Detecting systematic electoral interference from undervoting irregularities*

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Abstract. The field of election forensics employs inferential methods to detect anomalies in voting returns that are indicative of systematic irregularities. Theoretically, I propose to focus on a novel feature within voting returns that has so far received no attention by the literature. When multiple individual races form part of one general election—like the execution of separate vote processes for presidential candidates and parliamentary seats—I argue that systematic irregularities can be detected if protagonists of fraud fail to interfere into both races to equal extents. In particular, I remark that ballot box stuffing and the invalidation of votes can be detected from a systematic relation between the discrepancy in valid votes across the different races and its relation with the winner's vote share. Methodically, I provide a preliminary formulation of a parametric simulation model that constructs a case-specific distribution of this association under the absence of fraud, and estimates the degree of contamination that was present by comparing empirical data to distributions simulated under the presence of systematic interference. I demonstrate its use by showcasing the Ecuadorian general election held in 2017.

Keywords: *Electoral fraud, election forensics, voter turnout, undervoting, Monte Carlo simulation.*

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1 Introduction

In the last decade, increasing inquiry has been placed on the integrity of democratic elections (Levin and Alvarez 2012, Norris 2014). One crucial asset of electoral integrity is the execution of impartial and non-manipulated elections. When such are not executed, electoral fraud heavily disrupts the chain between the voters' will and the political system's responsiveness. Even simple allegations of fraud can depress civic participation (Alvarez et al. 2008), decrease trust in administering institutions, and spark post-electoral violence (Daxecker 2012). Although Simpser (2013) shows that regimes committing electoral manipulation may do so blatantly and without disguise to the public eye, perpetrators of fraud are highly incentivized to do so as a hidden and clandestine effort.

The field of election forensics employs inferential methods to detect anomalies in voting returns that are indicative of systematic irregularities. Up to this point, existing methods focus on the digits of vote count values and compare these to theoretical distributions expected if no manipulation was present (Pericchi and Torres 2011, Beber and Scacco 2012), infer irregular patterns from the distribution of vote-shares (Rozenas 2017), and use machine-learning algorithms to probabilistically detect fraud (Cantú and Saiegh 2011, Levin et al. 2016). Moreover, several contributions have focused on the multivariate distribution of vote shares and turnout (Filho et al. 2003, Myagkov et al. 2009). In particular, Klimek et al. (2012) propose a parametric model to estimate the extent to which ballot box stuffing may have influenced the published results. Their model investigates the relation between turnout and the winner's vote share over electoral precints, and, using statistical simulation, infer to which extent this relation deviates from clean processes. The richness of approaches to detect election irregularities shows that election fraud can be perpetrated in manifold ways, with its protagonists being able to choose from a whole menu of manipulation (Schedler 2002). It is thus of considerable importance to constantly expand the available toolkit to detect these practices.

This article expands earlier work on the statistical detection of election irregularities in two directions. Making a general substantive point that has received no systematic attention in the literature, I propose that forensic analyses can capitalize on the presence of multiple individual races—like the execution of separate vote processes for presidential candidates and parliamentary seats—if such are executed as part of one general election as perpetrators of fraud will likely fail to balance out their malpractice across races but interfere into these to unequal extends. Specifically, rather than focusing on pure turnout values, I argue that ballot box stuffing can be detected from a systematic relation between the *discrepancy* in turnout across the different races and its relation with the winner's vote share. Such discrepancies arise if out of all voters that turned out, a portion casts their vote in one (or some of the) race(s), but skips others, and has been referred to as 'undervoting' in the political science literature. Moreover, I extend prior work by Klimek et al. (2012) and provide a preliminary formulation of a parametric simulation model that constructs a case-specific distribution of this association under the absence of fraud, and estimates the degree of ballot box stuffing that was present by comparing empirical data to distributions simulated under the presence of systematic interference.

This document is structured as follows. Section 2 motivates the idea behind my method and considers the Ecuadorian general elections held in 2017. To understand why election irregularities should be detectable from systematic relationships between winner's vote shares and turnout discrepancies between multiple individual races, Section 3 sketches the main pathway of theoretical reasoning that underlines this project. Following, I develop my simulation approach and showcase it using Ecuador's 2017 (i) presidential vote and (ii) country-wide race for seats in regional assemblies, which both formed part of the general election. In a concluding section, I line out venues of future research. This document should not be perceived as a finished piece of work. Rather, it provides a momentary snapshot of the current state that this project is in.

