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Source: *Journal of Business & Economic Statistics*, Vol. 2, No. 4 (Oct., 1984), pp. 291-320
Published by: Taylor & Francis, Ltd. on behalf of American Statistical Association
Stable URL: https://www.jstor.org/stable/1391266
Accessed: 18-12-2019 00:17 UTC

© Journal of Business & Economic Statistics, Vol. 2, No. 4, October 1984

Issues Involved With the Seasonal Adjustment of Economic Time Series

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In the first part of this article, we briefly review the history of seasonal adjustment and statistical time series analysis in order to understand why seasonal adjustment methods have evolved into their present form. This review provides insight into some of the problems that must be addressed by seasonal adjustment procedures and points out that advances in modern time series analysis raise the question of whether seasonal adjustment should be performed at all. This in turn leads to a discussion in the second part of issues involved in seasonal adjustment. We state our opinions about the issues raised and review some of the work of other authors. First, we comment on reasons that have been given for doing seasonal adjustment and suggest a new possible justification. We then emphasize the need to define precisely the seasonal and nonseasonal components and offer our definitions. Finally, we discuss criteria for evaluating seasonal adjustments. We contend that proposed criteria based on empirical comparisons of estimated components are of little value and suggest that seasonal adjustment methods should be evaluated based on whether they are consistent with the information in the observed data. This idea is illustrated with an example.

KEY WORDS: Seasonal adjustment; Model-based seasonal adjustment; Seasonality; Signal extraction; Time series; Census X-11.

> When most consumers of seasonally adjusted data—and that includes nearly every economically literate person—are confronted by the question of why they prefer such a series to the original, the most common and natural reaction is that the answer is obvious. Yet on further reflection the basis for such a preference becomes less clear, and those who give the matter extensive thought often finish by becoming hopelessly confused. (Grether and Nerlove 1970, p. 685)

1. INTRODUCTION

The impact of seasonally adjusted data on modern U.S. society is pervasive. The Federal Reserve Board sets monetary policies based in part on seasonally adjusted data; presidential and congressional economic policies are influenced by seasonally adjusted economic indicators; and seasonally adjusted values are routinely reported by the news media. Although unadjusted figures are also published, they do not receive the attention of the adjusted data. Thus society is conditioned to expect and even demand seasonally adjusted data.

Even though the public appears for the most part to be comfortable with seasonally adjusted data, we doubt that many users understand the methods by which the data are produced. It may be too much to expect the statistically unsophisticated person to understand the procedures underlying seasonal adjustment, but even statistical experts are often mystified by these procedures, including the most widely used method, Census X-11. This method uses a set of moving averages to produce seasonally adjusted data; and although the basic idea of moving averages is simple enough, the method in which they are applied in the X-11 program is extremely complex. Moreover, the theoretical statistical underpinnings of X-11 and many other seasonal adjustment methods are not understood by many users. Thus many users of adjusted data merely trust that the adjustment procedure is providing useful data, and critics have advocated the abolishment of seasonal adjustment.

The purposes of this article are to express some ideas about seasonal adjustment, to attempt to clarify certain aspects of the subject, and to stimulate discussion in areas that need more attention. Our thinking on seasonal adjustment has been structured around three questions:

1. Why has seasonal adjustment been done in the past, and why have the current procedures evolved into their present forms?
2. Why should one do seasonal adjustment?
3. Given that seasonal adjustment is desirable, how should it be done?

In Section 3, we will attempt to answer the first question by giving a historical overview of developments in seasonal adjustment and relating them to developments in time series analysis. We shall see that seasonal adjustment was initially developed in the 1920's and 1930's as a tool for the analysis of seasonal economic time series in the absence of suitable statistical models for such series. The methods were developed empirically, using tools such as moving averages. Adequate models for seasonal series were not used until the 1950's, and they did not come into widespread use until after the publication of the time series book by Box and Jenkins in 1970 and the subsequent development of computer software for time series modeling.

In the 1950's, Julius Shiskin started doing seasonal adjustments on electronic computers at the Bureau of the Census, which permitted the adjustment of large numbers of time series. This advance also marked a transition for seasonal adjustment from a tool used by analyzers of data to a requirement of data publishers.

As time series models and related computer software have become widely used in recent years, seasonal adjusters have looked to time series modeling to solve some of the problems in seasonal adjustment. This search has led to the development of such approaches as the X-11 ARIMA method and various model-based methods. Considering, however, that seasonal adjustment developed as an analytic tool in the absence of suitable models for seasonal time series, and that it is now possible to adequately model many seasonal time series, it is not clear what is gained in general by seasonal adjustment. The use of models in connection with seasonal adjustment raises questions about whether seasonal adjustment should be done at all.

This leads us, in Section 4, to investigate the reasons for seasonal adjustment. In our view, reasons that have been given in the past for seasonal adjustment have tended to be too vague. We suggest that consumers of adjusted data should be concerned that simplifications resulting from seasonal adjustment not be at the expense of a significant loss of information. Seasonally adjusted data are useful to the statistically unsophisticated user only if information loss is small. We review the literature related to information loss in the seasonal adjustment process and contend that the results to date are inconclusive and that more research into this area is desirable.

Since the question of whether to do seasonal adjustment is a difficult one, and since seasonal adjustment is presently a requirement of data publishers, we also consider how one should do seasonal adjustment given that it is desirable. Methods of seasonal adjustment are determined by the assumptions made, explicitly or implicitly, about the components. We thus argue that it is essential to define rigorously the components being estimated. This has not been done in the past. We present an approach to defining the components and attempt to justify our definitions. A rigorous definition of the components makes it possible to examine critically the assumptions underlying an adjustment method and to compare the differences in assumptions for different methods.

Finally, we discuss the evaluation of seasonal adjustment procedures. Reviewing approaches that have been suggested, we argue that empirical comparisons based on criteria for a "good" adjustment are for the most part useless in evaluating competing methods. We recommend examining the assumptions underlying adjustment methods, which must remain subjective to an extent but can be partially checked against the data. We therefore believe that the most important criterion is that a seasonal adjustment method be consistent with the information about seasonality present in the data being adjusted. We present an approach to assessing whether this is the case.

We emphasize to the reader that we will not attempt to answer all questions involved with seasonal adjustment. Many of the issues involved are complex, some are nonstatistical, and there will always remain some arbitrary elements. We do feel, however, that insufficient attention has been given to several of these issues. We hope to shed new light on some of them and, perhaps most important, to stimulate further discussion and research, ultimately leading to a better understanding of seasonal adjustment.

2. DEFINITIONS AND NOTATION

Seasonal adjustment involves the decomposition of an observed time series, Z_t, into unobserved seasonal and nonseasonal components, S_t and N_t. The underlying decomposition is usually viewed as either additive, $Z_t = S_t + N_t$, or multiplicative, $Z_t = S_t \cdot N_t$. By taking logarithms, the multiplicative decomposition becomes additive; thus for the purpose of analysis, we shall use the additive decomposition. The nonseasonal component can be further decomposed into trend and irregular components if desired; however, we shall not consider this decomposition, for reasons of simplicity.

Many approaches to seasonal adjustment use symmetric moving averages in estimating S_t and N_t. A symmetric moving average of Z_t (of length $2M + 1$) is $(2M + 1)^{-1} \sum_{j=-M}^{M} Z_{t+j}$, or more generally $\sum_{j=-M}^{M} \alpha_j Z_{t+j}$ (sometimes called a weighted symmetric moving average), where $\alpha_j = \alpha_{-j}$ and $\sum_{j=-M}^{M} \alpha_j = 1$. In estimating S_t, it is relevant to use seasonal moving averages that use only values of a time series for the same calendar month. For t near enough to the end of the observed data so that not all of Z_{t+j} in $\sum_{j=-M}^{M} \alpha_j Z_{t+j}$ are available, either an asymmetric ($\alpha_j \neq \alpha_{-j}$) moving average is used, or the data are augmented with forecasts so that the symmetric moving average may be used.

We shall use the seasonal autoregressive integrated moving average (ARIMA) time series model (Box and Jenkins 1970):

$$(1 - \Phi_1 B^s - \cdots - \Phi_P B^{sP})(1 - \phi_1 B - \cdots - \phi_p B^p)$$
$$(1 - B^d)(1 - B^s)^D Z_t$$
$$\times (1 - \theta_1 B - \cdots - \theta_q B^q)(1 - \Theta_1 B^s - \cdots - \Theta_Q B^{sQ}) a_t$$

or

$$\Phi(B^s)\phi(B)(1 - B)^d(1 - B^s)^D Z_t = \theta(B)\Theta(B^s) a_t.$$

Here, B is the backshift operator ($BZ_t = Z_{t-1}$); the seasonal and nonseasonal autoregressive (AR) operators, $\Phi(B^s)$ and $\phi(B)$, have zeros outside the unit circle; the seasonal and nonseasonal moving average (MA) operators, $\Theta(B^s)$ and $\theta(B)$, have zeros outside or on the unit circle; and the a_t's are independent and normally distributed with zero mean and variance σ_a^2. For short, we will write this as $\phi^*(B)Z_t = \theta^*(B)a_t$, where $\phi^*(B) = \Phi(B^s)\phi(B)(1 - B)^d(1 - B^s)^D$ and $\theta^*(B) = \theta(B)\Theta(B^s)$. We assume that we are dealing with monthly time series, so $s = 12$; however, our remarks apply equally well to series of other seasonal periods such as quarterly ($s = 4$) series.

When Z_t follows the ARIMA$(p, d, q)x(P, D, Q)_{12}$ model given above, its spectral density, $f_Z(\lambda)$, is given by

$$f_Z(\lambda) = \frac{\sigma_a^2 \theta^*(e^{i\lambda})\theta^*(e^{-i\lambda})}{2\pi \phi^*(e^{i\lambda})\phi^*(e^{-i\lambda})} \quad \lambda \epsilon [-\pi, \pi]$$
$$= \frac{\sigma_a^2}{2\pi \Pi(e^{i\lambda})\Pi(e^{-i\lambda})},$$

where

$$\Pi(B) = \phi^*(B)/\theta^*(B).$$

The model $\Pi(B)Z_t = \sum_{j=0}^{\infty} \Pi_j Z_{t-j} = a_t$ is the infinite autoregressive form of the ARIMA model. Strictly speaking, $f_Z(\lambda)$ expressed here is not correct when $d > 0$ or $D > 0$, since then Z_t is nonstationary and does not have a spectral density. As defined in this case, however, $f_Z(\lambda)$ is still useful in theoretical manipulations if one is careful to make sure the end results are correct. In particular, spectral densities defined in this way are useful in doing signal extraction, which is used in model-based seasonal adjustment. Bell (1984) discussed the assumptions under which such results are correct.

Notice that $f_Z(\lambda)$ given here is well defined when $d > 0$ for all $\lambda \epsilon [-\pi, \pi]$ except for $\lambda = 0$, and when $D > 0$ except for $\lambda = 0$ and for the seasonal frequencies $\lambda = k\pi/6k = \pm 1, \ldots, \pm 6$. The denominator in $f_Z(\lambda)$ is zero for these λ, and at these values we will define $f_Z(\lambda)$ to be $+\infty$.

Our use of ARIMA models in this discussion of seasonal adjustment does not imply that we could not have used other types of time series models. ARIMA models are widely used and are convenient for our purposes, but our comments would generally apply when other types of time series models are used. We are more interested in drawing distinctions between time series modeling and seasonal adjustment than between different approaches to time series modeling.

3. HISTORICAL PERSPECTIVES

To investigate the first question posed (concerning past justification for and evolution of seasonal adjustment procedures), it is useful to examine the historical development of both seasonal adjustment and time series analysis. By comparing the development of both, we can see how seasonal adjustment and time series analysis dealt with various problems presented by economic time series and why seasonal adjustment might have been preferred to other methods of analysis. We shall also review model-based adjustment methods to see why empirical methods of adjustment may have been preferred to these and to understand what recently proposed model-based methods may have to say about seasonal adjustment today.

In considering the historical development of seasonal adjustment, we must admit that tradition doubtless played an important role. Many seasonal adjusters, even those currently practicing, may have studied unobserved components in time series because this was the traditional approach, not worrying about whether techniques other than seasonal adjustment might better serve their ultimate objectives. To shed some light on issues surrounding seasonal adjustment today, we will examine what options were available to early seasonal adjusters and ask how the choices made among available methodologies could have been justified, though these alternatives may not have been seriously considered by some people.

3.1 Historical Development of Seasonal Adjustment

This discussion of developments in seasonal adjustment concentrates on work done in the United States. This is partly justified by the fact that the Census X-11 seasonal adjustment method is today the most widely used method; therefore, it is relevant to look at the progression of events leading up to X-11, most of which took place in the United States. BarOn (1973) and Burman (1979) discussed seasonal adjustment methods used in other countries, and Dagum (1978) and Nerlove, Grether, and Carvalho (1979) gave historical discussions of seasonal adjustment from somewhat different points of view than the one given here. Pierce (1980a) discussed recent work in seasonal adjustment.

Nerlove, Grether, and Carvalho (1979) pointed out that the idea that an observed time series comes from several unobserved components is an old one that came originally from astronomy and meteorology and became popular in economics in England during the period 1825–1875. They also gave an extensive discussion of the work of Dutch meteorologist Buys Ballot (1847), who is frequently cited as an early seasonal adjustment reference. For our purpose, it is appropriate to begin our survey somewhat later.

3.1.1 1920's and 1930's

There was a substantial amount of work on seasonal adjustment in the 1920's and early 1930's, much of it inspired by the work of Persons (1919). He viewed time series as being composed of (a) a long-time tendency or secular trend, (b) wave-like or cyclical movements, (c) seasonal movements, and (d) residual variation. He presented a method, called the link-relative method, for isolating the components and used detrended and seasonally adjusted data to construct business indexes. (For a concise description of Persons's method, see Persons 1923, pp. 714–716.) Persons was not the first to do seasonal adjustment or to specify the four basic components. (Persons (1919) referred to a 1910 study by E. W. Kemmerer in which seasonal adjustment was done. Yule (1921) said that the four components were fixed by 1914 and quoted March (1905) as saying that one must distinguish "des changements annuels, des changements polyannuels (décennaux par exemple), des changements seculáires, sans parler des périodes plus courtes qu'une année," which roughly translates to the seasonal, cyclical, secular trend, and residual components.) Persons may, however, have been the first to come up with a method that people felt could adequately decompose economic series. At any rate, his work led to an explosion of interest in seasonal adjustment.

Several important concepts regarding seasonal components and adjustment became fixed in the 1920's and early 1930's. These included (a) the idea that seasonality changes over time, (b) the need to account for trends and cycles when estimating the seasonal component, (c) the impossibility of describing trends and cycles by explicit mathematical formulas, and (d) the need to deal with extreme observations.

Changing seasonality was noted as early as 1852 by Gilbart (1852; quoted by Kuznets 1933), who found it in the circulation of bank notes. Persons observed, "Although we wish to ascertain if a systematic variation exists it is not accurate to think of seasonal variation (or, for that matter, the other types of fluctuations) as being exactly the same year after year" (1919, p. 19). Persons used fixed seasonal factors, however, when adjusting, probably because he did not see a convenient way to produce varying seasonal factors. According to King (1924), the first researchers to adjust data with varying seasonal factors were Sydenstricker and Britten of the U.S. Public Health Service, who were investigating causes of influenza. Their graphical method is briefly described in Britten and Sydenstricker (1922). King (1924) modified Sydenstricker and Britten's method, retaining some graphical elements but also using moving medians (taking the median of successive sets of $2M + 1$ data points), and reemphasized the need to account for changing seasonality. Snow (1923) suggested fitting straight lines to each quarter (or month) separately and checking for varying seasonality by examining the lines to see whether they were parallel. Crum (1925) gave a general discussion of varying seasonality and modified Persons's link-relative method to handle changing seasonality. Other methods of dealing with changing seasonality were suggested by Hall (1924), Gressens (1925), Clendenin (1927), and Joy and Thomas (1928). Kuznets (1932) suggested a method to detect and adjust for changes in seasonal amplitude from year to year assuming the seasonal pattern remained constant. Mendershausen (1937) reviewed efforts made theretofore to deal with changing seasonality.

