# Coordinated Online In-Vehicle Navigation Guidance Based on Routing Game Theory

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**This study proposed a coordinated online in-vehicle routing mechanism for smart vehicles with real-time information exchange and portable computation capabilities. The proposed coordinated routing mechanism was modeled as a pure-strategy atomic routing game and was implemented by a sequentially updating distributed algorithm. This study showed the existence of an equilibrium joint routing decision for the routing game and the convergence of the distributed algorithm to an equilibrium; this equilibrium was based on the assumption that individual smart vehicles were selfish players who tried to minimize their own travel time and shared their route choices with other smart vehicles directly or indirectly. Numerical experiments based on the Sioux Falls, South Dakota, network indicated that the proposed distributed algorithm converged very quickly; thus the algorithm has great potential for online applications. Moreover, the proposed coordinate routing mechanism outperformed traditional independent routing mechanisms under various traffic regimes. The mechanism reduced travel time for both the overall system and the majority of individual vehicles; this achievement was the core idea of intelligent transportation systems. These positive effects were more significant in congested traffic condition than in free traffic. The experiments also indicated that even with imperfect information provision, the proposed approach could work well for convergence as well as travel time reduction in systems and individual vehicles.**

In recent years, the integration of wireless communication and onboard computation facilities into smart vehicles and roadside transport infrastructures has established vehicle-to-vehicle and vehicle-to-infrastructure information exchange, which together form connected vehicle systems (CVS). Many previous and ongoing studies have shown that CVS have great potential to address main challenges in transportation systems, such as traffic safety, mobility, and environmental issues, and thus have attracted attention from both academia and industry. For example, CVS can ensure that individual travelers are aware of nearby threats and hazards so that they can drive more consciously. Also, CVS can collect environment-relevant real-time transportation data to support and facilitate green transportation choices by transportation system users and operators.

This study is concerned with the use of CVS to provide individual drivers access to real-time traffic information from other vehicles or roadside infrastructure, allowing adaptive route choice decisions en route to avoid traffic congestion for both individual users and the traffic network. Prompted by the access to real-time traffic information, many studies have sought to develop online in-vehicle routing guidance (*1–11*), which helps individual vehicles avoid traffic congestion. According to literature reviews (*12, 13*), these efforts can be classified into two categories: (*a*) reactive guidance systems (*1–6*) provide route guidance to individual vehicles that is based on a snapshot of traffic conditions at the time the guidance is provided but without performing any predictions and (*b*) predictive guidance systems (*7–11*) give routing recommendations that are based on future traffic demands and make traffic condition predictions through traffic assignment or simulators.

Reactive and predictive systems share two common characteristics. First, they all focus on user optimality (individual vehicle's travel time) but leave system optimality (system travel cost) free, although there are a few exceptions. For example, Jahn et al. attempted to improve system performance by slightly sacrificing fairness among the vehicles (*6*). Hawas and Mahmassani (*14*) and Hawas (*15*) proposed a cooperative decentralized architecture to provide routing guidance, emphasizing network control. Second, all systems assume that each vehicle independently and selfishly seeks the shortest path according to real-time information without cooperation. Kaufman et al. present a slightly different concept, which proposes an iterative procedure of traffic flow forecasts and traffic guidance until a certain fixed point is reached (*7*). Traffic prediction based on traffic assignment implicitly involves coordination between individual vehicles, and it can potentially mitigate the chance of system traffic congestion, but the extensive computation load prevents online application.

Along with the above observation, this study defines all these routing mechanisms (both reactive and predictive guidance) as independent routing mechanisms (IRMs). IRMs can benefit the minority but not the majority, which is the expectation from a marketing perspective. That is, because vehicles with real-time information access are a minority of traffic flow over a network, their movement will not affect traffic flow significantly. Then, they can take advantage of real-time traffic information to avoid traffic congestion. However, as real-time information is reachable to a majority of vehicles over the network, IRMs may cause network traffic congestion oscillation because of the possibility that a high volume of vehicles is routed to a link, which is not crowded when route decisions are made but becomes congested when these vehicles arrive. Then, a large volume of vehicles may be rerouted to another, currently uncongested link. Eventually, even worse congestion results. A possible solution to the weakness of the IRM is system routing mechanisms (SRMs), which achieve system optimality through a central authority systematically determining an individual vehicles's route choice. SRMs may sacrifice some individuals' interests and may lack fairness and so are not consistent with the nature of driver route choice in transportation reality and cannot be applied in practice.

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The state of the art indicates that neither IRMs nor SRMs adapt to well-informed transportation systems toward the goal of balancing system performance and individual routing behavior. A new routing mechanism is needed to compensate for this deficiency, one that comprehensively incorporates real-time traffic information provision.

This study proposes a coordinated online in-vehicle routing mechanism (CRM) built on a well-connected platform such as CVS. Specifically, smart vehicles about to make routing decisions en route form a routing coordination group, in which they exchange their tentative route choices (e.g., taking route *i*) and other traffic information (e.g., traffic volume of road *a*). This information will be collected, aggregated, and disseminated in a coordination group by certain central communication units (or direct information exchange among the vehicles). According to the aggregated route choice information (such as the number of vehicles that will take link *j*), smart vehicles in a coordination group repropose (or adjust) their route choices, which are again collected, aggregated, and disseminated in a coordination group. This process continues until certain equilibrium is achieved (i.e., no single vehicle has the incentive to adjust its route proposal further). Then the resultant route choices become final. If each trip is a multiple routing decision process, one vehicle can join in a routing coordination group multiple times to make routing decisions until it completes a trip. Depending on how individual vehicles repropose their route choice according to the updated traffic information, the process will end up with different joint route choices. This paper focuses on the best response to the updated path travel time (i.e., always choosing the shortest path). The privacy issues related to the information exchange among vehicles can be addressed with privacy protection algorithms (*16*).

CRM is a traffic management mechanism for shaping the routing decisions of a flock of smart vehicles so that the travel time both networkwide and of individual vehicles is reduced. Thus it is different from routing algorithms that focus on the development of the shortest path algorithms for an individual vehicle (*17–19*). The closest work in the literature is that of Du et al. (*20*), which also proposed a coordinated routing mechanism but assumed that vehicles choose probability distribution among a few candidate routes. Additionally, the CRM is different from traditional traffic assignment in two aspects. First, the CRM works on microscopic online route choices of individual vehicles, while traffic assignment focuses on macroscopic traffic flow assignment. Second, the CRM is modeled by an atomic game concerning discrete individual vehicles' decisions, while traffic assignment is a nonatomic game that considers aggregated traffic demand as a continuing flow. These differences lead to different sets of analytical methodologies applicable to them.

The main contributions of this paper are as follows: (*a*) to propose a coordinated online in-vehicle routing mechanism and model it as a pure-strategy atomic routing game, which generates the route choices by balancing user optimality and system optimality in travel time; (*b*) to design a sequentially updating distributed algorithm (SUDA) for the CRM and prove its convergence to an equilibrium joint routing decision among smart vehicles in coordinated groups, taking advantage of the distributed computation recourse in vehicles; and (*c*) to conduct experiments to show the efficiency and applicability of the CRM implemented by SUDA. The proposed research is an early response to the new technologies of CVS and is an initial effort to introduce online coordinated routing and distributed algorithms into intelligent transportation systems. It is a significant intellectual and practical contribution to the development of a new generation of in-vehicle guidance systems and sheds new light on this field.

