Comparison of Seasonal Adjustment Approaches through State Space Representation

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Abstract

Two approaches for seasonal adjustment are used predominantly by statistical agencies throughout the world, namely X-12-ARIMA which uses moving averages to separate seasonal and irregular effects from the trend-cycle and TRAMO-SEATS which uses explicit parametric time series models to do so. State space models can also be considered for seasonal adjustment although they have not been widely adopted in official statistics at this point. We consider several state space models including one to approximate each of the predominant methods and empirically compare the methods using real datasets from Statistics Canada to gain insight into the different decompositions. The components of the model are contrasted to illustrate their differences, and the steps required to tune a state space model to approximate them are elaborated.

1 Introduction

Seasonal adjustment is a procedure that aims to remove predictable patterns in order to help analysts of the data to make fair comparisons between reference periods. In practise, these predictable patterns such as seasonal movements can mask the underlying signal, particularly if their magnitudes are large. Important signals that may be of interest to users can include the detection of turning points, as well as estimates of the pace and direction of changes between two time points.

In this paper, we empirically compare several available methods for seasonal adjustment using real datasets from Statistics Canada to gain insight into the different decompositions, and consider several state space models including one to approximate each of the predominant methods for seasonal adjustment.

The paper is organized as follows: in section 1 the general context around seasonal adjustment is provided, and several specific methods are detailed in section 2. Section 3 gives an empirical comparison of the results from each method, using a limited number of datasets from Statistics Canada's statistical programs. Section 4 presents suggestions from the literature to develop more comparable state space models, and details the current work to modify the basic structural model to approximate existing methods. The paper concludes with a summary discussion of results and the identification of several areas for future work in Section 5.

1.1 Seasonal Adjustment

To apply seasonal adjustment, the observed (unadjusted) series is decomposed into a number of components, and the seasonally adjusted series is constructed by adding together a subset of the components. A brief description of each component is provided below:

Trend-cycle (TC_t): This represents the smoothed version of the time series and indicates its general pattern or direction. The trend-cycle can be interpreted as the long-term movement in the time series, the result of different factors (or determinants) that condition long-run changes in the data over time. As its name suggests, the trend-cycle also reflects periodic expansions and contractions in economic activity, such as those associated with the business cycle.

Seasonal (S_t): These represent regular movements or patterns in time series data that occur in the same month or quarter every year. On the basis of past movements of the time series, these regular patterns repeat themselves from year to year. These seasonal patterns are fairly stable in terms of timing, direction and magnitude. Often these seasonal effects relate to well-established yearly variations in economic activity, such as the increase in retail sales in the lead up to Christmas, or increases in construction employment in the spring. Seasonal effects identify these regularly occurring patterns in the data.

Calendar Effects (C_t): Aside from seasonal effects, other systematic calendar-based effects can influence the level of economic activity in a specific period. The most important of these are the trading-day effects. These effects can be present when the level of economic activity varies depending on the day of the week. For example, retail sales are usually higher on Saturdays than on any other day of the week. Consequently, a five-Saturday month is more likely to result in higher retail sales than a month with only four Saturdays. Another common example of a calendar effect is the date of Easter, which can be expected to increase retail sales in March or April depending on the month in which it occurs. This particular calendar effect is referred to as a moving holiday effect.

Irregular (I_t): This component includes unanticipated movements in the data that (1) are not part of the trend-cycle, and (2) are not related to current seasonal factors or calendar effects. The irregular component could relate to unanticipated economic events or shocks (for example, strikes, disruptions, unseasonable weather, etc.), or can simply arise from noise in the measurement of the unadjusted data (due to sampling and non-sampling errors).

Ultimately, the seasonally adjusted series can be seen as the original series with seasonal and calendar effects removed, or, equivalently a new series that is constructed by adding the trend-cycle and irregular for each reference period.

Figure 1: Relationship between components and seasonally adjusted series

$$\boxed{\begin{aligned} SA_t &= Y_t - S_t - C_t \\ &= TC_t + I_t \end{aligned}}$$

Users are often interested in the movement including the irregular as it can help to estimate the effect of specific events such as strikes or weather events. Over the longer term, the trend-cycle component can be used to identify turning points and inform policy decisions by government. In cases where the users are interested in the underlying long-term direction of the data, it is recommended that the trend-cycle component alone be used as it is more robust against month-to-month short-term effects. For more detail on the interpretation of seasonally adjusted data, see Statistics Canada (2012).