The early writers discovered it was necessary to adjust data for the effects of trend before, or at the same time as, estimating the seasonal component. (For example, direct estimation of seasonal effects, using complete calendar year data with an upward trend, results in seasonal factors that are too low in January and too high in December. Furthermore, seasonality in a series makes direct estimation of trend difficult.) We will refer to this problem (which eventually led to iteration between trend and seasonal estimation—something currently done in X-11) as nonseasonal nonstationarity. Several different approaches to this problem were used. Some authors made simple transformations of the data to remove trend, then obtained seasonal estimates and converted these to estimates of seasonal effects in the original series. In this group, Persons (1919) took the ratio of each monthly value to the preceding value—"link relatives"—and Robb (1929) took second differences of the original data. Other authors estimated trend first and then removed it, usually by division (i.e., $Z_t/\hat{T}_t$); this is called the *ratio to trend* approach. Here Falkner (1924) used a straight line trend; King (1924), a trend curve drawn freehand; and Joy and Thomas (1928) and Macauley (1931), moving average trend estimates (*ratio to moving average* method). Carmichael (1927) suggested a hybrid approach, taking first or second differences of the ratio of the data to a trend estimate. Finally, some authors (Snow 1923 and Clendenin 1927) estimated the trend separately for each series of values for a particular calendar month to simultaneously deal with both trend and seasonality.

Although there was initially some use of specific trend functions such as the linear trends of Snow (1923) and Falkner (1924) previously mentioned, it was generally felt by the 1930's that one should not specify a functional form for the trend. The prevailing attitude was reflected by Macauley: "The type of smooth curve which might be expected to appear in any particular time series if the series were unaffected by the minor or temporary factors which give rise to seasonal and erratic fluctuations is not necessarily representable throughout its length by any simple mathematical equation" (1931, p. 38). Thus it was natural for Macauley and others to consider using moving averages and actuarial graduation formulas to obtain trends, rather than using explicit functions of time.

Finally, there was concern about the influence of extreme observations. For example, Falkner objected to the use of monthly means in seasonal adjustment primarily for this reason, stating, "The arithmetic average is peculiarly subject to extreme items, and it is for that reason that a monthly seasonal index obtained by this method may be governed more by an exceptional irregular deviation than by the systematic seasonal movement" (1924, pp. 168–169). Concern about the effects of outliers led Persons (1919) and others to use medians instead of means in deriving seasonal factors (some replaced moving averages by moving medians). Crum (1923a) suggested using medians or trimmed means, and Falkner (1924) and Joy and Thomas (1928) also advocated the use of the latter. These trimmed means involved considerable trimming, with the mean being computed using as few as two or three observations. Although the need to deal with extreme observations was recognized early, the problem of how to do it has continued to the present day.

3.1.2 *Impact of Computers on Seasonal Adjustment*

The next major development in seasonal adjustment did not come until 1954, when Julius Shiskin started doing seasonal adjustments (Method I) on the Univac 1 computer at the Bureau of the Census (see Shiskin 1957, 1978). Method II was introduced in 1955, with successive variants culminating in the development of X-11 in 1965 (Shiskin, Young, and Musgrave 1967). Soon after Shiskin's efforts in 1954, other organizations in the United States and abroad began using the Census method or developing their own computer methods. As a result of the interest in doing seasonal adjustment on electronic computers, a conference on the subject was held in Paris in 1960 (Organization for Economic Cooperation and Development 1960).

One of the objectives in doing seasonal adjustment on computers was to increase the number of series that could be adjusted. Shiskin stated that in 1954,

> Principal users of current economic series—for example, the chairman of the Council of Economic Advisers and the chief economist of the National Industrial Conference Board—complained that many of the monthly series published by the government were not adjusted for seasonal variations at all; that many others were adjusted by crude methods; and that for still others the seasonal adjustments did not reflect the most recent experience. (1957, p. 245)

He further noted that this was "attributable primarily to the huge amount of computation required and to the large costs involved" and that "the large-scale digital electronic computer has brought an end to this situation." With electronic computers, literally thousands of time series could be seasonally adjusted by government agencies. This capability had important implications for the procedures that were developed: The calculations required could now be complicated, but the amount of time that could be spent in determining how best to adjust each particular series was reduced. This was something of a reversal of the situation prior to computers. The adjustment methods developed (including X-11) were basically complex modifications of previously used methods that attempted to incorporate automatically, at least to a degree, the professional judgment that was previously required. This automation helped lend an air of objectivity to the seasonal adjustment process, so seasonal adjusters would not be accused of tampering with the data, a consideration that has become even more important in recent years. In this respect, the situation today is much different from that of the 1920's, when some people advocated freehand smoothing (e.g., King 1924) as part of their adjustment method.

Another important development in seasonal adjustment methodology facilitated by computers was the use of regression techniques to account for trading day variation. Important work on this was done by Eisenpress (1956), Marris (1960), and Young (1965), whose approach was incorporated into the X-11 program. Before this work, adjustments for trading day effects were generally based on a priori evidence or opinions about the proportion of activity occurring on each day of the week; Young (1965) discussed some of the difficulties with such an approach. Holiday effects, too, are important in some series and have been considered for many years (e.g., see Joy and Thomas 1927 and Homan 1933). Even today, however, adjustments for holiday effects tend to be made on an ad hoc basis, although recently Hillmer, Bell, and Tiao (1983a) suggested a modeling approach for dealing with holiday effects in seasonal adjustment.

3.1.3 Recent Developments

In recent years, there have been many attempts to improve the seasonal adjustment process. The most important recent development is the X-11 ARIMA method of Dagum (1975), which involves forecasting the data one year ahead using an ARIMA model. The forecasted values are used as if they were actual data so that the filters used in adjusting current data are closer to the symmetric filter that will eventually be used when more data are available. Similar approaches using autoregressive models were investigated by Geweke (1978a) and by Kenny and Durbin (1982). The idea of forecasting the series for this purpose is not new; it was recommended by Macauley (1931, pp. 95–96). Statistics Canada and the U.S. Federal Reserve Board use X-11 ARIMA. In the United States the Bureau of Labor Statistics also uses it on many of their series, and the Bureau of Economic Analysis applies it to some of their series, too. It remains to be seen what action the Bureau of the Census will take. Eventually (typically after three years) the X-11 ARIMA adjustments converge to the X-11 adjustments, so discussion of the characteristics of X-11 is relevant to X-11 ARIMA as well.

3.2 Historical Development of Time Series Analysis and Its Relation to Seasonal Adjustment

In considering historical developments in time series, we are interested in the question of why people used seasonal adjustment as an analysis technique rather than other time series methods. The developments mentioned here were chosen with this in mind. We concentrate on relevant developments in time series modeling but also mention some important developments in spectral analysis and signal extraction. In reviewing the history of time series analysis, it is useful to keep in mind the following essential problems presented by data being seasonally adjusted: (a) changing seasonality, (b) nonseasonal nonstationarity (trends and cycles), (c) the impossibility of describing seasonality, trends, and cycles by simple mathematical functions of time, and (d) outliers.

3.2.1 Time Series Modeling

The first important developments in time series modeling were the introduction in 1927 of autoregressive models by Yule and moving average models by Slutsky. Yule (1927) discussed properties of autoregressive models, introduced partial autocorrelations, and fit low order models to Wolfer's annual sunspot series by least squares. In his 1927 paper, Slutsky (1937) introduced moving average models and investigated how these models could lead to cyclical series. Wold (1938) was the first to fit moving average models to data, and he introduced the important innovations representation for stationary series and solved the prediction problem.

During the 1940's, progress was made in the area of inference for time series models. Mann and Wald (1943) derived asymptotic theory for parameter estimation in autoregressive models. Champernowne (1948) suggested the use of least squares estimates for autoregressive models and autoregressive models with regression terms, although he did not derive properties of the estimators. Cochrane and Orcutt (1949) suggested autoregressive filtering or differencing of the dependent and independent variables when using a regression model with autocorrelated errors. The asymptotic theory for sample autocorrelations was developed by Bartlett (1946) and Moran (1947).

Whittle (1952) seems to have been the first to use high lags in time series models to account for seasonality. Using a model discrimination procedure, he arrived at a model of the form

$$Z_t - \phi_1 Z_{t-1} - \phi_2 Z_{t-2} - \phi_8 Z_{t-8} = a_t$$

for the Beveridge (1921, 1922) wheat price series. In Whittle (1953a, 1954a) he used the model

$$Z_t - \phi_1 Z_{t-1} - \phi_{22} Z_{t-22} = a_t$$

for six-month sunspot data with a cycle of 22 periods. At about the same time, Whittle (1953a,b) derived properties of approximate maximum likelihood parameter estimates for a general model including autoregressive moving average models as a special case. He then (Whittle 1954b) obtained results for simultaneous estimation of regression and time series parameters. In an effort to find simpler procedures than Whittle's, Durbin suggested another approach and obtained results for moving average models (1959), models with regression terms and autoregressive errors (1960a), and mixed autoregressive moving average models (1960b). Walker suggested still another approach and obtained results for moving average models (1961) and autoregressive moving average models (1962).

More recently, the publication of the book by Box and Jenkins (1970) and the development of suitable computer software have led to the growing popularity and widespread use of ARIMA models in the analysis of time series data. ARIMA models use nonseasonal and seasonal differencing to deal with nonseasonal and seasonal nonstationarity. Although differencing was suggested many years before in other contexts by Carmichael (1927), Robb (1929), and many others (e.g., see the literature on the "variate-difference method," Tintner 1940), and seasonal differencing was even considered by Yule (1926), Box and Jenkins popularized it as part of a modeling procedure for nonstationary series. Furthermore, for ARIMA models to be useful in the analysis of seasonal time series, lags as high as the seasonal period are needed. Other than Whittle's attempts in the 1950's, this type of model appears not to have been used prior to the introduction of the multiplicative seasonal model by Box and Jenkins. The multiplicative seasonal model provided a representation involving relatively few parameters, which was a good approximation for many seasonal time series.

Finally, approaches to handling outliers when modeling time series were presented by Fox (1972), Abraham and Box (1979), Denby and Martin (1979), Martin (1980), Chang (1982), (see also Hillmer, Bell, and Tiao 1983a), and Bell (1983). For outliers with an assignable cause, the intervention analysis of Box and Tiao (1975) is relevant. Historically, outliers have presented more of an obstacle to time series modeling than they do today. Still, more work needs to be done on outliers both for time series modeling and seasonal adjustment.

3.2.2 *Spectral Analysis*

Spectral analysis actually became available before time series modeling and the work on seasonal adjustment discussed earlier, with the introduction of the periodogram by Schuster (1898). Since spectral analysis can be used to look for periodic components in time series, it would seem to be useful to investigators of economic cycles. Beveridge (1921,1922), in fact, used the periodogram to look for cycles in a detrended series of wheat prices. Fisher (1929) suggested a significance test for detecting periodicity in a time series. Daniell (1946), Bartlett (1950), and Tukey (1950) suggested smoothed periodogram spectral estimators, and many other spectral estimators have been developed since then. Furthermore, spectral analysis has become more practical in recent years with the advent of electronic computers and improved computational techniques, especially the fast Fourier transform (Cooley and Tukey 1965). Spectral analysis for nonstationary time series was investigated by Priestley (1965) and Hatanaka and Suzuki (1967). For a more extensive historical survey of spectral analysis, see Robinson (1982).

Despite Beveridge's work, spectral analysis was not widely used on economic time series in the early days of seasonal adjustment. One problem, as noted by Kendall (1945), was that people used the periodogram to look for exact periodicities, but economic cycles are not exactly periodic. This problem was overcome with the development of improved spectral estimators and a better understanding of spectral analysis. A more permanent problem was identified by Crum (1923b), who criticized use of the periodogram on economic series, saying that seasonality influences the appearance of cycles in the periodogram, thus making them more difficult to detect. (Crum advocated seasonal adjustment.) In modern terms, this is known as "leakage." A typical approach today to doing spectral analysis with seasonal series is to remove or reduce the seasonal effects by prefiltering the data, which leads right back to seasonal adjustment.

3.2.3 *Signal Extraction*

The signal extraction problem is to estimate the signal S_t in $Z_t = S_t + N_t$ when the observations Z_t contain "noise" N_t. Kolmogorov (1939,1941) and Wiener (1949) independently solved this problem for stationary time series, obtaining $\hat{S}_t$ to minimize $E[(S_t - \hat{S}_t)^2]$ for any linear function, $\hat{S}_t$, of the observations, Z_t. Hannan (1967), Sobel (1967), Cleveland and Tiao (1976), and Bell (1984) extended this result to nonstationary time series. Identifying S_t and N_t as the seasonal and nonseasonal components, signal extraction can be used, in conjunction with suitable models for Z_t, S_t, and N_t, to do seasonal adjustment. This approach has been taken in recent years by a number of authors, whom we discuss in the next section.

3.3 Model-Based Seasonal Adjustment

Some early authors criticized the popular empirical approaches to seasonal adjustment. For example, Snow criticized Persons's approach, saying, "The method of allowing for seasonal variations seems cumbersome and the logic of it is not clear" (1923, p. 334). Fisher said,

> To the student of mathematics it appears strange that economists and statisticians have adopted such rather primitive methods in measuring seasonal variations when, as a matter of fact, more elegant and also more practical mathematical tools, requiring a far smaller amount of tedious arithmetical calculations than the methods of the gifted academic schoolmen, have been available for more than half a century. (1937, p. 179)

(The more elegant and more practical tools Fisher refers to are the orthogonal polynomials of J. P. Gram and the quasi-systematic error theory of T. N. Thiele.) This dissatisfaction with the empirical nature of many seasonal adjustment methods led these and later authors to investigate the use of time series models to do seasonal adjustment. We shall refer to such methods of seasonal adjustment as model-based methods.

Model-based methods of seasonal adjustment generally use an additive decomposition, $Z_t = S_t + N_t$, or an additive decomposition for some transformation of Z_t (such as ln Z_t), and use explicit statistical models (or spectral densities) for Z_t, S_t, and N_t. The model for Z_t can be estimated from observed data, but since S_t and N_t cannot be observed, their models depend on arbitrary assumptions (see Sec. 4 of this article). The various methods differ in the type of model fit to the observed Z_t's and in the assumptions used in specifying models for S_t and N_t. S_t and N_t are estimated either directly when fitting the model for Z_t (as in regression methods) or after fitting the model for Z_t by using signal extraction theory.

Regression methods provided the first model-based approaches to seasonal adjustment. The basic approach consists of specifying functional forms for the trend and seasonal components that depend linearly on some parameters, estimating the parameters by least squares, and subtracting the estimated seasonal component from the data. The most popular specifications use polynomials in time for the trend component and seasonal means for a stable seasonal component (with modifications for handling changing seasonality). The error terms are generally assumed to be white noise, although Rosenblatt (1965) pointed out that the regression residuals tend to be autocorrelated and that this should be allowed for.

Regression methods of seasonal adjustment were proposed by Hart (1922), Snow (1923), Fisher (1937), Mendershausen (1939), Cowden (1942), Jones (1943), Hald (1948), Eisenpress (1956), Hannan (1960,1963), Lovell (1963,1966), Jorgenson (1964,1967), Rosenblatt (1965), Henshaw (1966), and Stephenson and Farr (1972). These efforts seem to have had little effect on the way U.S. government agencies perform seasonal adjustment. It may be that the regression approach was doomed from the start, since it requires explicit specification of the mathematical forms of the trend and seasonal components. We have indicated that as early as the 1930's, seasonal adjusters felt that this could not be done effectively.