The rest of the paper is organized as follows. This study first defines the CRM and then models the routing decision process as a pure-strategy atomic routing game. Next, the existence of an equilibrium solution for the game is demonstrated, and a SUDA is designed to search for an equilibrium joint routing decision. The convergence of the SUDA in finite steps is mathematically proved. Then numerical experiments are conducted and the results and insights are presented. The paper ends with a summary including the key contributions and results of the study.

## CRM AND PURE-STRATEGY ATOMIC ROUTING GAME

This study considers a transportation network denoted by a directed graph, $G(N, A)$, where $N$ is the set of nodes and $A$ is the set of arcs. Let the number of arcs be $l = |A|$ and the total number of nodes be $n = |N|$. Suppose at some point that there are $m$ smart vehicles, labeled $v = 1, \ldots, m$, in a coordinated group. These vehicles are en route and are about to make a routing decision within a short period. Each vehicle, ν, has a specific origin–destination (O-D) pair given by $(o_v, d_v) \in N \times N$ with a set of $k_v$ possible paths connecting them, denoted $P_v \triangleq \{p_v^1, \ldots, p_v^{k_v}\}$. Two vehicles may have the same O-D pair. This study considers that two pieces of real-time traffic information are available to all vehicles: (*a*) real-time link flow volume information and (*b*) real-time aggregated information on the other vehicles' tentative route choices. Given (*a*) and (*b*), individual vehicles following the CRM can predict the travel time on each link $\ell$ if $f_\ell$ more vehicles decide to take it. The link travel time function of traffic flow is denoted $C_\ell(f_\ell)$, $f_\ell = 0, \ldots, n$. Assume $C_\ell(f_\ell)$ increases as $f_\ell$ increases, which represents the congestion nature of the road links. The travel time for each vehicle depends not only on its own route choice but also on the other vehicles' route choices. In particular, let the route choice of each vehicle $v$ be denoted by $p_v \in P_v$, $v = 1, \ldots, m$. The total number of vehicles choosing link $\ell$ is given by

$$f_\ell(p_1, p_2, \ldots, p_m) = \sum_{v=1}^{m} y_v^\ell \qquad (1)$$

where

$$y_v^\ell = \begin{cases} 1 & \text{if } \ell \in p_v \\ 0 & \text{otherwise} \end{cases}$$

Therefore, the route travel time of vehicle $v$ on route $p_v$ given all other vehicles' route choices $(p_{-v})$ $[T_v(p_v, p_{-v})]$ is formulated as

$$T_v(p_v, p_{-v}) = \sum_{\ell \in p_v} C_\ell(f_\ell(p_v, p_{-v})) \qquad v = 1, \ldots, m \qquad (2)$$

The CRM proposes an autonomous negotiation process involving all $m$ vehicles in the coordination group, assuming each vehicle tries to minimize its own travel time given other vehicles' route choices are known. Specifically, the vehicles that are about to make their route choice decisions in a short time slot form a coordination group, in which they negotiate future route choice by proposing and reproposing their tentative route choices according to the aggregated route choice information from other vehicles. This process continues until an equilibrium is achieved. This coordinated routing process in a coordination group forms a routing game among the vehicles. This routing game is a pure-strategy atomic game. The resolution of this game is an equilibrium joint routing decision, namely, a combination of the vehicles' route choices, $p^* \triangleq \{p_v^*\}_{v=1}^m$, in which for all vehicles $v = 1, \ldots, m$,

$$T_v(p_v^*, p_{-v}^*) \leq T_v(p_v, p_{-v}^*) \qquad \forall p_v \in P_v \tag{3}$$

That is, no vehicles have the incentive to deviate from their current route choice unilaterally.

## EXISTENCE OF EQUILIBRIUM IN JOINT ROUTING DECISION

This section first demonstrates the existence of an equilibrium joint routing decision as vehicles in a coordination group making a route choice following the CRM. The routing game is a special case of the congestion game first proposed by Rosenthal (*21*). Therefore, an equilibrium joint routing decision of the proposed mechanism can be explored by an equivalent integer programming model, given in Equation 4, in which the decision variables $x_v^i$ are defined as follows:

$$x_v^i = \begin{cases} 1 & \text{if vehicle } v \text{ takes path } p_v^i \\ 0 & \text{otherwise} \end{cases}$$

$y^\ell$ = number of vehicles taking link $\ell$

equivalent integer program (EIP):

$$\min F(y^\ell) = \sum_{\ell=1}^{l} \sum_{z=0}^{y^\ell} C_\ell(z)$$

subject to

$$\sum_{i=1}^{k_v} x_v^i = 1 \qquad \forall v = 1, \ldots, m$$

$$y^\ell = \sum_{v=1}^{m} \sum_{p_v^i \ni \ell} x_v^i = 0 \qquad \forall \ell = 1, \ldots, l$$

$$x_v^i \in \{0,1\} \qquad \forall v = 1, \ldots, m; i = 1, \ldots, k_v$$

$$y^\ell \geq 0 \qquad \forall \ell = 1, \ldots, l \tag{4}$$

where $F(\cdot)$ is the potential function and $z$ is an index used in the sum. Theorem 1 states the existence of the equilibrium joint routing decision. Its proof is available elsewhere (*21*) and is not repeated here.

Theorem 1. An equilibrium joint routing decision always exists for the routing game formed by the vehicles making routing decisions following the CRM. Moreover, let $(\bar{x}, \bar{y})$ be an optimal solution of the integer program in Equation 4, where $\bar{x} = \{\bar{x}_v^i\}_{v=1,\ldots,m; i=1,\ldots,k_v}$ and $\bar{y} = \{\bar{y}^\ell\}_{\ell=1,\ldots,l}$ define

$$p_v^* = p_v^i \quad \text{if } \bar{x}_v^i = 1 \qquad \forall v = 1, \ldots, m \tag{5}$$

Then $p^* = \{p_v^*\}_{v=1}^m$ is an equilibrium joint routing decision.

Remark 1. According to Theorem 1, the objective function of the EIP in Equation 4 is actually a potential function of the routing game. The integer program in Equation 4 is only of theoretical or conceptual importance. It cannot be used in actual computing of an equilibrium of the routing game because of the nonlinearity in the objective function and possibly the exponential number of variables. This study proposes a distributed algorithm to search for the solution to the CRM.

## SEQUENTIALLY UPDATING DISTRIBUTED ALGORITHM

This study proposes a SUDA, in which vehicles sequentially update their route choices locally according to other vehicles' route choices and then propose it for negotiation until an equilibrium joint route decision among all vehicles, representing the optimal user route choices, is agreed on.

### SUDA Procedure

The procedure of the overall distributed algorithm is presented in Equation Box 1.