2 Seasonal Adjustment Methods

The need for seasonal adjustment has been recognized for approximately 100 years and a variety of methods have been proposed to decompose the data into components. Some of the methods are directly based on parametric models such as TRAMO-SEATS and state-space models, and are non-parametric, such as X-12-ARIMA which uses techniques such as moving averages to estimate the components.

2.1 The X-12-ARIMA method for seasonal adjustment

The first method we present is X-12-ARIMA. This method is the result of incremental developments to the original Census I method (Shiskin (1957). These developments ultimately resulted in the X-12-ARIMA method detailed in Findley et al (1998). Enhancements were gradually implemented over this period of time with important contributions from the United States Census Bureau and Statistics Canada as well as other organisations.

The X-12-ARIMA method consists of two main steps. The first step is based on a RegARIMA model which includes regressors to represent trading day effects, moving holiday effects and outliers. Once this model has been estimated, it is used to forecast future observations and back-cast earlier observations which are included in the extended series that is then used in the second step. The second step of X-12-ARIMA involves several iterations of an algorithm to apply moving averages to estimate components, and detect and treat extreme observations. Another desirable feature of the method is that it can be applied in a semi-automatic way, by using system generated parameters initially, and applying updates to parameters to improve the results for individual series, for example those with components that are dynamic. This approach achieves reliable results in general, without an extreme amount of manual resources.

In the case of Statistics Canada, this is the method used throughout the agency for seasonally adjusting estimates to be published or used internally, as pointed out in Statistics Canada's quality guidelines for seasonal adjustment (Statistics Canada, 2009, pp 63-67). This method has been used by many organizations throughout the world over time for seasonal adjustment. In particular, at Statistics Canada, the X-12-ARIMA method for seasonal adjustment has historically performed very well and Statistics Canada has developed a great deal of experience and expertise in applying the method in practise to real data. Of particular value are the transparency and intuitiveness for users, and the robustness of the method to limit the influence of individual observations. Nonetheless, some competing methods based on models are appealing as they bring certain advantages, most notably in automation, production of quality indicators, and the potential to integrate other time series processes within a unified framework. While the method does not produce variance estimates directly, the variance of the seasonally adjusted series can be estimated using for example, linearization or resampling methods.

2.2 The TRAMO-SEATS method for seasonal adjustment

TRAMO-SEATS is another method for seasonal adjustment originally developed by Burman (1980) and detailed in Gomez and Maravall (1997) that is currently widely used by statistical agencies. The pre-adjustment for calendar effects and outliers is very similar to that of X-12-ARIMA. The most significant difference between these methods is the decomposition between trend-cycle, seasonal and irregular components. The TRAMO-SEATS algorithm fits an ARIMA model to each of the three components, using a canonical decomposition to maximize the variance of the irregular. These estimated ARIMA models are then used to derive optimal filters to estimate each component. This yields a method that is fully automatic, although some key parameters can be adjusted such as which calendar effects to include and which periods are to be considered outliers. Literature on the method indicates that the results are reliable for the vast majority of cases, however it is not clear what actions can be taken for series where the results are not satisfactory (those that are highly irregular, or include moving seasonality). The method also produces a model-based variance estimate for each component along with the seasonally adjusted series, although it does not include a variance component resulting from measurement error in the input series, which could be important, especially in cases where the series is comprised of estimates from a sample survey.

2.3 State Space Models for Seasonal Adjustment

State space models offer an alternative to these methods that is flexible and transparent, and are expressed in a framework that can be applied to other time series problems. While state space models cover a broad range of models, the model considered in this work is the basic structural model, as described in Durbin and Koopman (2001). This model includes a seasonal and trend component in the measurement equation, as well as a white noise error term whose realizations are interpreted as the irregular component. A local level model is used to represent the trend-cycle and a trigonometric form is used for the seasonal component. The variance components dictate to what degree each component changes from period to period or year to year. The measurement and transition equations are provided below.