2 A motivating example

Remarkably, while being left rather untouched by the academic literature, peculiar patterns of undervoting have been repeatedly discussed in public media outlets. For instance, following the US-American midterm elections in November 2018, the Florida U.S. senate race was considered to close to call even days after the initial election, which led Florida's secretary of state to order a machine recount of all cast votes, and the National Republican Senatorial Committee to sue two counties for their handling of ballots (FiveThirtyEight 2018). What seemed peculiar to many observers was that for these counties, more than 20,000 fewer votes were reported in the U.S. Senate race than in the governor race, with this pattern emerging specifically in those counties where final vote shares were remarkably close.

This election was not the first case in which patterns of undervoting have been called into question. To illustrate the idea behind my method, consider the Ecuadorian general election held on February 19, 2017. In the Ecuadorian case, five individual races formed part of the electoral process. Voters directly elected (i) their head of government in a presidential race, (ii) the members of the country's national assembly, (iii) parliamentary members of 24 regional assemblies, (iv) Ecuador's five representatives in the Andean parliament—the deliberative body of the Andean community—and lastly (v) cast votes in a nation-wide referendum consisting of seven individual questions. In the published election data, disaggregated voting returns are made avaiable over 1,306 electoral districts.[1] Sometimes, these districts only consist of a singular polling station, for instance for Ecuadorian citizens voting in particular foreign countries. Usually however, a variety of polling stations is aggregated to an electoral district, with an average of 9,741 voters bein eligible to cast their vote in an individual district. Since all voting procedures are executed separately for male and female citizens, this results in n=2,612 individual data points over the 1,306 electoral districts. In the analysis here, I focus on the election of the country's head of government and the MEPs of its 24 regional assemblies and, in the following, refer to these as the *presidential* and *parliamentary race.*

A central finding developed in the field of election forensics is that over a large number of entities within a given election, both turnout and vote shares can be described by unimodal Gaussian distributions. While their means and variances depend on the political context and can vary substantially over countries, both distributions should behave strictly orthogonal to each other. That is, under clean processes, while both individual distributions can vary in shape, values across both variables should not be systematically associated with each other. This asset forms an underlying assumption of several forensic approaches and has been emprirically shown to hold for election data returned from a variety of countries (Klimek et al. 2012). Contrary, skew distributions in which especially the highest

[1] Data has been downloaded from the official election homepage www.cne.gob.ec.

values in turnout are associated with highest values in winner's support are indicative of fraud, as these patterns emerge from artifically stuffing the ballot box with votes for the winning candidate or destroying votes for its opponent.

Consider the left panel of Figure 1 which plots the share of valid votes over the n=2,612 electoral districts against the vote share of Lenin Moreno, the candidate of Ecuador's governing party *Alianza Pais* who won the presidential race. As one can easily derive from the figure, both univariate distributions of valid vote shares ($\bar{x} = 0.88, sd_x = 0.06$) and winner's support ($\bar{x} = 0.41, sd_x = 0.13$) are adequately characterized by two orthogonal Gaussian processes. Now, consider the right panel of Figure 1. This plot portrays winner Moreno's vote shares against a variable quantifying the *difference* in valid vote shares that have been reported between the first-order presidential race and the second-order race for seats in regional parliaments, with positive values indicating that more votes were casted in the presidential contest. Here, orthogonality is clearly violated as higher values for this *discrepancy* are substantially associated with larger vote shares for the winner of the presidential race obtaining a correlation of $r = 0.38$. This pattern is especially evident at the upper right and lower left tail of this bivariate distribution. Thus, it can be suspected that out of those voters that have been reported to cast votes in the presidential race but *not* in the parliamentary run, a large portion is documented in favor of the winner of the presidential race.

