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STAT 424

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Investigating the Effects of Factors on Sleep Quality

1. Introduction

In modern society, sleep quality is being challenged like never before. With the proliferation of electronic devices and the accelerated pace of life, more and more people are beginning to face sleep disturbances, such as difficulty falling asleep, frequent awakenings at night, or early awakenings. Research has shown that lifestyle factors such as room temperature, lighting environment, use of screen devices before bedtime, and bedtime can have a significant impact on the depth and efficiency of sleep. However, due to significant individual differences, these effects may manifest themselves differently in different populations.

This project started with the idea of "improving individual sleep quality" and systematically analyzed the effects of four common factors on sleep performance: room temperature, screen exposure before bedtime, bedtime, and whether or not to sleep with the lights on, in a 16-night sleep experiment. The experiment was conducted in a 2^4 full factorial design with two levels of each factor and a controlled variable experiment with myself as the only participant.

Table 1: Experimental Factors and Levels

Factor	**Variable**	**Level**	
		–	+
Room Temperature(°F)	A	65 (cool)	72 (warm)
Screen Exposure Before Sleep	B	No screen for 1 hour before bed Screen	use until sleep
Bedtime	C	Sleep before 11 PM	Sleep after 12 AM
Light	D	off	on

During the experiment, I used two different wearable sleep monitoring devices (Figure 1, Figure 2) and combined them with self-scoring to record and evaluate the quality of sleep each night from multiple dimensions. With the data obtained from the experiment, I hope to identify the factors that have the most significant impact on sleep quality, and then adjust my personal work and rest habits, so as to provide a scientific basis for improving the quality of sleep and the sense of well-being in life. At the same time, this project can also provide some reference and inspiration for people with similar problems.

Figure 1: Apple watch with "AutoSleep" App

Figure 2: Oura Ring

2. Experimental Design

2.1 Planning Matrix

A planning matrix (Table 2) was constructed to represent all 16 combinations of the four experimental factors in a 2^4 full factorial design. Each row in the matrix corresponds to one treatment condition applied on a specific night. The matrix specifies the assigned levels for each factor: room temperature (65°F or 72°F), screen exposure before sleep (no phone or use phone), bedtime (before 11 PM or after 12 AM), and light condition during sleep (off or on). This structured layout ensured comprehensive coverage of all possible factor combinations, providing a balanced framework for analyzing both individual and interactive effects on sleep quality.

Table 2: Planning Matrix

Run	Room Temperature(°F)	Screen Exposure Before Sleep	Bedtime	Light
1	65	No phone	before 11 PM	off
2	65	No phone	before 11 PM	on
3	65	No phone	after 12 AM	off
4	65	No phone	after 12 AM	on
5	65	Use phone	before 11 PM	off
6	65	Use phone	before 11 PM	on
7	65	Use phone	after 12 AM	off
8	65	Use phone	after 12 AM	on
9	72	No phone	before 11 PM	off
10	72	No phone	before 11 PM	on
11	72	No phone	after 12 AM	off
12	72	No phone	after 12 AM	on
13	72	Use phone	before 11 PM	off
14	72	Use phone	before 11 PM	on
15	72	Use phone	after 12 AM	off
16	72	Use phone	after 12 AM	on

2.2 Response Variable: score

To evaluate sleep quality under each treatment condition, I collected three types of scores each night (Figure 3), combining both objective and subjective measures. The first score was generated using the Apple Watch paired with the sleep tracking app AutoSleep (Figure 1), which calculates a composite score (out of 100) based on physiological signals such as heart rate, respiratory rate, and body temperature. The second score came from the Oura Ring (Figure 2), a wearable sleep-monitoring ring that uses its proprietary algorithm to assess sleep stages, duration, and recovery, also providing a score out of 100. Finally, each morning upon waking, I conducted a self-assessment of my sleep experience, rating my restfulness and how I felt upon waking on a scale from 1 to 10. These three scores provided a multifaceted perspective on my sleep quality, allowing for a more comprehensive analysis of the effects of the tested factors. Importantly, my target score for average sleep score is 85.

