

# Question about MLE

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2019/9/16

Hi Aaron,

This .pdf file is to illustrate my question about MLE's(or likelihood) computation.

When we talked about MAP, to compute it, we need to know the likelihood function for our parameters at first.

For the expression in SSM,

- $y_t = \mu_t + \gamma_t + \epsilon_t$  , where  $\epsilon_t \sim N(0, \sigma_y^2)$
- $\mu_{t+1} = \mu_t + \phi_t$  , where  $\phi_t \sim N(0, \sigma_\mu^2)$
- $\gamma_t = \sum_{j=1}^{s/2} \gamma_{jt}$
- $\gamma_{j,t+1} = \gamma_{jt} \cos \lambda_j + \gamma_{jt}^* \sin \lambda_j + \omega_{jt}$
- $\gamma_{j,t+1}^* = -\gamma_{jt} \sin \lambda_j + \gamma_{jt}^* \cos \lambda_j + \omega_{jt}^*$  , where  $\omega_{jt}, \omega_{jt}^* \sim N(0, \sigma_\gamma^2)$

In Bayesian, if I want to compute the MAP of one parameter  $\theta$ , I need to know the posteriori of it, which is essentially equal to compute the product of the prior of  $\theta$  and the likelihood of it.

Now, the prior will be given by us manually. My problem is how to get the likelihood expression of  $\theta$  without other parameters.

In our problem, if I write down the likelihood expression of  $\sigma_y^2$ , it will be

$$L(\sigma_y) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{y_i - \mu_i - \gamma_i^2}{2\sigma_y^2}\right)$$

that is to say:

$$L(\sigma_y) \propto \frac{1}{\sigma_y^2} \exp\left(-\frac{\sum_{i=1}^n y_i - \mu_i - \gamma_i^2}{2\sigma_y^2}\right)$$

But we only know the series  $\{y_t\}$  which is our data but don't know the information of trend series  $\{\mu_t\}$  and  $\{\gamma_t\}$ .

Therefore, my question is how to fix this?(Similarly for other variances)