Cache Memories

COMP402127: Introduction to Computer Systems
https://xjtu-ics.github.io/

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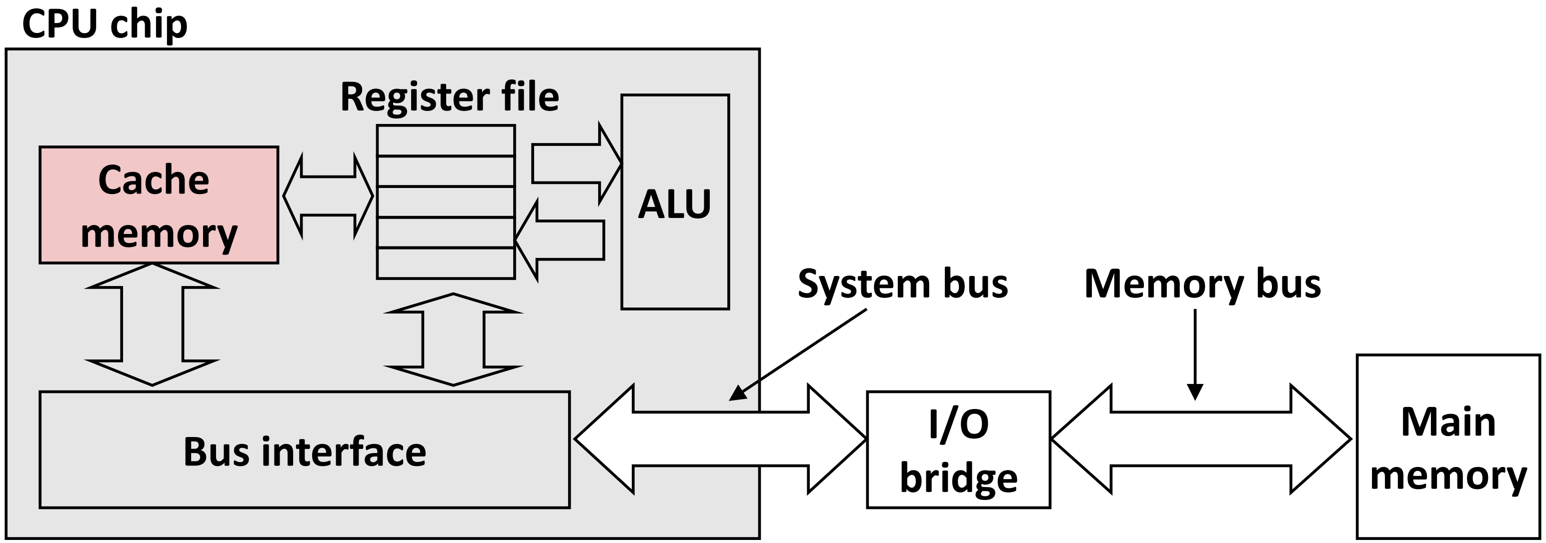
Today

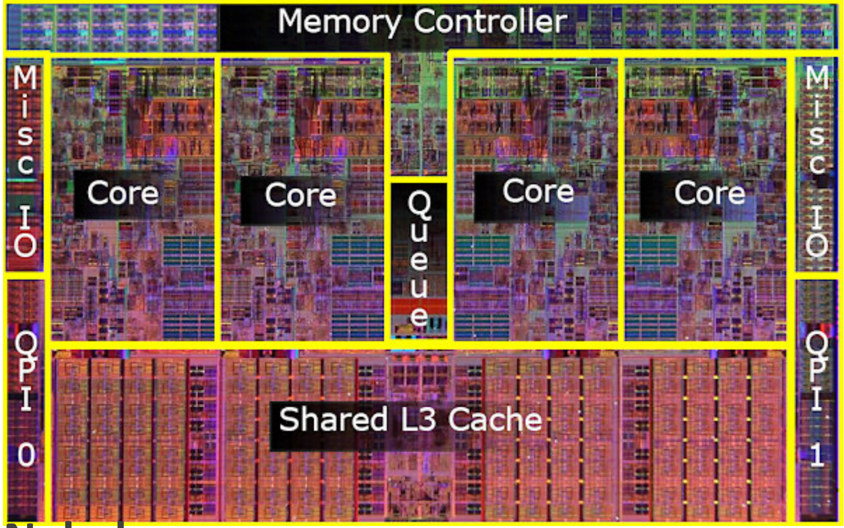
- **Cache memory organization and operation**
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Cache Memories

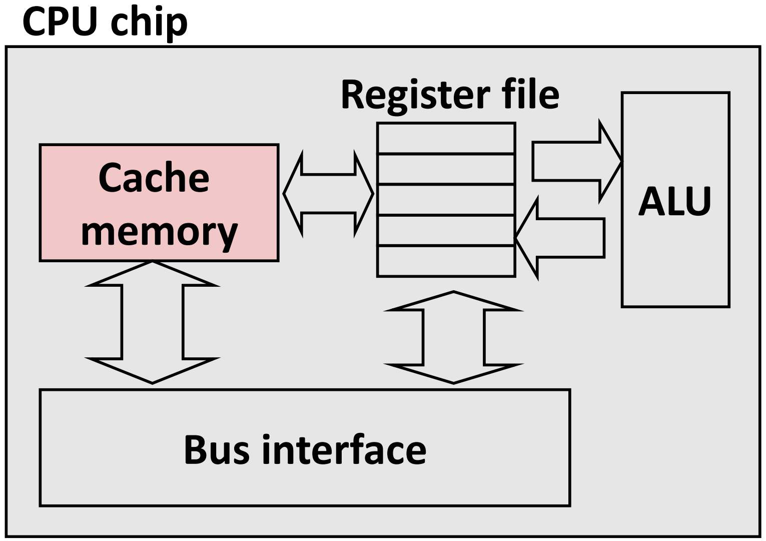
- Cache memories are small, fast SRAM-based memories managed automatically in hardware
  - Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