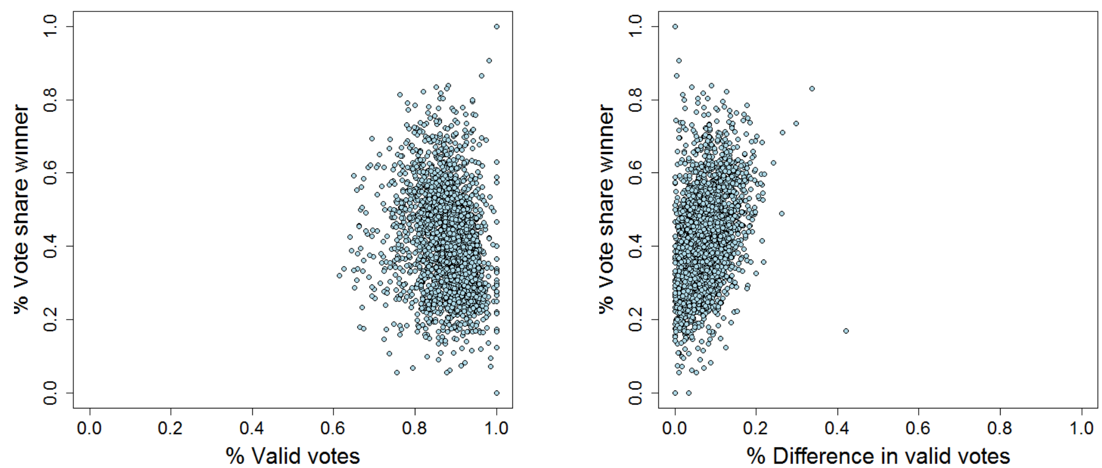
While the known orthogonality of vote and turnout that is inherent to clean election data is satisfied, it is this relation between the *difference* in valid vote (or turnout) shares and support rates that seems odd and forms the interest of this article. So far, no contribution has lined out how this relation is expected to look like under clean and tainted processes. The intuitive suspicion that if turnout and winner's vote share should be orthogonal under no interference, so should the relation between turnout *discrepancy* and vote shares drives the following endeavor.

3 The relation between turnout discrepancies and winner's vote shares with and without interference

3.1 Theoretical considerations

To understand why the relation between the winner's vote share and discrepancies in turnout between several races should be a suitable quantitiy of interest to indicate fraudulent behavior at all, this section outlines some theoretical considerations that form the basis of this article. Let us first set up some general vocabulary. Consider a scenario where one election consists of two individual Races A and B. I denote Race B as the *baseline race* in which less votes were cast, and Race A as the *primary race* that received a higher turnout share over all analyzed precints. The idea that the peculiar patterns outlined above can be driven by falsification of results is based on the possible explanations why different turnout rates for Races A and B are observed in official voting returns in the first place, and the resulting implications for their association with the vote share of the winner in Race A.

Let us review the possible explanations under *clean* vote processes in which no falsification was present. First, observed turnout differences could be resulting from poor ballot designs in which a proportion of voters simply overlook one of the indidvidual races. Poor ballot design was suspected to



(a) Multivariate distribution of valid votes and winners' vote share in presidential race.

(b) Multivariate distribution of the difference in valid votes between both races and winner's vote share in the presidential race.

Figure 1: Ecuador 2017, multivariate relations in voting returns with and without deviations from orthogonality over n=2,612 electoral precints.

be the driving force behind undervoting patterns in past publically discussed cases like Florida 2018, where the space rewarded to one individual race was pushed to the far bottom left of the ballot, where voters might have simply skimmed over it (FiveThirtyEight 2018). This explanation is especially plausible for second-order races that received less attention by media outlets prior to election day, with this race not being present at the top of voters' minds. Consider for instance the Ecuadorian case with five different individual races, resulting in an arguably complex manner of vote casting at the ballot box. Possibly, particular races were simply not noticed on the ballot paper.

Second, errors in vote counting procedures can lead to discrepancies in turnout rates. One, it is likely that a certain share of votes is failed to be taken into account due to human failure to correctly identify all votes. Two, if such were used, it is possible that errors with vote-tabulating machines cause them to sometimes not read in people's votes for a particular race. Such errors can be rather randomly distributed over all races that are part of the ballot paper, or systematically underreport turnout shares for certain races if technical inconsistencies cumulate at specific places on the sheets.

