Stat 346 Homework 2

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1 Problem 1, KNN #2.10

2 Problem 2

2.1 Part a

The confidence interval for β_1 is defined as follows:

$$\beta_1 \pm t_{1-\alpha\backslash 2,n-1}(s(b_1)) \tag{1}$$

Substituting the provided values, we get:

$$3 \pm 2.09302 \tag{2}$$

2.2 Part b

$$H_0 : \beta_1 = 4 \tag{3}$$

$$H_a : \beta_1 \neq 4 \tag{4}$$

Based on this, we can set up a test statistic:

$$t^* = \frac{b_1 - 4}{s(b_1)} = -1 \tag{5}$$

How does one calculate the p value?
Ultimately, I think we fail to reject H_0.

3 Problem 3

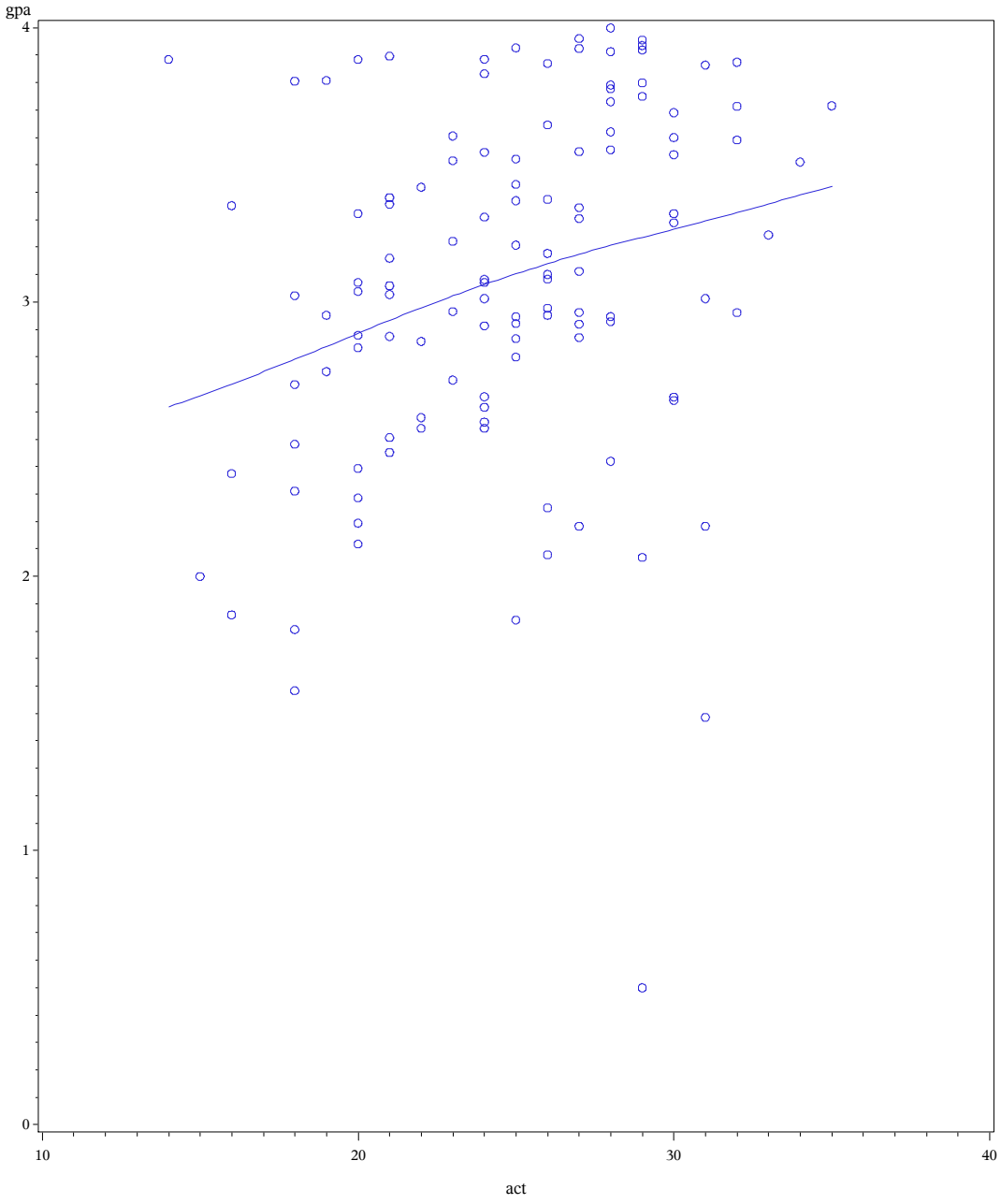
3.1 Part a

The scatterplot is shown in Figure 3.1 With sufficient smoothing, the relationship is vaguely linear.

