PaperReading

linzy

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Contents

1 Glueball

1.1 Gluons in glueballs: Spin or helicity

Title: Gluons in glueballs: Spin or helicity. [19]

这篇文章提出了一些新的关于胶子自旋的处理方法的讨论，通过它的讨论结果构造了一种哈密顿量模型，并得到了两胶子体系的能谱结构，并将其与格点的结果进行比较。值得一提的是，这篇文章中提到了关于瞬时相互作用对于哈密顿量的不可或缺的意义。

这篇文章中提到了一些关于胶球系统和QCD等的一些基础概念：

Like other effective approaches of QCD, potential models still have difficulties to cope with gluonic hadrons. Assuming that glueballs are bound states of valence gluons with zero current mass, it is readily understood that the use of a potential model, intrinsically non covariant, could be problematic in this case.

在处理涉及到胶子的强子时，很多有效模型都遇到了困难。这里还提到了一点有趣的结论：胶球是0流质量的价胶子。

The main challenge for this kind of model is actually to find a way to introduce properly the more relevant degree of freedom of the gluon: spin or helicity.

对于胶子来说，描述它的关于自旋的自由度选择自旋还是螺旋度是一个很关键的问题。

As quantum chromodynamics (QCD) is built on the nonabelian SU(3)-color group, it allows for purely gluonic bound states called glueballs.

量子色动力学建立在SU（3）的颜色群上，所以允许一种仅包含胶子的基态，叫做胶球（glueball）。

On the other hand, pure gauge QCD has been investigated by lattice QCD for many years, leading to a well established glueball spectrum below 4 GeV [2, 3].

在这篇文章的参考文献2,3中，介绍了由格点计算的胶球能谱。

As a bound state of two gluons can only have $C = +$, it is rather natural to assume that the lightest glueballs are mainly two-gluon states (the more constituent gluons are present, the more the glueball should be heavy).

这里提到了很重要的一点，关于胶子系统的宇称。由于宇称的限制，最轻的胶子系统应当包含两个胶子。但是关于如何确定整个系统的J^{PC}宇称，这是一个非常重要但是复杂的问题，甚至我认为这作为一个量子数应当在构建基态时就进行处理，需要进一步对其进行研究。

Moreover, it is interesting to mention some results of the Coulomb gauge study of Ref. [8]. In this approach, a Fock space expansion of glueball states in terms of quasigluons can be performed, and it appears that the influence of the three- and four-gluon components on the low-lying $C = +$ glueballs is negligible: The two-gluon component is dominant as intuitively expected.

对于在福克空间中展开的胶球系统，他们采取库伦规范。为什么有这样的要求？在参考文献8中也许有这个问题的答案。在福克空间展开中，三胶子，四胶子组分仅占了很小的比例，主要是二胶子的系统。

In this picture, the valence gluon is a posteriori massive, because it is confined into a glueball. Let us note that, more generally, both quarks and gluons can gain a constituent mass from renormalization theory (in the Coulomb gauge approach of Ref. [14], massless gluons gain a constituent mass of about 0.7 GeV at zero momentum). But, other studies keep the assumptions of Ref. [10] and state that a valence gluon has to be a priori considered as massive, that is with a non zero current mass [10, 15].

在参考文献10,14,15中提出了对于胶子质量的两种不同的处理方式，也是本文的核心讨论部分。

Moreover, the glueball spectrum is now far better known than at the time of Ref. [9] thanks to lattice QCD calculations [2, 3].

参考文献2,3,9中介绍了格点计算胶球质量谱的结果。

It is of great phenomenological interest to be able to express a given helicity state in terms of states of given orbital angular momentum L and intrinsic spin S.

关于角动量和自旋的波函数是一个非常引人关注的点。这篇文章中同样花了较小的篇幅介绍相关波函数的处理和作用，但是并没有给出计算结果。

In particular, the necessity of adding instanton-induced forces should not be seen as a flaw of the model, but rather as a way to be more coherent with other studies showing that instantons contribute in glueballs.

