Challenge 5

There are exactly ten ways of selecting three from five, 12345:

$$123, 124, 125, 134, 135, 145, 234, 235, 245, \text{and } 345$$

In combinatorics, we use the notation, $\binom{5}{3} = 10$.

In general, $\binom{n}{r} = \frac{n!}{r!(n-r)!}$, where $r \leq n$, $n! = n \times (n-1) \times \ldots \times 3 \times 2 \times 1$, and $0! = 1$.

It is not until $n = 23$, that a value exceeds one-million: $\binom{23}{10} = 1144066$.

How many, not necessarily distinct, values of $\binom{n}{r}$ for $1 \leq n \leq 100$, are greater than one-million?

Solution

```
challenge_V <- function(countOnly = \binom{}{}, upperLim = 1e6, n, r){
# Step 1
A <- upper.tri(matrix(NA,nrow = n, ncol = r), diag = F)
# Step 2
idx <- which(A == T, arr.ind = T)
# Step3
return(length(which(choose(idx[,"col"],idx[,"row"]) > upperLim)))
```

Step 1: Create an upper triangular matrix

$$A = \begin{bmatrix} F & T & T & T & \ldots & T \\ F & F & T & T & \ldots & T \\ F & F & F & T & \ldots & T \\ F & F & F & F & \ldots & T \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ F & F & F & F & \ldots & F \end{bmatrix}$$

A is an upper triangular matrix containing boolean values, meaning that every element is "FALSE" except the elements above the diagonal, which are "TRUE". The row and column index pairs will be the n and r in the binomial coefficient $\binom{n}{r}$ of each element. Since any binomial coefficient is 1 if

Challenge was taken from the Project Euler.

n and r are the same, the diagonal is excluded, since the inclusion criteria, so to say, is that the binomial coefficient is $> 1 \cdot 10^6$.

Step 2: Extract the pairs of index, where element is equal to "TRUE".

Using the `which()` function with the argument `arr.ind`, a 2D array of row and column indeces is created. Here an example of the output for a matrix of order 5×5.

Rows	Columns
1	2
1	3
1	4
2	3
2	4
⋮	⋮
4	5

Step 3: Compute the binomial coefficient and count elements that exceed $1 \cdot 10^6$.

$$\boldsymbol{B} = \begin{bmatrix} 0 & \binom{1}{2} & \binom{1}{3} & \binom{1}{4} & \dots & \binom{1}{n} \\ 0 & 0 & \binom{2}{3} & \binom{2}{4} & \dots & \binom{2}{n} \\ 0 & 0 & 0 & \binom{3}{4} & \dots & \binom{3}{n} \\ 0 & 0 & 0 & 0 & \dots & \binom{4}{n} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

$\boldsymbol{R} = \{b_{ij} | b_{ij} \in \mathbb{R} \wedge b_{ij} > 1 \cdot 10^6\}$, where $i, j = 1, 2, \dots, 100$ and $i \leq j$. The cardinality of the set $\boldsymbol{R}$, is the result of the challenge. $|R| = 4075$