Measurement equation:

$$y_t = \mu_t + \gamma_t + \varepsilon_t \qquad \varepsilon_t \sim N\left(0, \sigma_y^2\right)$$

Transition equations:

$$\mu_{t+1} = \mu_t + n_t + \delta_t \qquad \delta_t \sim N\left(0, \sigma_\mu^2\right)$$
$$n_{t+1} = n_t + \varphi_t \qquad \varphi_t \sim N(0, \sigma_n^2)$$

$$\gamma_t = \sum_{j=1}^{s/2} \gamma_{jt}$$
$$\gamma_{jt} = \gamma_{jt} \cos \lambda_j + \gamma_{jt}^* \sin \lambda_j + \omega_{jt}, \qquad \omega_{jt}, \omega_{jt}^* \sim N\left(0, \sigma_\gamma^2\right)$$
$$\gamma_{jt}^* = -\gamma_{jt} \sin \lambda_j + \gamma_{jt}^* \cos \lambda_j + \omega_{jt}^*$$

We point out here that the variance in the transition equation for the trend level, σ_μ^2, has been set to zero to increase the smoothness of this component. The model can be estimated with the Kalman filter, which is described and developed in detail in Durbin and Koopman (2001). The Kalman filter includes a filtering pass where each state and variance is estimated based only on the past observations, and a smoothing pass where the information from all available observations (past and present and future) are used in the estimation for each reference period. In order to initialize the Kalman filter, a prior distribution is required on the initial states and variances. If little or no information is available on this, we can specify a diffuse prior with a very large variance so that the effect of the prior is limited to the initial observations, after which the filtered values are based almost entirely on the observed data. The model lends itself naturally to variance estimation, and in fact, the measurement error can be reflected by providing it to the model as another error component with known variance.

We now turn our attention to the component variances in the model, and their impact on the results. The model, including the variance, is generally estimated using maximum likelihood methods or the Kalman filter. A larger value of the variance for a given term gives more flexibility for that component to absorb movements from the data. A larger variance in the measurement equation (σ_y^2) would imply a larger irregular component. A larger variance in the level or slope of the trend term (σ_μ^2 and σ_n^2) would imply a more variable trend component. Finally, a larger variance in the seasonal component model (σ_γ^2) would imply more movement from year-to-year in the estimated seasonal pattern, in effect responding more to the observed values, and less bound by the seasonal pattern that is observed in other years.

3 Empirical Comparison of the Methods

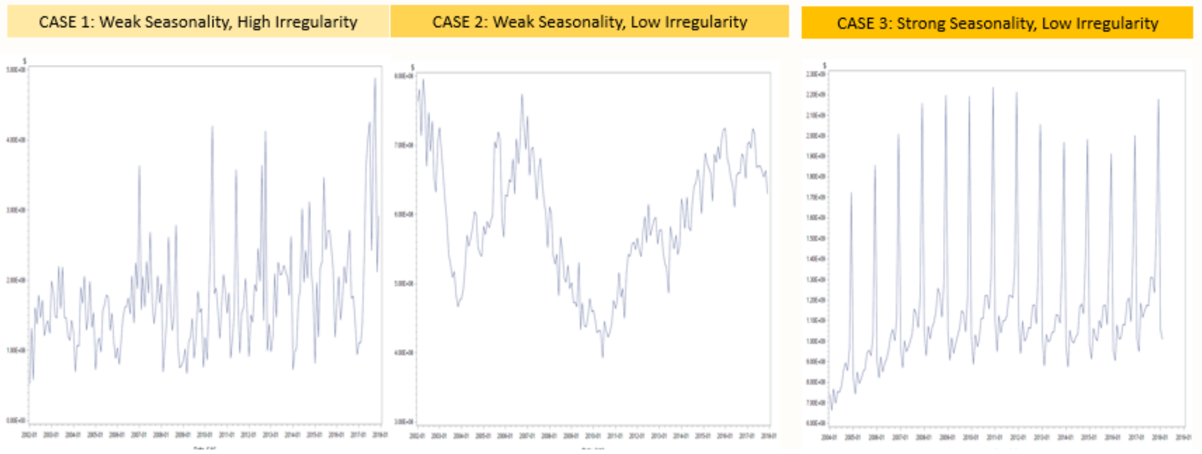
In order to better understand the differences between these methods, an empirical comparison was carried out using real datasets from Statistics Canada's programs. Each dataset is a monthly economic indicator where a seasonally adjusted estimate is produced on a monthly basis. Three series were chosen to show a range of possibilities for situations faced in developing seasonally adjusted estimates.

Case 1: This series represents the dollar value of building permits for a specific geography and building type in Canada. This is a case where the raw data series displays a large amount of irregularity, and only a weak seasonal pattern. As a result, this a series that is quite difficult to adjust, and it is expected that the choice of seasonal adjustment method may have a large impact on the resulting seasonally adjusted series.