加入瞬时相互作用项是一种更接近物理的要求而非单纯的模型上的调整。

1.2 The glueball spectrum from an anisotropic lattice study

Title: The glueball spectrum from an anisotropic lattice study. [20]

1.3 The Status of Glueballs.

Title: The Status of Glueballs. [22]

1.4 吕晓睿-郑州暑期学校

PPT中介绍了一些BESIII的实验，其中包括了J/Ψ的衰变中可能的与胶球相关的部分。（P31）

https://indico.ihep.ac.cn/event/24901/contributions/199839/attachments/94226/123556/0824%E5%90%95%E6%99%93%E7%9D%BF-LYU_%E9%83%91%E5%B7%9E%E8%AE%B2%E4%B9%A0%E7%8F%AD2025.pdf

BESIII的介绍，其中有关于胶球的实验的详细的信息。

https://indico.ihep.ac.cn/event/19694/contributions/133803/attachments/69026/82586/BJLiu_BESIII500.pdf

1.5 Calculations of Glueball Wavefunction in Lattice QCD

Title: Calculations of Glueball Wavefunction in Lattice QCD [26].
格点计算胶球的波函数的文章。

1.6 Glueball phenomenology and the relativistic flux tube model

Title： Glueball phenomenology and the relativistic flux tube model[5].

1.7 Glueball Calculations in Large-Nc Gauge Theory

Title: Glueball Calculations in Large-Nc Gauge Theory [8].
用光前方法计算胶球的文章。

1.8 Pure glueball states in a Light-Front holographic approach

Title: Pure glueball states in a Light-Front holographic approach [23]

Satvir推荐的glueball的文章，它们采用了Light-Front holographic的方法进行计算。

其中提到了Lattice的结果，0^{++}也是在1710附近的。然而soft wall model等会将$f_0(500)$作为基态进行fit。

还有一篇专门计算标量胶球的内容类似的文章：https://arxiv.org/pdf/2002.11720

1.9 On Glueballs, Pomeron, and Odderon in Holographic Quantum Chromodynamics: From Hadron Spectroscopy to Collider Phenomenology

Title: On Glueballs, Pomeron, and Odderon in Holographic Quantum Chromodynamics: From Hadron Spectroscopy to Collider Phenomenology[12]

Satvir推荐的关于glueball的综述文章，其中详细介绍了如何通过HolographicQCD计算胶球的过程。

基于QCD的Holographic方法，被称为ADS/QCD模型。它希望在一个五维时空里做一个超对称的理论，并且将其在QCD的思维时空中尽可能的组装起来。根据破坏共形性质方法的不同，可以区分其中的两种主要模型： hard-wall模型和 soft-wall 模型。这种方法得到的结果也适合用来计算FF，PDF,GPD等结构函数。实验上对相关结果的支持主要来自于小x区域的深度非弹实验。

1.10 Discovery of a Glueball-like particle X(2370) at BESIII

Title: Discovery of a Glueball-like particle X(2370) at BESIII [13]

天才推荐的文章。tensor胶球在BESIII的实验中得到了一些可能的数据，大多数是来自于J粒子的衰变。

还可以看到关于这篇文章的工作的一些汇报：

https://indico.ihep.ac.cn/event/23322/contributions/166226/attachments/86033/110266/7th%20Prof.%20Jin%20X2370-J50.pdf

1.11 The Glueball in a Chiral Linear Sigma Model with Vector Mesons

Title: The Glueball in a Chiral Linear Sigma Model with Vector Mesons [15]

线性σ模型对胶球的计算。这里它们在研究$f_0(1370)$,$f_0(1500)$,$f_0(1710)$等。这篇文章的重点在于σ模型，这可能和自发对称性破缺有关系。后面还需要仔细阅读模型相关的介绍。

1.12 Dispersive analysis of glueball masses

Title:Dispersive analysis of glueball masses [17]