It is important to note that both of these explanations are driven by processes which are independent of whether the votes that have been counted or neglected were cast in favor or against the winner's candidancy or political party. This holds even under a third possible explanation in which a portion of those citizens that turned out at the ballots *consciously* decided to vote in Race A, but to skip Race B. Under the absence of special political circumstances that systematically lets one specific supporter group skip one of the individual races—like the exclusion of an oppositional party which leads supporters of this party to only vote for their presidential candidate but not to choose an alternative force in the parliamentary race—also here, discrepancies in turnout should not, or at least only weakly, depend on whether skipping voters cast their vote in favor of the winning candidate in Race

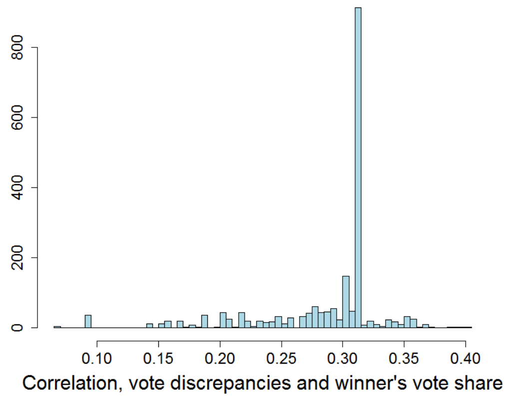


Figure 2: Distribution of correlation coefficients between turnout discrepancy and winner's vote share for s=2,000 artificial scenarios under no systematic interference for the electoral system of Ecuador.

A. Hence, *out of those additional votes that were exclusively cast in Race A, the percentage of those documented for the winner resembles the winner's vote support in the race overall.* For the motivating example outlined in Section 2, this expected percentage under clean processes is thus 41%.

However, let us now consider the scenario of *electoral fraud.* Under interference, votes are artifically added for one political fraction, may these be the winning candidate (or party) in Race A or in Race B. In a balanced fraud approach, for every vote that is added to Race A, one vote is simultaneously added to Race B. In an unbalanced fraud approach, these numbers differ. When multiple races are executed and fraudulent practices are committed to an extent that is detectable from voting returns, it is almost certain that such are either (i) restricted to one of the individual races or (ii) are committed in both to unequal extends. The reason for this is that, as outlined in the literature, fraud is often a hidden and clandestine effort that can come with severe consequences for its protagonists if their actions become public (Lehoucq 2003). Therefore, perpetrators of fraud have the incentive to falsify as few as possible in order to fullfill their goals. Even in scenarios in which local agents blatantly use fraud with few disguise to signalize loyality to their principal (Rundlett and Svolik (2016)), it is unlikely that these are aware of or aiming at balancing out their activities over the different races that are taking place. It is this feature that opens the door for my detection method. If fraudulent activities are not balanced, for instance when many more votes are added for the winner in Race A than in Race B, these induce a positive association between the difference in turnout over the two races and the winner's vote share in Race A, *while the percentage of those votes documented for the winner out of the additional votes that were exclusively cast in Race A exceeds the winner's vote support in the race overall.* For the motivating example outlined in Section 2, interference would thus lead to percentages higher than 41%.

3.2 Empirical associations under the absence of systematic interference

As substantively derived, when fraudulent practices are not balanced over individual races A and B, stuffing the ballot box with votes for the winner (or contrary invalidating votes of the opponent) should be diagnosable from the relation between the winner's vote share in Race A and discrepancies in turnout between the two races such as documented in Figure 1b. Naively, one might perform a simple correlational analysis and perform a T-test whether the correlation coefficient is significantly different from zero. This article deviates from such a strategy for two reasons.

First, this test would rely on the assumption that for this association between, the theoretical distribution of the T-statistic qualifies as a suitable baseline that empirical values can be compared against. Although several recent advances have been made in the understanding of statistical regularities that are embedded in voting returns (Filho et al. 2003, Mantovani et al. 2011, Camacho et al. 2003), this is a theoretical assumption that may or may not be the case.

Second, it can be shown that even under the absence of any systematic interference—thus if the percentage of all voters that skipped the baseline Race B and voted for the winner in Race A resembles exactly the winner's vote support—the genuine relation between the winner's vote share and discrepancies in turnout needs not to reduce to zero. Rather, its natural value under no fraud depends on a whole range of country-specific characteristics like the (i) number and sizes of analyzed precints and their interaction with (ii) precint-specific baseline turnout rates in Race B and the (iii) vote shares that were registered in these for the winner of Race A. This point calls for further clarification. Therefore, I construct a data structure that mimics Ecuador's country-specific characteristics and set the number and sizes (defined by the total number of eligible voters) of analyzed precints to its true empirical values. Following, I simulate artifical data over s=2,000 iterations varying (i) precint-specific turnout rates in Race B and (ii) vote shares in Race A when sampling data from two orthogonal Gaussian distributions. Similar to the empirical voting returns, each distribution takes on a standard deviation of 0.1, but in each iteration, its two means are randomly chosen from a vector of {0.4, 0.41, 0.42,...,0.9}. In line with a scenario under the strict absence of fraud, a random number of votes is added to Race A out of which exactly 41% are set in favor of the winner, and the correlation between turnout discrepancy and the winner's vote share in Race A is constructed. Figure 2 portrays the distribution of correlation coefficients that emerges from these s=2,000 random permutations of means for the two orthogonal distributions. As can be seen, for Ecuador's electoral precints and their respective number of eligible voters, a vast amount of substantive correlations can emerge under the scenario of no interference at all, by merely varing the figures for turnout and vote shares over the differently sized electoral precints.

