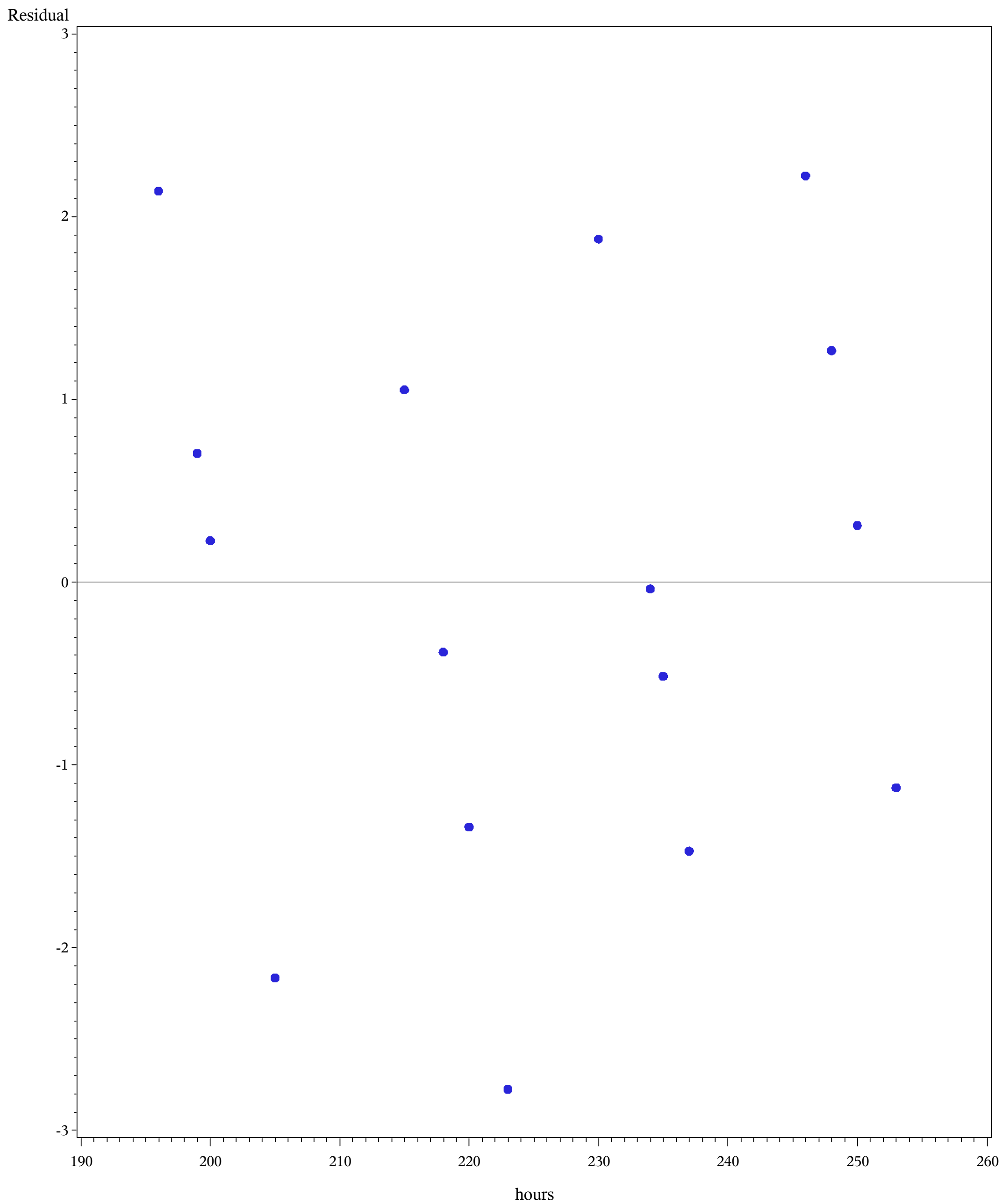
Plastic Hardness
Residuals vs. time



Plastic Hardness
Normal Probability

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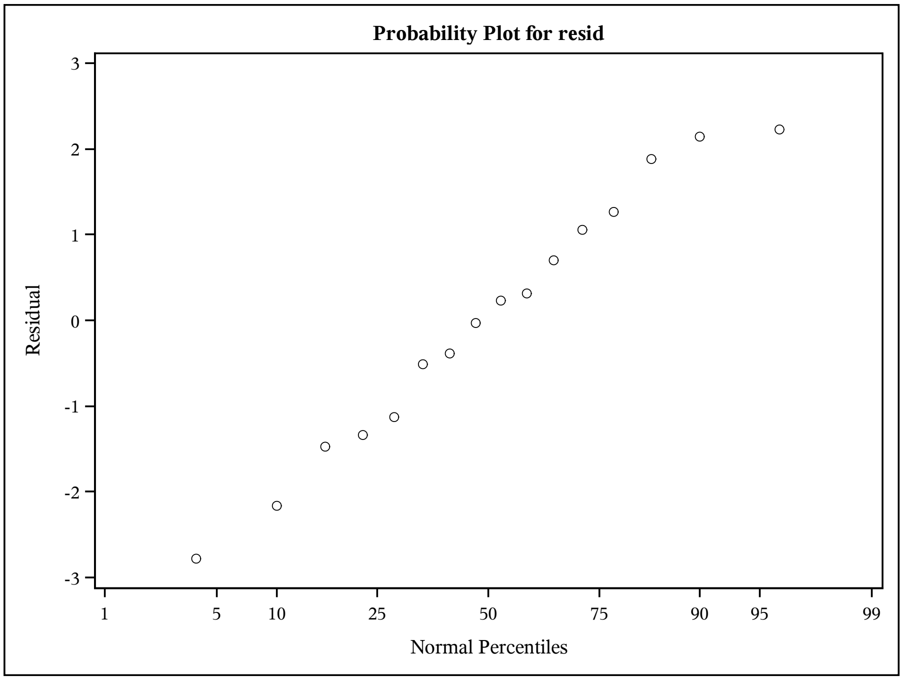

2.3 Part c

$\hat{Y} = 37.7759$
$s\{\hat{Y}\} = 0.5851$
$\hat{Y} \pm t(1 - 0.05/2, 16 - 2)s\{\hat{Y}\} = 2.145 * 0.5851$
$(36.5209, 39.0309)$

2.4 Part d

$R^2 = 1 - \frac{SSE}{SSTO} = 1 - \frac{34.342802}{1280} = 0.9731$

2.5 Part e

1. General Linear Test
2. ANOVA
3. T-test

3 Problem 3

3.1 Part a

	ORIGINAL REGRESSION	REGRESSION WITH OUTLIER
Fitted Regression Equation	$\hat{Y} = 2.11405 + 0.03883X$	$\hat{Y} = 3.04977 + 0.00090502X$
R-Square	0.0726	0.0011
MSE	0.38829	0.41822
$SE\{b_1\}$	0.01277	0.00250
P-Value	0.0029	0.7178

3.2 Part b

Plots 3.2 and 3.2 for the original dataset. Plots 3.2 and 3.2 for the edited dataset. The residuals plot is much more extreme.