Recently there has been considerable interest in using either stochastic models or spectral estimates to do seasonal adjustment by signal extraction. The first such model-based approach to seasonal adjustment was that of Hannan (1964), who filtered the data to remove trends and chose a model for the seasonal component, consisting of trigonometric terms at the seasonal frequencies multiplied by independent time series following first-order autoregressive models. These models were stationary but the approach was extended to nonstationary (random walk) models by Hannan (1967) and Hannan, Terrell, and Tuckwell (1970), where the approach is described in detail (see also Sobel 1971). The method required ad hoc specification of the relative magnitude of the seasonal and nonseasonal spectral densities near the seasonal frequencies.

Methods based on spectral estimation were suggested by Melnick and Moussourakis (1974) and Geweke (1978b). Melnick and Moussourakis estimated the spectrum of the data after detrending it with a least squares straight line and then determined empirically the neighborhoods of the seasonal frequencies that they assumed contained all of the seasonal power. They used spectral ordinates outside these neighborhoods in estimating the (detrended) nonseasonal spectrum within the neighborhoods and thus obtained their seasonal adjustment filter. Geweke estimated the spectrum of the original data at the seasonal frequencies by the periodogram ordinates and at other frequencies by smoothing the periodogram while leaving out the seasonal ordinates. The spectrum of the nonseasonal component was estimated by smoothing the periodogram with ordinates at and near the seasonal frequencies left out. He also used this approach with spectral density matrices to do multivariate seasonal adjustment via multivariate signal extraction—simultaneously seasonally adjusting several time series.

Several authors have suggested seasonal adjustment methods that involve fitting an ARIMA model (possibly with deterministic terms) to Z_t and using this along with some assumptions to determine models for S_t and N_t. Pierce (1978) suggested using ARIMA models and deterministic terms to allow for both stochastic and deterministic trends and seasonality. After estimating and removing the deterministic effects, he filtered the resulting series (the filter usually included differencing) to remove stochastic trends and specified a seasonal ARMA (1, 1) model for the filtered stochastic seasonal component when stochastic seasonality was present. This model was identified using assumptions, including an assumption that the variance of the seasonal be the minimum value consistent with the model. Wecker (1978) suggested an extension to Pierce's approach. Box, Hillmer, and Tiao (1978) started with the model

$$(1 - B)(1 - B^{12})Z_t = (1 - \theta_1 B)(1 - \theta_{12} B^{12})a_t$$

and derived models for the seasonal, trend, and irregular components consistent with this overall model, using certain assumptions including an assumption that the variance of the irregular component should be maximized, which in turn minimizes the variance of both the seasonal and the trend components. This approach was later extended to more general ARIMA models by Burman (1980) and by Hillmer and Tiao (1982), who discussed some properties of the approach (see also Hillmer, Bell, and Tiao 1983a). Cleveland (1979) fit ARIMA models to the observed data after removing seasonal means and used simple ARIMA models for the components. He chose the component models' moving average parameters in an attempt to make these models approximately consistent with the model for the original series (the autoregressive parameters were determined by assumptions).

The preceding methods all involved determining ARIMA models for the components and then using signal extraction theory to estimate them. Brewer, Hagan, and Perazelli (1975) took a different approach, fitting an ARIMA model to Z_t and then decomposing interpolated values of Z_t (estimates of Z_t, using the data other than the observation at t) into seasonal and trend-cycle components. This was done by considering a seasonal-trend-cycle-irregular decomposition of the filter that produces one-step-ahead forecasts. A modification of this approach was later suggested by Brewer (1979). Roberts (1978a) suggested a related method in which part of the fitted ARIMA model is identified as a seasonal adjustment filter.

The final model-based approach we shall mention involves specifying parametric models for the components, which leads to a model for Z_t subject to constraints. Estimating the model for Z_t subject to the constraints also yields models for S_t and N_t that can then be used to perform seasonal adjustment by signal extraction. Engle (1978) used ARIMA models for the components; but finding estimation of the model for Z_t subject to the constraints to be computationally burdensome, he relaxed some of them. Others used models for the components that made the constrained estimation somewhat simpler. Abrahams and Dempster (1979) used fractional Brownian motion for the trend component and a modification of this for the seasonal component. Fractional processes generalize the idea of differencing a time series to stationarity, thus providing a generalization of ARIMA models (see Granger and Joyeux 1980 for a discussion). Akaike (1980) took a smoothness priors approach (related to that of Schlicht 1981), which led to ARIMA models for the components, and used an information criterion to select from among alternative models. Kitagawa and Gersch (1984) further developed this approach, extending it to allow a wider variety of ARIMA component models.

3.4 Summary and Conclusions

Seasonal adjustment developed in the early part of this century out of a tradition of looking for unobserved components in time series. Early seasonal adjusters found that their time series contained nonstationary trends and changing seasonality and that this behavior could not be described by explicit mathematical functions of time. They empirically developed seasonal adjustment methods, using such tools as moving averages to deal with these problems. Some early authors criticized the empirical nature of the early adjustment methods. Time series models capable of dealing with the series being adjusted, however, were not available at that time; thus early attempts at modeling and model-based adjustment failed.

In the 1950's, Whittle began using models suitable for the sort of time series being seasonally adjusted. Widespread use of such models followed the publication of the book by Box and Jenkins in 1970. While these models were being developed, government agencies started using electronic computers to seasonally adjust large numbers of time series. This made model-based methods impractical by comparison, at least until the recent development of computer software for use in modeling time series.

Whereas seasonal adjustment was originally done as part of the analysis of time series data by statisticians and economists, computerized seasonal adjustment has come to serve the needs of government officials, business managers, and journalists—on the whole, a statistically unsophisticated group with little interest in time series modeling. Furthermore, the responsibility for performing seasonal adjustments has shifted from the analyzers of the data to the publishers of the data.

In recent years, with the further development of time series models and associated computer software, seasonal adjusters have looked to time series models to improve seasonal adjustment methods. Examples are the X-11 ARIMA method, which is now being used by several government agencies, and the recently proposed stochastic model-based methods. We shall see in the next section, however, that if one can model a time series, then it is not clear what is gained by arbitrarily decomposing the series into seasonal and nonseasonal components. Thus the use of modeling in connection with seasonal adjustment raises the basic question of whether seasonal adjustment should be done at all.

4. CURRENT ISSUES IN SEASONAL ADJUSTMENT

Although seasonal adjustment has become a well-established practice for the historical reasons discussed in Section 3, it is time to take a fresh look at seasonal adjustment and seasonal adjustment methods. Thus in this section, we will address the second and third questions listed in the Introduction—those regarding the why and how of seasonal adjustment today. We will not dwell on technical details but, rather, hope to stimulate discussion about some of the broader issues. We will express some opinions about the issues raised and attempt to provide the reasoning that shaped our opinions. Our hope is not that everyone will agree with our opinions but, rather, that readers will see that there are a number of important issues that require extensive thought and discussion before they can be satisfactorily resolved.

4.1 Reasons for Seasonal Adjustment

In Section 3, we noted that seasonal adjustment was developed in the 1920's and 1930's as a tool for analyzing seasonal economic time series in the absence of suitable statistical and economic models for such series. In recent years, as new modeling procedures have become available, the reasons for doing seasonal adjustment have become less clear. Reasons given for seasonal adjustments have typically been rather vague, but they seem to follow three main themes: (a) to aid in doing short-term forecasting, (b) to aid in relating a time series to other series, external events, or policy variables, and (c) to achieve comparability in the series values from month to month.

Shiskin argued that adjusted data are useful in short-term forecasting when he said,

> A principal purpose of studying economic indicators is to determine the stage of the business cycle at which the economy stands. Such knowledge helps in forecasting subsequent cyclical movements and provides a factual basis for taking steps to moderate the amplitude and scope of the business cycle. (1957, p. 222).

He went on to say that knowledge of the seasonal pattern in sales of products "is needed by all companies to determine the level of production that is most efficient" and suggested that forecasts of a series can be obtained by taking forecasts of annual totals and allocating these to months in proportion to the seasonal factors. Burman said that the most common purpose of seasonal adjustment "is to provide an estimate of the current trend so that judgmental short-term forecasts can be made" (1980, p. 321).

Several authors have argued that seasonal adjustment is useful because seasonality in a series can obscure the relationships between the time series and other series, external events, or policy variables. It is hoped that seasonal adjustment will make these relationships easier to investigate and, in the case of relationships with policy variables, make them easier to exploit. With regard to using adjusted data in relating several series, Burman said that seasonal adjustment "may be applied to a large number of series which enter an economic model, as it has been found impracticable to use unadjusted data with seasonal dummies in all but the smallest models" (1980, p. 321). Furthermore, Granger saw a possible advantage in that "by using adjusted series, one possible source of spurious relationship is removed" (1978a, p. 39). An example of the use of seasonally adjusted data to examine the effect of external events on a series was provided by BarOn (1978), who related several seasonally adjusted economic series to unusual external events. Finally, governments use seasonally adjusted data in setting policy variables designed to control various aspects of their economies. According to Dagum, "The main causes of seasonality, the climatic and institutional factors, are exogenous to the economic system and cannot be controlled or modified by the decision makers in the short run" (1978, p. 10). Thus the nonseasonal component may be what can be controlled, to some degree, by government intervention, and so seasonally adjusted data are useful because they "provide the basis for decision making to control the level of the economic activities" (Dagum 1978, p. 14). Note that for some series, however, seasonality may also be controllable. For example, the Federal Reserve Board has effectively removed seasonality from interest rates through monetary policy.

The third reason given for seasonally adjusting data is that it makes values comparable from month to month. This may be true; but do we really want comparability, or should observations for different months be regarded differently? For instance, atmospheric temperature data are highly seasonal, but people seem comfortable with the original data. We suspect the desire for comparability has something to do with the two points discussed above—forecasting series and relating series to other series, external events, or policy variables.

4.1.1 Justification for Signal Extraction

Seasonal adjustment may be viewed as a signal extraction problem. In both cases, we observe $Z_t = S_t + N_t$, where S_t and N_t are unobserved components that we wish to estimate using the observed series Z_t. In signal extraction, S_t and N_t are "signal" and "noise," whereas in seasonal adjustment they are "seasonal" and "nonseasonal." Z_t can be a transformation of the original series, such as the logarithm, in which case we can view the decomposition as multiplicative. To put the issues regarding justification of seasonal adjustment in perspective, let us consider how one might justify doing signal extraction in general. That is, if we observe Z_t, why should we try to estimate S_t and N_t? To answer this in any given situation, we must consider three basic questions:

1. Is there reason to believe that the observed data Z_t are generated as $Z_t = S_t + N_t$?
2. Given $Z_t = S_t + N_t$, are we really interested in S_t and N_t, rather than Z_t or something else related to Z_t?
3. Given that we are interested in S_t and N_t, how can we estimate them?

For signal extraction to be appropriate, we must be able to answer adequately these three questions. In connection with the third question, it should be noted that standard signal extraction results on estimating the components require that the models for S_t and N_t be known.

Much of the original motivation for studying signal extraction came from problems in the field of communications engineering. In this field there are physical reasons that imply $Z_t = S_t + N_t$ (see e.g., Blanc-Lapierre and Fortet 1965, chap. 13, sec. 7). Here, S_t is an emitted signal, and the received signal, Z_t, is corrupted by noise, N_t. The problem is to produce an estimate, $\hat{S}_t$, as close as possible to the emitted signal S_t by attempting to remove the noise N_t. It is obvious that in communications engineering (a) the decomposition $Z_t = S_t + N_t$ makes sense and (b) the interest is in the signal S_t rather than the observed data Z_t. Furthermore, Yaglom noted:

1. The model for Z_t is calculated (or estimated) from observed data.
2. The model for N_t "can be determined by using the same measuring device and the same observer ... to make a series of measurements of any quantity whose value is known precisely, e.g., which equals zero because of the conditions of the experiment" (1962, p. 127).

Thus the models for Z_t and N_t, and hence for $S_t = Z_t - N_t$, can be obtained, so standard signal extraction results can be used to estimate the components. Therefore, the use of signal extraction methods in communications engineering is sensible.

Consider now how seasonal adjustment of economic time series fits into the framework of the three questions. Question 1 can always be answered affirmatively, in that the decomposition $Z_t = S_t + N_t$ is always mathematically possible. Whether Z_t was actually produced—by certain economic forces generating S_t and N_t separately and then combining them (additively or otherwise) to get Z_t—is another question. Some writers have regarded the seasonal, trend, and irregular components as arising from different economic factors. In particular, Mendershausen (1937,1939) advocated this point of view and attempted to model seasonality in terms of meteorological and social variables. Factors generating seasonality in financial data are discussed in a report to the Board of Governors of the Federal Reserve System (1981). Trading day adjustments, as done today, provide a causal explanation for some of the seasonality in economic series. Moreover, the idea that the nonseasonal component is subject to control through manipulation of policy variables, whereas the seasonal component is not, relates to the idea of S_t and N_t being generated separately. Today, however, little emphasis is placed on physical causes when adjustment is actually done; so without physical justification we view the decomposition as a mathematical one.

For seasonal adjustment, the answer to question 2 depends on the components' ultimate use and our ability to define the components precisely. For instance, if the purpose is short-term forecasting of Z_t, then S_t and N_t are not of direct interest, and some would argue that seasonal adjustment is unnecessary. We shall argue in Section 4.2 that the components have not been precisely defined. Until a more rigorous definition of the components is provided, it is difficult to justify the proposition that the components are of interest as ends in themselves.

Question 3 is not difficult to answer if the components can be precisely defined. If not, then it is difficult to construct estimators, since we do not know what is being estimated. With precisely defined components, it seems logical to use signal extraction theory to estimate them.

4.1.2 *Justification of Seasonal Adjustment*

We favor modeling series in terms of the original data, accounting for seasonality in the model, rather than using adjusted data. Others have voiced similar opinions. For example, Watts stated, "I have yet to be convinced that seasonal adjustment is the best thing to do to a series. I believe, rather, that the aim of time series model building should be to develop forecasting models that yield white-noise residuals" (1978, p. 307). Furthermore, Roberts said, "it appears to me that seasonal adjustments can be only a source of trouble to a statistician interested in forecasting unadjusted values" (1978b, p. 164) and "surely the route to better scientific understanding is to incorporate the seasonality directly into multivariate models that are formulated in terms of unadjusted data so the source, transmission, and effects of seasonal variations can be better understood" (p. 164). Some econometricians have argued that knowledge of a series' underlying economic structure can provide an understanding of the nature of seasonality in specific time series (see Crutchfield and Zellner 1963, Plosser 1978, and Wallis 1978). This knowledge permits the incorporation of seasonality directly into an economic model, eliminating the need to work with seasonally adjusted data. In fact Plosser (1978) argued that use of adjusted data could lead one to misspecified models, misleading inferences about parameters, and poor forecasts.

In light of these remarks and the previous discussion, it is relevant to ask whether seasonal adjustment can be justified, and if so, how? It is important to remember that the primary consumers of seasonally adjusted data are not necessarily statisticians and economists, who could most likely use the unadjusted data, but people such as government officials, business managers, and journalists, who often have little or no statistical training. We thus offer the following *possible* justification for seasonally adjusting time series:

> Seasonal adjustment is done to simplify data so that they may be more easily interpreted by statistically unsophisticated users without a significant loss of information.

We say "possible" justification because its validity has not yet been established. They key phrase is "without a significant loss of information." Obviously, many people have found seasonally adjusted data simpler to use than unadjusted data; but to establish that the above justification is valid, we need to know that the amount of information lost in adjusting is not excessive. In general, there will be some information loss from seasonal adjustment, even when an adjustment method appropriate for the data being adjusted can be found. The situation will be worse when the seasonal adjustment is based on incorrect assumptions. If people will often be misled by using seasonally adjusted data, then their use cannot be justified.