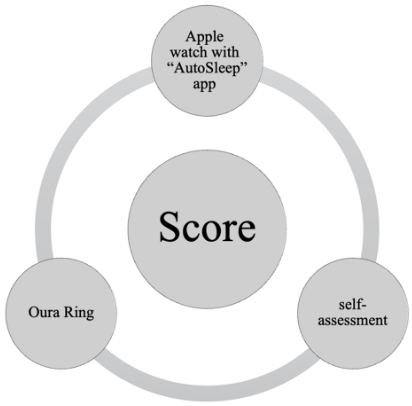

Figure 3: Composition of the Score

2.3 Measurement of Location and Dispersion

To comprehensively assess the effects of each treatment combination on sleep quality in terms of both "location" and "dispersion" dimensions, two key statistics were calculated for each treatment condition. First, the mean total score was used to measure the average level of sleep quality under the combination (Equation 1). Second, the logarithmic variance Second, the logarithmic variance $\ln s^2$ was used to measure the stability of the scores, i.e., the magnitude of fluctuations. It is calculated by first finding the sample variance s^2 (Equation 2) and then taking the natural logarithm.

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \textbf{(1)}$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \qquad \textbf{(2)}$$

Since the variance s^2 is always positive and skewed in distribution, its natural logarithm $\ln s^2$ is used in this paper to measure volatility. This treatment brings the data closer to a normal distribution for subsequent modeling and analysis, and also makes the results easier to compare and interpret numerically and graphically.

2.4 Main and Interaction Effects

Before optimization, the effect of each factor on the response needs to be analyzed. The main effect indicates the effect of a change in the level of a single factor on the mean of the response (Equation 3) . The interaction effect indicates the additional effect of multiple factor combinations on the response (Equation 4). These effects help identify which factors or combinations have a significant effect on sleep quality and provide a basis for subsequent optimization.

$$\mathrm{ME(A)} = \bar{\mathrm{Z}}(\mathrm{A}+) - \bar{\mathrm{Z}}(\mathrm{A}-) \qquad \textbf{(3)}$$

$$INT(A_1, A_2, \dots, A_k) = \frac{1}{2} INT(A_1, A_2, \dots, A_{k-1} | A_K +) - \frac{1}{2} INT(A_1, A_2, \dots, A_{k-1} | A_K -) \qquad \textbf{(4)}$$

2.5 Statistical Model

To analyze the effects of experimental factors on sleep quality, I used a linear summation model. The model assumes that the total response can be expressed as a combination of main effects, interaction effects, and random errors (Equation 5, 6).

$$y = \mu + \sum_i ME_i + \sum_{i<j} INT_{ij} + \cdots + \varepsilon \qquad \textbf{(5)}$$

$$\begin{aligned} y = \mu &+ \beta_A x_A + \beta_B x_B + \beta_C x_C + \beta_D x_D + \beta_{AB} x_A x_B + \beta_{AC} x_A x_C + \beta_{AD} x_A x_{D+} \beta_{BC} x_B x_C + \beta_{BD} x_B x_D \\ &+ \beta_{CD} x_C x_D + \beta_{ABC} x_A x_B x_C + \beta_{ABD} x_A x_B x_D + \beta_{ACD} x_A x_C x_D + \beta_{BCD} x_B x_C x_D \\ &+ \beta_{ABCD} x_A x_B x_C x_D + \epsilon \qquad \textbf{(6)} \end{aligned}$$

2.6 Nominal-the-Best Optimization

This data structure is used in this study to support the analysis of "Nominal-the-Best" type of problems, where the goal is not only to improve the average response, but also to minimize its fluctuation around the desired target value t. In this framework, a quadratic loss function is used to measure the deviation from the target value. In this framework, a quadratic loss function is used to measure the deviation from the target value (Equation 7). According to the loss expectation (Equation 8) the magnitude of the loss depends on the sum of the squares of the variance of the response variable and its mean deviation from the target value.

$$L(y,t) = c(y-t)^2 \qquad \textbf{(7)}$$

$$E[L(y,t)] = c \cdot Var(y) + c \cdot (E[y]-t)^2 \qquad \textbf{(8)}$$

Based on this, the study used a two-step optimization strategy:

i. Select levels of some factors to minimize Var(y).
ii. Select the adjustment factor (level of a factor not in i.) to move E(y) closer to t.