16:58 Wednesday, February 12, 2014 **1**

GPA and ACT

Scatter plot of ACT Score vs. GPA with Smoothing Line N=80



3.2 Part b

GPA and ACT

Scatter plot of ACT Score vs. GPA with Smoothing Line N=80

The REG Procedure

Model: MODEL1

Dependent Variable: gpa

Number of Observations Read	120
Number of Observations Used	120

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	3.58785	3.58785	9.24	0.0029
Error	118	45.81761	0.38828		
Corrected Total	119	49.40545			

Root MSE	0.62313	**R-Square**	0.0726
Dependent Mean	3.07405	**Adj R-Sq**	0.0648
Coeff Var	20.27049		

Parameter Estimates							
Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**	**95% Confidence Limits**	
Intercept	1	2.11405	0.32089	6.59	<.0001	1.47859	2.74951
act	1	0.03883	0.01277	3.04	0.0029	0.01353	0.06412

The estimated regression equation is

$$\hat{Y} = 2.11405 + 0.03883 * \hat{X} \tag{6}$$

For the slope, the 95% confidence limits are 0.01353 and 0.06412. Since they are both positive, we can conclude with confidence > 95% that there is a positive linear relationship between GPA and ACT scores.

3.3 Part c

A significance test for the slope, b_1 uses the hypothesis

$$H_0 : b_1 = 0 \tag{7}$$

$$H_a : b_1 \neq 0 \tag{8}$$

Our test statistic is 3.04 with 118 DOF and the resulting p-value is 0.0029. Since p < 0.05, we reject H_0, meaning that there is a significant linear relationship between GPA and ACT scores.

3.4 Part d

The lowest X value in the data set is 14, so an ACT score of 0 (the intercept) would not be within the scope of the model.

3.5 Part e

Our mean response for an ACT score of 21 is a GPA of 2.92948. The 95% confidence interval is $(2.7826, 3.0763)$.

3.6 Part f

The prediction is the same as for Part E, 2.92948. However, the confidence interval is $(1.6868, 4.1721)$.

3.7 Part g

Percent of variance accounted for by ACT score:

$$\frac{3.58785}{49.40545} * 100 = 7.26205\% \tag{9}$$

Correlation: 0.26948

3.8 Part h

I do not think that ACT scores make a good predictor for GPA. The confidence intervals on predictions are very wide, and ACT score accounts for only 7% of the variance in GPA.