4.1.3 Loss of Information From Seasonal Adjustment

There has been some work on the consequences of using seasonally adjusted data. It has concentrated on how seasonal adjustment affects (a) forecast accuracy and (b) relating one series to another.

Makridakis and Hibon (1979) forecast 111 time series by various methods and compared the overall accuracy of the forecasts produced by different methods. They used methods that handled seasonal series directly (such as ARIMA modeling) and nonseasonal methods applied to seasonally adjusted data. With the latter methods, forecasts were reseasonalized by applying seasonal factors. Makridakis and Hibon used their own method of seasonal adjustment, which produced fixed seasonal factors. Their results do not permit direct assessment of the effects of seasonal adjustment on forecast accuracy because (a) the forecast results for seasonal and nonseasonal series were not separated and (b) most of the methods used directly on the seasonal series were not used in nonseasonal form with the adjusted data. Still, they found the methods that used seasonally adjusted data did somewhat better than the methods that handled seasonality directly—including forecasting with ARIMA models. Their results may have been influenced by the use of constant seasonal factors and measures of forecast accuracy that aggregate over series differing in how accurately they may be forecast (thus giving undue influence to series that are inherently difficult to forecast).

Plosser (1979) forecast five economic time series with seasonal ARIMA models and forecast the X-11 adjusted series with nonseasonal ARIMA models. Instead of reseasonalizing the forecasts of the adjusted data, he converted the monthly forecasts to annual totals to compare forecast accuracy. He used fully revised seasonally adjusted values, which could favor the use of the adjusted data because they are obtained by using future values of the series. He found that the seasonal ARIMA models performed substantially better on two series, slightly better on two series, and slightly worse on one series. These results seem to be inconclusive, since direct comparisons were not made in the Makridakis and Hibon paper and Plosser examined only five series.

There is, however, an important aspect of forecasting not considered in these two studies. This is the estimation of forecast error variances and the subsequent provision of forecast intervals for the future observations. There are well-established procedures for estimating forecast error variances and obtaining forecast intervals when using ARIMA or other time series models (Box and Jenkins 1970, chap. 5). Use of seasonally adjusted data in forecasting, however, whether the forecasting is done formally through a model or informally, seems to preclude estimation of forecast error variances and production of forecast intervals. This is obviously true for forecasting the unadjusted data, but it is also true if one wishes to forecast the adjusted data (though we question why anyone would want to do this). Future adjusted values depend on future values of the seasonal components through the future unadjusted data; hence forecast error variances for adjusted data should allow for errors in forecasting the seasonal component, but there is no way to do this with adjusted data. These problems will not be solved if, as has been recommended, government agencies start publishing standard errors for seasonally adjusted data, because it is not clear how to use these to produce forecast error variances.

Seasonally adjusted data have been used in relating time series (as in econometric modeling) presumably on the assumption that their use would eliminate the need to deal explicitly with seasonality in the model without altering the relationships between the series. We now survey some of the work that has been done on the consequences of using adjusted data for this purpose. A more detailed discussion of some of this work is given by Nerlove, Grether, and Carvalho (1979, pp. 162–171).

Lovell (1963,1966) and Jorgenson (1964,1967) investigated regression approaches to seasonal adjustment and the appropriateness of seasonally adjusting time series before subsequently using them in a regression analysis. Lovell (1963) showed that prior adjustment by regressing the dependent and explanatory variables on seasonal dummy variables can be appropriate, since this gives the same results as including the seasonal dummy variables in a regression with the unadjusted data. He also noted that adjusting effectively uses up some degrees of freedom and that results with the adjusted data should be modified accordingly. Jorgenson (1964) discussed optimal (i.e., minimum mean squared error) estimation of the seasonal component (in a regression model). Their subsequent papers (Lovell 1966 and Jorgenson 1967) point out that the optimal estimate of the seasonal component is not generally appropriate for adjusting series prior to relating them in a regression model.

Sims (1974) considered the estimation of a distributed lag relation between the nonseasonal components of two time series, y and x, when they are observed with seasonal noise added. He observed that the estimated lag distribution can be biased (especially if a smooth, one-sided—rather than long, two-sided—lag distribution is estimated) and that seasonal adjustment of both y and x by a linear filter that removes seasonality in x can reduce the bias. He constructed adjustment filters for this purpose, noting that official procedures (or seasonal differencing or removal of seasonal means) may not be suitable. He found that if y and x are adjusted with different filters, then the bias may be reduced; but it may be made much worse, so it is usually safer to use the same filter on y and x. The exception to the rule occurs when the seasonal components of y and x are unrelated, in which case optimal (i.e., minimum mean squared error) adjustment of x alone will remove the bias.

Wallis (1974) also observed that adjusting y and x with different filters can distort the lag relationships between them, so using the same filter is safer. He further observed that using the filter that reduces to white noise the residuals in the distributed lag regression of y on x will produce efficient estimates, since this is equivalent to doing generalized least squares. He then used simulated time series to verify his conclusions regarding the effect of seasonal adjustment on estimated relations between series and to ascertain that a linear filter approximation to X-11 that he had devised behaved similarly in this respect to X-11 itself.

An additional point was made by Granger (1978a). He showed that if the seasonal components of two series are correlated and the nonseasonal components are independent, then the adjusted series will be correlated if both series are adjusted separately with linear filters. Thus the adjusted series will exhibit a relationship even though the nonseasonal components are unrelated.

Newbold (1980) illustrated some problems that can arise when relating one adjusted series to another through a transfer function (distributed lag) model. In his example, nonseasonal models were inadequate for his adjusted series and led to distortions in the estimated transfer function and noise models. He remedied these problems by putting "anti-seasonal" terms (leading to negative correlations at seasonal lags) in his model. His example illustrates that it is dangerous to assume, at least without checking, that nonseasonal models will be appropriate for seasonally adjusted data, and he shows how one might proceed when a nonseasonal model is inappropriate.

From these studies, we might conclude that it is hard to say what effect using seasonally adjusted data has on forecast accuracy. Using seasonally adjusted data, however, has a severe disadvantage in forecasting, since it prevents estimation of forecast error variances and production of forecast intervals—something that can be done with the unadjusted data. Adjusted data can be useful in relating series; here, it is usually safer to use the same adjustment filter on all series, unless the seasonal components of the series are known to be unrelated. Sims (1974) and Wallis (1974) offer guidance, the latter pointing out that using X-11 (with standard options) on all series is close to using the same linear filter on all of them. It should be kept in mind, however, that the simplicity of using adjusted data is bought at some risk of biased or inefficient estimation of relationships between series, that degrees of freedom need to be modified if adjusted data are used, and that as illustrated by Newbold (1980), even the simplicity of adjusted data is sometimes illusory.

4.2 Defining the Components

It is surprising that so many people have provided estimates of seasonal, trend, cyclic, and irregular components without bothering to define what they were estimating. Statements made about the components have tended to be vague—really descriptions rather than definitions. For example, Falkner said, "*Seasonal variation* is that part of the fluctuation due to the persistent tendency for certain months of each year to be regularly higher than certain other months of the year" (1924, p. 167), and "*Secular trend* is the long-time tendency of the items of the series to grow or decline" (p. 167). Shiskin, Young, and Musgrave stated, "The seasonal component(s) is defined as the intrayear pattern of variation which is repeated constantly or in an evolving fashion from year to year" (1967, p. 1). Although few would argue with these statements, they are certainly not enough to define what is being estimated.

In recent years, there have been efforts in the direction of more mathematically precise definitions of the seasonal component based on spectral considerations. The first of these was by Nerlove, who defined seasonality as "that characteristic of a time series that gives rise to spectral peaks at seasonal frequencies" (1964, p. 262). Granger (1978a) gave a reasonably precise definition of when a series is seasonal and when it is strongly seasonal. He suggested taking intervals of width δ (for some small $\delta > 0$) about the seasonal frequencies $2\pi k/12$, $k = 1, \ldots, 6$; then he defined a seasonal time series as one whose spectral density has peaks somewhere in these intervals and a strongly seasonal time series as one whose spectral density integrated over all of these intervals almost equals the integral of the spectral density over $[0, \pi]$. The problem is that these definitions only tell us when a series has a seasonal component, not what the seasonal component is.

It is essential that the component models be precisely specified; otherwise it is not known what is being estimated in seasonal adjustment. We now present an approach to defining the seasonal and nonseasonal components for the additive decomposition $Z_t = S_t + N_t$. Z_t may, of course, be transformed data. We assume that trading day and other deterministic effects have been removed from Z_t. The definitions of the components are based on the following assumptions, grouped for purposes of discussion.

Basic Assumptions

1. $Z_t = S_t + N_t$.
2. $\{S_t\}$ and $\{N_t\}$ are independent of each other.

Harmless Assumptions

3. Z_t follows a known ARIMA model $\phi^*(B)Z_t = \theta^*(B)a_t$.
4. S_t follows an unknown ARIMA model $\phi_S(B)S_t = \theta_S(B)b_t$.
5. N_t follows an unknown ARIMA model $\phi_N(B)N_t = \theta_N(B)c_t$.
6. $\phi_S(B)$ and $\phi_N(B)$ have no common zeros.

Arbitrary Assumptions

7. $\phi_S(B) = 1 + B + \cdots + B^{11}$.
8. The order of $\theta_S(B) \leq 11$.
9. $\sigma_b^2 = \text{var}(b_t)$ is as small as possible, consistent with assumptions 1–8.

Under these assumptions, the results of Hillmer and Tiao (1982) can be used to show that the models for S_t and N_t are uniquely determined. (It is mathematically possible for the model for Z_t to be such that a decomposition according to these assumptions does not exist; however, we have rarely found this to happen in practice.) We then define the components S_t and N_t to be the unobserved time series satisfying these assumptions. This definition does not allow exact calculation of S_t and N_t from Z_t, nor should it; but it does tell us what models they follow, which allows us to use signal extraction theory to estimate them. It is vital to discuss why it might be reasonable to make the above assumptions.

The basic assumptions, 1 and 2, define the problem. Someone who does not want to make these assumptions is working on a different problem. (Actually, with respect to assumption 2, note that since S_t and N_t will be nonstationary, they will require starting values. We may want to allow these starting values to be correlated, and only assume that b_t and c_t are independent.) In assumption 3, it is assumed that an ARIMA model can be built from the observed data to approximate adequately the covariance structure of Z_t. This allows us to handle a wide range of time series, since data that are seasonally adjusted can often be modeled with ARIMA models. A larger class of models than pure ARIMA models is actually allowed, since it is assumed that deterministic effects, such as trading day variation, have been subtracted out. With regard to assumptions 4 and 5, if Z_t follows an ARIMA model, then it seems harmless to assume that S_t and N_t also follow ARIMA models. For all of the ARIMA models here, we assume that the autoregressive and moving average polynomials for a given model have no common zeros and the white noise series (a_t, b_t, c_t) have zero mean and constant variance. If assumption 6 does not hold, then the spectral densities of S_t and N_t will have peaks of similar intensity at the same frequency, which seems unreasonable.

Based on our experience with series that are seasonally adjusted, appropriate models for these series typically have

$$\begin{aligned}\phi^*(B) &= \phi(B)(1 - B)^d(1 - B^{12}) \\ &= \phi(B)(1 - B)^{d+1}(1 + B + \cdots + B^{11}),\end{aligned}$$

where $d \geq 0$ and $\phi(B)$ is of low order in B (say, ≤ 3). Given assumptions 1–6, Findley (in press) has shown that $\phi^*(B) = \phi_S(B)\phi_N(B)$, so for the above $\phi^*(B)$, we let

$$\phi_S(B) = 1 + B + \cdots + B^{11}$$

and

$$\phi_N(B) = \phi(B)(1 - B)^{d+1},$$

which leads to assumption 7.

Hillmer and Tiao (1982) show that our choice for $\phi_S(B)$ leads to a spectral density for the seasonal component that has infinite peaks at the seasonal frequencies and relative minima between them. Furthermore, assumption 7 implies that summing S_t over 12 consecutive months produces a stationary series with mean zero, which is consistent with the general belief (as in X-11) that in an additive decomposition the seasonal component should sum to something near zero over a year. The nonseasonal component will be nonstationary, and its spectral density will have an infinite peak at zero frequency. Thus assumption 7 leads to reasonable seasonal and nonseasonal component models.

One case not addressed in our choice of autoregressive operators in assumption 7 is that of models with stationary seasonal autoregressive operators. For models including a factor $1 - B^{12}$, we invariably find seasonal moving average terms to be more appropriate than seasonal autoregressive terms. We have modeled a few series without a $1 - B^{12}$, using instead a seasonal autoregressive operator, $1 - \phi_{12}B^{12}$, where ϕ_{12} is not near 1. We have chosen not to adjust such series because the seasonal pattern of the data tends to change quickly—the highest month could become the lowest month after four or five years. A similar choice was made by Hannan (1964). It is possible to make a different choice of $\phi_S(B)$ and still use the preceding framework if a set of rules replacing assumption 7—for specifying $\phi_S(B)$ and $\phi_N(B)$ given $\phi^*(B)$—is provided.

Hillmer, Bell, and Tiao (1983a) noted that assumption 8 implies that the forecast function in the model for S_t follows a fixed annual pattern that sums to zero over 12 consecutive months. In contrast, if the order of $\theta_S(B)$ exceeds 11, then the forecast function for the seasonal component will change its annual pattern. The forecastable change in the seasonal pattern should be part of the trend and hence included in N_t.

Given assumptions 1–8, Hillmer and Tiao (1982) showed that σ_b^2 must lie in some known range $[\bar{\sigma}_b^2, \tilde{\sigma}_b^2]$ and that the models for S_t and N_t are uniquely determined once a choice of σ_b^2 is made. They called the decomposition corresponding to a σ_b^2 in $[\bar{\sigma}_b^2, \tilde{\sigma}_b^2]$ an admissible decomposition, with corresponding admissible seasonal and nonseasonal components; and they called the decomposition corresponding to the choice $\sigma_b^2 = \bar{\sigma}_b^2$ the *canonical decomposition.* Thus we have defined the seasonal component to be the *canonical seasonal component*, $\bar{S}_t$, corresponding to the choice $\sigma_b^2 = \bar{\sigma}_b^2$. The *canonical nonseasonal component*, $\bar{N}_t$, is then $Z_t - \bar{S}_t$. Hillmer and Tiao (1982) demonstrated that choosing $\sigma_b^2 = \bar{\sigma}_b^2$ minimizes var$[(1 + B + \cdots + B^{11})S_t]$, making the seasonal pattern as stable as possible. In addition, they established that for any other choice of σ_b^2, the corresponding seasonal component, S_t', can be written $S_t' = \bar{S}_t + e_t$, where e_t is white noise. Thus any admissible seasonal component is the sum of the canonical seasonal component (which follows as stable a pattern as possible and is as predictable as possible) and white noise (which is totally unpredictable and nonseasonal). We see no reason to add white noise to $\bar{S}_t$ when defining the seasonal component.

Assumptions 1–9 lead to precise definitions of the seasonal and nonseasonal components. If we have built a model for the observed data Z_t and assumptions 1–9 are made, then we know the models for $\bar{S}_t$ and $\bar{N}_t$ and can use signal extraction theory to estimate these components. This is the approach to seasonal adjustment taken in Burman (1980), Hillmer and Tiao (1982), and Hillmer, Bell, and Tiao (1983a). Of course, assumptions other than 1–9 can be made about S_t and N_t, even while remaining consistent with the model for Z_t; in particular, a choice of σ_b^2 in $[\bar{\sigma}_b^2, \tilde{\sigma}_b^2]$ other than $\bar{\sigma}_b^2$ could be used. Different assumptions will lead to different definitions and models for the components, which, when used in signal extraction theory, will lead to different methods of seasonal adjustment.