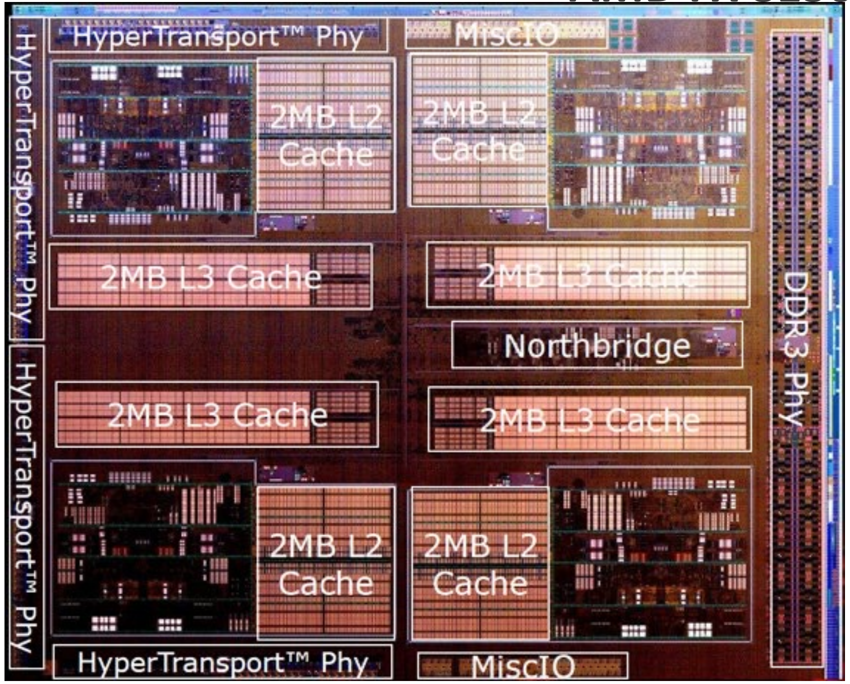
What it Really Looks Like



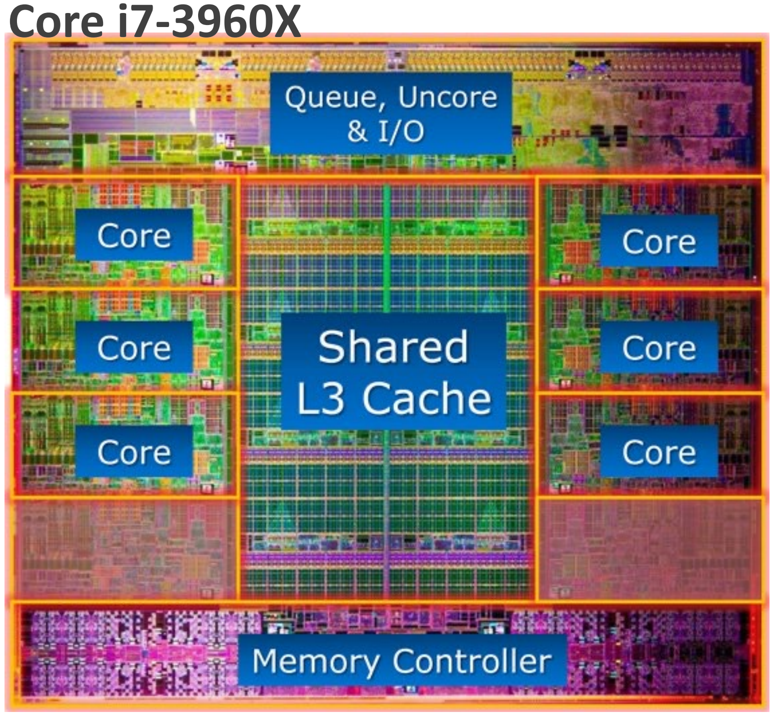
Nehalem



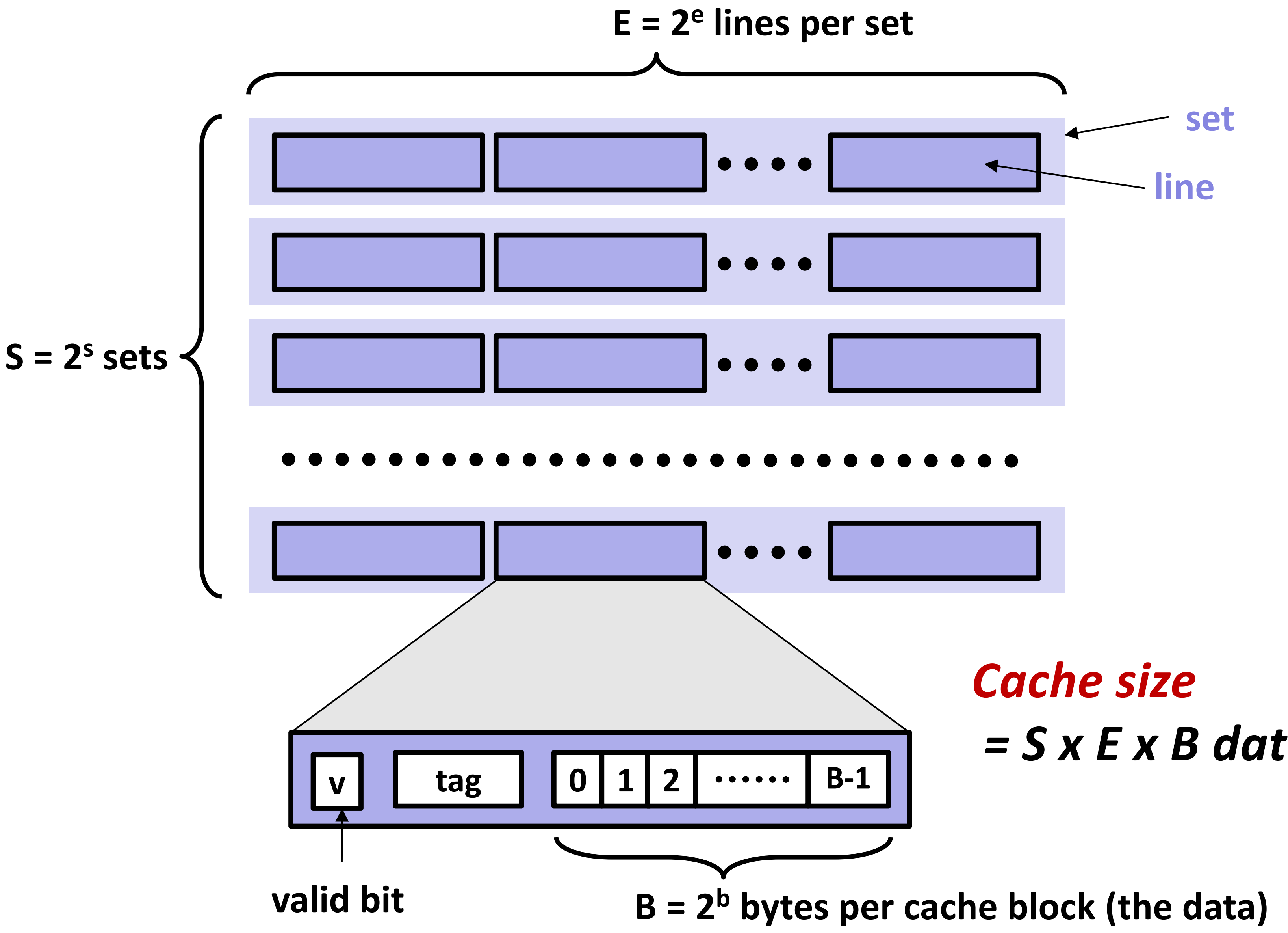
AMD FX 8150



Core i7-3960X



General Cache Organization (S, E, B)