In the SUDA procedure, mod$(n, m)$ means the remainder of the division $n/m$. It generates the index of a vehicle, which will conduct a route update in iteration $n$. The travel cost of $p_\omega^{*,n}$ is based on the link cost $c^{\ell,n}$ defined by Equation 8. If the travel costs of $p_\omega^{*,n}$ and $p_\omega^n$ are the same, then the vehicle does not change its route choice; otherwise $p_\omega^{n+1} \leftarrow p_\omega^{*,n}$. In the synchronization step, all other vehicles receive vehicle ω's new route choice (which could be the same as before), either directly from vehicle ω or from a central information collection facility.

The process for finding new route $p_\omega^{*,n}$ is given below. At each iteration, only one vehicle is allowed to update its route choice by solving a shortest path problem, given all other vehicles' route choices are known and fixed. After a new route choice is made, this information will be proposed and shared with other vehicles instantaneously. Suppose at the current iteration, $n$, it is vehicle ω's turn to update its route; the current route choice of another vehicle, $v$, is given by $p_v^n$. Let $y_{-\omega}^{\ell,n}$ be the total number of vehicles choosing link $\ell$ excluding ω itself; that is,

$$y_{-\omega}^{\ell,n} = \sum_{v \neq \omega} u_v^{\ell,n} \tag{6}$$

EQUATION BOX 1

Initialization. Each vehicle picks a route $p_\omega^1$ and then synchronizes

For $n = 1, 2, \ldots,$

If $\omega = \text{mod}(n, m)$, then vehicle ω finds the shortest path $p_\omega^{*,n}$.

If the cost of $p_\omega^{*,n}$ is strictly less than the cost of $p_\omega^n$, then $p_\omega^{n+1} \leftarrow p_\omega^{*,n}$,

otherwise, $p_\omega^{p+1} \leftarrow p_\omega^n$.

Synchronize.

If convergence criterion is satisfied, then exit.

where

$$u_v^{\ell,n} = \begin{cases} 1 & \text{if } \ell \in p_v^n \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The cost on each link $\ell$ is

$$c^{\ell,n} = C_\ell\left(y_{-\omega}^{\ell,n} + 1\right) \tag{8}$$

and vehicle $\omega$ then finds a shortest path $p_\omega^{*,n}$ with the cost on each link $\ell$ being $c^{\ell,n}$.

One unit of extra volume (value 1) is added to the total number of other vehicles traversing a link $\ell$; if vehicle $\omega$ decides to take a link $\ell$, then the total number of vehicles taking this link becomes $y_{-\omega}^{\ell,n} + 1$. This point is crucial for the convergence proof. However, this technical detail may not be consistent with the transportation fact that a single vehicle itself will not change link traffic volume or travel time very much, and this one unit of extra volume can be ignored. This study states that Equation 8 reflects mathematical rigor. It is consistent with the atomic game theory proposed to model the routing mechanism in this study, as well as the proof of the algorithm convergence below. The transportation fact is reflected by the travel time function (such as the Bureau of Public Roads function), which may not change significantly by increasing one unit of extra volume. Numerical experiments also have been conducted, and it was found that counting or not counting this one unit of extra volume does not significantly affect the convergence of the proposed algorithm in practice.

The procedure of the algorithm indicates that the communication center is responsible only for information collecting, aggregating, and disseminating without routing guidance provision; thus it does not require a lot of computational resources. However, individual vehicles conduct route choice calculation (the shortest path search) locally; thus the major computational load of the SUDA is distributed to local vehicles. Moreover, contemporary smart devices can carry out this kind of calculation without any difficulty since the shortest path problem can be solved by very efficient polynomial algorithms. Remark 2 summaries the advantage of the SUDA.

Remark 2. The advantages of the SUDA are threefold. First, it possesses good scalability. When many smart vehicles are involved, the per-iteration computation load of individual vehicles as well as the communication center do not change. Second, it is economical. Since the communication center does not require a lot of computational resources, it does not require a lot of investment. Smart vehicles are equipped with communication and computational facilities, so no additional cost is needed to implement this distributed architecture. Third, it is robust. If the communication center is down, the vehicles can still use vehicle-to-vehicle communication, and the SUDA can still be performed, although the control is more complex.

## Convergence of SUDA

This section explores the convergence performance of the SUDA. Lemma 1 provides its termination criterion, followed by the proof for its validation.

Lemma 1. If no vehicle changes its route in $m$ consecutive iterations, then the SUDA terminates, and all the vehicles end up taking their equilibrium route choice.

Proof. In $m$ consecutive iterations, each vehicle has exactly one iteration in which it gets a turn to update its route choice. $n$ is assumed to be the first iteration in the $m$ consecutive iterations, and the route choice of each vehicle $\omega$ is given by $p_\omega^n$. Note the definition of $c^{\ell,n}$ and that the vehicle does not change its route choice, that is, for each vehicle the cost of the currently chosen path is minimal given other vehicles' current path choices.

$$T_\omega\left(p_\omega^n, p_{-\omega}^n\right) \le T_\omega\left(p_\omega, p_{-\omega}^n\right) \qquad \forall p_\omega \in P_\omega;\ \omega = 1, \dots, m \tag{9}$$

Therefore the joint route choice $(p_\omega^n)_{\omega=1}^m$ satisfies the equilibrium condition (3). ■

This section provides a rigorous argument to prove the convergence of the SUDA stated in Theorem 2. The idea of proving this convergence theorem is to show that the value of the potential function (the objective function of the EIP in Equation 4) decreases as the algorithm proceeds. Since there are only a few feasible solutions, the algorithm is assumed to stop at an optimal solution of EIP (i.e., equilibrium decision among vehicles under the CRM) in a finite number of iterations.

Theorem 2. The proposed distributed algorithm converged at an equilibrium joint routing decision in a finite number of iterations, with the termination condition in Lemma 1.

Proof. For iteration $n$, let $\omega = \text{mod}(n, m)$. Therefore, it is vehicle $\omega$'s turn to update its route choice. Before vehicle $\omega$ updates its route choice, each vehicle $v$'s route choice is given by $p_v^n$; according to $u_v^{\ell,n}$ and $c^{\ell,n}$ defined by Equations 7 and 8, the total number of vehicles (including vehicle $v$) choosing link $\ell$ is given by

$$y^{\ell,n} = \sum_{v=1}^{m} u_v^{\ell,n} = \sum_{v \ne \omega} u_v^{\ell,n} + 1 = y_{-\omega}^{\ell,n} + 1 \tag{10}$$

and the value of the potential function ($F^n$) is given by

$$F^n = \sum_{\ell=1}^{1} \sum_{z=0}^{y^{\ell,n}} C_\ell(z) = \sum_{\ell=1}^{l} \sum_{z=0}^{y_{-\omega}^{\ell,n}} C_\ell(z) + \sum_{\ell \in p_\omega^n} C_\ell\left(y_{-\omega}^{\ell,n} + 1\right) \tag{11}$$