4 Problem 4

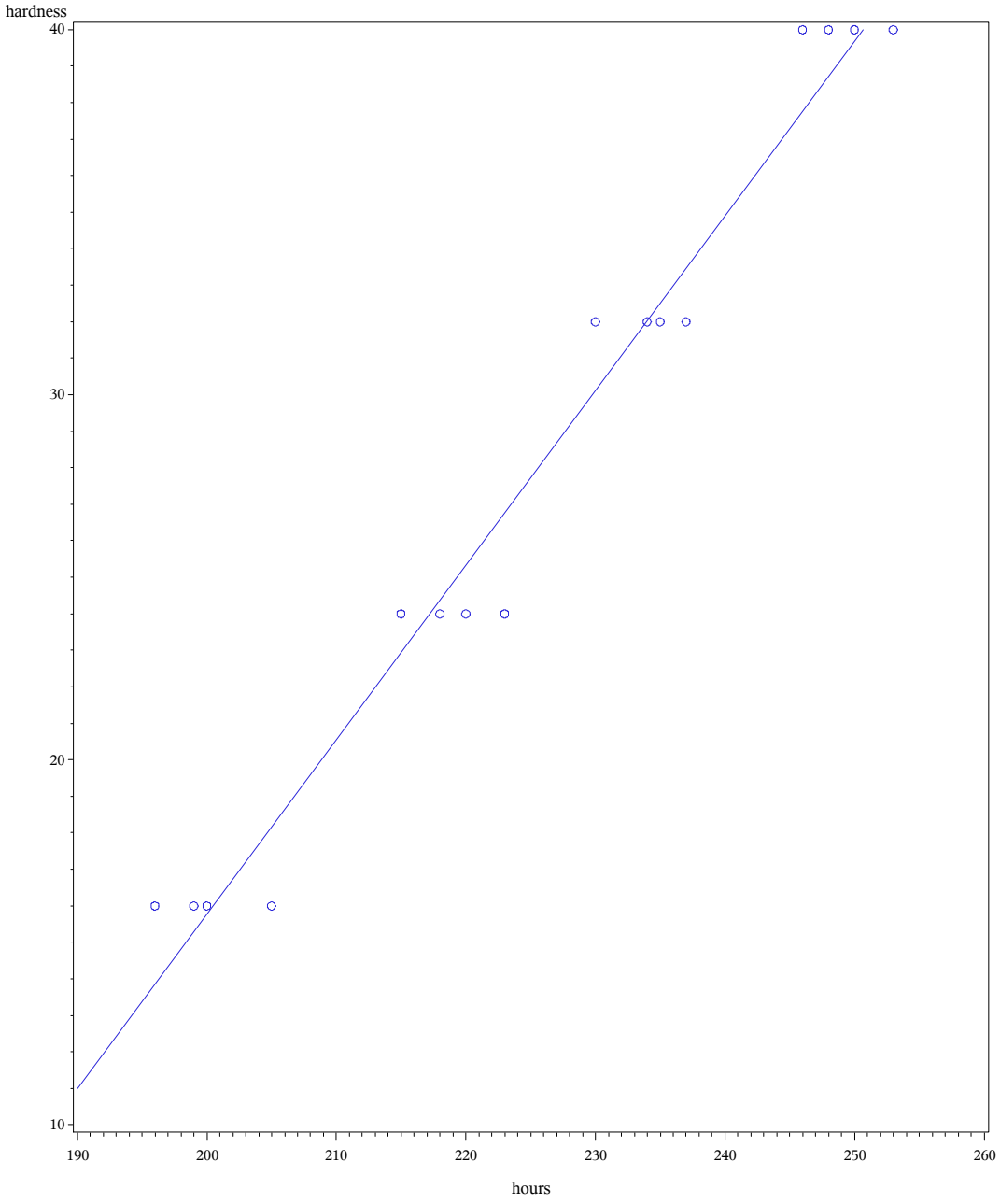
4.1 Part a

As shown in Figure 4.1, the data definitely follows a linear pattern, but seems to be sorted into buckets. This may demonstrate a nonlinear relationship between hardness and time, or it may be a limitation of the measurement method for hardness.

12:44 Thursday, February 13, 2014 **1**

Plastic Hardness

Scatter plot of hardness vs time with regression line



4.2 Part b

Plastic Hardness

Scatter plot of hardness vs time with regression line

The REG Procedure

Model: MODEL1

Dependent Variable: hardness

Number of Observations Read	16
Number of Observations Used	16

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	1245.57198	1245.57198	506.51	<.0001
Error	14	34.42802	2.45914		
Corrected Total	15	1280.00000			

Root MSE	1.56817	**R-Square**	0.9731
Dependent Mean	28.00000	**Adj R-Sq**	0.9712
Coeff Var	5.60059		

Parameter Estimates							
Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**	**95% Confidence Limits**	
Intercept	1	−79.89337	4.81005	−16.61	<.0001	−90.20990	−69.57683
hours	1	0.47833	0.02125	22.51	<.0001	0.43275	0.52392

Estimated regression equation:

$$\hat{Y} = -79.89337 + 0.47833 * \hat{X} \tag{10}$$

The slope 95% confidence interval is $(0.43275, 0.52392)$.

4.3 Part c

As shown in Figure 4.3, the prediction band is wider to account for the extra variation due to the fact that new observations will probably not fall on the regression line.