This method is both "good" and "stable", and can provide an operational reference for real-life sleep optimization strategies.

3. Data Collection

After completing the experimental design and variable setup, I began a 16-night sleep experiment. Each night, the treatment combinations were strictly executed according to the pre-setup in 2.1, and the three scores were recorded at the end of sleep according to the setup in 2.2 (Score 3 need to multiple 100 to make sure the unit is consistent). Finally, $\bar{y}$ and $\ln s^2$ were calculated using R. I obtained the complete experimental data (Table 3), a matrix that faithfully reflects the changes in my sleep quality under different combinations of lifestyle habits.

Table 3: Medel Matrix of Score Response

Run	Room Temperature (°F)	Screen Exposure Before Sleep	Bedtime	Light	score			$\bar{y}$	s^2	$\ln s^2$
	A	B	C	D	1	2	3			
1	–	–	–	–	84	82	80	82.00	4.00	1.39
2	–	–	–	+	81	83	70	78.00	49.00	3.89
3	–	–	+	–	64	69	70	67.67	10.33	2.34
4	–	–	+	+	59	61	70	63.33	34.33	3.54

5	–	+	–	–	68	69	80	72.33	44.33	3.79
6	–	+	–	+	62	63	70	65.00	19.00	2.94
7	–	+	+	–	71	67	60	66.00	31.00	3.43
8	–	+	+	+	65	63	60	62.67	6.33	1.85
9	+	–	–	–	88	87	80	85.00	19.00	2.94
10	+	–	–	+	87	81	80	82.67	14.33	2.66
11	+	–	+	–	80	78	70	76.00	28.00	3.33
12	+	–	+	+	74	77	70	73.67	12.33	2.51
13	+	+	–	–	82	75	80	79.00	13.00	2.56
14	+	+	–	+	69	74	70	71.00	7.00	1.95
15	+	+	+	–	71	73	60	68.00	49.00	3.89
16	+	+	+	+	66	67	60	64.33	14.33	2.66

To further explore the effects of the factors and their interactions on sleep performance, I built two linear regression models in R according to the settings of 2.4 and 2.5, using $\bar{y}$ and $\ln s^2$ as response variables, respectively. The model covers all main effects and interactions (Table 4). It not only demonstrated the direction and strength of each factor's effect on average sleep quality, but also revealed which combinations of factors might cause an increase or decrease in volatility. These statistical results served as the theoretical basis for the subsequent “Nominal-the-Best” optimization step.

Table 4: Factorial Effects for Sleep Experiment

Effect	$\bar{y}$	$\ln s^2$
A	5.33	-0.08
B	-7.50	0.06
C	-9.17	0.18
D	-4.41	-0.21
AB	-1.25	-0.16
AC	2.50	0.39
AD	3.33	-0.52
BC	2.58	-0.03
BD	-1.17	-0.86
CD	1.00	-0.40
ABC	-2.50	0.48
ABD	-5.83	0.67
ACD	8.33	0.11
BCD	1.08	0.06
ABCD	3.52	-0.08

4. Data Analysis

4.1 half-normal plot

To identify the factors and interaction terms that had a significant effect on sleep quality, I plotted two half-normal plots (Figures 4, 5) to assess the mean (location) and logarithmic variance (dispersion) of the response variables, respectively.