This discussion points out the arbitrariness inherent in seasonal adjustment. Different methods produce different adjustments because they make different assumptions about the components and hence estimate different things. This arbitrariness applies equally to methods (such as X-11) that do not make their assumptions explicit, since they must implicitly make the same sort of assumptions as we have discussed here. (The assumptions implicit in additive X-11 with standard options are investigated by Cleveland and Tiao (1976), by Burridge and Wallis (1984), and in Sec. 4.3.4.) Unfortunately, there is not enough information in the data to define the components, so these types of arbitrary choices must be made. We have tried to justify our assumptions but do not expect everyone to agree with them. If, however, anyone wants to do seasonal adjustment but does not want to make these assumptions, we urge them to make clear what assumptions they wish to make. Then the appropriateness of the various assumptions can be debated. This debate would be more productive than the current one regarding the choice of seasonal adjustment procedures, in which no one bothers to specify what is being estimated. Thus if debate can be centered on *what* it is we want to estimate in doing seasonal adjustment, then there may be no dispute about *how* to estimate it.

4.3 Evaluating Seasonal Adjustments and Seasonal Adjustment Methods

Given the arbitrary nature of seasonal adjustment, people have found it difficult to decide when a "good" adjustment has been done or when one method is "better" than another. In this section, we discuss the problems with approaches that have been used to evaluate adjustments and adjustment methods, including criteria for evaluating adjustments, simulation studies, and revisions comparisons. Finally, we make some suggestions about how seasonal adjustment evaluation might be approached.

4.3.1 Criteria for Evaluating Seasonal Adjustments

Various criteria have been proposed for assessing the adequacy of a seasonal adjustment and for deciding when one method does a better job of adjusting a series than another. Attempts at designing such criteria have failed, so today there are no accepted standards by which adjustments can be judged.

Criteria proposed for evaluating seasonal adjustments have generally reflected properties that were thought to be desirable for nonseasonal components. These criteria have been phrased in both spectral and time domain terms. It has been thought that a method performed adequately if the adjusted series exhibited properties similar to those of the "true" nonseasonal component, and the performances of different adjustment methods have been compared, based on this belief. Unfortunately, although the suggested criteria may reflect desirable properties for the nonseasonal component of a series, this does not mean that they reflect desirable properites for the adjusted series, which is an *estimate* of the nonseasonal component. Anderson (1927) emphasized long ago that the estimated components are not the same as the true components. Furthermore, even if models for Z_t, S_t, and N_t are known, the true underlying components cannot be calculated; and the best estimates of the components will behave differently enough from the true components to make the proposed criteria of little or no value in evaluating seasonal adjustments. To substantiate this, we cite two examples.

First, Nerlove (1964) suggested various spectral criteria that a good adjustment should satisfy, including (a) high coherence between original and adjusted series, except at seasonal frequencies, (b) minimal phase shifts in the cross spectrum between the original and adjusted series, and (c) removal of peaks at the seasonal frequencies in the spectral density of the original series, without producing dips at these frequencies or greatly affecting the spectral density at other frequencies. Subsequently, Grether and Nerlove (1970) investigated empirically (by simulating series from known component models) and theoretically the performance of the optimal (i.e., minimum mean squared error linear) method of adjustment. They discovered that the optimal method did not look good in terms of these criteria. It reproduced all of the undesirable features that Nerlove (1964) had noted for X-11. Since the minimum mean squared error linear estimator is a reasonable choice if it is available, they concluded that the criteria in Nerlove (1964) left much to be desired.

Second, Granger (1978a) reviewed some criteria that could be used for evaluating adjustments, including (a) and (c) of Nerlove (1964), which he referred to as "highly desirable." In their discussions of Granger's paper, both Sims (1978) and Tukey (1978) showed that the spectral properties he suggested have unreasonable parallels in other situations and that the minimum mean squared error linear adjustment need not satisfy these properties. (Wecker (1978) made similar comments about why "overadjustment," the production of dips at seasonal frequencies in the spectrum of the adjusted series, should not be regarded as a problem.) Granger than responded, "The criteria I suggested have been shown to be impossible to achieve in practice, and thus, should be replaced by achievable criteria. However, I am at a loss to know what these criteria should be" (1978b, p. 55).

Empirical studies comparing the performance of different adjustment methods on various sets of data using the previously proposed criteria are of little value in determining which methods of adjustment are "better" than others. We doubt that useful criteria that are functions of the adjusted data only can be found. There may be a role, however, for the previously mentioned criteria. Since these criteria are reasonable when applied to the true nonseasonal component, they may be useful in evaluating the assumptions made about the components by adjustment methods. Thus in our approach to defining the components discussed in Section 4.2, we used some criteria to evaluate the properties of the assumed underlying component models. These and other criteria might be applied to the assumptions underlying other seasonal adjustment methods. Efforts would be better spent evaluating the assumptions underlying adjustment methods, rather than trying to evaluate methods by looking at adjusted data.

4.3.2 Simulation Studies

Another approach that has been suggested for evaluating seasonal adjustment methods is to check their performance on simulated series. The S_t and N_t components are generated and an adjustment method applied to $Z_t = S_t + N_t$ to see how accurately the method estimates the components. In general, little will be learned from such studies.

The basic problem with this approach is that the results depend heavily on what the adjustment methods being considered are actually estimating. This can vary considerably from method to method. If one method makes assumptions about S_t and N_t that are similar to those used in generating them and a second method makes different assumptions, then the first method will estimate the components more accurately than the second. This phenomenon is reflected in the results of Godfrey and Karreman (1967). Comparing different methods on simulated data will merely verify that the methods make different assumptions.

To illustrate the preceding remarks, we generated S_t and N_t series of length 900 from each of the following two models, the rationale for which will become apparent.

1. Min Seasonal Model

$$\begin{aligned}(1 + B + \cdots + B^{11})S_{1t} = (1 &+ 1.45B + 1.50B^2 \\ &+ 1.44B^3 + 1.24B^4 \\ &+ .99B^5 + .72B^6 \\ &+ .45B^7 + .23B^8 \\ &+ .002B^9 - .11B^{10} \\ &- .43B^{11})b_{1t}, \\ &\qquad b_{1t} \text{ iid } N(0, .0107)\end{aligned}$$

$$\begin{aligned}(1 - B)^2 N_{1t} = (1 - 1.38B + .39B^2)d_{1t}, \\ d_{1t} \text{ iid } N(0, .8223)\end{aligned}$$

Table 1. A Simulation Experiment With Model-Based Seasonal Adjustment

	Model Used	
Case	*To Generate Data*	*To Construct $\hat{S}_{It}$, $\hat{N}_{It}$*
A	Min seasonal	Min seasonal
B	Min seasonal	Max seasonal
C	Max seasonal	Max seasonal
D	Max seasonal	Min seasonal

2. Max Seasonal Model

$$(1 + B + \cdots + B^{11})S_{2t} = (1 + 1.10B + 1.10B^2 + 1.05B^3 + .99B^4 + .96B^5 + .94B^6 + .94B^7 + .86B^8 + .80B^9 + .83B^{10} + .87B^{11})b_{2t},$$

$$b_{2t}\ iid\ N(0, .4422)$$

$$(1 - B)^2N_{2t} = (1 + .01B - .98B^2)\, d_{2t},$$

$$d_{2t}\ \text{iid}\ N(0, .0740)$$

For both of these models, the resulting model for the sum $Z_{it} = S_{it} + N_{it}$ is the same and is given by

$$(1 - B)(1 - B^{12})Z_{it} = (1 - .4B)(1 - .8B^{12})a_{it},$$

$$a_{it}\ \text{iid}\ N(0, 1).$$

Actually, the Min Seasonal Model corresponds to making assumptions 1–9 given in Section 4.2 (lowest possible σ_b^2), and the Max Seasonal Model makes assumptions 1–8 and then chooses the maximum possible σ_b^2. The series were generated in such a way that in fact the same series was obtained from both models; that is, we have $Z_t = S_{it} + N_{it}$, $i = 1, 2$. The following model was identified and estimated for the observed data Z_t:

$$(1 - B)(1 - B^{12})Z_t = (1 - .41B)(1 - .85B^{12})a_t,$$

$$\sigma_a = .967$$

Using signal extraction theory and the estimated model for Z_t, S_{it} and N_{it} ($i = 1, 2$) were estimated from Z_t under two assumptions:

1. that the true model for S_{it} had minimum σ_b^2 and
2. that the true model for S_{it} had maximum σ_b^2.

Thus, as can be seen in Table 1, there are four cases. In cases A and C, the correct models for S_{it} and N_{it} (within parameter estimation error) have been used, and in cases B and D, incorrect models have been used. The error series

$$e_{it} = S_{it} - \hat{S}_{it} = \hat{N}_{it} - N_{it}$$

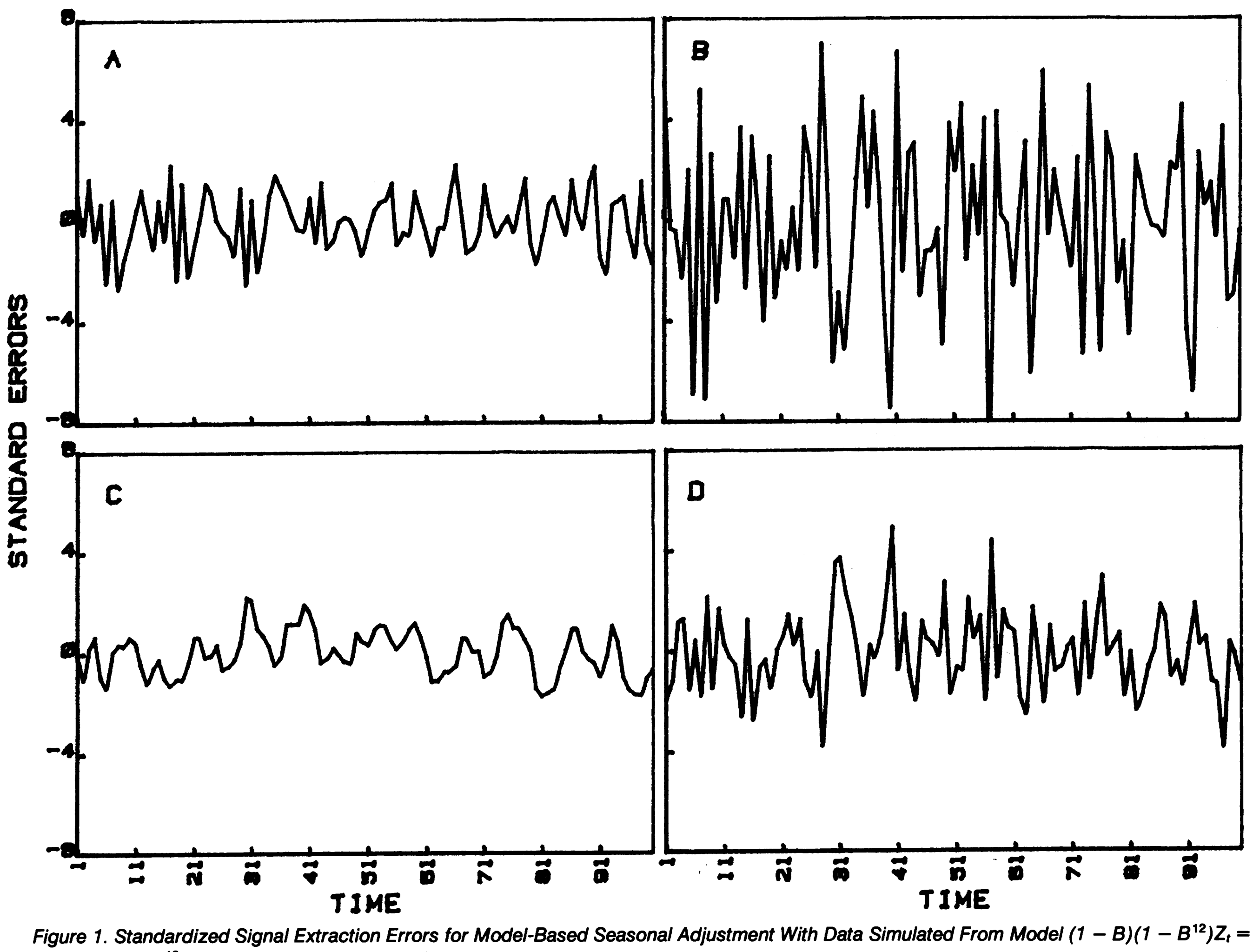

Figure 1. Standardized Signal Extraction Errors for Model-Based Seasonal Adjustment With Data Simulated From Model $(1 - B)(1 - B^{12})Z_t = (1 - .4B)(1 - .8B^{12})a_t$. (A, B, C, and D refer to Table 1.)

were computed in each of the four cases, and the e_{it}'s were standardized by dividing them by their standard deviation from signal extraction theory under the correct model. The results are shown in Figure 1 for the middle 100 observations.

When the correct model is used, as in Figure 1 A and C, the standardized e_{it}'s vary about zero reasonably within ± 2 limits. When the incorrect model is used, however, as in Figure 1 B and D, the e_{it}'s are considerably larger. This does not tell us that either the Min Seasonal or Max Seasonal method of adjustment is better; it merely illustrates how the accuracy of the estimator depends heavily on what is being estimated.

4.3.3 Revisions

Most seasonal adjustment methods are based on symmetric two-sided filters. When the observation for the current time period is adjusted, future observations are not available; thus near the end of the series, one-sided filters must be used. As more observations become available, one can come closer to using the symmetric filter. The changes in the seasonally adjusted values as additional observations are added are called revisions.

Many researchers who have conducted empirical studies of seasonal adjustment methods have used measures of the magnitude of revisions as one criteria for evaluating the different methods. This makes sense when comparing adjustment methods that give the same final adjustment, such as X-11 and X-11 ARIMA, or X-11 in year-ahead and concurrent modes. In this case the different methods are all shooting at the same target value—the final X-11 adjustment. Comparisons of the magnitudes of total revisions (changes from the initial to the final adjustment) reflect how close the initial adjustments come to the target. Since the final adjustment is, presumably, better than the earlier adjustments (or we would not bother to revise as additional data became available), lower total revisions are better. Studies comparing total revisions for X-11 and modifications to X-11 that still yield the same final adjustment were done by Dagum (1978), Geweke (1978a), Kenny and Durbin (1982), and McKenzie (1984).

There is a fundamental problem, however, with using revisions as a standard of comparison when the methods being compared produce different final adjustments and thus estimate different nonseasonal components. In this case the magnitude of revisions can be greatly affected by the choice of nonseasonal components. This choice should be based on information in the data and beliefs about seasonality (see Secs. 4.2 and 4.3.4), not on the magnitude of revisions. In the extreme, one could use a method based exclusively on one-sided filters, which leads to no revisions—an approach that has seldom been adopted.

To illustrate the dependence of the size of revisions on the final adjustment used, we shall consider additive X-11 with standard options, the model-based method of Section 4.2 (min seasonal model), and the max seasonal model variant of this discussed in Section 4.3.2. Suppose these methods are used by applying their symmetric filters to data extended with minimum mean squared error forecasts and backcasts. This minimizes the mean squared revisions (MSR) (Geweke 1978a and Pierce 1980b), so differences in MSR between the methods for a given model for Z_t are only due to the different final adjustment targets. Using the results of Pierce (1980b), we computed mean squared total revisions for the particular case where Z_t follows the model

$$(1 - B)(1 - B^{12})Z_t = (1 - \theta_1 B)(1 - \theta_{12}B^{12})a_t$$

for various values of θ_1 and θ_{12}, with $\sigma_a^2 = 1$. Table 2 presents some illustrative results. Notice that the magnitude of revisions for a given model depends dramatically on the relevant final adjustment. It also depends on the characteristics of the data, so that no one method gives the lowest MSR for all series. These results point out the inappropriateness of using total revisions to evaluate seasonal adjustment methods that give different final answers.