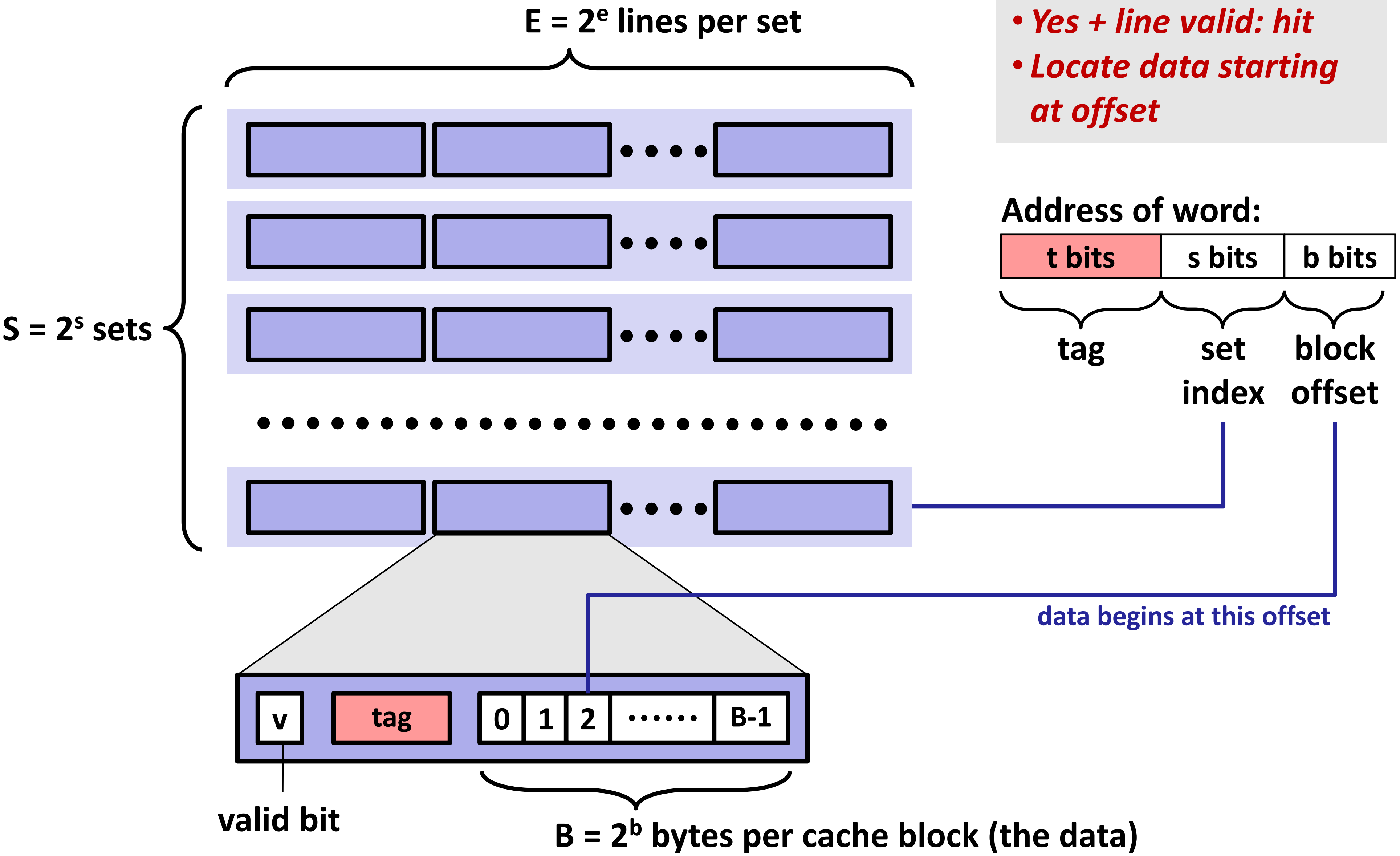


Cache size

= S x E x B data bytes

Cache Read

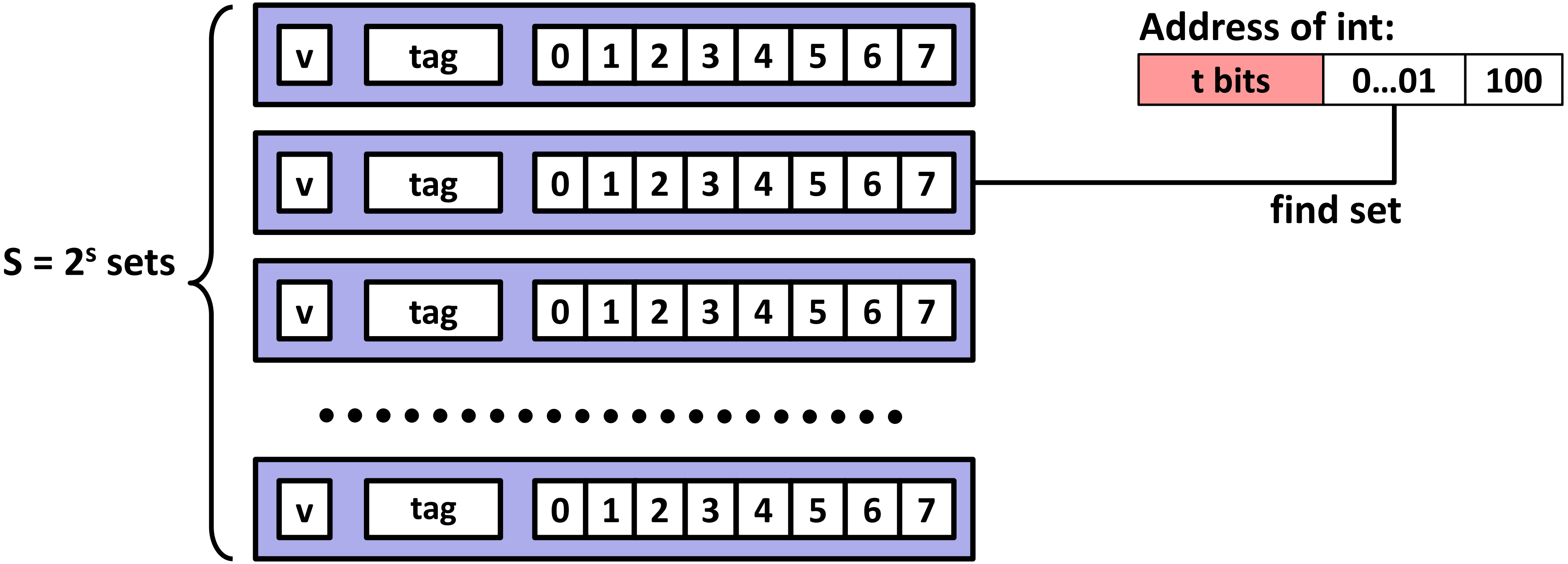
- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

Assume: cache block size B=8 bytes



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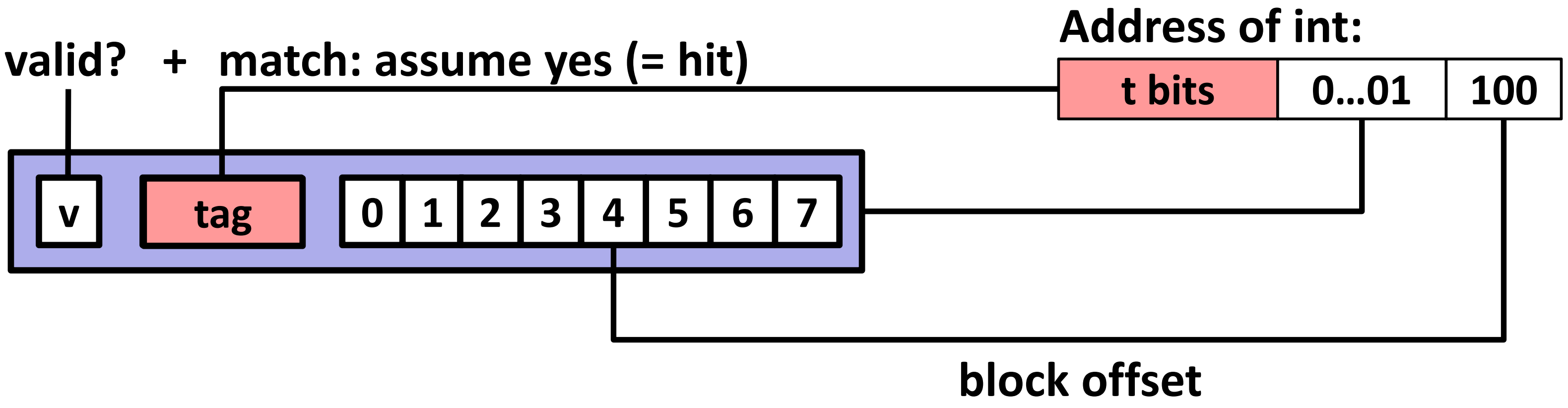
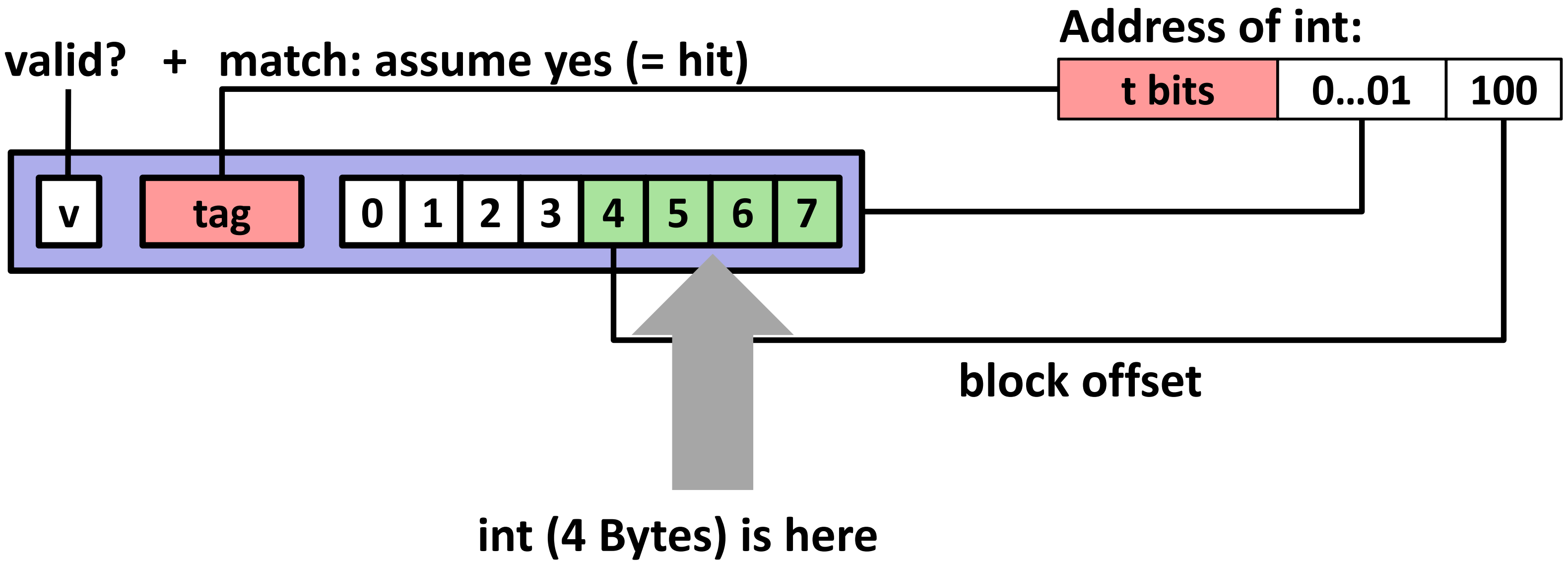


Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set
Assume: cache block size B=8 bytes



If tag doesn’t match (= miss): old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

4-bit addresses (address space size M=16 bytes)
S=4 sets, E=1 Blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

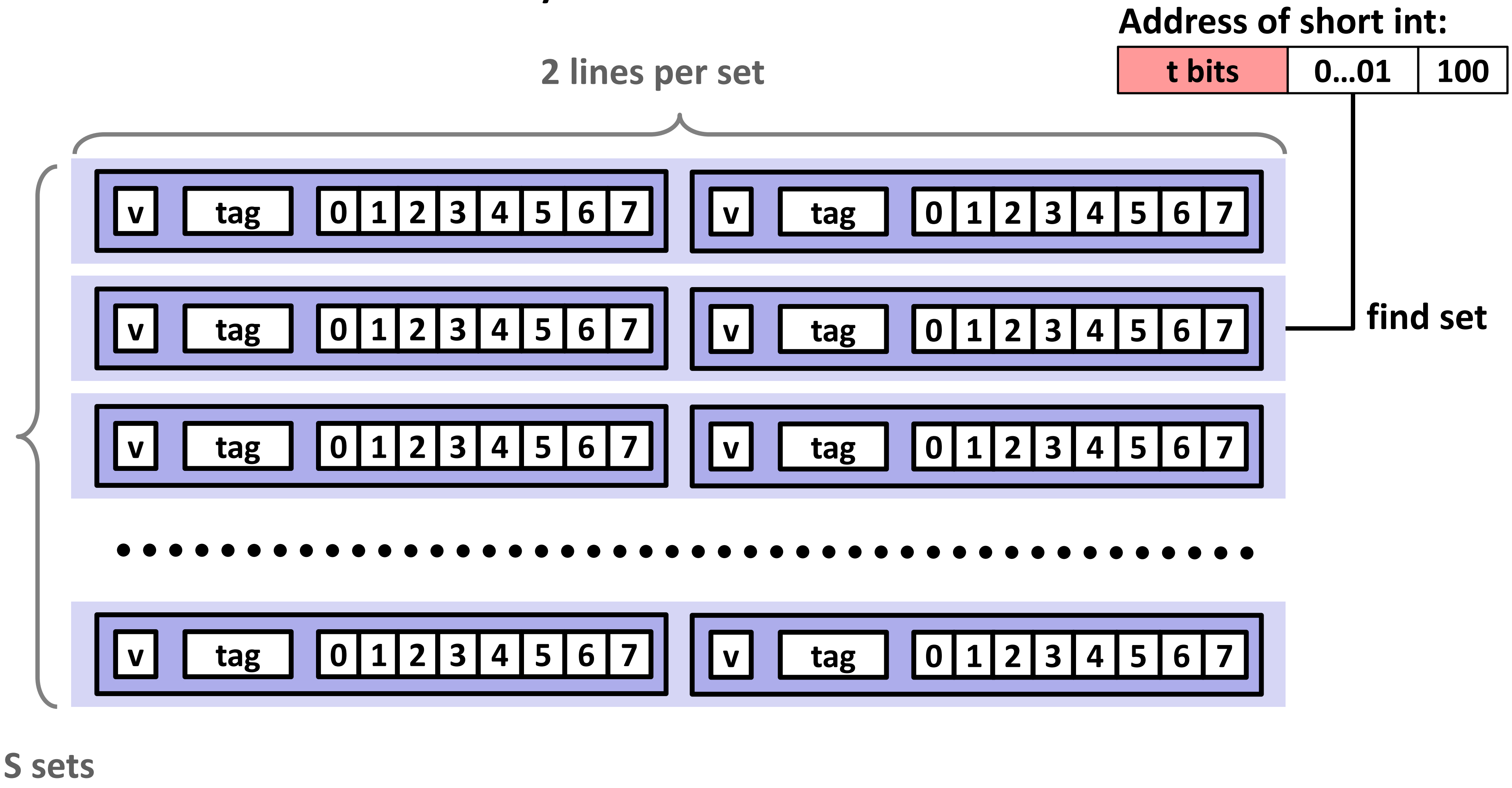
0	**$[0000_2]$,**	miss
1	**$[0001_2]$,**	hit
7	**$[0111_2]$,**	miss
8	**$[1000_2]$,**	miss
0	**$[0000_2]$**	miss

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1	0		
Set 2	0		
Set 3	1	0	M[6-7]