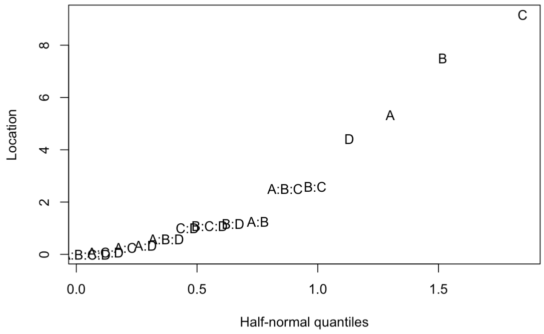

Figure 4: Half-normal Plot of Location Effects

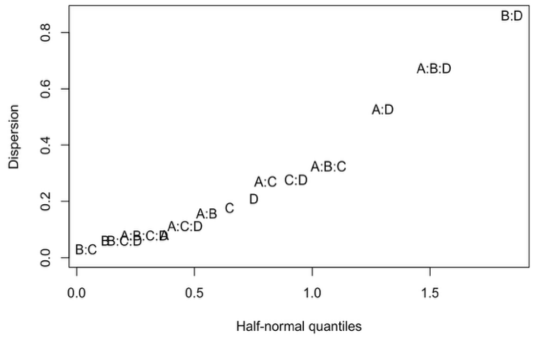

Figure 5: Half-normal Plot of Dispersion Effects

In Figure 4, factors C (bedtime), B (screen use), A (room temperature), D (light), B:C and A:B:C deviate significantly from the reference line, indicating that they have a significant effect on average sleep quality. This implies that going to bed earlier, reducing screen exposure, having the right room temperature and turning off light may help improve overall sleep levels.

Figure 5 shows the results of the log-variance analysis, which is used to look at the volatility of sleep quality. the interaction terms B:D, A:B:D and A:D deviate significantly from the straight line, suggesting that these combinations have a significant effect on the stability of sleep, while a single factor has a relatively small effect on the volatility.

4.2 Refit the Model

After identifying the main influence terms, I re-fitted linear regressions to $\bar{y}$ and $\ln s^2$, respectively (Tables 5, 6).

In the location model, A (room temperature), B (screen), C (bedtime), and D (light) all significantly affected the means, with A being a positive effect and the rest negative, suggesting that appropriate temperature contributes to sleep quality, whereas screen exposure, late bedtime, and turning on the light may reduce scores. The interaction terms B:C and A:B:C also reached significant levels.

In the discrete model, A:D, B:D, and A:B:D significantly affected sleep volatility, and the negative effects of B:D and A:D suggest that certain combinations may contribute to sleep stability.

Table 5: Refit Location Model

```
##
## Call:
## lm(formula = y.bar ~ A + B + C + D + B:C + A:B:C, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.5417 -0.7083 -0.2917  1.0417  2.2083
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   72.292      0.395 182.998  < 2e-16 ***
## A              2.667      0.395   6.750 8.36e-05 ***
## B             -3.750      0.395  -9.493 5.51e-06 ***
## C             -4.583      0.395 -11.602 1.03e-06 ***
## D             -2.208      0.395  -5.590 0.000339 ***
## B:C            1.292      0.395   3.270 0.009687 **
## A:B:C         -1.250      0.395  -3.164 0.011472 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.58 on 9 degrees of freedom
## Multiple R-squared:  0.9728, Adjusted R-squared:  0.9547
## F-statistic: 53.71 on 6 and 9 DF,  p-value: 1.535e-06
```

Table 6: Refit Dispersion Model

```
##
## Call:
## lm(formula = ln.s.2 ~ A:D + B:D + A:B:D + A:D, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.64732 -0.37225 -0.00846  0.33809  0.67955
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.8552     0.1190  23.989 1.65e-11 ***
## A:D          -0.2638     0.1190  -2.216  0.04673 *
## D:B          -0.4305     0.1190  -3.617  0.00353 **
## A:D:B         0.3372     0.1190   2.833  0.01509 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4761 on 12 degrees of freedom
## Multiple R-squared:  0.6844, Adjusted R-squared:  0.6055
## F-statistic: 8.675 on 3 and 12 DF,  p-value: 0.002474
```

$$\hat{y} = 72.292 + 2.667x_A - 3.750x_B - 4.583x_C - 2.208x_D + 1.292x_Bx_C - 1.250x_Ax_Bx_C \quad \mathbf{(9)}$$

$$\hat{z} = ln(\sigma^2) = 2.8552 - 0.2638x_Ax_D - 0.4305x_Bx_D + 0.3372x_Ax_Bx_D \quad \mathbf{(10)}$$