3.3 Part c

Yes, you could construct a sequence diagram over the dates, or perhaps school years, at which the GPA and ACT scores were collected.

4 Problem 4

4.1 Part a

As shown in Plot 4.1, the relationship is not linear.

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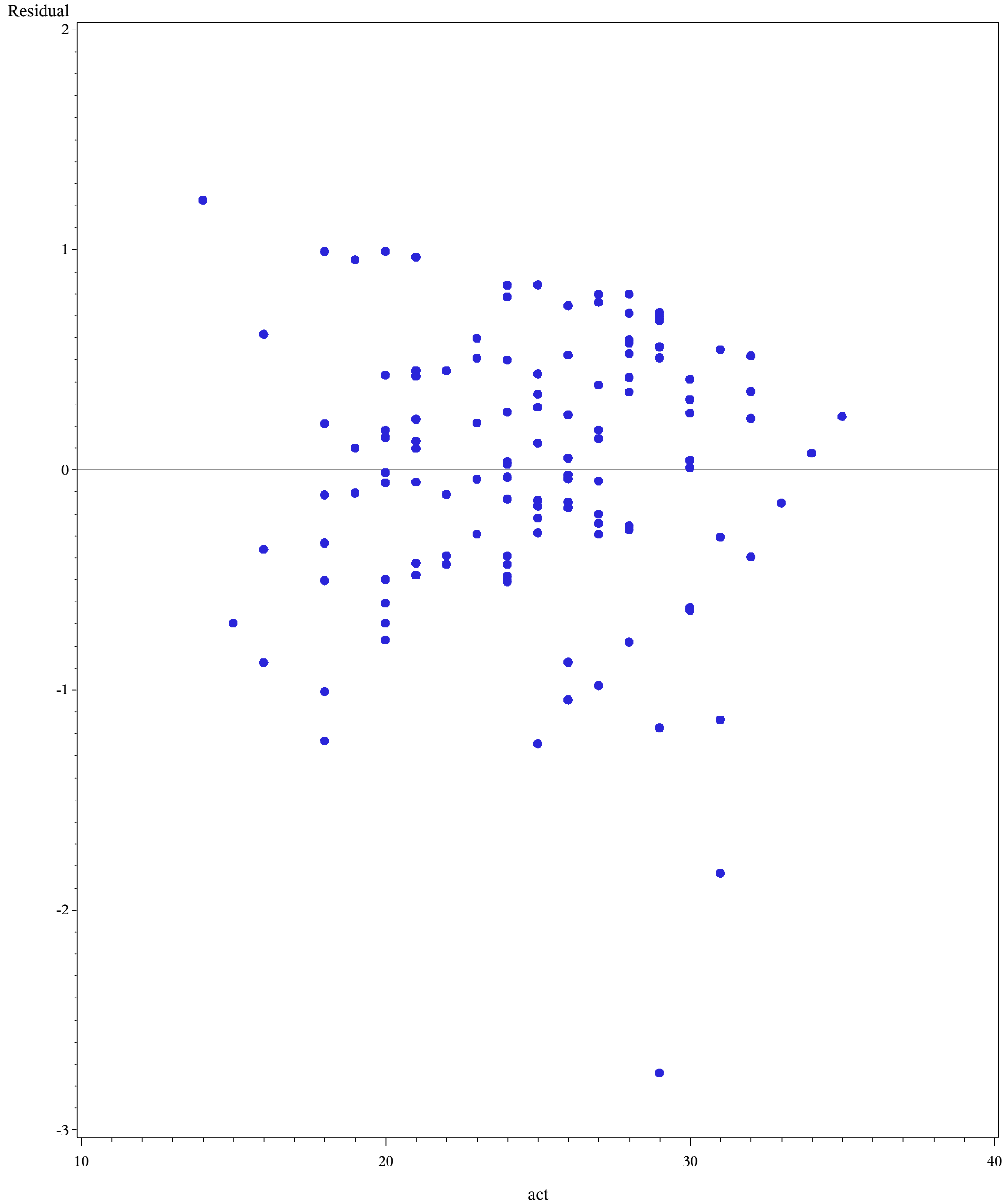
GPA and ACT
Residuals vs. time



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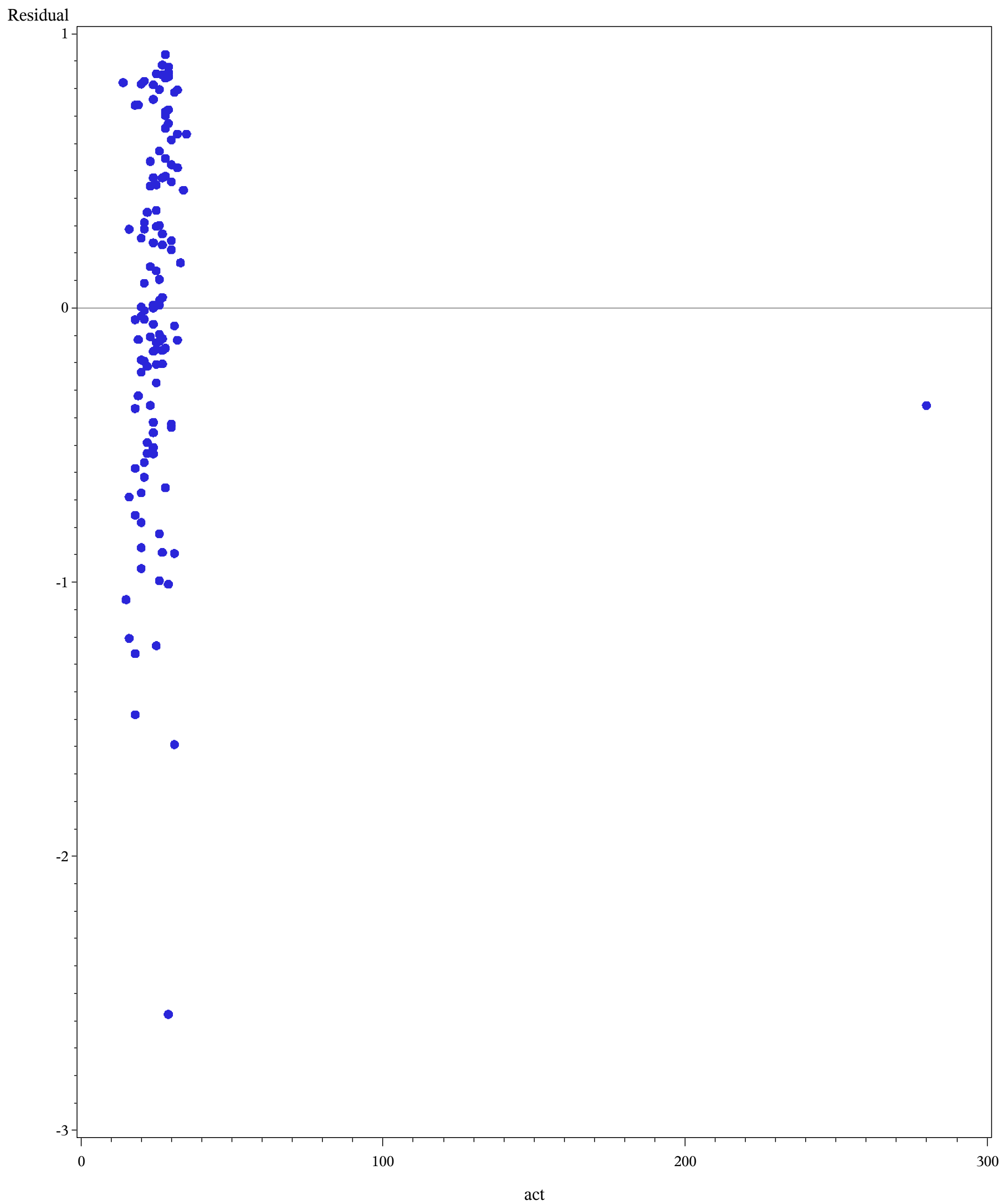
GPA and ACT with typo