利用算符乘积展开（OPE）计算glueball能谱结构的文章。在0.6GeV处发现了一个较小的峰，它认为这是最轻的meson：$f_0(500)$中含有较少的胶球的成分。

1.13 Light Scalar Isoscalar Mesons and the Glueball

Title:Light Scalar Isoscalar Mesons and the Glueball [9]

对f系列的介子进行详细分析的文章。

Convention部分介绍了f的定义：Thus the names for scalar isoscalar mesons which consist of a quark and an antiquark ($q\bar{q}$) and which have the quantum numbers of the vacuum, i.e. spin J = 0, parity P = +, charge conjugation-parity C = +1 and isospin I = 0 are as follows:

If the object is composed of $u\bar{u} + d\bar{d}$, or $s\bar{s}$ or mixtures of the two components it should be called f_0, f_0'

$f_0(500)$的宽度非常大，难以获得一个关于它的准确的质量。这一点可以在这篇文章的fig1中很直观的体现出来。

In the quark model by Gell-Mann [2] and Zweig [3] two isoscalar quark-antiquark ($q\bar{q}$) states are expected: $u\bar{u} + d\bar{d}$ and $s\bar{s}$. The physical states are mixtures of these two states with an unknown mixing angle.

这个观点可能比较有启发性，对于轻介子的物理态，它们是两种理论上的态以某种混合角混合得到的。那么对于胶球而言，我们需要证明它异于上面说的混合态。

The predictions have converged during the past decade to a scalar glueball (JPC = 0++) with a mass of 1.6 GeV/c2 as lowest state.

从05年开始似乎从Lattice到Sum rule的方法就不认为有很低质量的胶球了，普遍认为都在1.5GeV以上。那么$f_0(500)$似乎不能算是胶球？

Having tentatively merged f0(600) and f0(1370) into one resonance, one realizes that a monster resonance has been created with a mass and width of about 800 MeV/c2

这篇文章最后认为它可能$f_0(500)$和其它的态可以合并为一个新的共振态。

1.14 SCALAR GLUEBALL: ANALYSIS OF THE (IJP C = 00++) WAVE

Title: SCALAR GLUEBALL: ANALYSIS OF THE (IJP C = 00++) WAVE [3]

1997年的文章，使用了propagator matrix (D-matrix) technique，认为$f_0(1300)$，$f_0(1500)$是一个可能的候选态。

1.15 Revisiting Scalar Glueballs

Title: Revisiting Scalar Glueballs [6]

15年的文章，支持1710是可能的基态。

整理了到15年为止Latiice对胶球的计算

1.16 Scalar Mesons below 1 GeV

Title: Scalar Mesons below 1 GeV

https://pdg.lbl.gov/2023/reviews/rpp2023-rev-scalar-mesons.pdf#:~:text=In%20this%20note%2C%20we%20discuss%20the%20light%20scalars,and%20f0%28980%29%20Abstract%20both%20with%20I%20%3D%200.

PDG中详细介绍标量介子，包括$f_0(500)$ 的文章。

1.17 Scalar mesons above and below 1 GeV

Title: Scalar mesons above and below 1 GeV [7]

1.18 Scalar isoscalar mesons and the scalar glueball from radiative J/ψ decays

Title: Scalar isoscalar mesons and the scalar glueball from radiative J/ψ decays [25]
主要是介绍BESIII中的J/ψ衰变的实验数据的文章。
Introduction部分写得非常好。有关于f介子的介绍和胶球历代各种方法的理论计算。

1.19 THE LOW LYING GLUEBALL SPECTUM

Title: THE LOW LYING GLUEBALL SPECTUM [27]
同样是QCD Hamiltonian的方法，另外Intro写得不错。

1.20 Glueballs from bound state equations

Title: Glueballs from bound state equations [14]
BSE方法计算胶球基态的文章，2022年的比较新并且和格点的结果有比较。