We might also consider how the behavior of yearly revisions—the changes in the adjusted values as each additional year of data is added—depends on the final adjustment underlying a method. It might be argued that first-year revisions are relevant, since some users will not be concerned about revisions more than a year or two after the initial adjustment and thus will not be concerned with the actual final adjustment. (For X-11 with standard options, the final adjustment is effectively obtained three years after the initial adjustment; see Young 1968. For the model-based methods considered—min and max seasonal—the filters can be quite long, so the final adjustment comes much later or is effectively never achieved.) Using the assumptions and model given above, Hillmer, Bell, and Tiao (1983a) found that theoretical first-year MSR's were smaller for the min seasonal model-based method when $\theta_{12} > .4$ and smaller for X-11 when $\theta_{12} < .4$, with the difference being more pronounced the further θ_{12} was from .4. (The max seasonal method was not considered.) These theoretical calculations were confirmed empirically by

Table 2. Mean Squared Total Revisions When $(1 - B)(1 - B^{12})Z_t = (1 - \theta_1 B)(1 - \theta_{12}B^{12})a_t(\sigma_a^2 = 1)$

θ_1	θ_{12}	X-11	Model-Based (min seasonal)	Model-Based (max seasonal)
.3	.5	.123*	.133	.177
.5	.9	.059	.032*	.114
.9	.7	.130	.079	.049*

* Minimum across the row.

studying the first-year revisions of 76 times series that were modeled and adjusted by both approaches. They found that the model-based approach gave substantially lower first-year revisions and argued that this was because the estimated values of the seasonal moving average parameter (θ_{12}) for the 76 series were almost always substantially larger than .4. Since the model-based method effectively uses longer filters than does X-11 when $\theta_{12} > .4$ and shorter filters when $\theta_{12} < .4$, this leads to the conclusion that longer filters lead to smaller first-year revisions. The filters used to adjust the most recent observations for both methods are modifications of the symmetric filter used for the final adjustment, and the lengths of the filters used for recent data correspond to the lengths of the filters used for the final adjustments. Thus the final adjustment underlying a method has a profound effect on first-year revisions.

To reemphasize our point: These results illustrate that it is inappropriate to use measures of revisions to judge the relative merits of seasonal adjustment methods giving different final adjustments. The decision about which final adjustment is appropriate should be based on information in the data, beliefs about seasonality, and when possible, on the objectives of the seasonal adjustment. Therefore, in choosing a seasonal adjustment method it is important that attention be concentrated on what is being estimated—the target—rather than on revisions. Evaluating seasonal adjustment methods that yield different final adjustments by using revisions is like judging a parameter estimator by how rapidly it converges as the sample size increases, even if it converges to the wrong value.

4.3.4 Consistency With the Data

Consider the ideal situation, in which we know the spectral densities for Z_t, S_t, and N_t: $f_Z(\lambda)$, $f_S(\lambda)$, and $f_N(\lambda)$, respectively. From $Z_t = S_t + N_t$, the spectral densities (and hence the models) are constrained by the relation

$$f_Z(\lambda) = f_S(\lambda) + f_N(\lambda). \qquad (1)$$

The minimum mean squared error estimator, $\hat{N}_t$, of the nonseasonal component is obtained by applying a symmetric linear filter, $W_N(B)$, to the observed data:

$$\hat{N}_t = W_N(B)Z_t, \qquad (2)$$

where

$$W_N(e^{-i\lambda}) = \sum_{-\infty}^{\infty} W_{N,k}e^{-i\lambda k}$$
$$= f_N(\lambda)/f_Z(\lambda) = 1 - f_S(\lambda)/f_Z(\lambda).$$

(Bell (1984) discusses the assumptions under which (4) provides the minimum mean squared error (linear) estimator of N_t when S_t or N_t or both are nonstationary.)

Notice that any two of $f_Z(\lambda)$, $f_S(\lambda)$, $f_N(\lambda)$, and $W_N(B)$ are sufficient to determine the other two, using (1) and (2), but no one of them is sufficient to determine the other three.

In practice, although we will not know $f_Z(\lambda)$, we can at least approximate it by modeling Z_t. Let $\hat{f}_Z(\lambda)$ be our estimate of $f_Z(\lambda)$. Now suppose that we have a linear filter, $W_N(B)$, to be used in adjusting Z_t. From (2), the implied spectral densities for S_t and N_t are

$$\hat{f}_S(\lambda) = \hat{f}_Z(\lambda)[1 - W_N(e^{-i\lambda})],$$
$$\hat{f}_N(\lambda) = \hat{f}_Z(\lambda)W_N(e^{-i\lambda}). \qquad (3)$$

By examining $\hat{f}_S(\lambda)$ and $\hat{f}_N(\lambda)$ we can investigate the assumptions that are implicit when Z_t is adjusted with $W_N(B)$.

Suppose $W_N(B)$ results from signal extraction theory for some set of models for Z_t, S_t, and N_t, which we assume are expressed in infinite autoregressive form as

$$\Pi_Z(B)Z_t = a_t \quad \Pi_S(B)S_t = b_t \quad \Pi_N(B)N_t = c_t. \qquad (4)$$

$W_N(B)$ then satisfies the equation

$$W_N(e^{-i\lambda}) = f_N(\lambda)/f_Z(\lambda) = \frac{\sigma_c^2/\Pi_N(e^{i\lambda})\Pi_N(e^{-i\lambda})}{\sigma_a^2/\Pi_Z(e^{i\lambda})\Pi_Z(e^{-i\lambda})}.$$

We cannot say adjustment with $W_N(B)$ implies the models in (4) because if all of the models in (4) are replaced by the models

$$\alpha(B)\Pi_Z(B)Z_t = a_t,$$
$$\alpha(B)\Pi_S(B)S_t = b_t,$$
$$\alpha(B)\Pi_N(B)N_t = c_t, \qquad (5)$$

where $\alpha(B) = 1 + \sum_1^\infty \alpha_j B^j$ has all of its zeros on or outside the unit circle, then the adjustment filter is

$$\frac{f_N(\lambda)/\alpha(e^{i\lambda})\alpha(e^{-i\lambda})}{f_Z(\lambda)/\alpha(e^{i\lambda})\alpha(e^{-i\lambda})} = W_N(e^{-i\lambda}).$$

Thus the adjustment filter stays the same. This reflects the fact that $W_N(B)$ alone cannot determine the models for Z_t, S_t, and N_t. If we have an estimated model $\hat{\Pi}_Z(B)Z_t = a_t$ and an estimate $\hat{\sigma}_a^2$, however, then setting $\alpha(B) = \hat{\Pi}_Z(B)/\Pi_Z(B)$ in (5) leads to implied models for S_t and N_t:

$$\hat{\Pi}_S(B)S_t = \hat{b}_t \qquad \hat{\Pi}_N(B)N_t = \hat{c}_t$$
$$\hat{\Pi}_S(B) = \frac{\hat{\Pi}_Z(B)\Pi_S(B)}{\Pi_Z(B)} \qquad \hat{\Pi}_N(B) = \frac{\hat{\Pi}_Z(B)\Pi_N(B)}{\Pi_Z(B)}$$
$$\hat{\sigma}_b^2 = (\sigma_b^2/\sigma_a^2)\hat{\sigma}_a^2 \qquad \hat{\sigma}_c^2 = (\sigma_c^2/\sigma_a^2)\hat{\sigma}_a^2 \qquad (6)$$

Of course, if (4) uses the estimated model for Z_t, then $\Pi_Z(B) = \hat{\Pi}_Z(B)$, $\sigma_a^2 = \hat{\sigma}_a^2$, and the models in (6) are the same as those in (4). The implied spectral densities for

S_t and N_t are obtained from the relation

$$\hat{f}_S(\lambda) = \hat{\sigma}_b^2/2\pi\hat{\Pi}_S(e^{i\lambda})\hat{\Pi}_S(e^{-i\lambda}),$$
$$\hat{f}_N(\lambda) = \hat{\sigma}_c^2/2\pi\hat{\Pi}_N(e^{i\lambda})\hat{\Pi}_N(e^{-i\lambda}). \quad (7)$$

The preceding discussion suggests an approach to evaluating the suitability of any linear adjustment method for a particular time series. The overriding consideration is that any method of seasonal adjustment should be consistent with the information in the data, which is summarized, at least approximately, by the estimated model (spectral density) for Z_t. If the implied models (spectral densities) for S_t and N_t in (3), (6), and (7) are seen to be unreasonable, such as if the model for N_t is seasonal, we would conclude that seasonal adjustment using $W_N(B)$ is *inconsistent* with the information in the data. If the implied models (spectral densities) appear reasonable, we would say that seasonal adjustment with $W_N(B)$ is *consistent* with the information in the data. This leads us to propose the following criterion for evaluating a method of seasonal adjustment with respect to a given set of data:

> A method of seasonal adjustment should be consistent with an adequate model for the observed data.

This condition is not sufficient for a "good" seasonal adjustment, since although a method may satisfy the condition for a given set of data, it does not follow that the resulting seasonal adjustment is "good." Since many different seasonal adjustments can be consistent with an adequate model for the data (see Sec. 4.3.2), judgments about whether a method that is consistent with the data is "good" must either be made subjectively, or be based on additional information, such as the intended use of the adjusted data. The criterion is necessary for a good seasonal adjustment, however, in that any method not consistent with the information contained in an adequate model for a given set of data is certainly bad for that set of data. Application of the criterion depends on arbitrary judgments regarding the adequacy of the fitted model for Z_t and the reasonableness of the implied models (spectral densities) for S_t and N_t. Even with these difficulties, application of the criterion can be informative and sometimes the conclusions will be obvious, as we shall illustrate with an example.

We should point out that (3) may not be defined at $\lambda = k\pi/6$, $k = 0, \pm 1, \ldots, \pm 6$, since $\hat{f}_Z(\lambda)$ may well be $+\infty$ at these frequencies, whereas $W_N(e^{-i\lambda})$ or $1 - W_N(e^{-i\lambda})$ may be zero at any given one of these frequencies. Depending on $\hat{f}_Z(\lambda)$ and $W_N(B)$, it may be sensible to set $\hat{f}_S(\lambda) = +\infty$ at the seasonal frequencies and $\hat{f}_N(\lambda) = +\infty$ at $\lambda = 0$. This problem does not arise if $W_N(B)$ corresponding to models (4) and (6) are used. We present our criterion as a general approach to evaluating the consistency of a seasonal adjustment method with a model for the data and hope to investigate the computational considerations further.

Example. We evaluate the use of the X-11 method (additive version with standard options) on the series Z_t = employed nonagricultural males, aged 20 and older (ENM20) from January 1965 through August 1979 (data from the Bureau of Labor Statistics). Young (1968) found a linear filter that approximates additive X-11. Cleveland and Tiao (1976) then found approximately the same filter results from signal extraction theory, using the following models for S_t and N_t:

$$(1 - B^{12})S_t = (1 + .64B^{12} + .83B^{24})b_t,$$
$$(1 - B)^2N_t = (1 - 1.252B + .4385B^2)c_t,$$
$$\sigma_c^2/\sigma_b^2 = 24.5 \quad (8)$$

(They actually gave models for the trend (T_t) and irregular (I_t). The model for $N_t = T_t + I_t$ can be obtained from these.) The models in (8) lead to a model for Z_t:

$$\begin{aligned}(1 - B)(1 - B^{12})Z_t &= (1 - .337B + .144B^2 \\ &\quad + .141B^3 + .139B^4 \\ &\quad + .136B^5 + .131B^6 \\ &\quad + .125B^7 + .117B^8 \\ &\quad + .106B^9 + .093B^{10} \\ &\quad + .077B^{11} - .417B^{12} \\ &\quad + .232B^{13} - .001B^{20} \\ &\quad - .003B^{21} - .004B^{22} \\ &\quad - .006B^{23} + .035B^{24} \\ &\quad - .021B^{25})a_t \\ &= \eta(B)a_t; \\ &\qquad \sigma_a^2/\sigma_b^2 = 43.1. \quad (9)\end{aligned}$$

Table 3. Autocorrelations of Residuals From Model (10) for ENM20

k	1	2	3	4	5	6	7	8	9	10	11	12
$r_k(\hat{a})$	.00	.00	.11	−.06	.02	.13	−.07	.07	.02	−.02	.06	.09
k	13	14	15	16	17	18	19	20	21	22	23	24
$r_k(\hat{a})$	−.03	.04	−.01	−.13	−.08	−.02	.10	−.08	−.08	−.01	−.03	−.05
k	25	26	27	28	29	30	31	32	33	34	35	36
$r_k(\hat{a})$	−.02	−.01	−.01	.01	−.12	−.02	.00	−.14	−.02	−.03	−.14	−.07

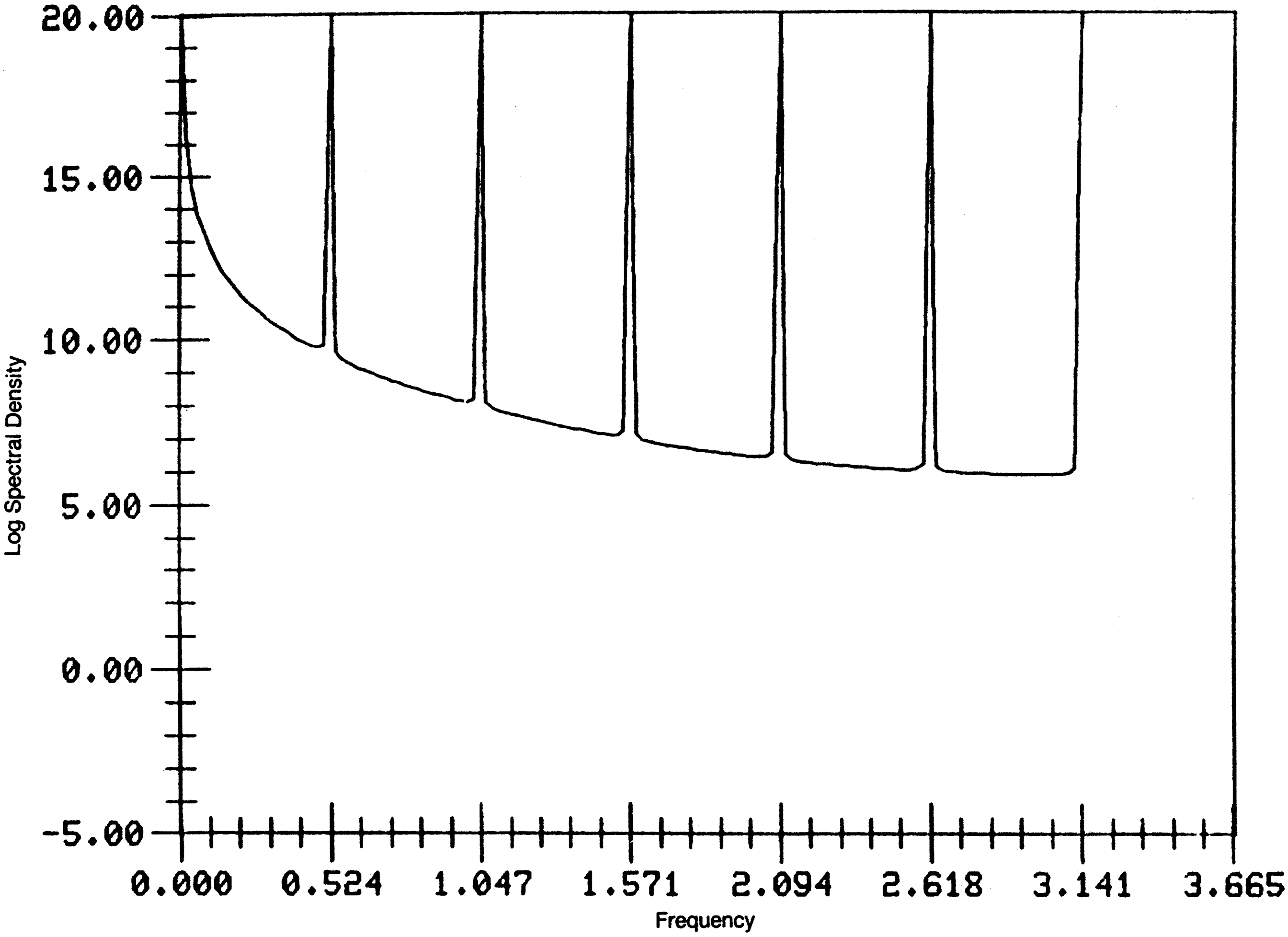

Figure 2. Spectral Density for ENM20 From Model (10) (log $\hat{f}_Z(\lambda)$).

