

Hi Aaron,

I am writing to give you two points I want to talk about tomorrow.

I. Out of reality, for different time series data, we may need different kinds of state space models, so I hope we can talk about how to find one corresponding state space model given one time series. I already did some reading, the main source is Section 3.4 of Book <time series analysis by state space methods>

<https://www.oxfordscholarship.com/view/10.1093/acprof:oso/9780199641178.001.0001/acprof-9780199641178-chapter-3>

For example, how to find that of 'airline model' ( $ARIAM(0,1,1) \times (0,1,1)_{12}$ )? In the report of Durbin, he mentioned

Now consider what happens when seasonality is added to the picture. Starting with the random walk plus noise model (3), add a seasonal term  $\gamma_t$  giving

$$y_t = \mu_t + \gamma_t + \epsilon_t .$$

If the seasonal pattern were constant over time, the  $\gamma_t$ s would satisfy the condition  $\gamma_t + \gamma_{t-1} + \dots + \gamma_{t-s+1} = 0$ , where  $s$  is the number of 'months' per 'year'. To allow the seasonal pattern to change over time, we add a white noise term  $\omega_t$  and obtain the structural model

$$y_t = \mu_t + \gamma_t + \epsilon_t, \quad \mu_t = \mu_{t-1} + \eta_t, \quad \gamma_t = -\gamma_{t-1} - \dots - \gamma_{t-s+1} + \omega_t . \quad (9)$$

This satisfies the general state space forms (1) and (2).

Now take first differences and first seasonal differences of (9). We find

$$\Delta \Delta_s y_t = \eta_t - \eta_{t-s} + \omega_t - 2\omega_{t-1} + \omega_{t-2} + \epsilon_t - \epsilon_{t-1} - \epsilon_{t-s} + \epsilon_{t-s-1} ,$$

which is a stationary time series with non-zero autocorrelations at lags 1, 2,  $s-1$ ,  $s$  and  $s+1$ . Consider the BJ model, which in expanded form is

$$\Delta \Delta_s y_t = v_t - \theta_1 v_{t-1} - \theta_s v_{t-1} + \theta_1 \theta_s v_{t-s-1} .$$

This is the famous 'airline model' of Box and Jenkins which has been a good fit to many economic time series containing trend and seasonal effects. It has non-zero autocorrelations at lags 1,  $s-1$ ,  $s$  and  $s+1$ . Now the autocorrelation at lag 2 from model (9) arises only from  $\text{var}(\omega_t)$  which in most cases in practice is small. Thus when we add seasonal effects to the models we find again a close correspondence between the state space and the BJ models. A slope term  $\beta_t$  can be added to (9) as in (8) without affecting the conclusions.

But this model is not accurate(does not reflect coefficients), shall we talk about the specific expression?

II. I am learning some packages(basically is pomp, some with BayesianTools) related to state space methods and could talked about them.