1.21 Glueballs in Radiative J/ψ Decays

Title: Glueballs in Radiative J/ψ Decays [16]
分析BESIII的衰变的实验数据的，值得细看。

这里提到了胶球和拥有相同的量子数的介子会发生混合，可以通过下面的这种混合矩阵描述，通过衰变的数据可以分析得到具体的矩阵元。

$$\begin{array}{c} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{array} = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \begin{array}{c} |n\bar{n}\rangle \\ |s\bar{s}\rangle \\ |gg\rangle \end{array}$$

1.22 Glueballs from the Bethe-Salpeter equation

Title: Glueballs from the Bethe-Salpeter equation[24]

Bethe-Salpeter equation (BSE)计算胶球。得到的结果在1.6左右。可以用来了解这个方法。和DSE可能有一些联系。

2 QCD Theory

2.1 Symmetries of baryons and mesons

Title:Symmetries of baryons and mesons [11]

在R. D. Field - Applications of Perturbative QCD 的附录D中引用了这篇文章，其中包含了最早的关于colorfactor的讨论。

文章中并未具体介绍如何计算某一个ColorFactor，这里讨论了重子和介子中存在的群结构。其内容接近于国科大粒子物理基础课程中对于对称性的介绍，认为系统中存在$U(3)\times U(3)$的对称性。

不是特别有用的文章，等有空以后再研究其技术细节。

2.2 Color in QCD

两个介绍具体颜色计算过程的PPT：

https://www.hep.phy.cam.ac.uk/~thomson/lectures/QCD.pdf

https://www.roma1.infn.it/people/luci/I2P/Colour_QCD.pdf

3 Structure Function

3.1 Polarized Parton Distributions and Future Neutrino Factories

Title:Polarized Parton Distributions from Charged–Current Deep-Inelastic Scattering and Future Neutrino Factories [10]

里面是关于PDF的讨论，重要的是详细的介绍了强子张量。

3.2 Parton Distribution Functions and their Generalizations

Title:Parton Distribution Functions and their Generalizations[18].

李阳推荐的综述，介绍了，PDF,TMF,GPD的一些信息。

4 GFF

4.1 Gravitational form factors and mechanical properties of quarks in protons: A basis light-front quantization approach

Title:Gravitational form factors and mechanical properties of quarks in protons: A basis light-front quantization approach [21]

Raj介绍的如何从波函数计算得到GFF的文章。

QCD中规范不变的张量？EMT的形式如下：

$$\begin{aligned}\theta^{\mu\nu} =& \frac{1}{2}\bar{\psi} i\left[\gamma^\mu D^\nu + \gamma^\nu D^\mu\right]\psi - F^{\mu\lambda a}F^{\nu}_{\lambda a} + \frac{1}{4}g^{\mu\nu}\left(F_{\lambda\sigma a}\right)^2 \\ &- g^{\mu\nu}\bar{\psi}\left(i\gamma^\lambda D_\lambda - m\right)\psi,\end{aligned} \tag{1}$$

将这个张量放入态矢中可以得到下面的形式：

$$
\begin{aligned}
\langle P',\Lambda'|\,\theta_i^{\mu\nu}(0)|P,\Lambda\rangle = & \bar{u}\left(P',\Lambda'\right)\left[-B_i\left(Q^2\right)\frac{\bar{P}^\mu\bar{P}^\nu}{M}+\left(A_i\left(Q^2\right)\right.\right. \\
& +B_i\left(Q^2\right)\Big)\frac{1}{2}\left(\gamma^\mu\bar{P}^\nu+\gamma^\nu\bar{P}^\mu\right) \\
& +C_i\left(Q^2\right)\frac{q^\mu q^\nu-q^2 g^{\mu\nu}}{M} \\
& \left.+\bar{C}_i\left(Q^2\right)M g^{\mu\nu}\right]u(P,\Lambda)
\end{aligned}
\tag{2}
$$

其中A的部分就是GFF。态矢可以在blfq的基矢上展开，最终可以得到由blfq方法计算得到的GFF。

4.2 Gravitational form factors of glueballs in Yang-Mills theory

Title: Gravitational form factors of glueballs in Yang-Mills theory [1]

Nowadays, there are multiple experimental candidates for various glueball states with allowed quantum numbers.