Case 2: This series represents the aggregate dollar value of manufacturing inventories for establishments within a specific manufacturing industry in Canada. This is a case where the seasonal pattern is more apparent, but the data still includes a large amount of irregularity. This is a fairly typical example of a series that we adjust at Statistics Canada, for which we would typically evaluate the effect of modifying parameters used in the X-12-ARIMA method to improve the quality of the seasonal adjustment.

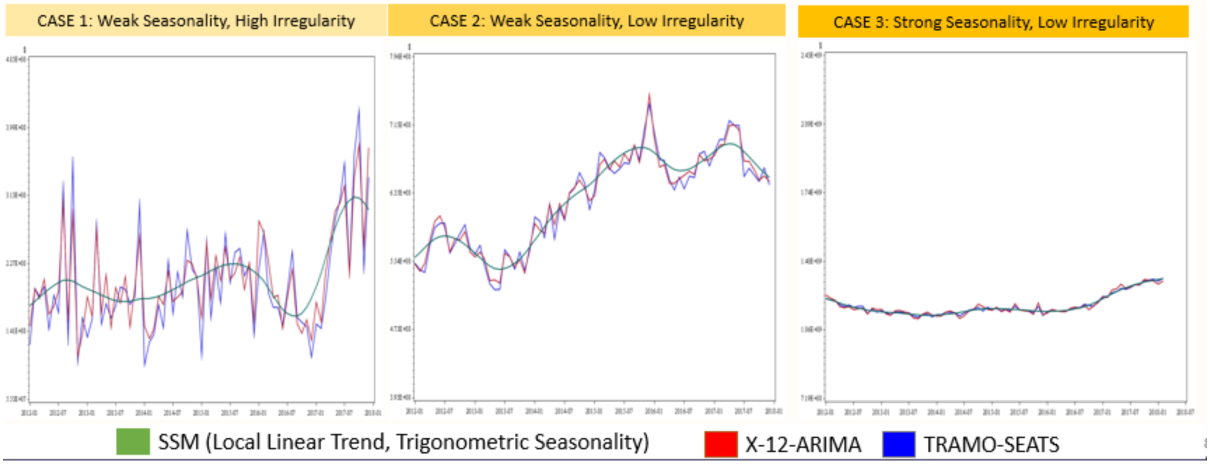
Case 3: This series represents retail sales of electronics retailers in Canada. This is an example of a series with an obvious and stable seasonal pattern, and a small degree of irregularity. This series can be seasonally adjusted reliably using an automatic X-12-ARIMA approach and would be expected to yield good results.

Figure 1 - Unadjusted Data used in Empirical Comparison: Case 1, 2 and 3



In order to isolate the differences between the decomposition by each method, the unadjusted data first had calendar and outlier effects removed using a RegARIMA adjustment similar to X12-ARIMA. This was done so that differences in the resulting seasonally adjusted series can be attributed uniquely to the decomposition between trend-cycle, seasonal and irregular components.

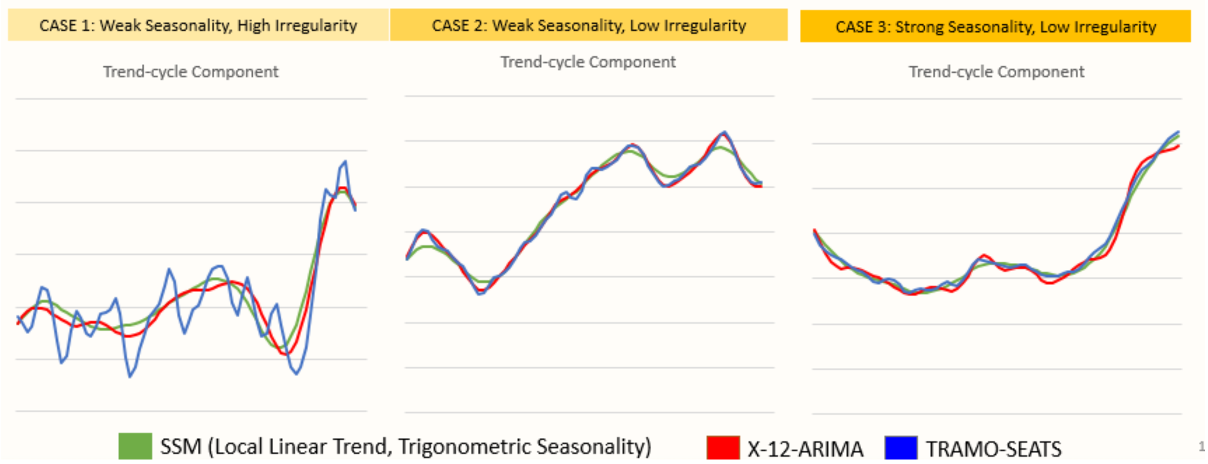
Figure 2 - Seasonally Adjusted Data used in Empirical Comparison: Case 1, 2 and 3