Residuals vs. time



GPA and ACT
Normal Probability

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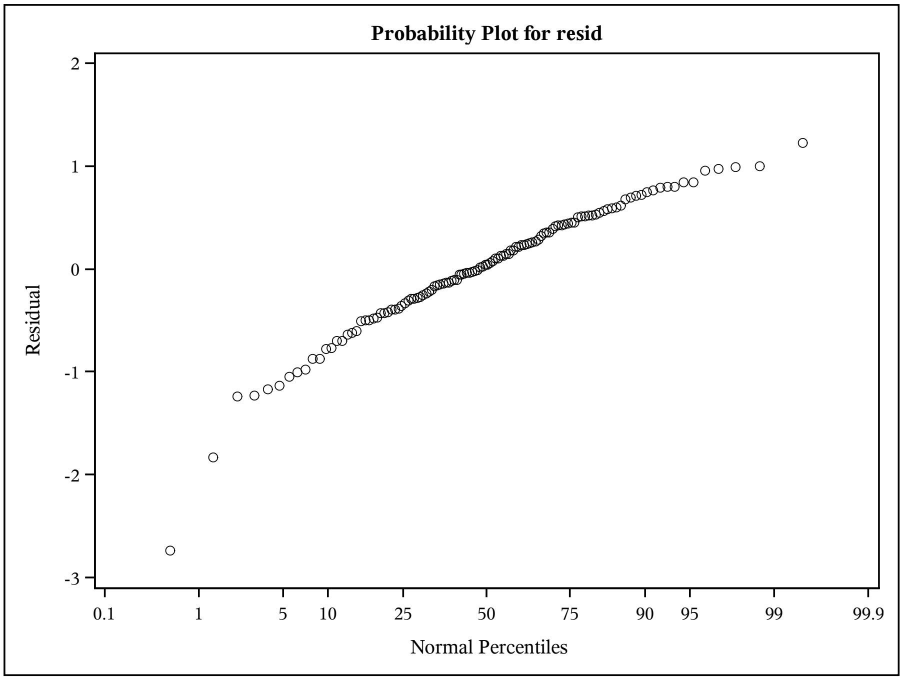
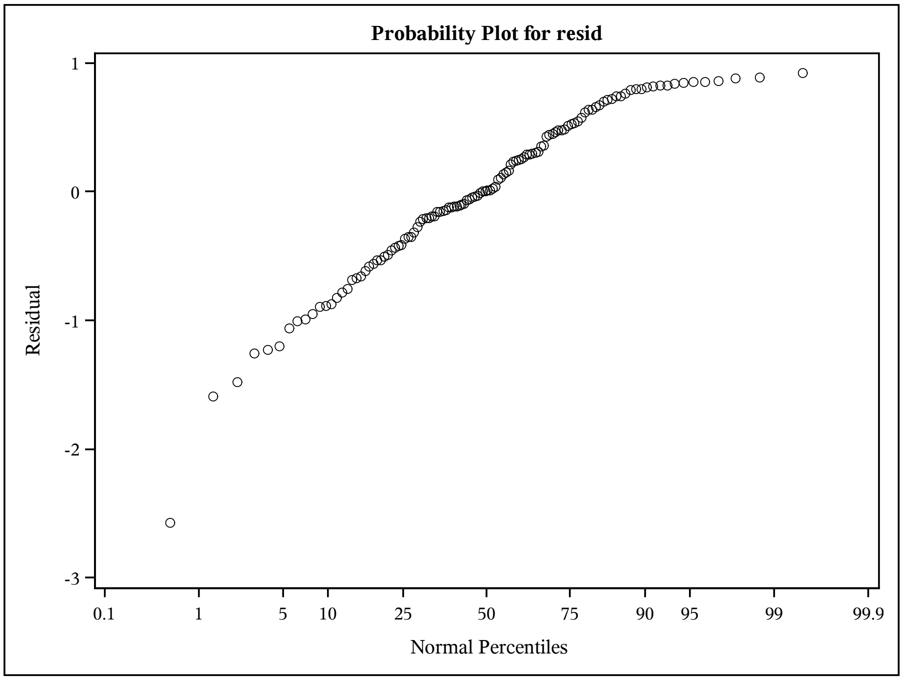


GPA and ACT with typo 01:31 Thursday, February 20, 2014 **1**
Normal Probability



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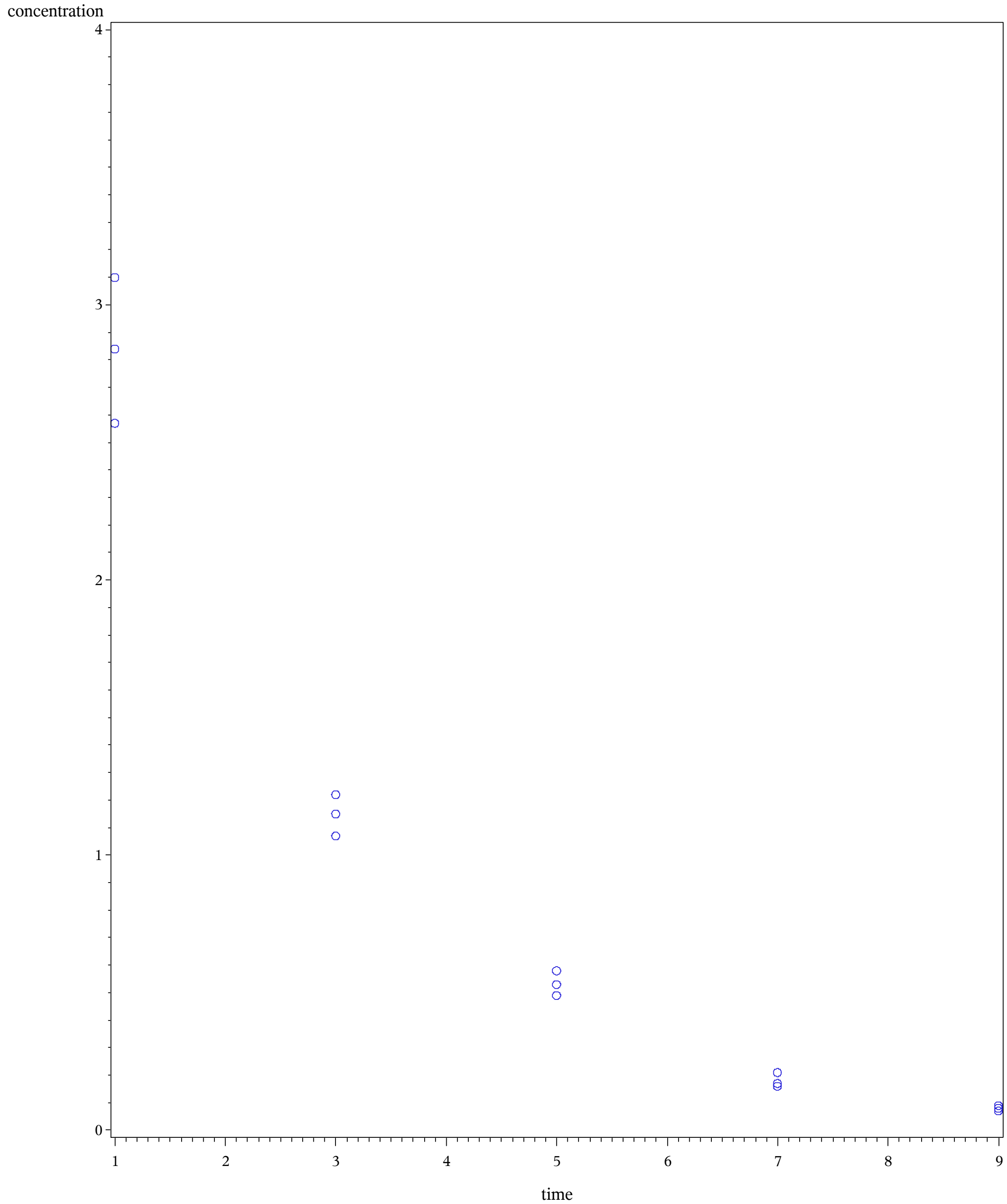
Solution Concentration
Scatterplot Concentration vs. Time