After vehicle $\omega$ updates its route choice by applying SUDA, vehicle $\omega$ switches to path $p_\omega^{*,n}$. $F^{n+1} \le F^n$ holds. Illustrating this claim, in each route update procedure, vehicle $\omega$ first finds a shortest path $p_\omega^{*,n}$ by using the link costs defined by Equation 8. If the cost of $p_\omega^{*,n}$ is the same as $p_\omega^n$, then no vehicle has changed its route choice, and hence $F^{n+1} = F^n$. If the travel time of $p_\omega^{*,n}$ is strictly less than $p_\omega^n$, it equally indicates that

$$\sum_{\ell \in p_\omega^{n+1}} C^{\ell,n} = \sum_{\ell \in p_\omega^{*,n}} C^{\ell,n} < \sum_{\ell \in p_\omega^n} C^{\ell,n} \tag{12}$$

Substituting $c^{\ell,n}$ by its definition in Equation 8,

$$\sum_{\ell \in p_\omega^{n+1}} C_\ell\left(y_{-\omega}^{\ell,n} + 1\right) = \sum_{\ell \in p_\omega^{*,n}} C_\ell\left(y_{-\omega}^{\ell,n} + 1\right) < \sum_{\ell \in p_\omega^n} C_\ell\left(y_{-\omega}^{\ell,n} + 1\right) \tag{13}$$

Only vehicle $\omega$ updates its route choice in iteration $n$; therefore, the value of the potential function can be calculated by Equation 14 after vehicle $\omega$ switches or updates its path from $p_\omega^n$ to $p_\omega^{n+1} = p_\omega^{*,n}$. According to the inequality implied by Equation 13, Equation 15 indicates that the claim $F^{n+1} \le F^n$ holds.

$$F^{n+1} = \sum_{\ell=1}^{l} \sum_{z=0}^{y_{-\omega}^{\ell,n}} C_\ell(z) + \sum_{\ell \in p_\omega^{n+1}} C_\ell\left(y_{-\omega}^{\ell,n} + 1\right) \tag{14}$$

$$< \sum_{\ell=1}^{l} \sum_{z=0}^{y_{-\omega}^{\ell,n}} C_\ell(z) + \sum_{\ell \in p_\omega^{n}} C_\ell\left(y_{-\omega}^{\ell,n} + 1\right) = F^n \tag{15}$$

Now, $F^{n+m}$ and $F^n$ are compared. If $F^{n+m} = F^n$, then there are $m$ consecutive iterations in which the route choices are not changed, and hence by Lemma 1 the algorithm converges to an equilibrium. Otherwise, $F^{n+m} < F^n$. Since there are finite possible joint route choices, the algorithm must terminate at the optimal solution of EIP in Equation 4. According to Theorem 1, the algorithm must also converge at an equilibrium joint routing decision. ■

## NUMERICAL EXPERIMENTS

This section conducts numerical experiments to demonstrate the performance of the SUDA and provide more insight. The objective is threefold: (*a*) test the convergence of the SUDA, (*b*) explore the impacts of smart vehicle penetration (the percentage of smart vehicles in the whole network flow) on the SUDA convergence and CRM performance, and (*c*) investigate the effect of imperfect information provision on the convergence of the SUDA and the performance of the CRM, considering communication failures that cause smart vehicles to exchange route choice information only with a subset of other smart vehicles (i.e., imperfect information provision).

All the experiments are set up with the following specifications. First, the Sioux Falls, South Dakota, city network, including 24 nodes and 76 links, is the test bed. Second, the BPR function in Equation 16 is used as the link travel time function:

$$C_\ell\left(y^\ell\right) = t_o\left(1 + \alpha\left(\frac{y^\ell}{C}\right)^\beta\right) \tag{16}$$

where $t_o$ represents free-flow link travel time and $C$ represents link capacity; $\alpha = 0.15$ and $\beta = 4$. Free-flow link travel time $t_o$ and link capacity $C$ are randomly generated in all experiments. Third, smart vehicles following the CRM and nonsmart vehicles (i.e., background flow) following the IRM (such as reactive guidance) are randomly generated with O-D pairs. All smart vehicles in the network form a coordination group. Fourth, the SUDA is implemented based on MATLAB R2013a without use of any existing transportation simulation software. Fifth, the Bellman–Ford–Moore shortest path algorithm is applied for shortest path searching. And sixth, the experiments are run on a computer with an Intel Xeon CPU E5-2603 0 1.8 GHz processor with 16.0 GB RAM.

### Convergence and Computational Load

The convergence of the SUDA is explored with experiments run for nine cases. In the experiments, the number of smart vehicles in a coordination group increases from 800 to 4,000 in increments of 400, and the background traffic including about 2,000 nonsmart vehicles is preloaded according to free-flow conditions. Thus, the experiments cover low, medium, and high traffic load regimes as more smart vehicles are loaded in the network. Figure 1 validates the convergence process of the SUDA. Consistent with the proof of Theorem 2, the convergence is indicated by the strict decrease