These graphs are shown in the same scales as Figure 1 in order to emphasise the smoothing effect of seasonal adjustment. The seasonally adjusted series are much smoother as repeating seasonal fluctuations have been removed. While in this scale the differences are not as apparent, we can immediately see some differences in the seasonally adjusted series from the three methods. In case 3, each of the adjustments removes a large amount of seasonality, and in fact, the three methods give quite similar results. Cases 1 and 2 show more of an impact of the choice of seasonal adjustment method. First of all, the state space model yields a seasonally adjusted series that is very smooth. In fact, the results contain very little irregular movement from month-to-month which is not typical of a seasonally adjusted series and resembles a traditional trend-cycle series more closely. The TRAMO-SEATS and X12-ARIMA methods give more similar results to each other, although there is somewhat more noise remaining after the TRAMO-SEATS adjustment.

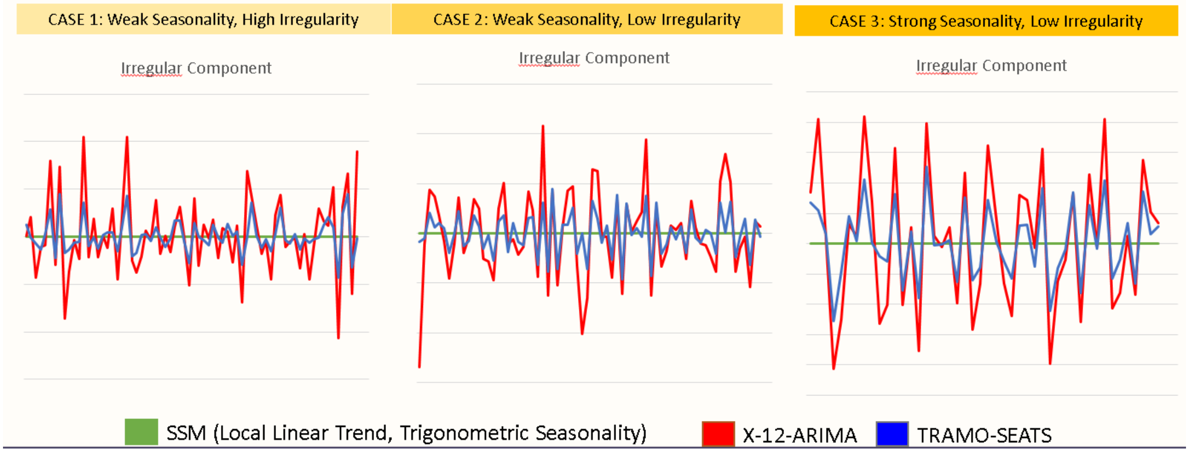
We now consider each component from the decomposition individually: the trend-cycle, irregular and seasonal.