4.2 Part b

In the plots in 4.2 and 4.2, we see a large skew towards smaller times. The boxplot especially shows a very nonlinear relationship.

4.3 Part c

The regression equation is

$$\hat{Y} = 2.57533 + 0.324X \tag{1}$$

The p value is < 0.0001, indicating a significant linear relationship if the SLR assumptions hold. The correlation is 0.8116, which is pretty good.

Solution Concentration

Regression

The REG Procedure

Model: MODEL1

Dependent Variable: concentration

Number of Observations Read	15
Number of Observations Used	15

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	12.59712	12.59712	55.99	<.0001
Error	13	2.92465	0.22497		
Corrected Total	14	15.52177			

Root MSE	0.47431	**R-Square**	0.8116
Dependent Mean	0.95533	**Adj R-Sq**	0.7971
Coeff Var	49.64901		

Parameter Estimates							
Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**	**95% Confidence Limits**	
Intercept	1	2.57533	0.24873	10.35	<.0001	2.03798	3.11269
time	1	−0.32400	0.04330	−7.48	<.0001	−0.41754	−0.23046

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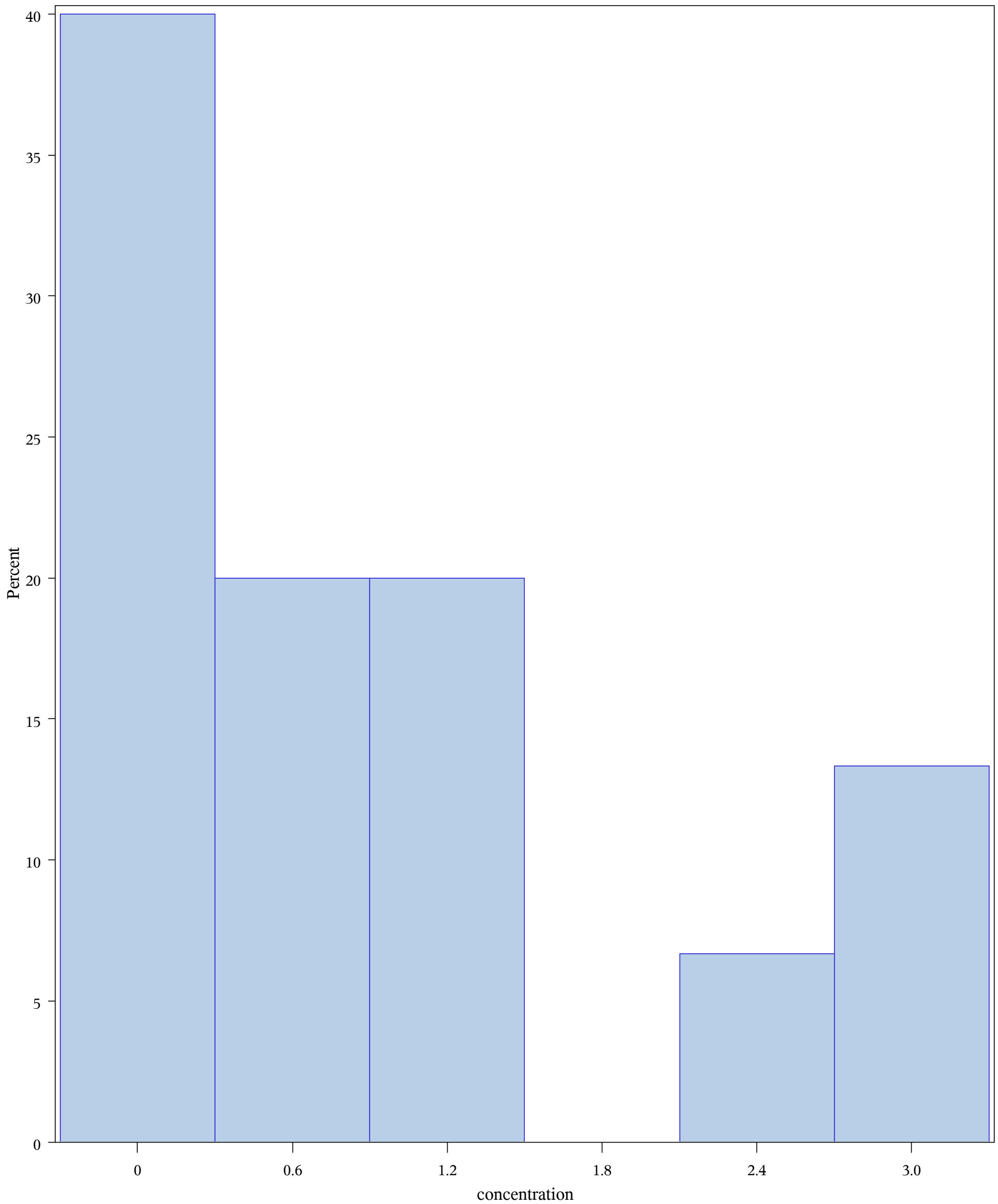
Solution Concentration