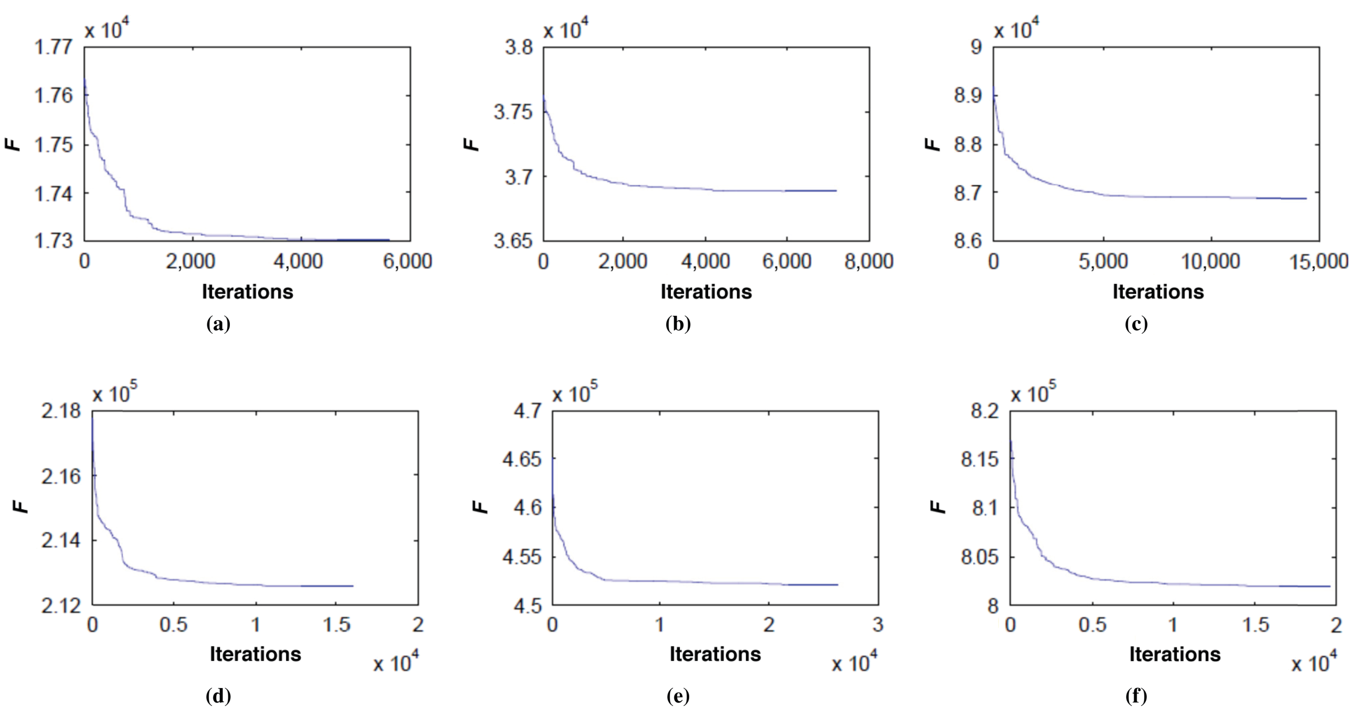


FIGURE 1 Convergence for SUDA: (*a*) $v$ = 800, (*b*) $v$ = 1,200, (*c*) $v$ = 1,600, (*d*) $v$ = 2,000, (*e*) $v$ = 2,400, and (*f*) $v$ = 2,800.

*(continued)*

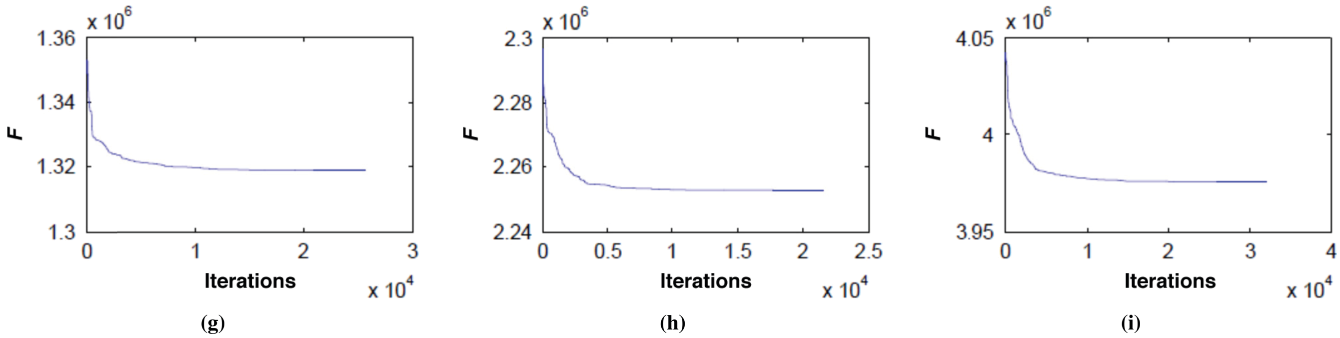


FIGURE 1 *(continued)* Convergence for SUDA: (*g*) $v$ = 3,200, (*h*) $v$ = 3,600, and (*i*) $v$ = 4,000.

of $F$ (the value of the potential function) until stopping at a constant value under various smart vehicle loads.

The computational performance of the SUDA is mainly captured from two aspects: central processing unit (CPU) running time and the average individual smart vehicle's route update times before reaching equilibrium. The experiments are run under traffic loads equal to 1,000, 1,500, 2,000, 3,000, 3,250, and 4,000 vehicles (including both smart and nonsmart vehicles) corresponding to the levels of service (LOS) from A to F. Thus, the experiments cover free flow (LOS A and B), low traffic congestion (LOS C and D), and high traffic congestion (LOS E and F) traffic. The traffic LOS here is calculated by the average of flow-to-capacity ratios over all links in the network. The experiment for each traffic regime is conducted multiple times (such as 20 times) to remove the effect of randomness. The corresponding results are plotted in Figures 1 and 2.

Figure 2 provides the results for the number of route updates per smart vehicle (denoted $I_n$) versus vehicle penetration for various traffic regimes.

Under a given traffic LOS, as more smart vehicles are involved in a coordination group (i.e., smart vehicle penetration increases), the $I_n$ value increases correspondingly. For example, the size of a coordination group increases from 400 to 4,000 under LOS F, corresponding to the smart vehicle penetration increase from 10% to 100% and the value of $I_n$ increase from 4 to 12.

The value of $I_n$ mainly factors the size of a coordination group rather than traffic LOS. The values of $I_n$ are close (about 3 or 4) under various LOS when a coordination group is relatively small, but they are significantly different when a coordination group is relatively large. This insight can be further illustrated. First, Figure 2 shows that $I_n$ increases more quickly under LOS F than under LOS B, although they have the same penetration increment rate, because a coordination group under LOS F is larger than it is under LOS B with the same penetration. Second, the values of $I_n$ are close if the size of a coordination group is the same under different LOS levels. For example, Figure 2 illustrates that $I_n$ is about 4 when a coordination group of 1,500 vehicles is formed under LOS B with 100% penetration or under LOS C with 75% penetration. It increases only slightly, to near 5, when this coordination group is formed under LOS D with 50% penetration or under LOS E and F with 40% penetration or so. These insights necessitate the study of a mechanism that optimizes the formation of coordination groups when the proposed CRM is applied to a large network.

Figure 3 provides results for CPU running time of the experiments versus smart vehicle penetration. The experimental results illustrate that the SUDA needs less than 7 s to reach the convergence under LOS F with 4,000 smart vehicles in the coordination group. Thus, the SUDA converges very quickly, although it needs more computational time to converge as more smart vehicles are involved in a coordination group. This is consistent with the results in Figure 2. In general, the computational experiments in Figures 1, 2, and 3 sustain the convergence of the SUDA and demonstrate the reasonable low computational load of the SUDA. These good characteristics of the

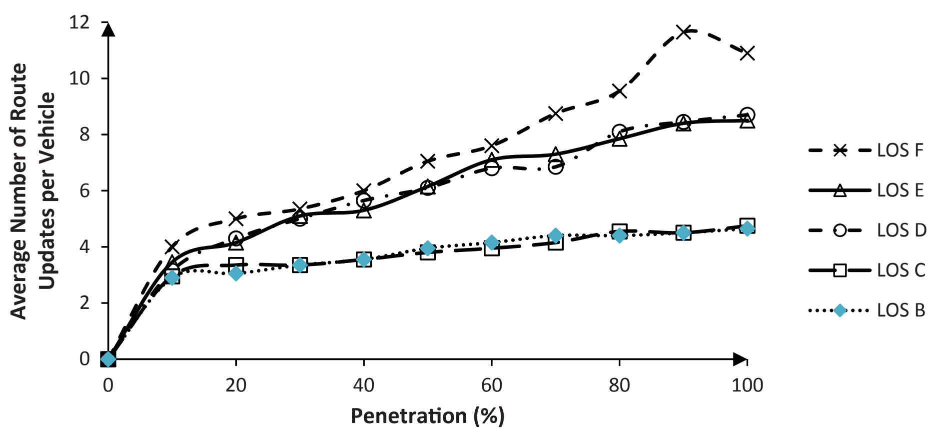


FIGURE 2 Average number of route updates per vehicle to reach equilibrium in SUDA (LOS B through F correspond to vehicle load of 1,500, 2,000, 3,000, 3,250, and 4,000, respectively).