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

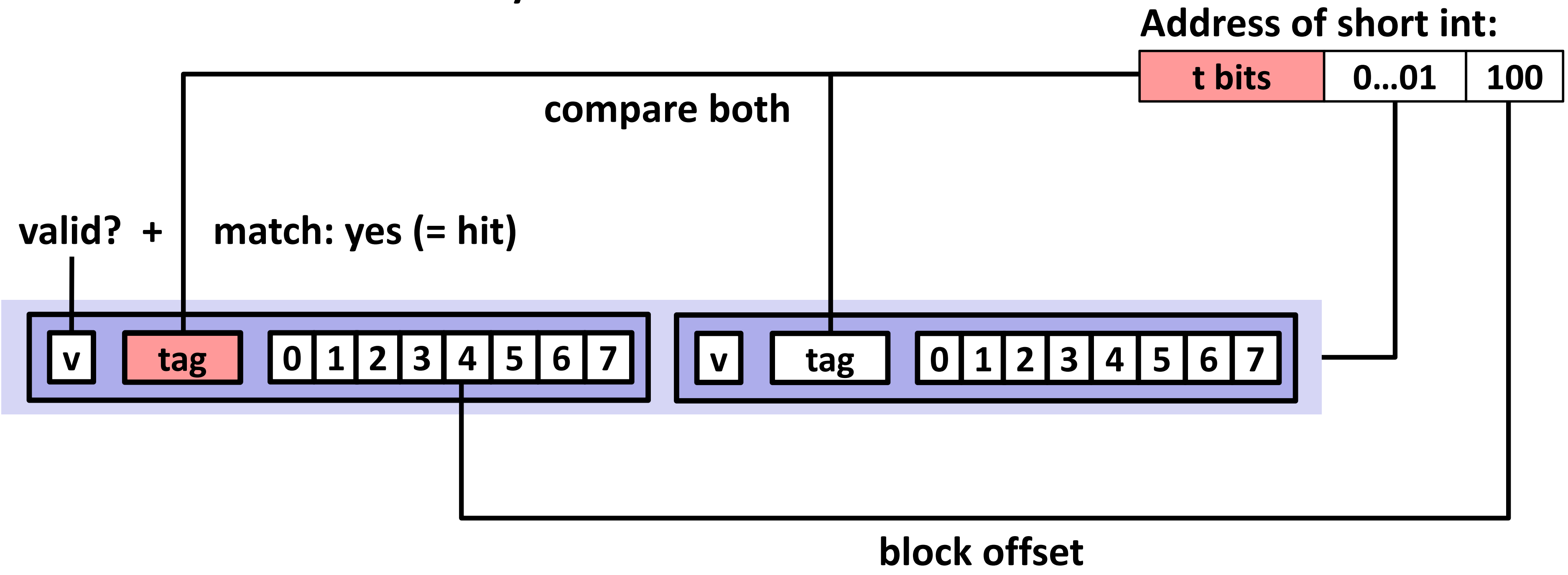
Assume: cache block size B=8 bytes



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

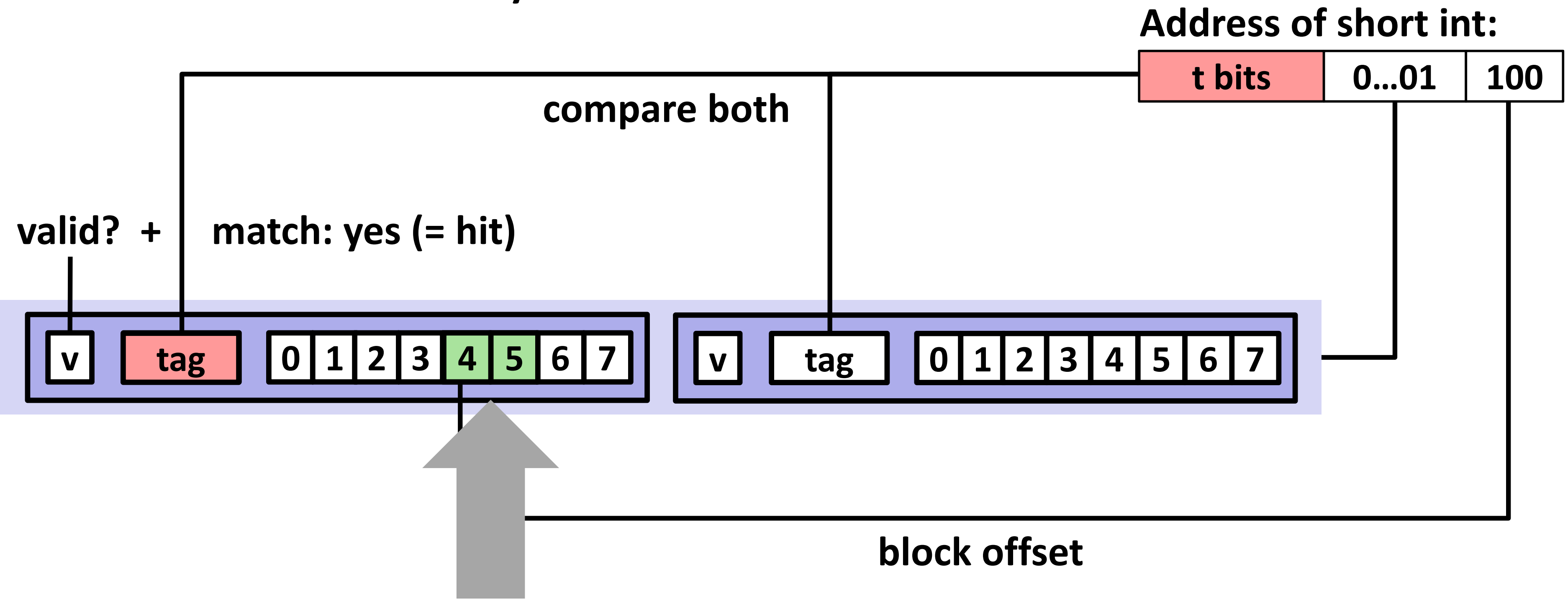
Assume: cache block size B=8 bytes



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes



short int (2 Bytes) is here

No match or not valid (= miss):

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), …

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

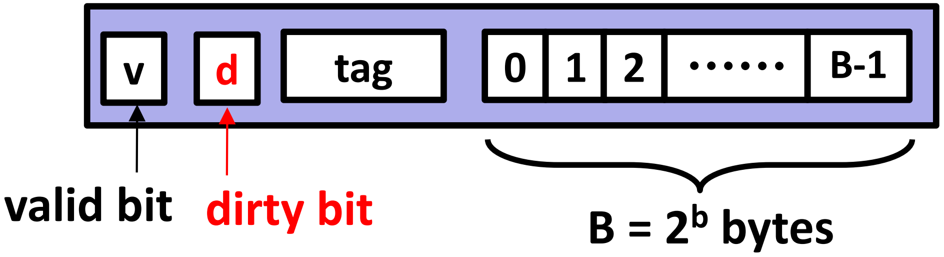
4-bit addresses (M=16 bytes)
S=2 sets, E=2 blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

0	$[00\underline{0}0_2]$,	miss
1	$[00\underline{0}1_2]$,	hit
7	$[01\underline{1}1_2]$,	miss
8	$[10\underline{0}0_2]$,	miss
0	$[00\underline{0}0_2]$	hit

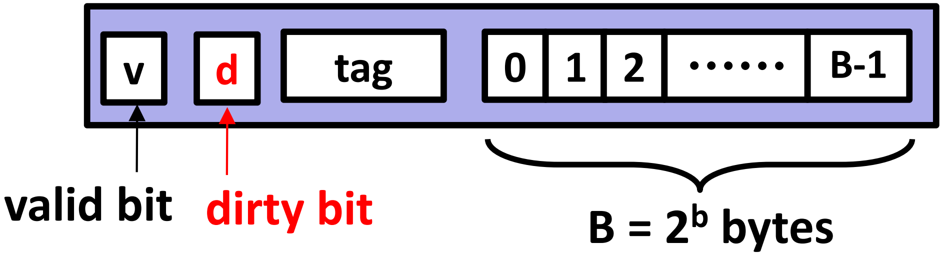
	v	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

What about writes?



- **Multiple copies of data exist:**
  - L1, L2, L3, Main Memory, Disk
- **What to do on a write-hit?**
  - Write-through (write immediately to memory)