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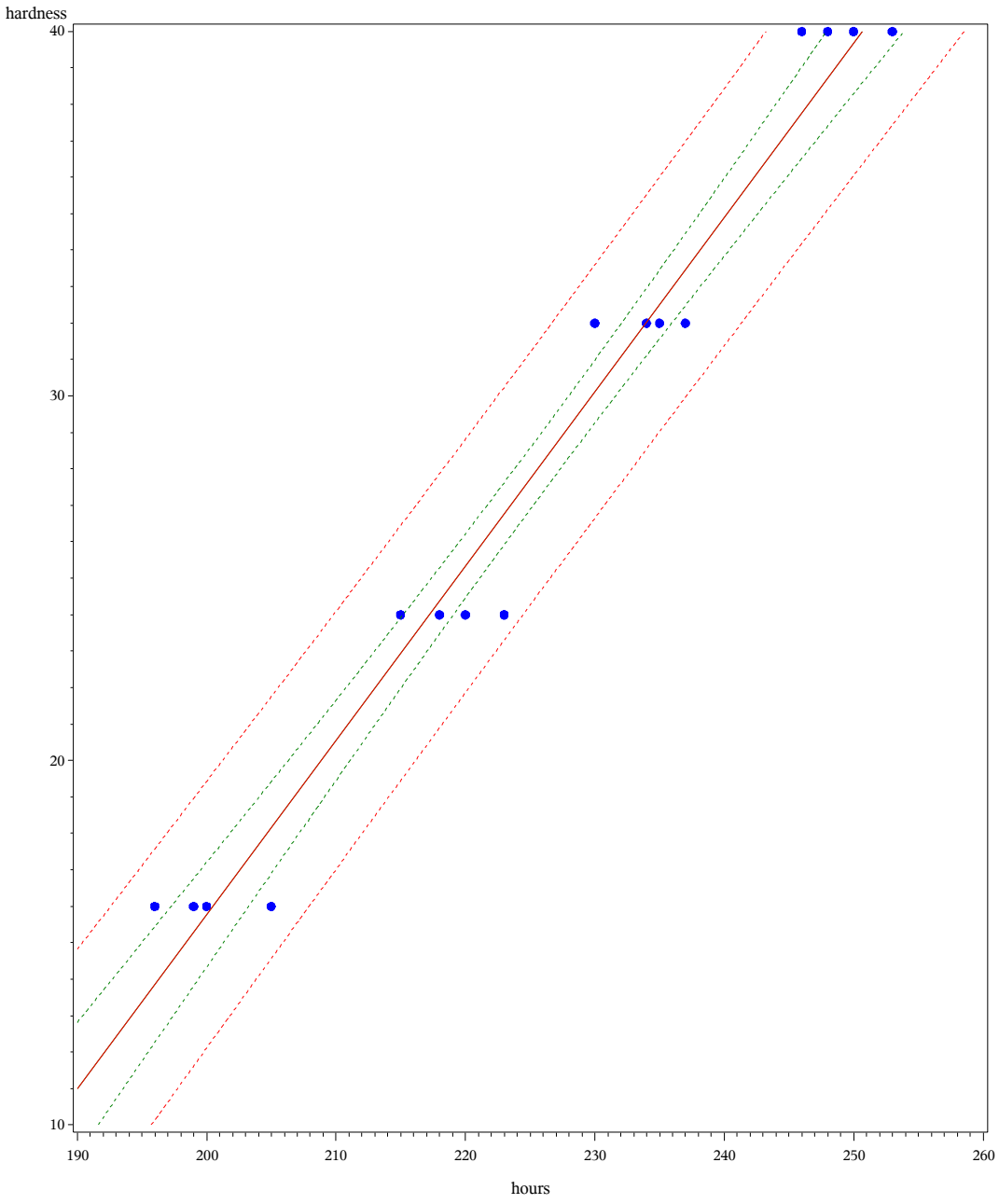
Plastic Hardness
Confidence and Prediction Bands



Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	45.20	45.20	43.88	<0.0001
Error	70	72.1	1.03		
Total	71	117.3			

Tab. 9: Problem 5 ANOVA Table

4.4 Part d

97.31031% of the variance in hardness is due to time. The correlation coefficient between hardness and time is 0.98646.

4.5 Part e

Time seems to be a very good predictor for plastic hardness.

5 Problem 5

$$H_0 : b_1 = 0 \tag{11}$$

$$H_a : b_1 \neq 0 \tag{12}$$

From the values in table 9, we can reject the null hypothesis and conclude that we have a reasonably good fit.