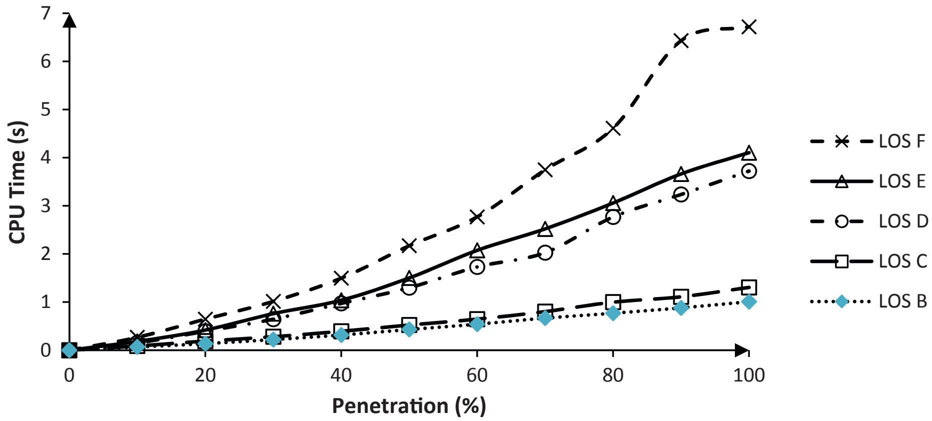


FIGURE 3 CPU time of SUDA.

SUDA indicate good potential for the proposed approach to be used in online in-vehicle routing guidance.

Remark 3. The presented computational performance of the SUDA can be further improved by optimizing several components of the algorithm. For example, implementing the shortest path algorithm by a more efficient language than MATLAB can reduce CPU time. Optimally choosing the size of the coordination group will reduce iterations of sequential route updates, which represents the bottleneck of the SUDA. Developing rigorous methods for the formation of the coordination group deserves a separate paper. The population and the size of a coordination group are closely related to network topology, the overlap among smart vehicles' candidate paths. The authors are working on a method that allows a portion of the vehicles to update their route choices simultaneously if their candidate paths do not have overlap or have a low degree of overlap. This enhancement could reduce the computational time significantly. However, the communication time is not counted in the proposed experiments. Future work on the formation of the coordination group and the enhancement of the SUDA will reduce communication time, too.

## Advantages of CRM over IRM

This section investigates the advantages of the proposed CRM over the traditional IRM under traffic LOS from A to F, in each of which a fixed number of vehicles is loaded into the network according to free flow with a smart vehicle penetration ranging from 0% to 100%, where 0% penetration indicates no smart vehicles are involved, and 100% penetration indicates that all vehicles are smart vehicles. A penetration in the range of (0%, 100%) represents the scenario in which smart vehicles and nonsmart vehicles coexist in the experiments. To indicate the effect of the CRM, each test will run smart vehicles by the CRM and then the IRM under the same traffic background, which is composed of nonsmart vehicles. Then the system optimality and user optimality are compared. User optimality is evaluated by smart vehicle travel time. System performance is evaluated by system travel time, $\sum_{\ell=1}^{l} f_\ell C_\ell(f_\ell)$, where $f_\ell$ represents traffic flow including both smart vehicles and nonsmart vehicles on link $\ell$.

Figure 4 provides the experiment results regarding system performance improvement under various LOS and smart vehicle

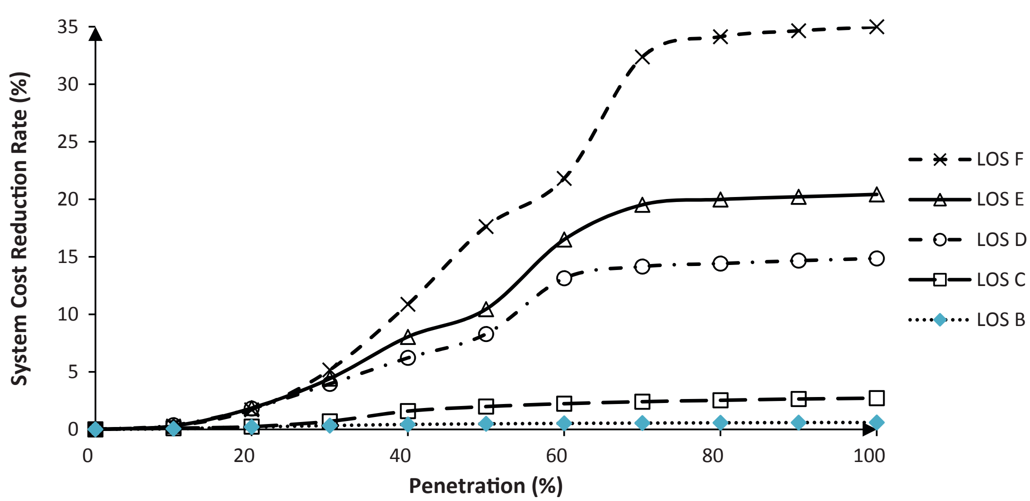


FIGURE 4 Benefit of CRM for system optimality.

penetrations. Although each experiment (featured by traffic LOS and penetration) was run 20 times, only the curve of the average values are presented. In addition, the result for LOS A is not presented since the effect is not significant. Figure 4 demonstrates the system cost reduction rate by the CRM from IRM under various smart vehicle penetrations in each traffic LOS. The results demonstrate that CRM outperforms IRM consistently under various traffic regimes. The effect is more significant as the smart vehicle penetration increases. Moreover, as traffic becomes more congested, the CRM is better for system optimality.

Figure 5 investigates the advantages of the CRM over the IRM from the user optimality perspective. Since the system cost is the sum of the individual travel time, the average travel time of individual vehicles also decreases by the same percentage when the CRM is compared with the IRM. For further demonstration of the advantages of the CRM, the percentage of smart vehicles whose travel time is reduced by the CRM from the IRM is examined. The results in Figure 5 indicate several good features of the CRM. First, the CRM always makes some smart vehicles save travel time under various traffic regimes and smart vehicle penetrations. It will have more benefits for user optimality as traffic congestion is higher. Second, under each LOS level, the variation of the benefit along smart vehicle penetrations presents a shape close to parabola, and the peak point of the benefit moves to a higher smart vehicle penetration as the traffic congestion level increases. This observation indicates that if a small portion of vehicles does not follow the CRM, the performance of the CRM will not be compromised much, and it may be improved.