4.3 Select the level

To simultaneously optimize the stability and mean performance of sleep scores, I used the Two-Step Procedure method for factor screening. The method first controls volatility by minimizing the variance of the response variable, and then selects an adjustment factor to regulate the mean on this basis.

Since BD is the most significant in the regression model, we first determine the level of BD. According to Figure 6, B should be chosen - and D should also be chosen - to make the dispersion as small as possible. Similarly, according to Equation 9, Figure 7, and Figure 8, A should choose +. Therefore, A, B, D = $(+, -, -)$ to make the value of σ^2 smaller.

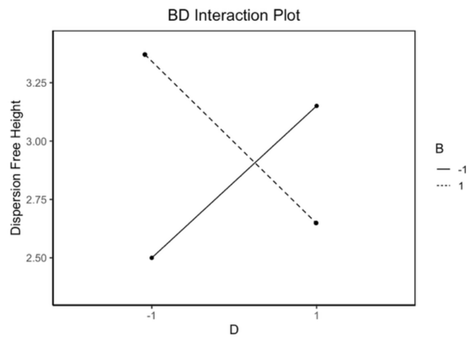

Figure 6: Screen Use VS. light

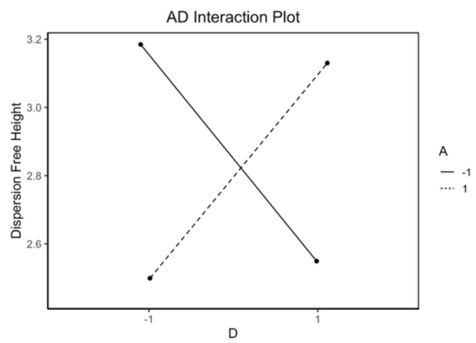

Figure 7: Room Temperature VS. light

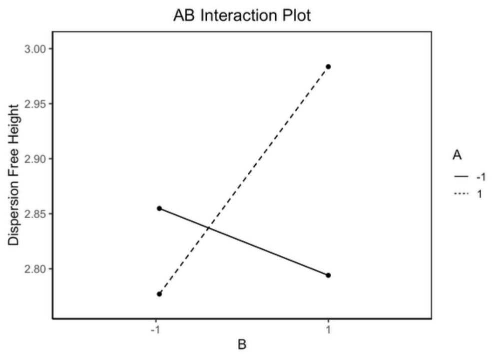

Figure 8: Room Temperature VS. Screen Use

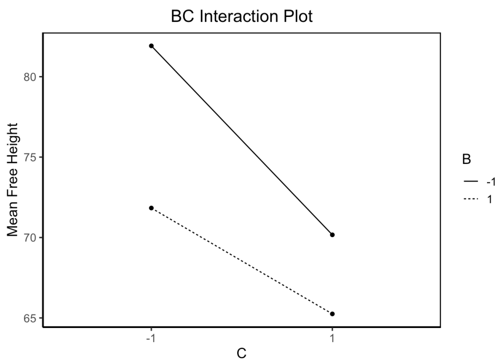

Figure 9: Screen Use VS. Bedtime

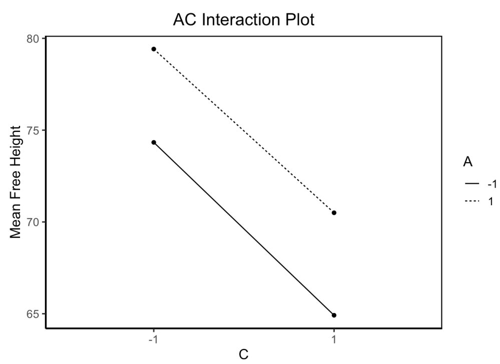

Figure 10: Room Temperature VS. Bedtime

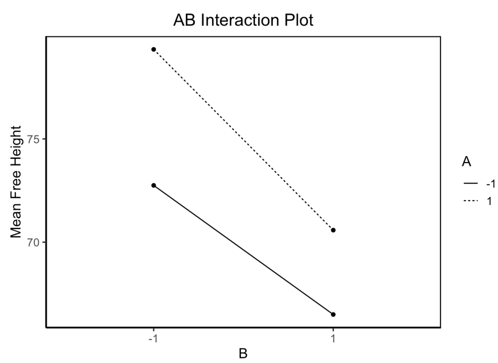

Figure 11: Room Temperature VS. Screen Use

4.4 Adjustment factor

After fixing the levels of A, B, and D, an adjustment factor needs to be further selected to enhance the mean value of sleep scores without introducing additional fluctuations. Equations 9 and 10 show that C (bedtime) significantly affects the mean in the location model, and there is no significant main effect or interaction term in the dispersion model, which is consistent with the definition of adjustment factor.

The conclusion of 4.3 can be carried into Equation 9 to obtain Equation 11 and Equation 12, and then the expected score value of 85 before the experiment can be carried into (Equation 13) to solve for x_C as -0.88(Equation 14). Meanwhile, it is consistent with the conclusion presented in Figure 9 and Figure 10. Therefore, A,B,C,D = $(+,-,-,-)$.

$$\hat{y} = 72.292 + 2.667(1) - 3.750(-1) - 4.583x_C - 2.208(-1) + 1.292(-1)x_C - 1.250(-1)x_C \quad (\mathbf{11})$$

$$\hat{y} = 80.917 - 4.625x_C(\mathbf{12})$$