目前胶球实验上的候选态有很多。

Information about the internal structure of hadrons may allow classification of observed hadron states as glueball-like objects. Features like their radius, or the momentum fraction carried by gluons, could serve as smoking-gun evidence of a hadron having predominantly gluonic degrees of freedom. Both of these quantities, as well as additional information like the energy distribution, are contained in their gravitational form factors (GFFs)

内部结构可能是确定胶球的决定性证据。GFF中包括很多相关的信息。

With a pure gauge action, the EMT contains only a gluonic contribution

$T^{\mu\nu}=2\,\mathrm{Tr}\left[-F_\alpha^\mu F^{\alpha\nu}+\frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right]$

对于胶球来说，EMT的形式会更简单一些。这也导致了它只有A和D两种GFF。

4.3 Gravitational form factors of charmonia

Title: Gravitational form factors of charmonia [28]

计算Charmonian的GFF文章。要作为计算胶球的文章的baseline。

GFF中的A可以由本征方程通过这种方式计算得到：

$$
\begin{aligned}
A\left(q_\perp^2\right)= & \sum_n\sum_{\{s_i\}}\int\left[\mathrm{d}x_i\,\mathrm{d}^2 k_{i\perp}\right]_n \\
& \times\sum_j x_j\psi_n\left(\left\{x_i,\vec{k}_{i\perp},s_i\right\}\right)\psi_n\left(\left\{x_i,\vec{k}_{i,j\perp},s_i\right\}\right),
\end{aligned}
\tag{3}
$$

在原文的公式53，引用了[168]参考文献。

4.4 Lattice evidence that scalar glueballs are small

Title: Lattice evidence that scalar glueballs are small [2]

GFF可以和半径联系起来：

$$r^2_{\text{mass,BF}} = \frac{1}{A(0)}\left[6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{3}{4m^2}(A(0)+2D(0))\right], \tag{4}$$

4.5 Light-Cone Representation of the Spin and Orbital Angular Momentum of Relativistic Composite Systems

Title: Light-Cone Representation of the Spin and Orbital Angular Momentum of Relativistic Composite Systems [4]

2000年最早的推导GFF的文章。

5 Review

5.1 Light front QCD in a transverse harmonic oscillator basis

李阳的综述，介绍了很多BLFQ的细节,特别是谐振子相关的。其中第六页开始介绍了动能项的部分。

https://github.com/Linzy2002/Docunment/blob/main/notes_LFQCD_HObasis_v8.pdf

5.2 A Friendly Introduction to the Light Front

系列讲座，内容非常的具体和基础。以后写毕业论文用得到。https://indico.knu.ac.kr/event/718/

更新了地址：

https://indico.knu.ac.kr/category/10/

5.3 Nucleon Structure from Basis Light-Front Quantization : Status and Prospects

Chandon写的综述文章，仔细介绍了blfq的工作成果。

其中在3.1部分有关于质心系得处理方法的详细介绍：

By choosing λ_L sufficiently large and positive, we elevate the excited CM states to higher energies while preserving the low-lying physical states. The complete effective Hamiltonian for diagonalization becomes:

$$H'_{\text{eff}} = H_{\text{eff}} - \left(\sum_i \boldsymbol{p}_{i\perp}\right)^2 + \lambda_L \left(H_{\text{CM}} - 2b^2 I\right). \tag{5}$$

在第二章有关于福克空间展开的介绍：

The Fock space expansion of the eigenstate, given in Eq. (3), provides a natural basis for solving the LF Hamiltonian eigenvalue equation, Eq. (1). This expansion leads to an infinite hierarchy of coupled integral equations, which must be truncated for practical computation. This structure is reminiscent of the Dyson–Schwinger and Bethe–Salpeter equations in covariant formulations. The Light-Front Tamm–Dancoff Approximation (LFTDA) achieves this truncation by limiting the Fock space to a finite number of partons [267].

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