Histogram



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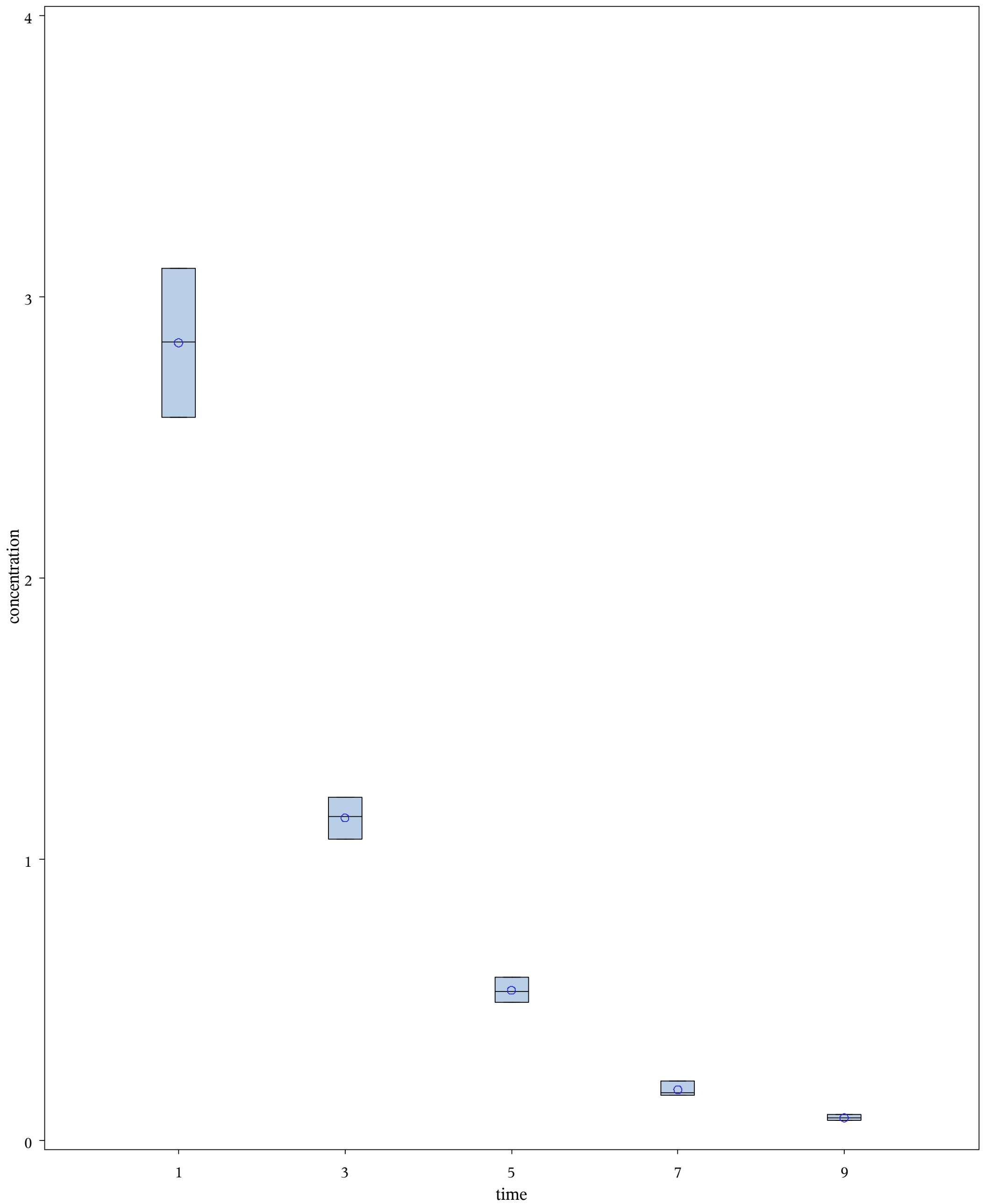
Solution Concentration
Box Plot



4.4 Part d

Both the normal probability plot and the residuals plot shown in plot 4.4 and 4.4 indicate that our errors are not normally distributed. This could be rectified by a transformation on Y.

4.5 Part e

Solution Concentration

Lack-Of-Fit

The RSREG Procedure

Response Surface for Variable concentration	
Response Mean	0.955333
Root MSE	0.474314
R-Square	0.8116
Coefficient of Variation	49.6490

Regression	DF	Type I Sum of Squares	R-Square	F Value	Pr > F
Covariates	1	12.597120	0.8116	55.99	<.0001
Linear	0	0	0.0000	.	.
Quadratic	0	0	0.0000	.	.
Crossproduct	0	0	0.0000	.	.
Total Model	1	12.597120	0.8116	55.99	<.0001

Residual	DF	Sum of Squares	Mean Square	F Value	Pr > F
Lack of Fit	3	2.767253	0.922418	58.60	<.0001
Pure Error	10	0.157400	0.015740		
Total Error	13	2.924653	0.224973		

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|	Parameter Estimate from Coded Data
Intercept	1	2.575333	0.248732	10.35	<.0001	2.575333
time	1	−0.324000	0.043299	−7.48	<.0001	−0.324000

With a lack of fit test, our hypothesis are

$$H_0 : E(Y) = 2.57533 + 0.324X \tag{2}$$

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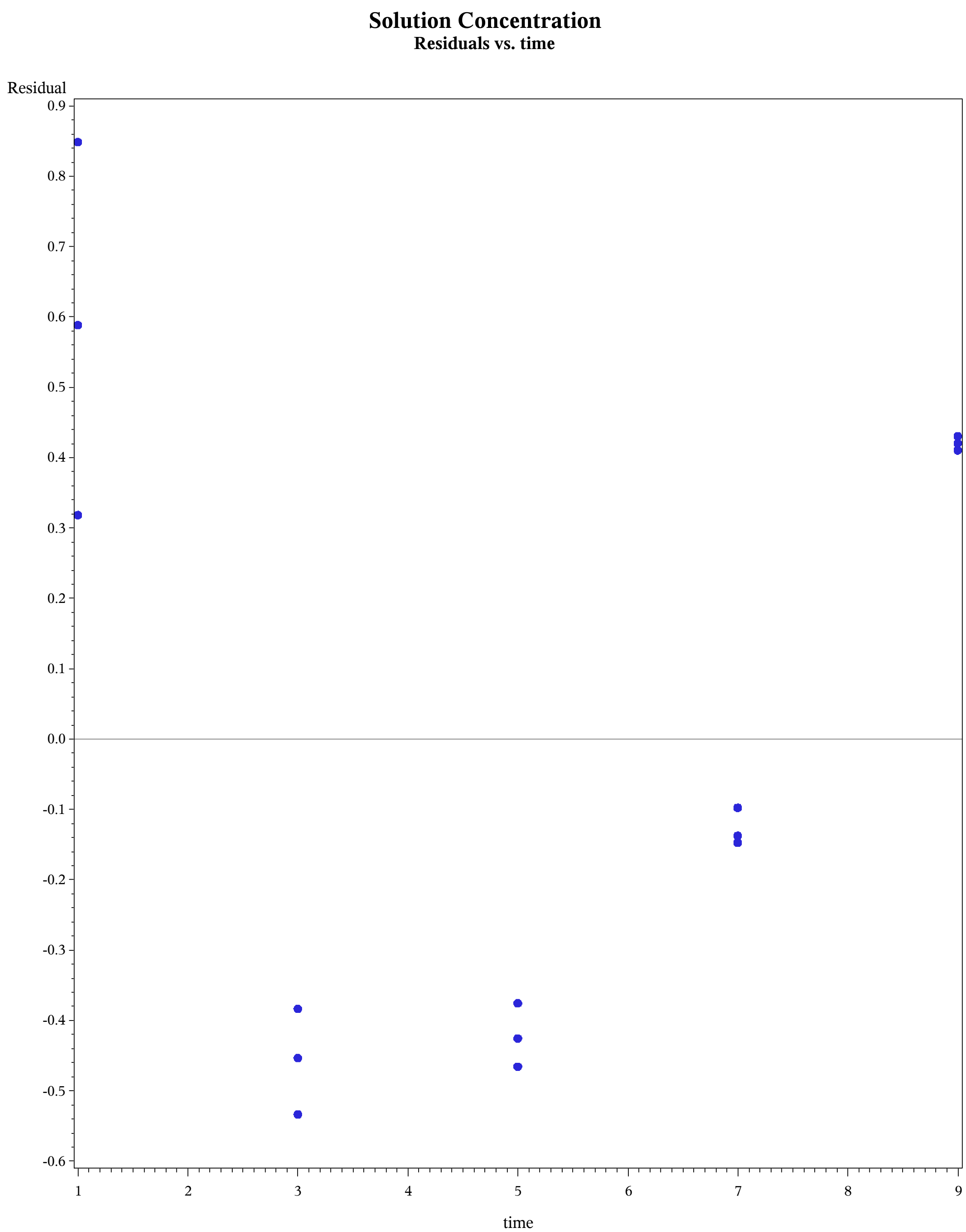