(For alternative models for X-11, see Burridge and Wallis 1984.) For ENM20, we obtained the model

$$\frac{(1 - .26B)(1 - B)(1 - B^{12})}{1 - .88B^{12}} Z_t = a_t$$

$$\hat{\sigma}_a^2 = 16{,}150. \qquad (10)$$

The sample autocorrelations of the residuals, $\hat{a}_t$6, from (11) are reported in Table 3. The statistics

$$Q_L = n(n + 2) \sum_{k=1}^{L} r_k(\hat{a})^2/(n - k)$$

(Ljung and Box 1978) are approximately distributed as X^2_{L-2} if the model is adequate. For this example, none of the $r_k(\hat{a})$'s is larger in magnitude than two standard errors (.16), and $Q_{12} = 10.2$, $Q_{24} = 20.6$, and $Q_{36} = 33.9$ are all insignificant. We proceed with the estimated model (10).

The logarithm of $\hat{f}_Z(\lambda)$, the estimated spectral density corresponding to (10), is plotted in Figure 2. It has infinite peaks (truncated at 20 for the graph) at $\lambda = 0$ and at the seasonal frequencies $\lambda = \pi k/6$, $k = 1, 2, \ldots,$ 6. From (6) and (8)–(10), the implied models for S_t and N_t are

$$\frac{(1 - .26B)(1 - B^{12})\eta(B)}{(1 - .88B^{12})(1 + .64B^{12} + .83B^{24})} S_t = \hat{b}_t$$

$$\hat{\sigma}_b^2 = 374.7 \qquad (11)$$

$$\frac{(1 - .26B)(1 - B)^2\eta(B)}{(1 - .88B^{12})(1 - 1.252B + .4385B^2)} N_t = \hat{c}_t$$

$$\hat{\sigma}_c^2 = 9{,}176.1 \qquad (12)$$

The implied spectral densities, $\hat{f}_S(\lambda)$ and $\hat{f}_N(\lambda)$, were obtained and their logarithms plotted in Figures 3 and 4. ($\hat{f}_N(\lambda)$ was computed directly, using (12), but $\hat{f}_S(\lambda)$ was obtained as $\hat{f}_Z(\lambda) - \hat{f}_N(\lambda)$ to satisfy (1). Because of the small number of significant digits provided by Cleveland and Tiao (1976), computing $\hat{f}_S(\lambda)$ directly from (11) would not have satisfied (1).) $\hat{f}_S(\lambda)$ has infinite peaks at the seasonal frequencies and may appear reasonable. (There is also an infinite peak at $\lambda = 0$ due to the $(1 - B)$ factor implied by the $1 - B^{12}$ in (8). It would not necessarily appear if another set of models,

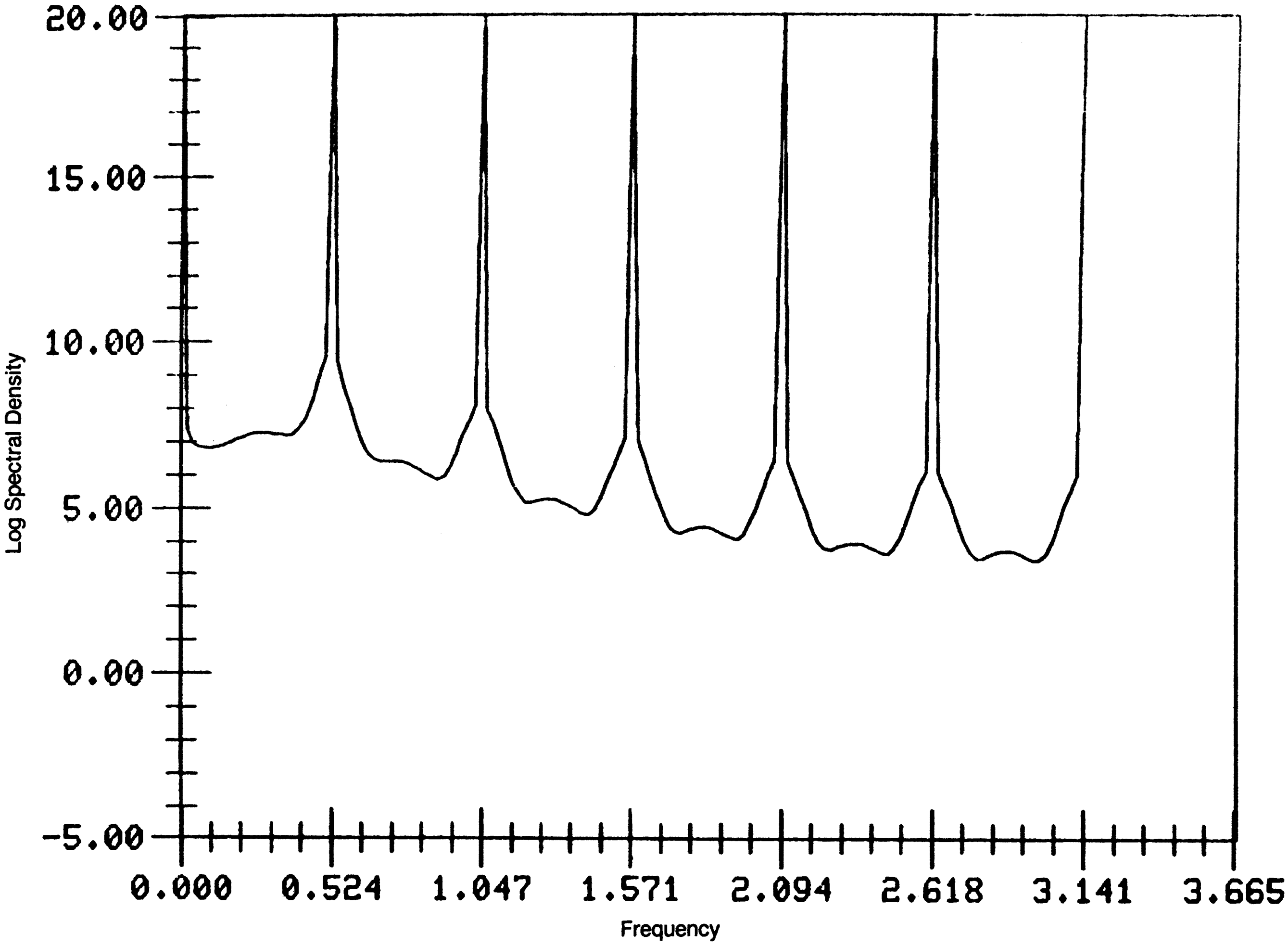

Figure 3. Seasonal Spectral Density for ENM20 Implied by X-11.

for example, Cleveland 1972 or Burridge and Wallis 1984, were used to approximate X-11.) $\hat{f}_N(\lambda)$, however, has (finite) dips at the seasonal frequencies, which is unreasonable. This is *not* the same as the overadjustment phenomenon referred to in Section 4.3.1, which has to do with dips in the spectral density of the adjusted data. Here we have dips in the implied spectral density for the underlying nonseasonal component, which is unreasonable. Thus we conclude that X-11 is inconsistent with the information in the data for this series.

To see why the dips arose in $\hat{f}_N(\lambda)$, notice that to a rough approximation, $\eta(B)$ in (9) is

$$\eta(B) \doteq (1 - .35B)(1 - .4B^{12}),$$

so (12) becomes

$$\frac{(1 - .26B)(1 - .35B)(1 - B)^2(1 - .4B^{12})}{(1 - 1.252B + .4385B^2)(1 - .88B^{12})} N_t \doteq c_t.$$

Thus $\hat{f}_N(\lambda)$ contains

$$\frac{(1 - .88e^{12i\lambda})(1 - .88e^{-12i\lambda})}{(1 - .4e^{12i\lambda})(1 - .4e^{-12i\lambda})} = \frac{(1.774[1 - .992\cos(12\lambda)]}{1.16[1 - .690\cos(12\lambda)]}.$$

This function is plotted in Figure 5. It is near zero at the seasonal frequencies because the $1 - .992\cos(12\lambda)$ in the numerator is quite small, whereas the $1 - .690\cos(12\lambda)$ in the denominator is at least .31 at all frequencies. This results in dips at the seasonal frequencies in $\hat{f}_N(\lambda)$. All of this follows from the estimate (.88) of the seasonal moving average parameter in (10) being considerably larger than .4, the value implicitly used by X-11. Thus this behavior can be expected whenever the estimate of θ_{12} is much greater than .4, which seems to be the case most of the time (see Sec. 4.3.3).

In contrast, we examine the canonical decomposition. For the model (10), the component models turn out to be

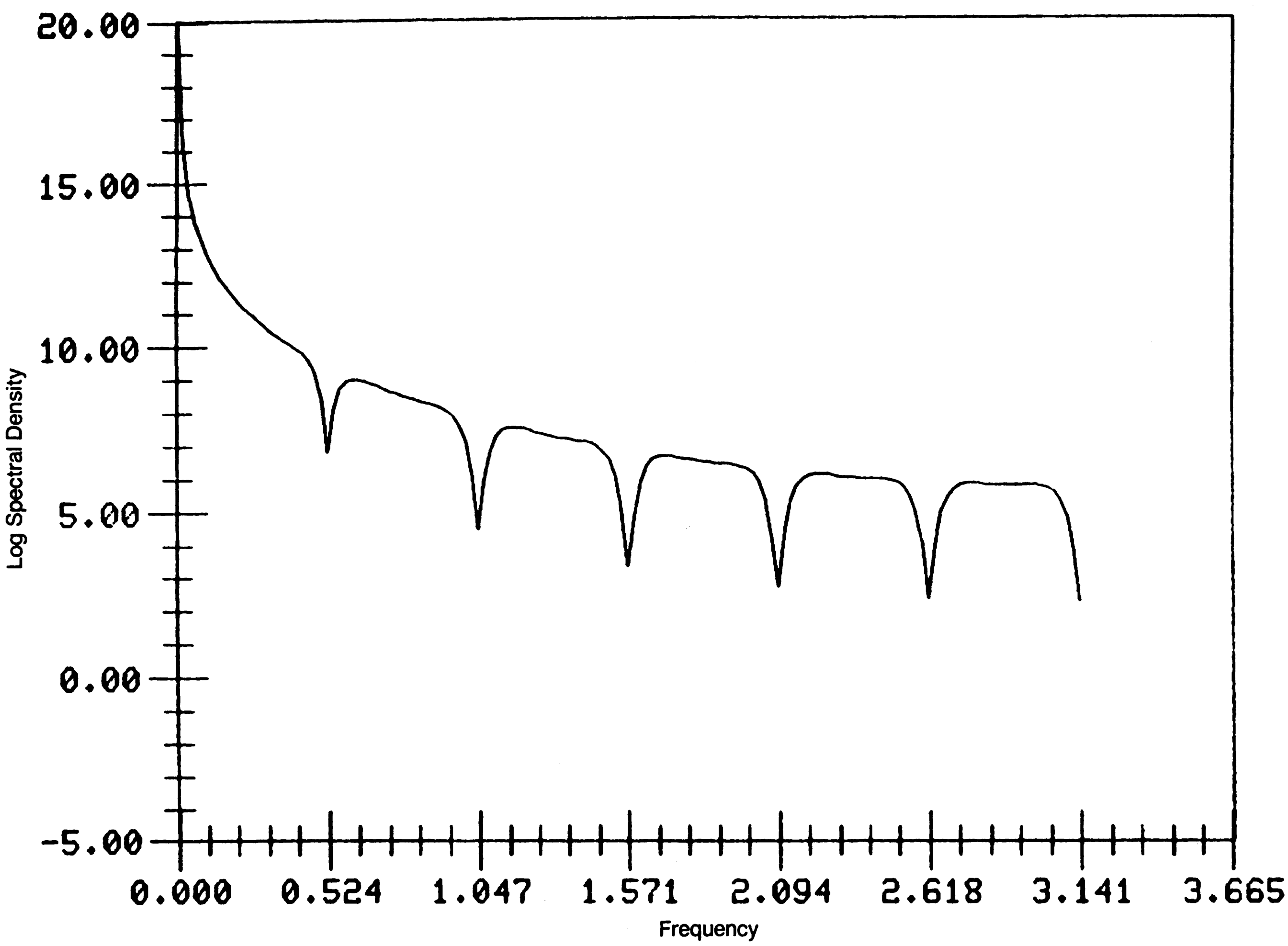

Figure 4. Nonseasonal Spectral Density for ENM20 Implied by X-11.

$$(1 + B \cdots + B^{11})S_t = (1 + 2.093B + 2.722B^2 + 2.977B^3 + 2.869B^4 + 2.581B^5 + 2.169B^6 + 1.670B^7 + 1.206B^8 + .745B^9 + .411B^{10} - .007B^{11})b_t,$$

$$\sigma_b^2 = 82.11,$$

$$(1 - .26B)(1 - B)^2N_t = (1 - .990B + .001B^2)c_t,$$

$$\sigma_c^2 = 14{,}412. \quad (13)$$

The logarithms of the implied spectral densities for S_t and N_t are plotted in Figures 6 and 7. $\hat{f}_S(\lambda)$ has infinite peaks at the seasonal frequencies and minima in between, as was noted in general in Section 4.2. $\hat{f}_N(\lambda)$ has an infinite peak at $\lambda = 0$ and decreases smoothly after that, with no dips or peaks at the seasonal frequencies. This is reasonable behavior for the implied spectral density of an underlying N_t series. Thus the canonical adjustment appears reasonable in this case, whereas additive X-11 with standard options does not.