4 A parametric simulation model

4.1 Model formulation

Against this background, as a central point of this work, I now present a preliminary formulation of a parametric simulation model to detect whether manipulations to the number of valid votes were executed from the correlation between turnout discrepancy and winner's vote share in Race A. In particular, this model is designed to (i) *test* the observed correlation against a null distribution that is constructed under no interference for the case-specific electoral system, vote, and turnout shares as well as to (ii) *estimate* the proportion of discrepant votes that was added to the winner's vote share. I denote each analyzed precint unit with the index i. To initiate the model, the following steps are applied.

I. *Incorporation of country- and election-specific characteristics.* Fix the number of electoral precints and their size (defined as their number of eligible voters) to those values that are underlying the analyzed election.

II. *Construction of the baseline distributions of turnout and winner's vote share in Race A.* Model turnout T_i and vote shares V_i in Race A by drawing i values from a multivariate Gaussian distribution. The means, standard deviations and covariances of the turnout and vote distributions are directly taken from their empirical distributions. Hence, this distribution is defined as

$$VT \sim N(\mu, \mathbf{\Sigma}), \tag{1}$$

where $\mu = \begin{bmatrix} \bar{x}_V \\ \bar{x}_T \end{bmatrix}$ and $\mathbf{\Sigma} = \begin{bmatrix} s_V^2 & s_V s_T \\ s_V s_T & s_T^2 \end{bmatrix}$.

III. *Construction of turnout discrepancy c_i between Race A and B.* For every precint, take the turnout in Race B as a baseline and draw the difference in turnout to Race A from a Gaussian distribution. Like in II, its mean and width are taken from the observed empirical values, coming down to

$$c \sim N(\mu, \sigma^2), \tag{2}$$

where $\mu = \bar{x}_c$ and $\sigma^2 = s_c$.

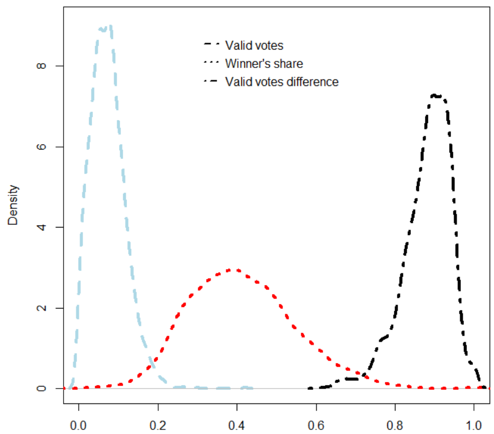
The first step of the above protocol ensures that the case-specific number of precints and electorate sizes are represented in the model. The second step guarantees that the overall dispersion of vote and turnout preferences of the country's population are correctly represented in the model. The third step guarantees that the actual scale of nominal turnout discrepancies that the estimation of the true empirical correlation is based on is accurately represented in the model.

In order to initialize my simulation method, I now subtract all discrepant votes c_i from Race A und exclusively take them away from the winner's votes.

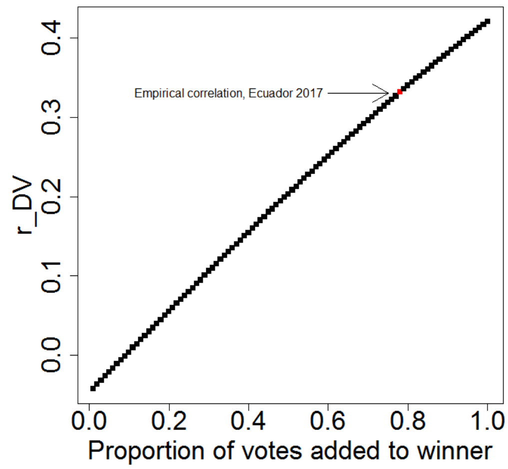
IVa. *Testing the empirical correlation against correlations stemming from zero interference.* For 1,000 iterations, add c_i votes per precint to Race A. In each iteration, draw the proportion of votes that is added to the winner's vote share from a Binomial distribution described by