  - Write-back (defer write to memory until replacement of line)
    - Needs a dirty bit (set if data has been written to)
- **What to do on a write-miss?**
  - Write-allocate (load into cache, update line in cache)
    - Good if more writes to the location will follow
  - No-write-allocate (writes straight to memory, does not load into cache)
- **Typical**
  - Write-through + No-write-allocate
  - **Write-back + Write-allocate**

Practical Write-back Write-allocate



- **A write to address X is issued**
- **If it is a hit**
  - Update the contents of block
  - Set dirty bit to 1 (bit is sticky and only cleared on eviction)
- **If it is a miss**
  - Fetch block from memory (per a read miss)
  - The perform the write operations (per a write hit)
- **If a line is evicted and dirty bit is set to 1**
  - The entire block of 2^b bytes are written back to memory
  - Dirty bit is cleared (set to 0)
  - Line is replaced by new contents

Why Index Using Middle Bits?

Direct mapped: One line per set

Assume: cache block size 8 bytes

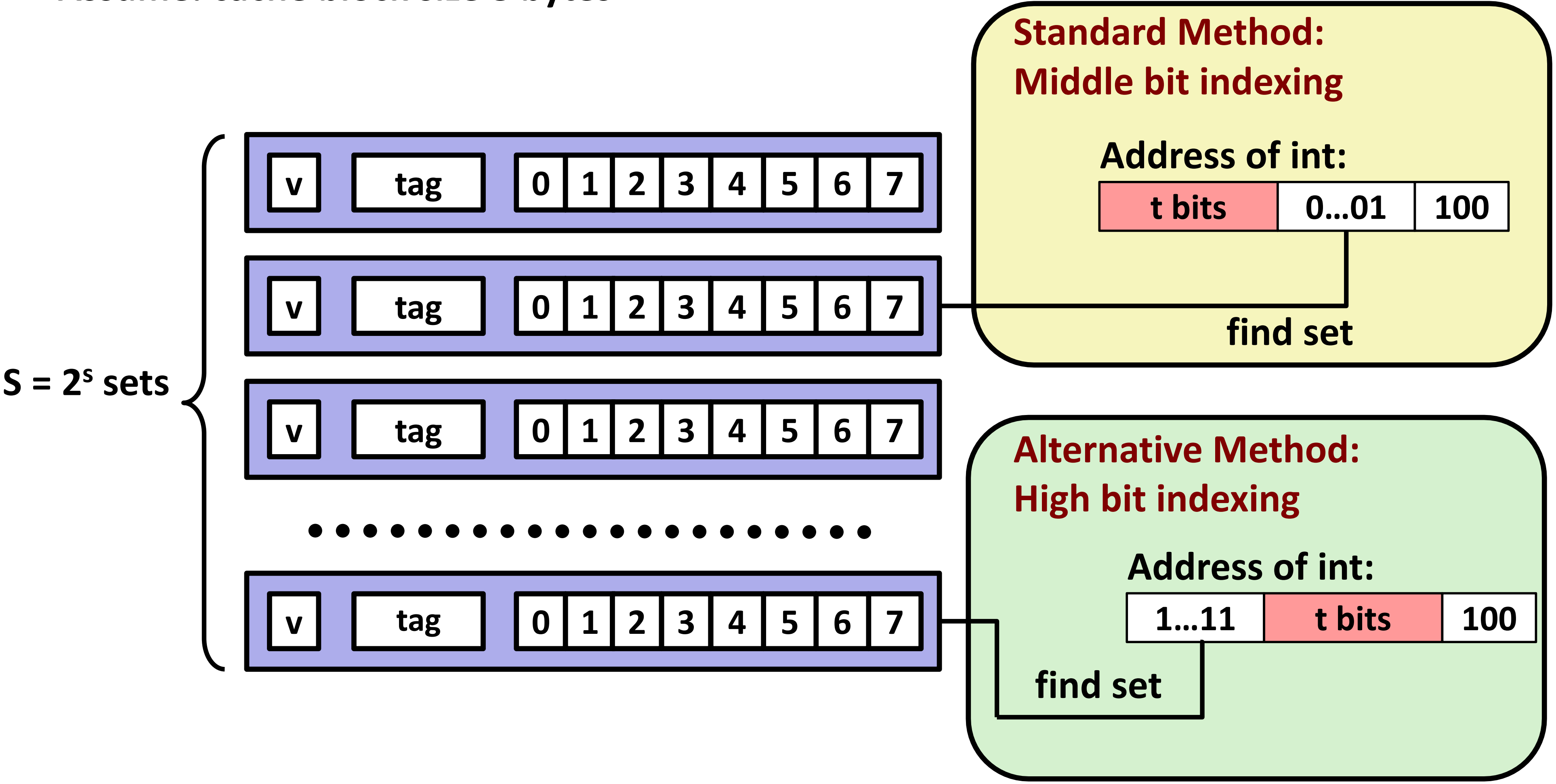
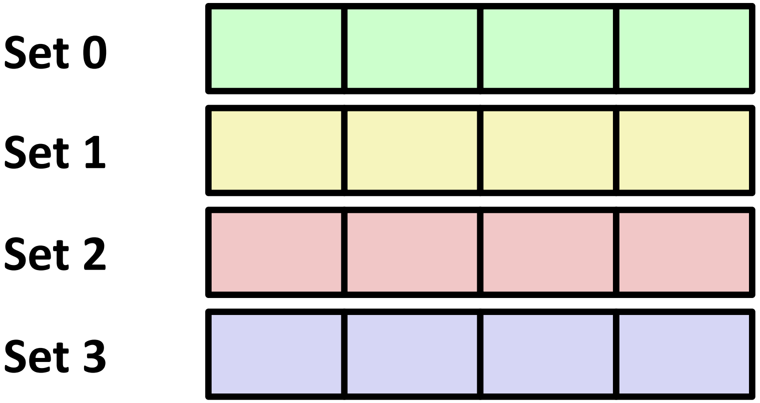