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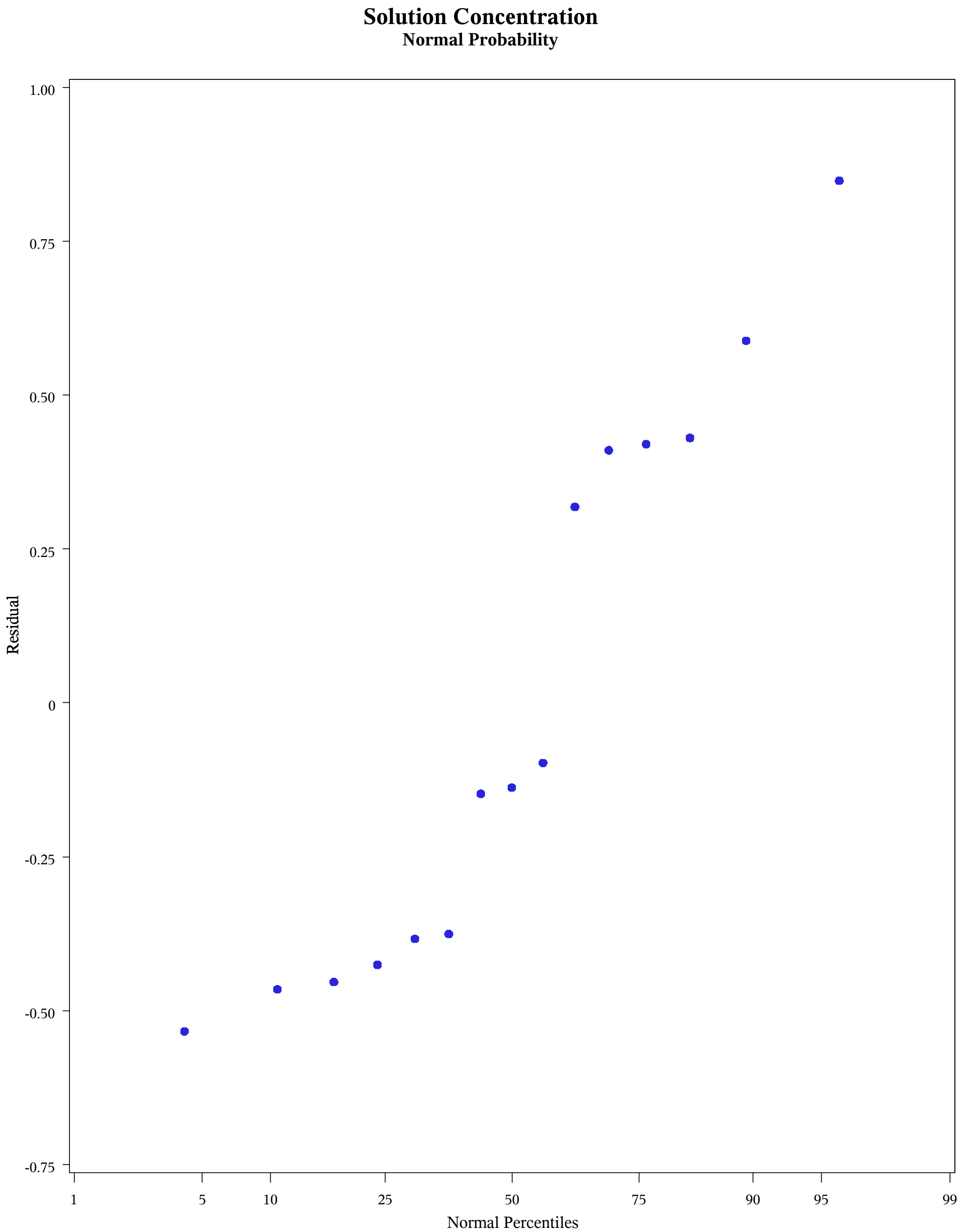

$$H_a : E(Y) \neq 2.57533 + 0.324X \tag{3}$$

To prove a linear relationship, we want to fail to reject H_0. Unfortunately, our p-value for this test are < 0.0001, indicating that we do reject H_0. Thus, we can conclude that the data is not accurately modeled by a linear relationship.

4.6 Part f

Running Box-Cox on the data, we get the following results:

Solution Concentration

Box-Cox Analysis

The TRANSREG Procedure

Box-Cox Transformation Information for time				
Lambda		**R-Square**	**Log Like**	
−3.00		0.88	−37.2367	
−2.75		0.88	−32.9674	
−2.50		0.89	−28.6758	
−2.25		0.90	−24.3345	
−2.00		0.91	−19.9035	
−1.75		0.92	−15.3228	
−1.50		0.94	−10.5023	
−1.25		0.95	−5.3101	
−1.00		0.97	0.4221	
−0.75		0.98	6.7164	
−0.50	+	0.99	12.2568	*
−0.25		0.99	12.8678	<
0.00		0.97	9.1486	
0.25		0.95	5.0555	
0.50		0.91	1.5428	
0.75		0.86	−1.4700	
1.00		0.81	−4.1511	
1.25		0.76	−6.6264	
1.50		0.71	−8.9803	
1.75		0.66	−11.2682	
2.00		0.61	−13.5262	

Box-Cox Transformation Information for time				
Lambda		**R-Square**	**Log Like**	
2.25		0.57	-15.7777	
2.50		0.54	-18.0375	
2.75		0.51	-20.3149	
3.00		0.48	-22.6154	
< - Best Lambda * - 95% Confidence Interval + - Convenient Lambda				

From this, we chose to transform the data with the following equation:

$$Y' = Y^{-0.25} \tag{4}$$

The resulting scatterplot of the transformed data, plot4.6, indicates a much stronger linear relationship.