4.3.5 Classification of Linear Seasonal Adjustment Methods

As a general aid to comparing linear methods of seasonal adjustment and assessing their consistency with observed data, we present a scheme for classifying them. Since model-based approaches are linear and Young (1968) and Wallis (1974) showed X-11 (and hence X-11 ARIMA) to be approximately linear, this scheme covers a large number of proposed adjustment methods. The linear filter used is the most important element in a linear adjustment method, so our classifications are based on the derivation of the various linear filters. Our scheme and some of the methods that fall in each group are as follows:

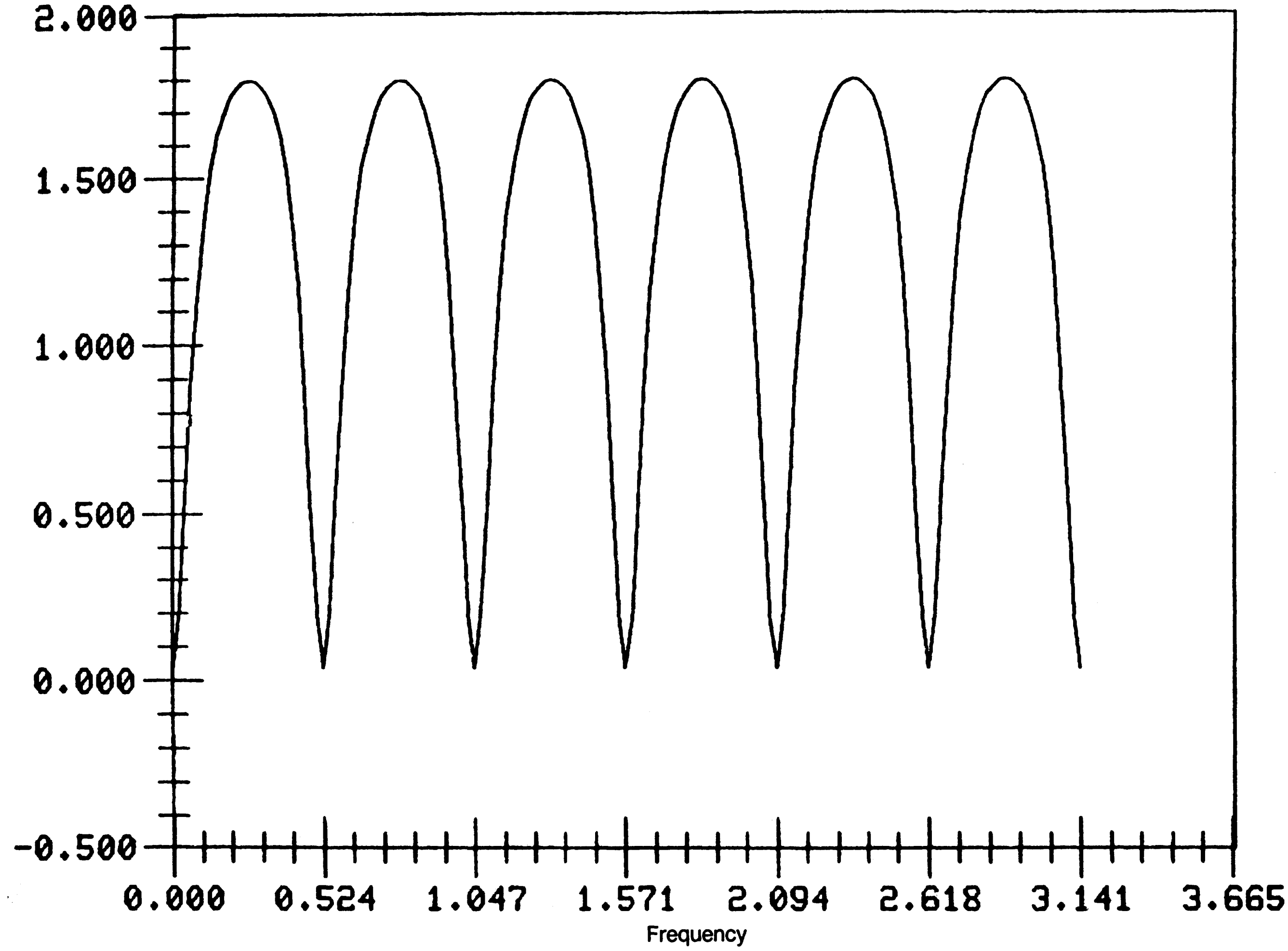

Figure 5. 1.7744[1 − .992 cos(12λ)]/1.16[1 − .69 cos(12λ)].

1. Methods that choose filters directly are (a) X-11, (b) X-11 ARIMA, and (c) SABL.
2. Methods that directly choose models for the components S_t and N_t are (a) Hannan, Terrell, and Tuckwell (1970), (b) Engle (1978), (c) Abrahams and Dempster (1979), (d) Cleveland (1979), (e) Akaike (1980), and (f) Kitagawa and Gersch (1984).
3. Methods that model the observed data and deduce models for the components from that model are (a) Melnick and Moussourakis (1974), (b) Brewer, Hagan, and Perazelli (1975), (c) Geweke (1978b), (d) Pierce (1978), (e) Cleveland (1979), (f) Burman (1980), and (g) Hillmer and Tiao (1982).

Actually, SABL is not really linear, since it uses moving M estimates instead of moving averages. According to Cleveland, Dunn, and Terpenning (1978), however, "the philosophy of its overall approach is exactly the same as that used in the X-11 procedure" (p. 203); so it can be viewed as a robust analogue of a linear method, and the considerations we will discuss here should apply to it. Cleveland (1979) used elements of both 2 and 3; his approach is to fit a model to Z_t, directly choose component models, and then set the parameters in the component models to approximate the overall model for Z_t.

It should be obvious that for methods in group 1, one would have to be extremely lucky to make a choice of filter that would imply reasonable component models and hence be consistent with an adequate model for Z_t. Section 4.3.4 illustrates this point for X-11. The use of nonstandard options in X-11 or other methods in group 1 may increase the chances that an adjustment method will be consistent with the data; but the number of options in such methods is necessarily limited and options are generally selected subjectively, not based objectively on a model for the data. Thus methods in group 1 are at a disadvantage when it comes to being consistent with the data.

The methods in group 2 afford the opportunity to begin with reasonable component models. Because the model for Z_t (up to parameter estimates) is determined

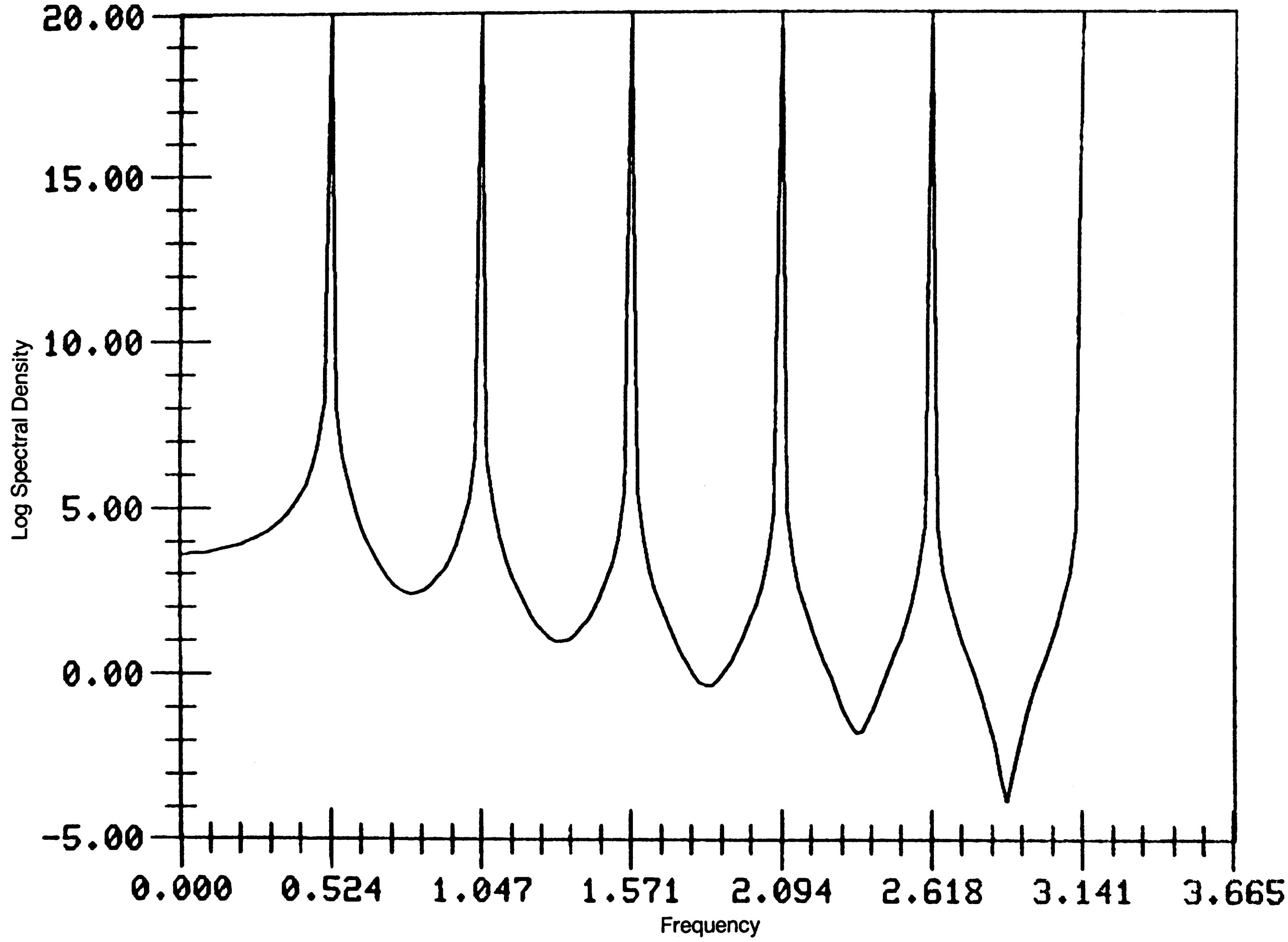

Figure 6. Canonical Seasonal Spectral Density for ENM20.

by the specified component models, it is important when using these approaches to perform diagnostic checks on the adequacy of the resulting model for Z_t. Even when the originally specified component models appear reasonable, if the model for Z_t is deficient in some way, then these component models may not be consistent with an adequate model for Z_t. To determine whether the resulting seasonal adjustment is consistent with the data, one would first have to find an adequate model for Z_t and then proceed in the manner discussed in Section 4.3.4.

Another point to consider about methods in group 2 is that the overall model for Z_t should be estimated subject to the constraints imposed by the component models. Depending on the complexity of the component models, this may be a difficult task—Engle (1978) was unable to estimate his model for Z_t subject to all of the constraints of his component models, whereas Akaike (1980), using simpler component models, was able to do this.

In striving for consistency with the data, methods in group 3 have a potential advantage, in that they begin with a model for the observed data. This advantage will be completely lost, however, if the starting point is an inadequate model for Z_t; hence diagnostic checking of the model is important here, too. The reasonableness of the assumptions leading from the model for Z_t to the component models should also be considered. Usually these assumptions are spelled out explicitly for these methods, which allows them to be readily evaluated.

In Section 4.3.4, we saw that a seasonal adjustment filter does not completely determine models for the components and Z_t. This makes it somewhat difficult to evaluate the assumptions being made about the components for methods in group 1, requiring an analysis like that of Section 4.3.4 for each series. Typically, methods in group 1 are applied without knowing what is being assumed. Regarding methods in group 2 and 3, there generally exist multiple sets of component models leading to the same model for Z_t. To avoid this

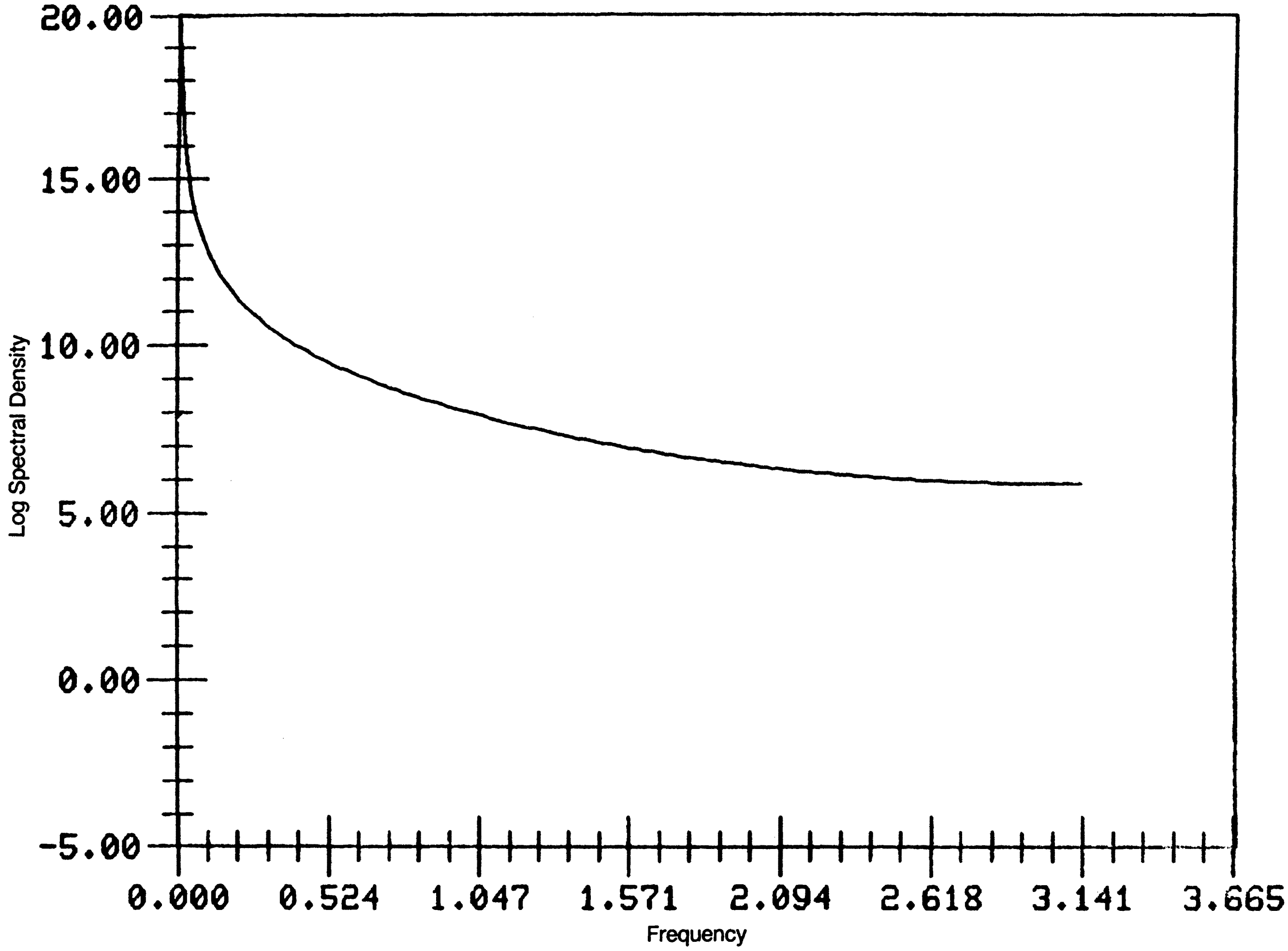

Figure 7. Canonical Nonseasonal Spectral Density for ENM20.

identification problem, a particular choice must be made. Problems arise when this process is given insufficient attention and the choice is not justified, which is why we attempted to justify our choices in Section 4.2. Again, methods in group 3 have a potential advantage here, in that this approach forces consideration of the range of possible component models consistent with the model for Z_t. Methods in group 2 often employ component models based on considerations other than the suitability of their expression of beliefs about seasonality—considerations such as simplicity of the resulting estimation of the model for Z_t.

In conclusion, we favor adjustment methods in group 3 because we believe that the model for Z_t is a logical starting point in developing an adjustment method that will be consistent with the data, and because we feel that acceptable assumptions, such as those offered in Section 4.2, can be made leading from the model for Z_t to component models.

ACKNOWLEDGMENTS

This article is based on work supported by National Science Foundation (NSF) Grant SES 81-22051, "Research to Improve the Government-Generated Social Science Data Base." The research was partially conducted at the U.S. Bureau of the Census while the authors were participants in the American Statistical Association/Census Bureau Research Program, which is supported by the Bureau of the Census and through the NSF grant. In addition, Hillmer was partially supported by NSF Grant SES 82-19336. Any opinions, findings, and conclusions or recommendations expressed here are those of the authors and do not necessarily reflect the views of the National Science Foundation, the American Statistical Association, or the Bureau of the Census. The authors would like to thank David Findley, Sandra McKenzie, Arnold Reznek, Arnold Zeller, an associate editor, and a referee for valuable comments and discussions.

[*Received June 1983. Revised January 1984.*]

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