$$85 = 80.917 - 4.625x_C(\mathbf{13})$$

$$x_C = -0.88 \quad (\mathbf{14})$$

Since the prediction point x_C = -0.88 still belongs to the valid range [-1,+1] of the experimental factor, the estimation is an interpolation with model rationality and explanatory power of the results, without additional caution about the risk of extrapolation.

5. Results & Reflection

In Conclusion, the modified 2^4 full factorial experiment systematically examined the effects of four daily habits on sleep quality. Its findings provided me with a set of optimization recommendations with practical guidance. Firstly, a slightly warmer room temperature not only helped improve overall sleep scores but also reduced fluctuations in scores to some extent. Secondly, not using electronic devices before bedtime significantly improves sleep stability. More importantly, going to bed as early as possible played a decisive role in improving average sleep performance, especially if other conditions remained stable. Furthermore, sleeping with the lights off significantly reduces the ups and downs of sleep states during the night and improves overall sleep quality. Therefore, this combination of habits helped me achieve higher quality and more consistent sleep.

This also indicates that by combining the experimental design principles learned in STAT 424 and through systematic modeling and analysis, we can achieve scientific and effective optimization and improvement in aspects of our life and health.

Appendix: R code

Data read

```r
# Read CSV file
data <- read.csv("sleep_study.csv", header = TRUE)

colnames(data) <- c("Run","A", "B", "C", "D", "score1", "score2", "score3")

data$y.bar <- rowMeans(data[, c("score1", "score2", "score3")])
data$s.2 <- apply(data[, c("score1", "score2", "score3")], 1, var)
data$ln.s.2 <- log(data$s.2)
head(data,3)
```

```
##   Run  A  B  C  D score1 score2 score3    y.bar      s.2   ln.s.2
## 1   1 -1 -1 -1 -1     84     82     80 82.00000  4.00000 1.386294
## 2   2 -1 -1 -1  1     81     83     70 78.00000 49.00000 3.891820
## 3   3 -1 -1  1 -1     64     69     70 67.66667 10.33333 2.335375
```