Figure 3 - Trend-cycle component from each decomposition: Case 1, 2 and 3



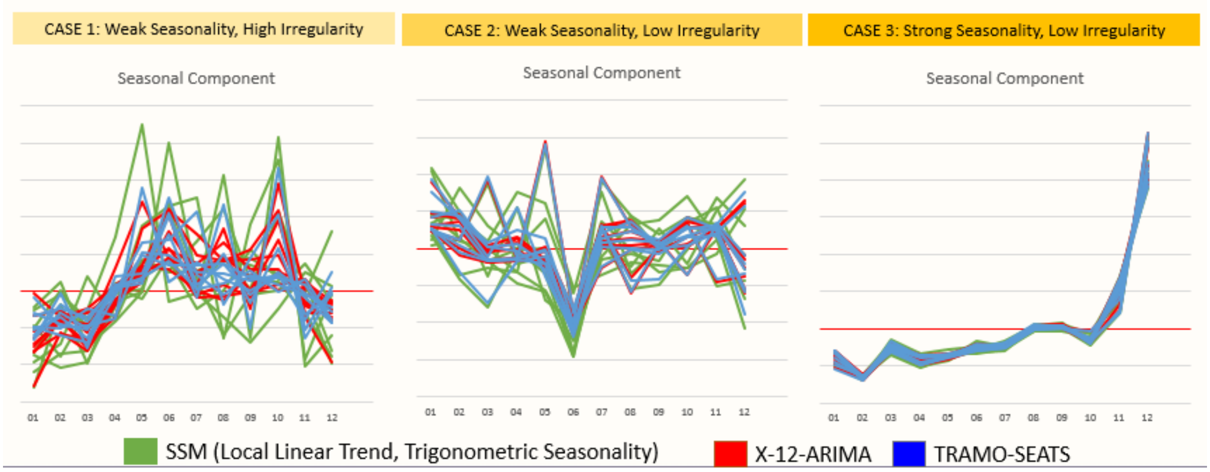
In case 3 the trend-cycle is very similar between the three approaches. This is also true of case 2, although some differences are observed between the state space model and the other two methods, where the state space model is generally smoother. In case 1, the trend-cycle component of the TRAMO-SEATS algorithm is much more volatile than the result for either X-12-ARIMA or state space model. This result could be problematic for identifying turning points and pace of growth of a series based on the estimated trend-cycle component.

Figure 4 - Irregular component from each decomposition: Case 1, 2 and 3



The irregular component is markedly different for the three methods. The irregular component from the state space model is very small, appearing almost as a flat line over time. The irregular components for the other two methods are larger in magnitude, with X12-ARIMA producing the largest irregular.

Figure 5 - Seasonal component from each decomposition - Case i, ii and iii

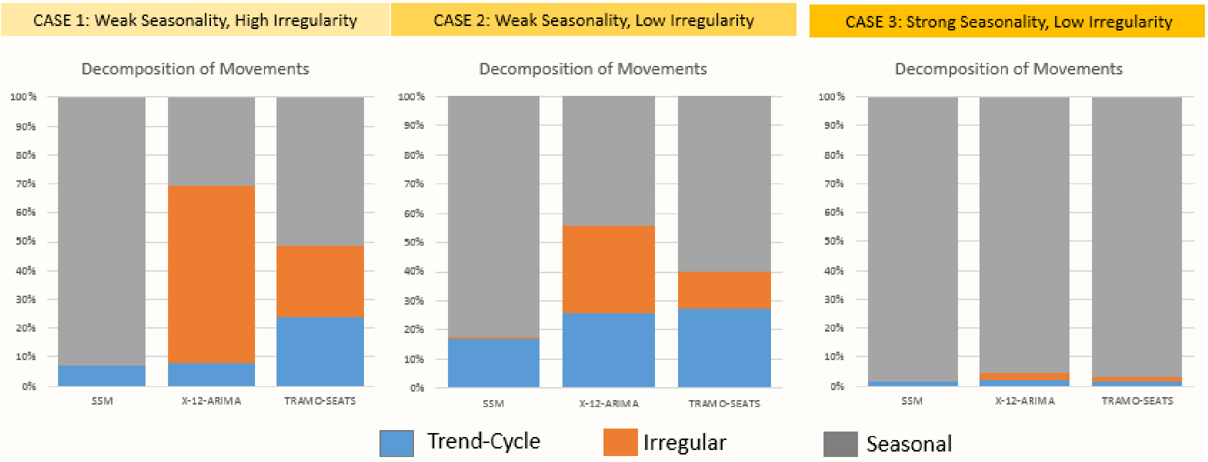


As expected, in case iii the seasonal components are very similar, essentially covering each other and are visually indistinguishable on the graphs. However, in cases i and ii, the seasonal component from the state space model is more variable than the other methods. The seasonal components for X-12-ARIMA and TRAMO-SEATS are quite similar, with slightly more stability from year-to-year in the X-12-ARIMA method.

To visually summarize the comparison, graphs were produced which display the amount of the total month-to-month variability that is absorbed into each component under each method based on the empirical study. The statistic represented in these three graphs is the relative share of the month-to-month movement over the entire series that is removed with the incremental removal of each component. For example, the percentage represented by the seasonal component is estimated as:

$SEAS = \frac{\sum_{t=2}^{N}((y_t - S_t) - (y_{t-1} - S_{t-1}))^2}{\sum_{t=2}^{N}(y_t - y_{t-1})^2}$, and the absorption of the irregular and trend-cycle components are calculated analogously by further removing the appropriate component from each term.