## Impact of Imperfect Information Provision in Coordination Group

In reality, communication among vehicles and between information communication centers and vehicles is affected by various factors such as traffic flow dynamics, roadside environment, communication interference, and bandwidth limits. These factors will cause packet dropout, time delay, and transmission failure during the information exchange among vehicles or between vehicles and information communication centers. As a result, some vehicles may not be able to submit their route choice decisions to the communication center successfully and in a timely manner. Accordingly, the communication center collected only imperfect information, which will be disseminated to individual vehicles. Because of this observation, this study considers imperfect information provision (i.e., a smart vehicle only receives information such as route choice decisions from a subset of other smart vehicles in a coordination group). The performance of the CRM and SUDA under imperfect information provision is difficult to study with mathematical rigor. This study therefore resorts to numerical experiments. The convergence of SUDA and the impact of information provision levels on CRM performance are investigated. In the experiments, the information provision level refers to the percentage of smart vehicles with which one smart vehicle can communicate and share route choice decisions among all smart vehicles in a coordination group. The smart vehicle penetration is fixed at 60% in each experiment to separate the effect of the penetration and the information provision level, although the experiments cover various smart vehicle penetrations.

Remark 4. Four remarks are made for the proposed experiment. First, the communication group of each smart vehicle remains constant until an equilibrium joint routing decision is reached, although it may be changed in the next round of routing decision making. Second, the experiments are run with the traffic LOS changing from A to F. Thus, multiple traffic regimes are covered. In each case, information provision level is increased from 0% to 100% (0% and 100% represents full and no information provision, respectively) with increments of 10%. The section from 10% to 90% represents imperfect information provision. Third, since there is no proof to guarantee the convergence of the equilibrium joint route decisions, the experiments choose other stop criteria, such as stopping at the same iteration when the SUDA converges given 100% information provision. Fourth, all the experiments are run 20 times to remove randomness. The average results are presented.

### *Observations for Convergence*

Because of space limitations, this study provides only the result of the convergence test under LOS A (1,000 smart vehicles are involved) by Figure 6, where information provision level ($I$) increases from 10% to 100%. The key observations are summarized as follows: (*a*) there is no guarantee of strictly decreasing $F$ (the value of potential function) until it comes to a steady value, so the convergence of SUDA proved previously does not hold for the SUDA under imperfect information provision; (*b*) the fluctuation of $F$-values demonstrates a declining

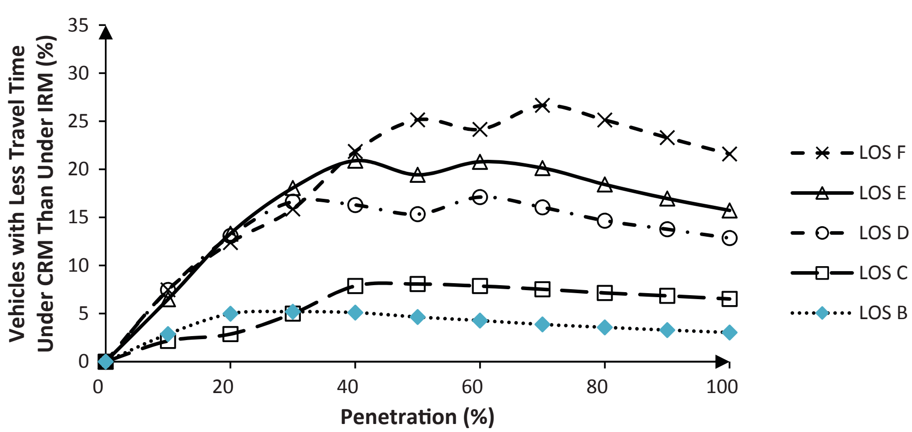


FIGURE 5 Benefit of CRM for user optimality.

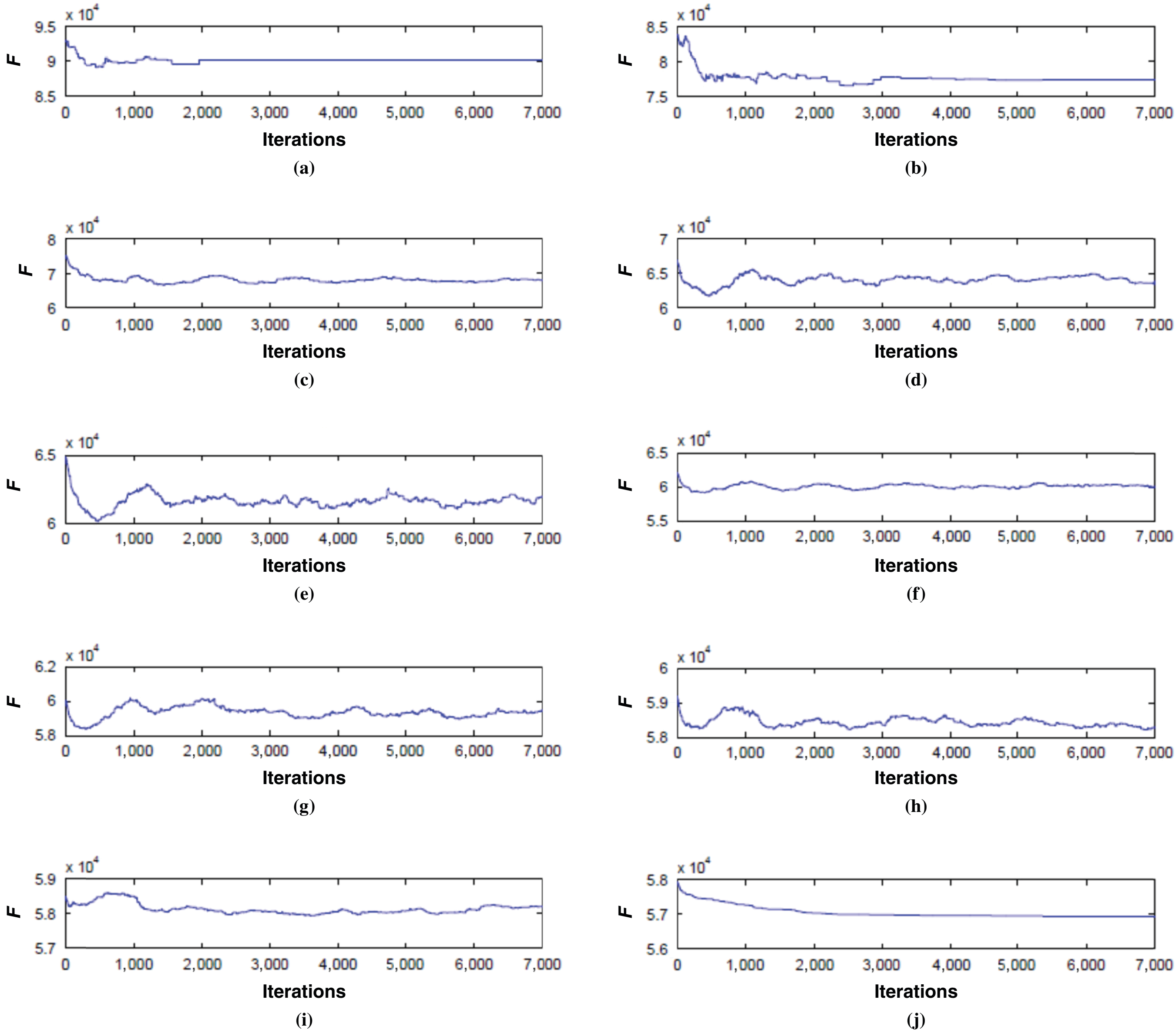


FIGURE 6 Convergence of SUDA by information provision levels: (*a*) $I$ = 0.1, (*b*) $I$ = 0.2, (*c*) $I$ = 0.3, (*d*) $I$ = 0.4, (*e*) $I$ = 0.5, (*f*) $I$ = 0.6, (*g*) $I$ = 0.7, (*h*) $I$ = 0.8, (*i*) $I$ = 0.9, and (*j*) $I$ = 1.

trend under all imperfect information provision levels; and (*c*) the SUDA converges (i.e., SUDA stops with a stable *F*-value) under some cases with relatively lower information provision levels (10% to 30%) or higher information provision level (70% to 100%) but fluctuates relatively severely in the case of medium levels of information provision, such as (40% to 60%).