Linear models of location and dispersion & half-normal plot

```r
# dispersion main effects
mod.dis <- lm(ln.s.2 ~ A*B*C*D, data)
theta.dis <- mod.dis$coefficients * 2
theta.dis <- theta.dis[!is.na(theta.dis)]
theta.dis <- theta.dis[2:16]
theta.dis
```

```
##           A           B           C           D         A:B         A:C
## -0.08109893  0.06001427  0.17725587 -0.20990155 -0.15658159  0.39300211
##         B:C         A:D         B:D         C:D       A:B:C       A:B:D
## -0.03045920 -0.52760388 -0.86103097 -0.39923598  0.48197552  0.67440056
##       A:C:D       B:C:D     A:B:C:D
##  0.11217573  0.06147200 -0.07950838
```

```r
# half-normal plots
halfnorm(theta.dis, labs = names(theta.dis), nlab = 15, ylab = "Dispersion")
```

```r
# dispersion main effects
mod.dis <- lm(ln.s.2 ~ A*B*C*D, data)
theta.dis <- mod.dis$coefficients * 2
theta.dis <- theta.dis[!is.na(theta.dis)]
theta.dis <- theta.dis[2:16]
theta.dis
```

```
##           A           B           C           D         A:B         A:C
## -0.08109893  0.06001427  0.17725587 -0.20990155 -0.15658159  0.39300211
##         B:C         A:D         B:D         C:D       A:B:C       A:B:D
## -0.03045920 -0.52760388 -0.86103097 -0.39923598  0.48197552  0.67440056
##       A:C:D       B:C:D     A:B:C:D
##  0.11217573  0.06147200 -0.07950838
```

```r
# half-normal plots
halfnorm(theta.dis, labs = names(theta.dis), nlab = 15, ylab = "Dispersion")
```

Refit linear models of location and dispersion

```r
# re-fit the dispersion model with significant factors only (change the BCQ to DEQ)
mod.dis.final <- lm(ln.s.2 ~ A:D + B:D + A:B:D + A:B:C+C:D+A:C + A:D, data)
summary(mod.dis.final)
```

```
## 
## Call:
## lm(formula = ln.s.2 ~ A:D + B:D + A:B:D + A:B:C + C:D + A:C + 
##     A:D, data = data)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -0.31425 -0.16079  0.01612  0.13663  0.31279 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  2.85515    0.06096  46.835 4.62e-12 ***
## A:D         -0.26380    0.06096  -4.327 0.001912 ** 
## D:B         -0.43052    0.06096  -7.062 5.91e-05 ***
## D:C         -0.19962    0.06096  -3.274 0.009613 ** 
## A:C          0.19650    0.06096   3.223 0.010433 *  
## A:D:B        0.33720    0.06096   5.531 0.000365 ***
## A:B:C        0.24099    0.06096   3.953 0.003339 ** 
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.2438 on 9 degrees of freedom
## Multiple R-squared:  0.9379, Adjusted R-squared:  0.8965 
## F-statistic: 22.66 on 6 and 9 DF,  p-value: 5.968e-05
```

```r
# re-fit the location model with significant factors only
mod.loc.final <- lm(y.bar ~ A + B + C + D + B:C + A:B:C, data)
summary(mod.loc.final)
```

```
## 
## Call:
## lm(formula = y.bar ~ A + B + C + D + B:C + A:B:C, data = data)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -2.5417 -0.7083 -0.2917  1.0417  2.2083 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)   72.292      0.395 182.998  < 2e-16 ***
## A              2.667      0.395   6.750 8.36e-05 ***
## B             -3.750      0.395  -9.493 5.51e-06 ***
## C             -4.583      0.395 -11.602 1.03e-06 ***
## D             -2.208      0.395  -5.590 0.000339 ***
## B:C            1.292      0.395   3.270 0.009687 ** 
## A:B:C         -1.250      0.395  -3.164 0.011472 *  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 1.58 on 9 degrees of freedom
## Multiple R-squared:  0.9728, Adjusted R-squared:  0.9547 
## F-statistic: 53.71 on 6 and 9 DF,  p-value: 1.535e-06
```