$$Binom(n = c_i, \theta = \bar{x}_V). \tag{3}$$

Step IVa constructs the country- and election-specific null distribution of the correlation coefficient under the scenario of zero interference. By comparing the observed correlation coefficient to this null distribution, one can test how likely it is that the empirical association stems from a process that is described by zero interference and $\theta = \bar{x}_V$ (in the motivating example, $\bar{x}_V = 0.41$).



(a) Empirical distributions of valid votes, Lenin Moreno's vote share in the presidential race, and turnout differences for Ecuador 2017.



(b) Simulated correlations between turnout discrepancy and winner's vote share that emerge from 100 theoretical fraud parameters.

Figure 3

IVb. *Estimating the true proportion of additional votes in Race A that was cast for the winner.* Construct a vector of one hundred possible binomial parameters θ were $\theta \in \{0, 0.01, 0.02, ..., 1\}$. For each parameter, repeat the 1,000 iterations, add c_i votes per precint to Race A, and draw the proportion of votes that is added to the winner's vote share from a Binomial distribution described by $Binom(n = c_i, \theta = \theta)$.

Step IVb constructs one hundred country- and election-specific distributions of the correlation coefficient under one hundred different scenarios for θ. By comparing the empirical correlation coefficient to the value that maximizes each simulated distribtion, one can estimate the proportion of discrepant votes that was added to the winner's vote share by choosing the value with the smallest distance. Now, let us turn to the application of this method for the case of Ecuador 2017.

4.2 Undervoting in Ecuador 2017

In Figure 3a, I first show the empirical distributions of (i) valid votes, (ii) Lenin Moreno's vote share for the presidential race and (iii) the differences in valid votes for the case of Ecuador 2017. Roughly, these data all collapse to an approxmiate Gaussian distribution, just as previously observed (Klimek et al. 2012). This observation validates the assumptions made be the steps I-III in the outlined simulation approach.

When performing step IVa, the null distribution emerging under zero interference and $\theta = 0.41$ takes on a mean of 0.241 with the minimum and maximum values that were emerging out of the 1,000 iterations being located at 0.238 and 0.244. Clearly, this is very distinct from the empirical correlation of 0.38 that was reported in Section 2 of this document. Due to this large deviation, one can reject the null hypothesis of no interference that exactly 41% of those votes that were additionally reported in the presidential race were contributing to the vote share of the winner like expected in a scenario of no systematic interference.

Finally, step IVb simulated these data 1,000 times for each of the one hundred different parameters of θ ranging from 0.01 to 1. Figure 3b displays the mean correlation coefficients that result under each scenario. As can be seen, a θ-parameter of 0.78 maximizes the likelihood to observe the empirical correlation of 0.38 that is emerging from the empirical voting returns and indicates deviances from clean processes such as outlined in Section 3.

These are the two central results of my simulation model. First, I have constructed a case-specific null distribution of the correlation coefficient of interest. By comparing the observed correlation to this distribution, one can confidently reject that the true proportion of votes that was added to the winner of Race A lies at 0.41. Subsequently, I have *estimated* this true proportion that is underlying the Ecuadorian voting returns by reverse-engineering it from the proposed model.

5 Conclusion

This document intented to provide a novel substantive argument on how electoral interference can be detected from published voting returns. In particular, it was argued that one can capitalize on the execution of multiple races within one general election and detect fraudulent mechanisms such as ballot box stuffing from analyzing differences in turnout and their relation with the winner's vote share. Methodically, this piece aimed to preliminary formulate a simulation model which quantifies evidence for fraudulent processes and to document its current state of development showcasing the Ecuadorian general election of 2017.

While this preliminary formulation is promising, a large chunk of work is still missing. This analysis did only focus on the case of Ecuador 2017 and diagnosed substantial deviations from relations that are expected to be observed under clean processes. In the future, data will need to be collected from a multitude of countries, with special emphasis on advanced industralized democracies where *no fraud* is supposed to be present. Only this confrontation will allow an assessment whether the proposed model yields 'oversensitive' conclusions that diagnose fraud in each individual case, or if it is merely detecting clear outliers in voting returns as it should.