Illustration of Indexing Approaches

- 64-byte memory
  - 6-bit addresses
- 16 byte, direct-mapped cache
- Block size = 4. (Thus, 4 sets; why?)
- 2 bits tag, 2 bits index, 2 bits offset

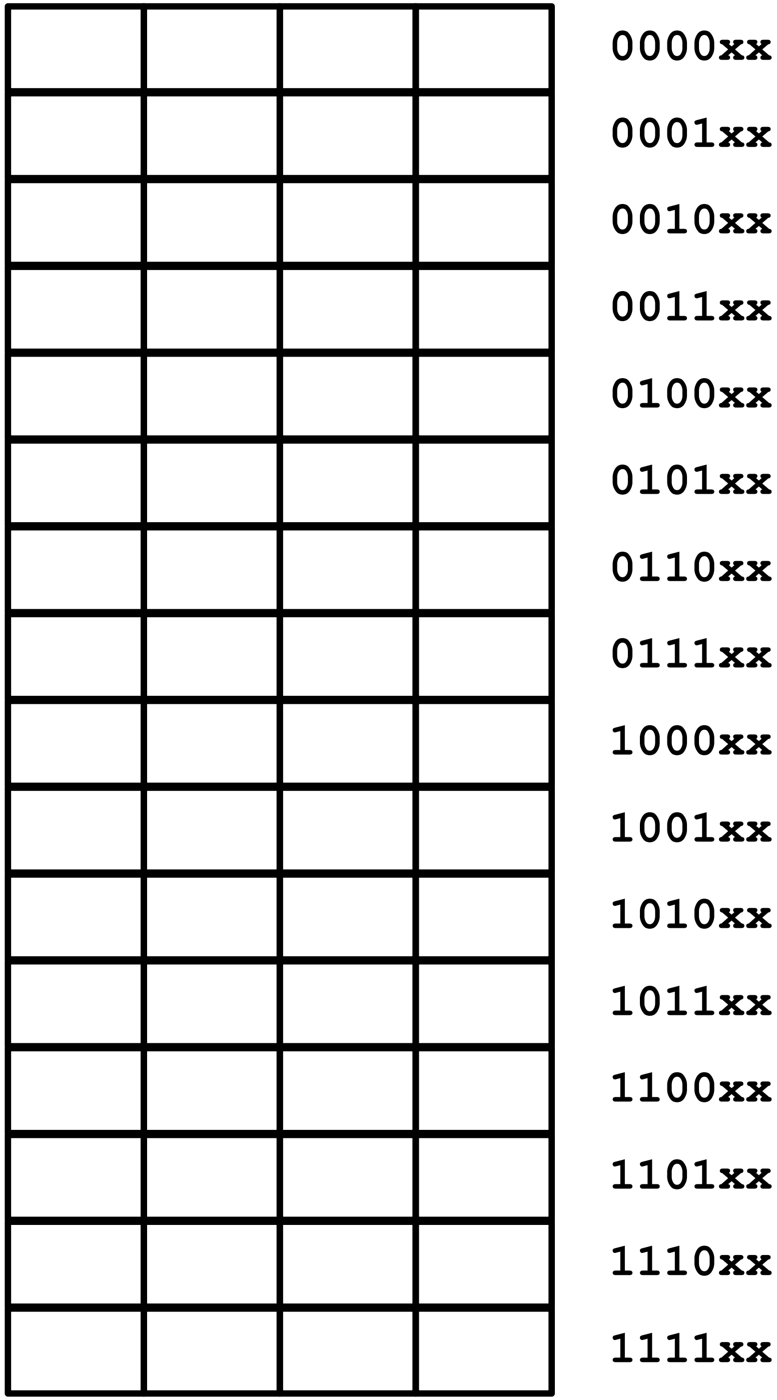


Set 0

Set 1

Set 2

Set 3



0000xx

0001xx

0010xx

0011xx

0100xx

0101xx

0110xx

0111xx

1000xx

1001xx

1010xx

1011xx

1100xx

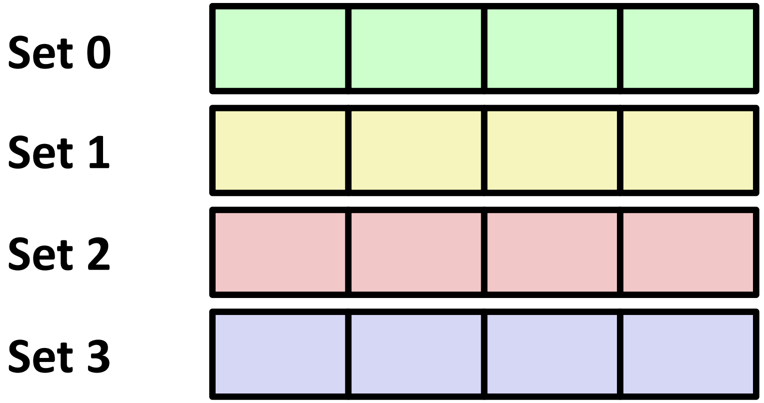
1101xx

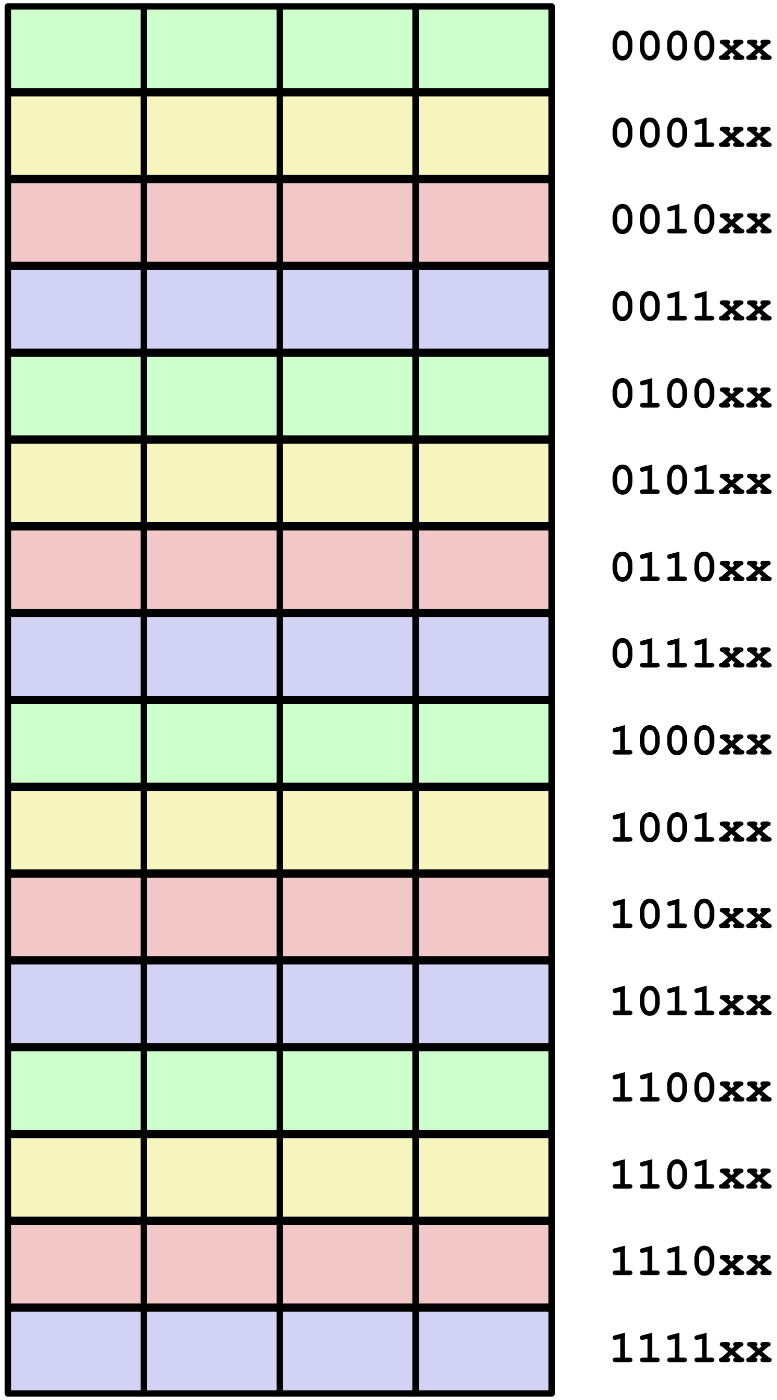
1110xx

1111xx

Middle Bit Indexing

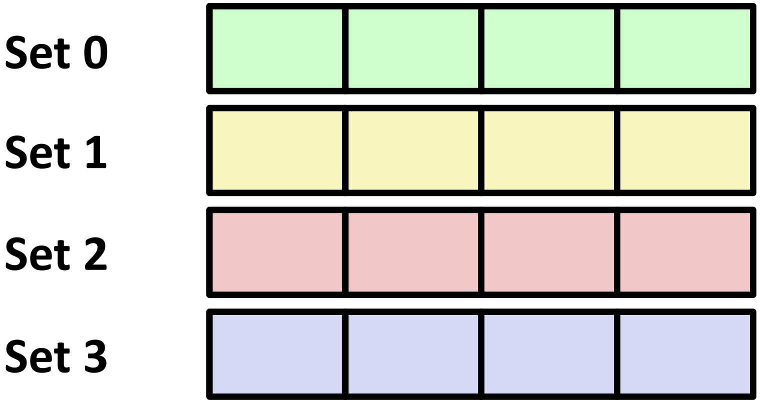
- Addresses of form TTSSBB
  - TT Tag bits
  - SS Set index bits
  - BB Offset bits
- Makes good use of spatial locality