Interaction plots

```r
# creates the B vs D interaction plot
ggplot(data, aes(x = factor(D), y = ln.s.2, linetype = factor(B), group = factor(B))) +
  stat_summary(fun = mean, geom = "line", color = "black") +
  stat_summary(fun = mean, geom = "point", color = "black") +
  labs(x = "D", y = "Dispersion Free Height", linetype = "B") +
  theme_classic(base_size = 14) +
  theme(
    panel.border = element_rect(colour = "black", fill = NA, size = 1),
    plot.title = element_text(hjust = 0.5)
  ) +
  ggtitle("BD Interaction Plot")
```

```r
# creates the A vs D interaction plot
ggplot(data, aes(x = factor(D), y = ln.s.2, linetype = factor(A), group = factor(A))) +
  stat_summary(fun = mean, geom = "line", color = "black") +
  stat_summary(fun = mean, geom = "point", color = "black") +
  labs(x = "D", y = "Dispersion Free Height", linetype = "A") +
  theme_classic(base_size = 14) +
  theme(
    panel.border = element_rect(colour = "black", fill = NA, size = 1),
    plot.title = element_text(hjust = 0.5)
  ) +
  ggtitle("AD Interaction Plot")
```

```r
# creates the A vs B interaction plot
ggplot(data, aes(x = factor(B), y = ln.s.2, linetype = factor(A), group = factor(A))) +
  stat_summary(fun = mean, geom = "line", color = "black") +
  stat_summary(fun = mean, geom = "point", color = "black") +
  labs(x = "B", y = "Dispersion Free Height", linetype = "A") +
  theme_classic(base_size = 14) +
  theme(
    panel.border = element_rect(colour = "black", fill = NA, size = 1),
    plot.title = element_text(hjust = 0.5)
  ) +
  ggtitle("AB Interaction Plot")
```

```r
# creates the B vs C interaction plot
ggplot(data, aes(x = factor(C), y = y.bar, linetype = factor(B), group = factor(B))) +
  stat_summary(fun = mean, geom = "line", color = "black") +
  stat_summary(fun = mean, geom = "point", color = "black") +
  labs(x = "C", y = "Mean Free Height", linetype = "B") +
  theme_classic(base_size = 14) +
  theme(
    panel.border = element_rect(colour = "black", fill = NA, size = 1),
    plot.title = element_text(hjust = 0.5)
  ) +
  ggtitle("BC Interaction Plot")
```

```r
# creates the A vs C interaction plot
ggplot(data, aes(x = factor(C), y = y.bar, linetype = factor(A), group = factor(A))) +
  stat_summary(fun = mean, geom = "line", color = "black") +
  stat_summary(fun = mean, geom = "point", color = "black") +
  labs(x = "C", y = "Mean Free Height", linetype = "A") +
  theme_classic(base_size = 14) +
  theme(
    panel.border = element_rect(colour = "black", fill = NA, size = 1),
    plot.title = element_text(hjust = 0.5)
  ) +
  ggtitle("AC Interaction Plot")
```

```r
# creates the A vs B interaction plot
ggplot(data, aes(x = factor(B), y = y.bar, linetype = factor(A), group = factor(A))) +
  stat_summary(fun = mean, geom = "line", color = "black") +
  stat_summary(fun = mean, geom = "point", color = "black") +
  labs(x = "B", y = "Mean Free Height", linetype = "A") +
  theme_classic(base_size = 14) +
  theme(
    panel.border = element_rect(colour = "black", fill = NA, size = 1),
    plot.title = element_text(hjust = 0.5)
  ) +
  ggtitle("AB Interaction Plot")
```