Last, future versions of this document will need to line out clearcut —theoretical and methodical— differences between the approach presented here and approaches that focus on the relation between vote shares and *turnout* rather than *turnout discrepancies over multiple races*. In which scenarios does a focus on the latter outperform current forensic methods? This is a substantial step that needs to be undertaken in upcoming formulations of this document, especially if different results are emerging from the two approaches.

However, summing up, this document did line out a promising venue for future research and can hopefully guide future endeavors in the detection of electoral fraud from peculiar patterns in undervoting.

References

Alvarez, R. M., Hall, T. E. and Hyde, S. D. (2008), Introduction: Studying Election Fraud, *in* R. M. Alvarez, T. E. Hall and S. D. Hyde, eds, 'Election Fraud: Detecting and deterring electoral manipulation', Brookings, Washington D.C., pp. 1–20.

Beber, B. and Scacco, A. (2012), 'What the Numbers Say: A Digit-Based Test for Election Fraud', *Political Analysis* **20**, 211–234.

Camacho, J., Park, K., Domany, E., Bahar, S. and Kantelhardt, J. W. (2003), 'Generalized Zipf ' s law in proportional voting processes', *Europhysics Letters* **63**(1), 131–137.

Cantú, F. and Saiegh, M. S. (2011), 'Fraudulent Democracy? An Analysis of Argentina ' s Infamous Decade Using Supervised Machine Learning', *Political Analysis* **19**(4), 409–433.

Daxecker, U. E. (2012), 'The cost of exposing cheating: International election monitoring, fraud, and post-election violence in Africa', *Journal of Peace Research* **49**(4), 503–516.

Filho, R. N. C., Almeida, M. P., Moreira, J. E. and Jr, J. S. A. (2003), 'Brazilian elections : voting for a scaling democracy', *Physica A* **322**, 698–700.

FiveThirtyEight (2018), 'Something Looks Weird In Broward County. Here's What We Know About A Possible Florida Recount'.
URL: *https://fivethirtyeight.com/features/something-looks-weird-in-broward-county-heres-what-we-know-about-a-possible-florida-recount/*

Klimek, P., Yegorov, Y., Hanel, R. and Thurner, S. (2012), 'Statistical detection of systematic election irregularities', *PNAS* **109**(41), 16469–16473.

Lehoucq, F. (2003), 'Electoral Fraud: Causes, Types, and Consequences', *Annual Review of Political Science* **6**(1), 233–256.

Levin, I. and Alvarez, R. M. (2012), 'Introduction to the Virtual Issue: Election Fraud and Electoral Integrity', pp. 1–7.

Levin, I., Pomares, J. and Alvarez, R. M. (2016), Using Machine Learning Algorithms to Detect Election Fraud, *in* 'Computational Social Science', Cambridge University Press, Cambridge, pp. 266–294.

Mantovani, M., Ribeiro, H., Moro, M., Picoli, S. and Mendes, R. (2011), 'Scaling laws and universality in the choice of election candidates Scaling laws and universality in the choice of election candidates', *EPL: A letters jounral exploring the frontiers in physics* **96**, 1–5.

Mebane, W. R. and Klaver, J. (2015), Election Forensics: Strategies versus Election Frauds in Germany.

Myagkov, M., Ordeshook, P. C. and Shakin, D. (2009), *A Forensics Approach to Detecting Election Fraud*, Cambridge University Press, pp. 1–289.

Norris, P. (2014), *Why Electoral Integrity Matters*, Cambridge University Press, New York.

Pericchi, L. and Torres, D. (2011), 'Quick Anomaly Detection by the NewcombBenford Law, with Applications to Electoral Processes Data from the USA, Puerto Rico and Venezuela', *Statist. Sci.* **26**(4), 502–516.

Rozenas, A. (2017), 'Detecting Election Fraud from Irregularities in Vote-Share Distributions', *Political Analysis* **25**, 41–56.

Rundlett, A. and Svolik, M. W. (2016), 'Deliver the vote! Micromotives and macrobehavior in electoral fraud', *American Political Science Review* **110**(1), 180–197.

Schedler, A. (2002), 'The Menu of Manipulation', *Journal of Democracy* **13**(2), 36–50.

Simpser, A. (2013), *Why Governments and Parties Manipulate Elections: Theory, Practice, and Implications*, Cambridge University Press, New York.