Figure 6 - Movement absorption to each component - Case 1, 2 and 3



In summary, the state space model leads to the seasonal component absorbing the vast majority of movement, with very little absorbed by the trend-cycle, and the irregular component. This would coincide with less stability in the seasonal pattern from year to year, as more of the individual movements from month-to-month are absorbed within the seasonal component. Conversely, for X-12-ARIMA and TRAMO-SEATS we see that a larger share of the movement in Cases 1 and 2 is absorbed into the irregular component (more so in X-12-ARIMA), with a reduction in the absorption into the seasonal component. This implies a more consistent seasonal pattern from year-to-year from these methods. Note also that in case 1, much more variability is absorbed into the trend-cycle component for TRAMO-SEATS, which clearly is not as smooth, as indicated by figure 3.

The large difference between the state space model and the other two methods has been previously noted in the literature. In fact, Durbin (2000) proposes that more work be done to develop state space models that behave more like the other seasonal adjustment methods. Without a formal definition for each component, this may be difficult, but a number of suggestions have been offered, including the following: consider a broader class of state space model instead of limiting ourselves to the basic structural model, using only the filtering pass of the Kalman filter to derive the seasonal term removed from the seasonally adjusted values, and simplifying the form of the seasonal state by removing higher frequency trigonometric components, allowing for the method to have a more crude but stable seasonal component. In this paper, we explore an option to constrain the relationship between the variance components to redirect the absorption of the movements away from the seasonal component, primarily towards the irregular component.

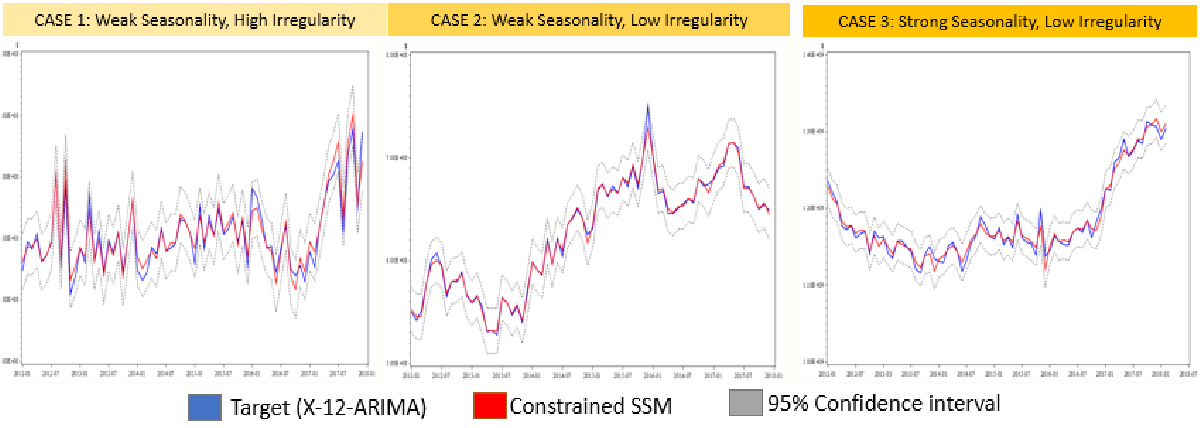
4 Approximations through State Space Representation

For this exploration, we have the benefit of having applied X-12-ARIMA and TRAMO-SEATS to each dataset, and each can act as a target for seasonal adjustment via state space models. In order to constrain the state space model to approximate each result, we construct a loss function consisting of the squared difference between the seasonal adjustment target and seasonally adjusted estimate from the state space model, and manipulate parameters to minimize this loss function.

$$\min_{\frac{\sigma_y^2}{\sigma_n^2},\frac{\sigma_\gamma^2}{\sigma_n^2}} \sum_{t=1}^{T}\left[z_t^{*(TARGET)} - z_t\left(\sigma_n^2,\frac{\sigma_y^2}{\sigma_n^2},\frac{\sigma_\gamma^2}{\sigma_n^2}\right)\right]^2$$