4.7 Part g

The transformed regression equation is

$$\hat{Y'} = 0.56720 + 0.1398X \tag{5}$$

The p-value for the slope is < 0.0001, also indicating a strong linear relationship assuming the SLR assumptions hold. The correlation value is 0.9725, much better than the non-transformed correlation.

4.8 Part h

The residual and normal probability plots in 4.8 and 4.8 indicate a much more normal distribution of error terms. The residual plot is not quite perfectly independently distributed, but it is much less erratic than the non-transformed plot.

4.9 Part i

Solution Concentration ** -0.25

Lack-Of-Fit Test

The RSREG Procedure

Response Surface for Variable transconcentration	
Response Mean	1.266211
Root MSE	0.071415
R-Square	0.9725
Coefficient of Variation	5.6401

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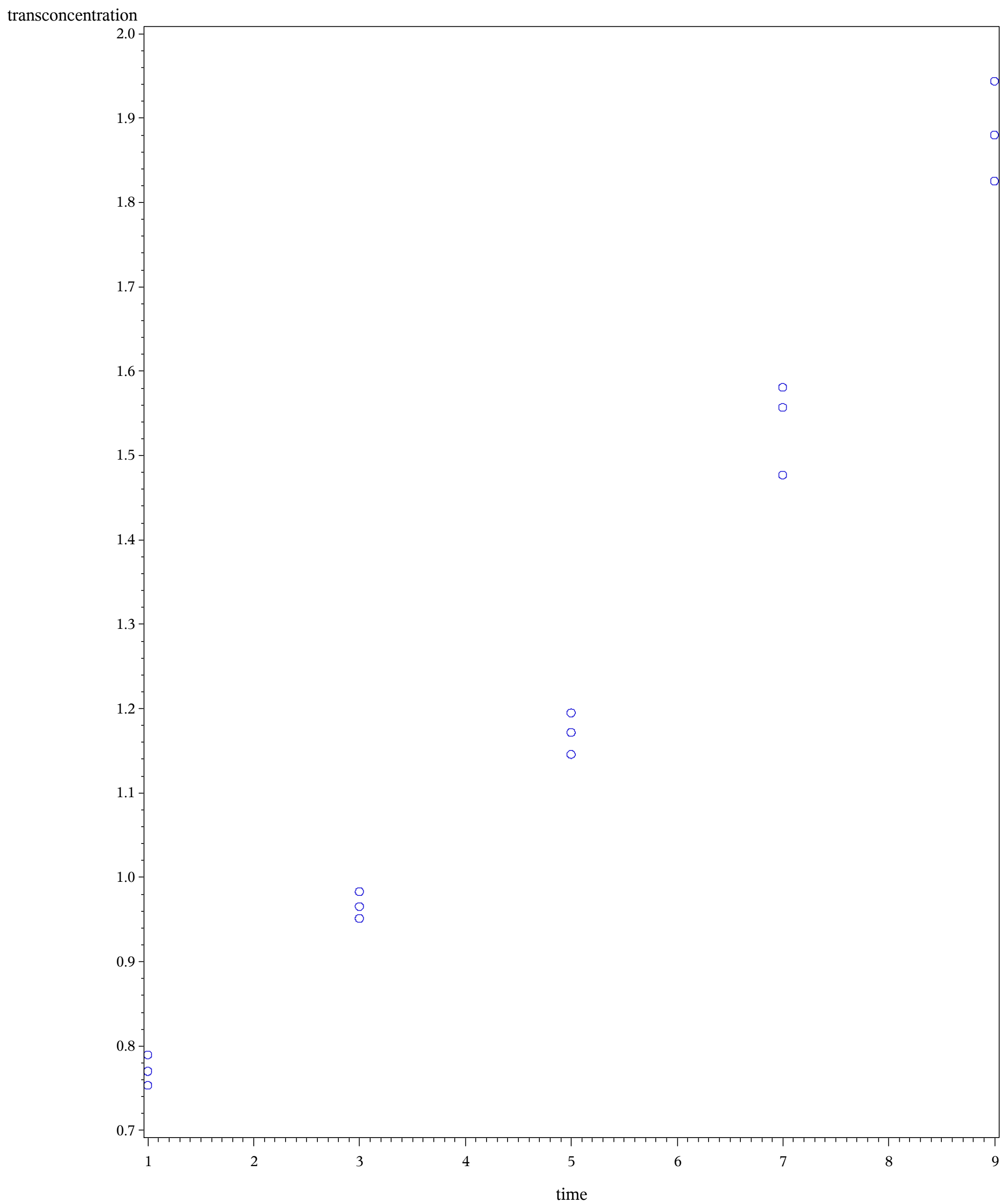
Solution Concentration ** -0.25
Scatterplot Concentration vs. Time



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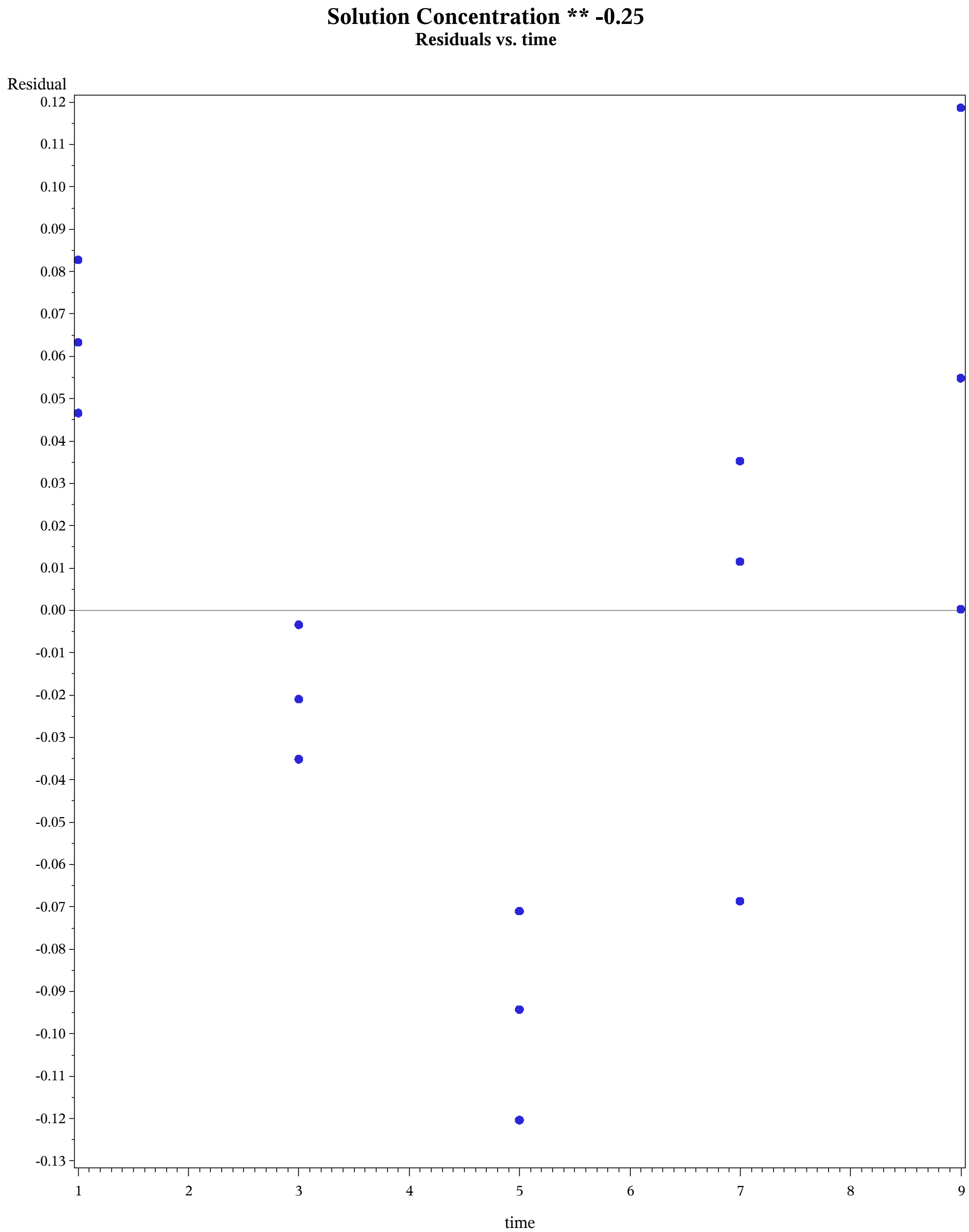