### *System Performance Improvement*

Figure 7 illustrates the reduction of system cost by the CRM from the IRM when imperfect information is provided under LOS B to F. The result under LOS A is insignificant, so it is not presented. Some critical observations can be made from Figure 7. First, the CRM will result in less system cost than will the IRM under different traffic regimes, although a small proportion of information is exchanged among smart vehicles in a coordinated group. Second, under each traffic regime, a higher level of information exchange will lead to more system cost reduction. Third, for a given information provision level in a coordinated group, the reduction of system cost by the CRM is more apparent under a congested traffic regime (such as LOS D, E, or F) than a sparse or free traffic regime (such as LOS B or C). These insights ensure the constant advantages of the CRM over the IRM under an imperfect information provision condition.

### *Travel Time Reduction for Individual Vehicles*

The percentage of smart vehicles that can benefit from the CRM compared with the IRM is examined. The results in Figure 8

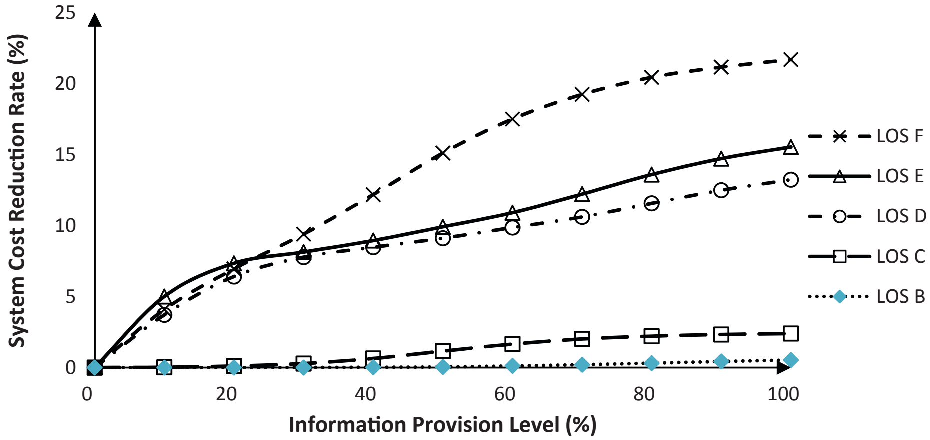


FIGURE 7 System performance comparison by information provision levels.

demonstrate that even with imperfect information provision, a significant percentage of smart vehicles obtain travel time reduction from the IRM by the CRM. Moreover, as expected, a higher information provision level benefits more smart vehicles. Finally, it is observed that the CRM with imperfect information provision benefits more vehicles as traffic congestion level is high. These observations further reinforce the advantages of the CRM over the IRM, especially under the congested traffic condition, which is the main focus of traffic control in general.

## CONCLUSION AND FUTURE WORK

This study proposed a coordinated in-vehicle routing mechanism that takes advantage of information exchange capability among smart vehicles in connected vehicle systems. The routing mechanism is coordinated in the sense that individual vehicles can exchange tentative routing choices as well as real-time traffic information and then make their own route choices accordingly. The proposed coordinated routing mechanism is modeled as a pure-strategy atomic routing game and is implemented by a SUDA, which is proved to converge to an equilibrium of the atomic routing game. Numerical experiments based on the Sioux Falls city network indicate that the distributed algorithm converges quickly with very low computation load under various smart vehicle penetration levels and traffic regimes; thus it possesses great potential for online application. Moreover, the proposed coordinated routing mechanism outperforms traditional IRM. It reduces travel time for both the overall system and individual vehicles. The positive effect becomes more significant as traffic congestion level is higher. Although it is difficult to give a rigorous proof for convergence of the SUDA under imperfect information provision, the numerical experiments indicated that it converges most of the time. In addition, the proposed coordinated routing mechanism outperforms traditional independent routing in both system optimality and user optimality. Hence the proposed algorithm can be expected to generate good routing decisions even with imperfect information.

This research has been an early effort to develop a new generation of routing guidance used in connected vehicle systems. It introduced the application of distributed systems and algorithms to traffic engineering. Potential future work stemming from this study includes improving the applicability of the CRM by further research on the formation of coordinated groups. The population size and spatial distribution of the group have significant impact on the computational time of the proposed method and hence should be controlled by certain mechanisms so that both coordination routing efficiency and computational efficiency can be sustained. Addressing this problem involves separate research, which is not the focus of this paper; it is proposed for future research. Another possible future work is to further investigate the impact of imperfect information and the robust coordinated

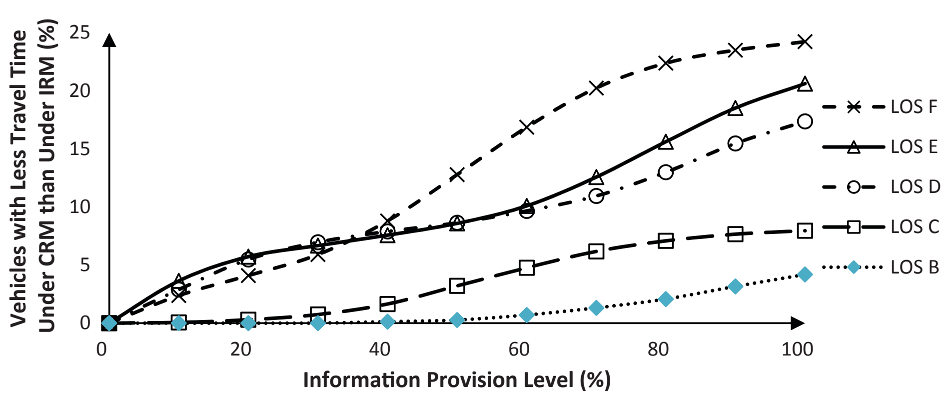


FIGURE 8 Percentage of smart vehicles that have less travel time under CRM than under IRM.

routing mechanism with imperfect information. Because of the frequent change of topology, traffic information propagated in CVS is usually subject to error, time delay, and many other imperfections. From a practice point of view, it is necessary to understand the impact of imperfectness in information and to generate robust, useful routing guidance. Both the above extensions consider more traffic reality in practice but bring in more challenges in distributed algorithm design and convergence analysis. These are proposed as future work.

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