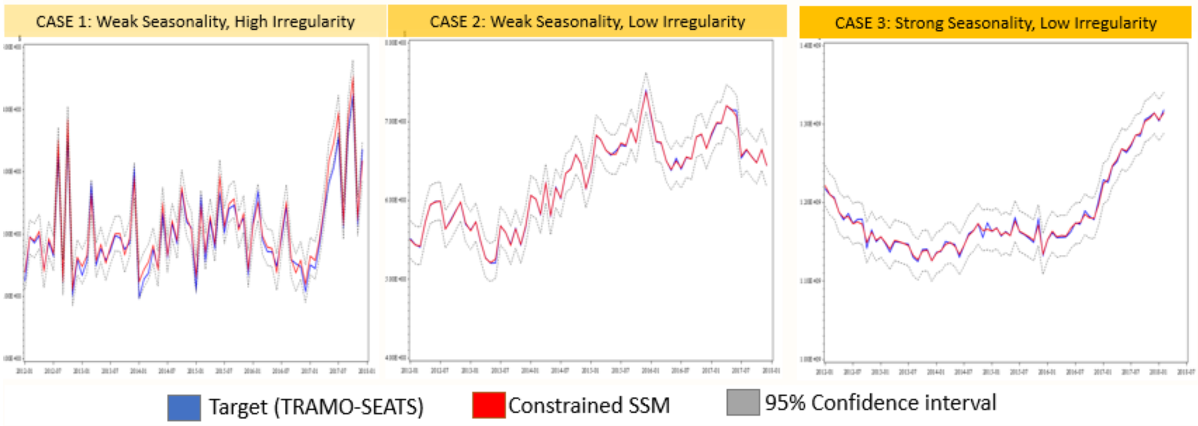
The specific parameters that we manipulate are the ratios of the state variances, and the optimum is found using a multi-stage grid search over the feasible region. From this approach we are able to estimate a constrained state space model to approximate the results of X-12-ARIMA and TRAMO-SEATS for each of the three series.

Figure 7 - Comparison of X-12-ARIMA adjustment and constrained state space model



We can see from these graphs that the constrained state space model gives very similar results to the target of X-12-ARIMA. In fact, the X-12-ARIMA target is within the 95% confidence interval of the state space model seasonal adjustment for virtually all data points. The overall effect is that much more of the month-to-month movement is captured in the seasonally adjusted series, than was the case for the unconstrained basic structural model.

Figure 8 - Comparison of TRAMO-SEATS adjustment and constrained state space model



Again, we see that the constrained state space model gives very similar results to the target. In this case, the TRAMO-SEATS target is within the 95% confidence interval of the state space model seasonal adjustment for virtually all data points.

We note that this exercise is equivalent to targeting the seasonal component alone since it is the only difference between the unadjusted (fixed) and the seasonally adjusted (target). This demonstrates that the basic structural model can be fine-tuned to yield very similar seasonally adjusted results. We do note that the estimated variances of the constrained state space model are quite large compared to the basic structural model with no constraints, which is expected since the parameters are intentionally moved away from their maximum likelihood estimates under the unconstrained model. Nonetheless, analysis of the constraints that were applied show that for both X-12-ARIMA and TRAMO-SEATS, the constraints that were required implied a large decrease in the variance of the seasonal component. In the case of X-12-ARIMA, the constraint also implied a large increase in the variance of the irregular. In the case of TRAMO-SEATS, while the seasonal component was approximated very well by the state space model, the irregular and trend-cycle components were not. The constrained SSM for TRAMO-SEATS included a very smooth trend-cycle, and an overly large irregular component. Further work is being considered to include each individual component in the loss function instead of implicitly targeting only the seasonal component.

5 Conclusions

In conclusion, the empirical study confirmed that in some cases the differences between the three methods are negligible, and in other cases, the differences are more important. In cases where the differences are large, the state space model tends to over-smooth in comparison with the other methods, which are broadly accepted based on a vast amount of user experience over time. As well, the options to modify the TRAMO-SEATS method are limited and it is unclear how to proceed for cases where the results are poor (those that cannot be accurately modelled).

The approach to constrain the state space model show some promise to produce more typical seasonally adjusted series. To further explore this option, we propose to investigate data-driven constraints, analogous to the automatic setting of X-12-ARIMA parameters based on diagnostics of the input data, such as signal to noise ratios. The option of seasonally adjusting using another method to provide a target to find constraints is not appealing in practise. At the same time, a different specification of the basic structural model in terms of seasonal and trend components may provide a more appealing solution. Other aspects would also need to be developed including a complete treatment of calendar effects and outliers, as is currently done in the pre-treatment step for X-12-ARIMA and TRAMO-SEATS. Further, the assumptions on the irregular need to be validated as they are assumed to be white noise in the state space model, whereas some autocorrelation is included in the X-12-ARIMA and TRAMO-SEATS methods which may be more realistic.

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