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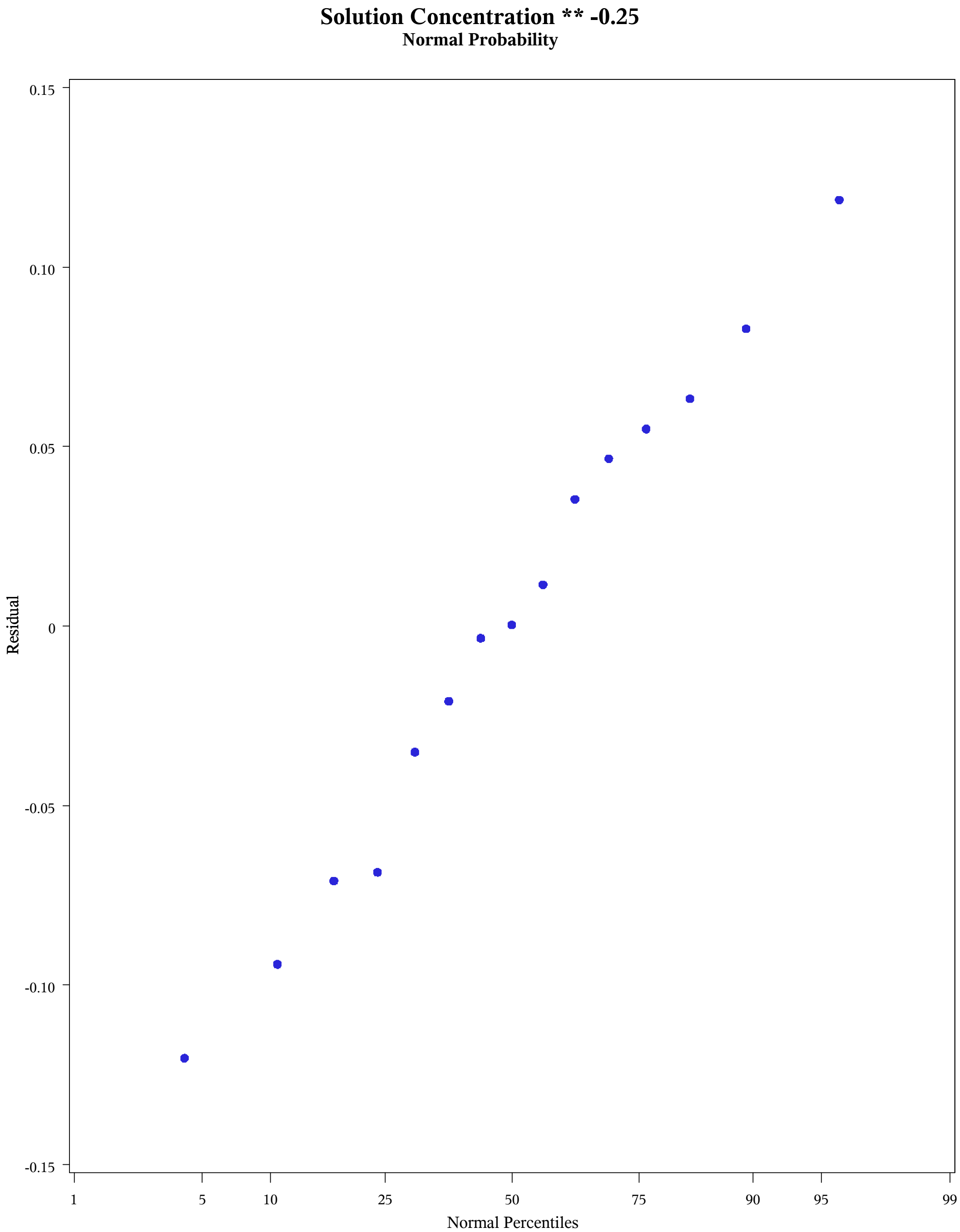

Regression	DF	Type I Sum of Squares	R-Square	F Value	Pr > F
Covariates	1	2.345379	0.9725	459.87	<.0001
Linear	0	0	0.0000	.	.
Quadratic	0	0	0.0000	.	.
Crossproduct	0	0	0.0000	.	.
Total Model	1	2.345379	0.9725	459.87	<.0001

Residual	DF	Sum of Squares	Mean Square	F Value	Pr > F
Lack of Fit	3	0.050971	0.016990	11.08	0.0016
Pure Error	10	0.015330	0.001533		
Total Error	13	0.066301	0.005100		

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|	Parameter Estimate from Coded Data
Intercept	1	0.567197	0.037450	15.15	<.0001	0.567197
time	1	0.139803	0.006519	21.44	<.0001	0.139803

To perform a lack-of-fit test on the transformed data, we start with the following hypotheses:

$$H_0 : E(Y) = 0.56720 + 0.1398X \tag{6}$$

$$H_a : E(Y) \neq 0.56720 + 0.1398X \tag{7}$$

The resulting p-value is 0.0016, indicating that we fail to reject H_0. Thus, we can conclude that our transformed data can be modeled accurately by linear regression.

5 Problem 5, KNN #3.23

Full Model:

$$Y_{ij} = \mu_j + \epsilon_{ij} \tag{8}$$

Degrees of freedom: $n - c$

Reduced Model:

$$Y_{ij} = \beta_1 X_j + \epsilon_{ij} \tag{9}$$

Degrees of freedom: $n - 2$