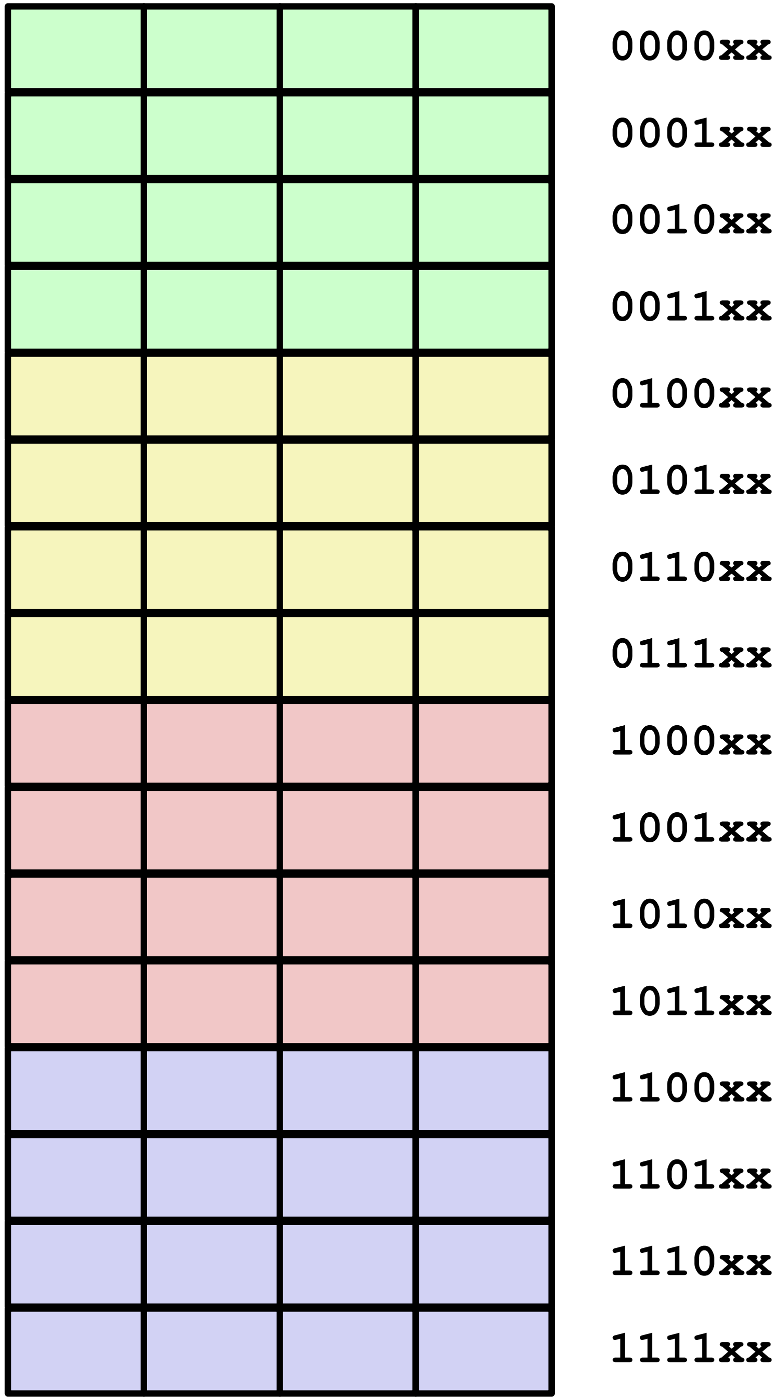




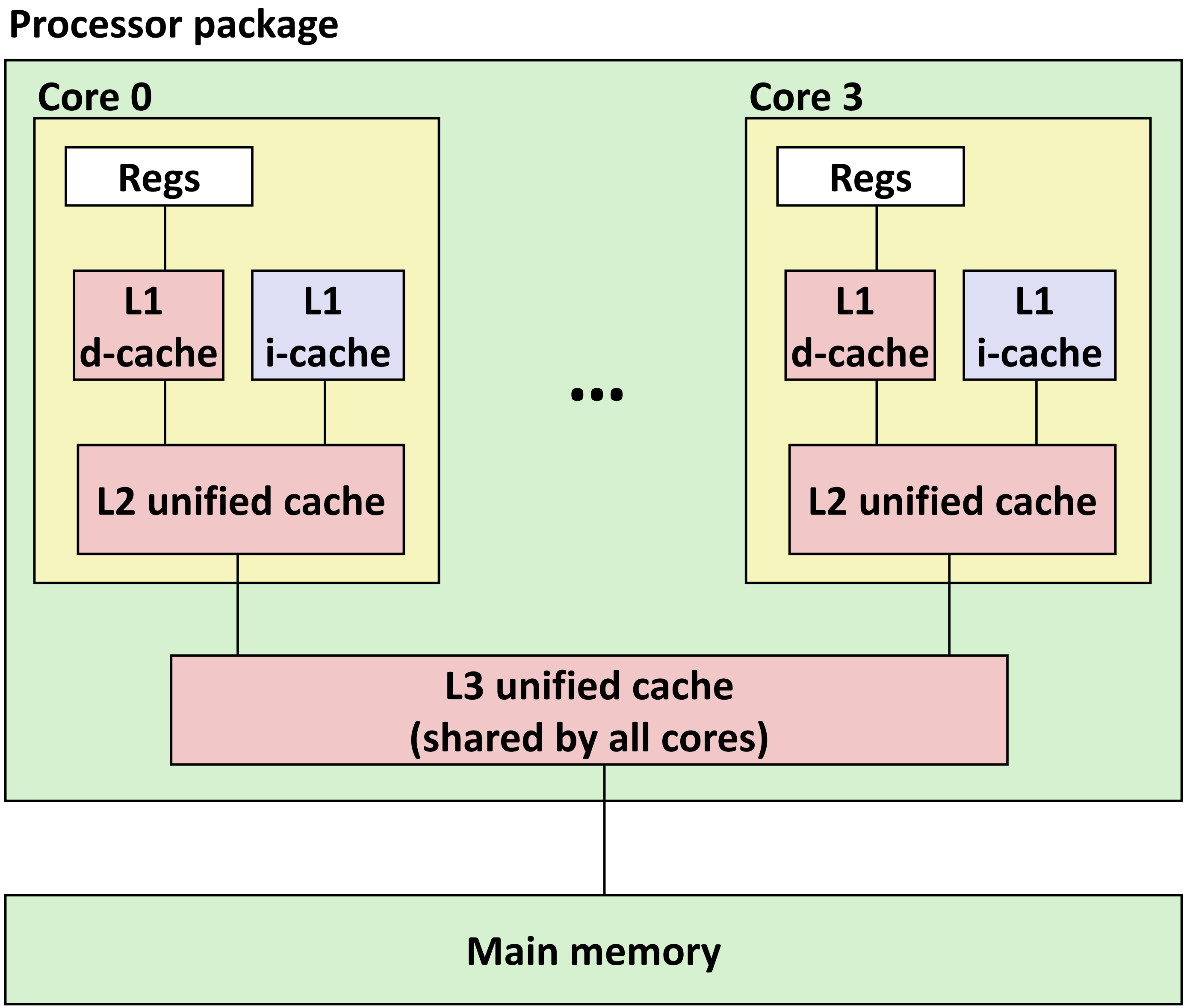
High Bit Indexing

- Addresses of form SSTTBB
  - SS Set index bits
  - TT Tag bits
  - BB Offset bits
- Program with high spatial locality would generate lots of conflicts





Intel Core i7 Cache Hierarchy



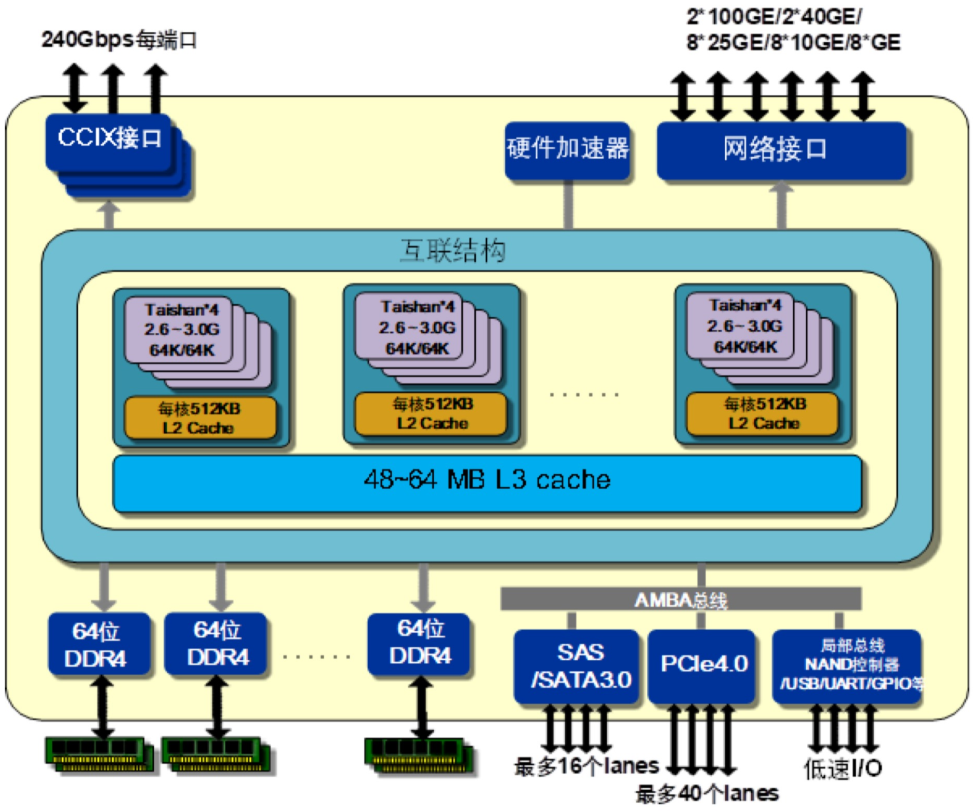
L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

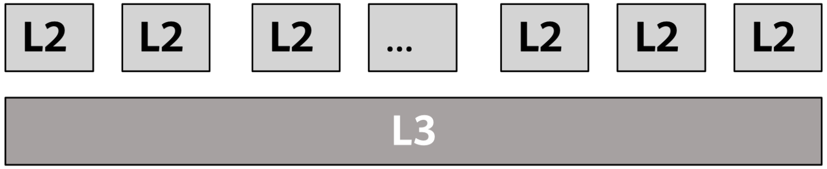
Kunpeng 920 Cache Hierarchy



- 集成最多64×自研核
  - 指令集兼容ARMv8.2, 最高主频达3.0GHz
  - 每核集成64KB L1 I/D缓存
  - 每核独享512KB L2缓存，单芯片共享48-64MB L3缓存
- 8×DDR4控制器@2933MT/s
- 集成PCI-e/SAS接口
  - 支持PCI-e 4.0，向下兼容PCI-e 3.0/2.0/1.0
  - 支持x16,x8,x4,x2,x1 PCI-e 4.0, 集成20 PCI-e控制器
  - 支持16×SAS/SATA 3.0控制器
- 支持CCIX接口，支持加速器的缓存一致性
- 支持2×100G RoCE v2, 支持25GE/50GE/100GE标准NIC
- 支持2P/4P扩展
- 封装大小: 60mm×75mm

Kunpeng 920 Cache Hierarchy

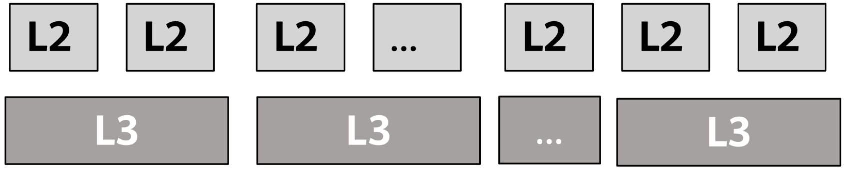
- Share Cache：对所有的L2来说L3 cache是共享的，一个进程可以使用整个L3的容量



L2 L2 L2 ... L2 L2 L2

L3

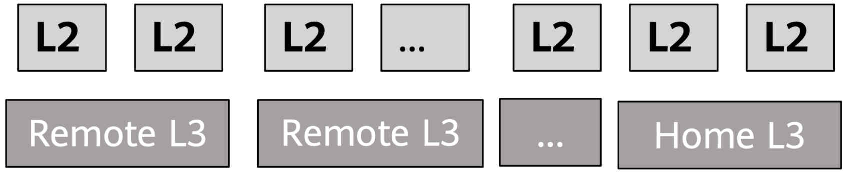
- Private Cache：有N个Private的L3，每个Private L3只缓存对应的L2的数据。即一个进程只能使用对应的部分L3的容量，无法使用全部L3的容量，L3和L3之间不通信



L2 L2 L2 ... L2 L2 L2

L3 L3 ... L3

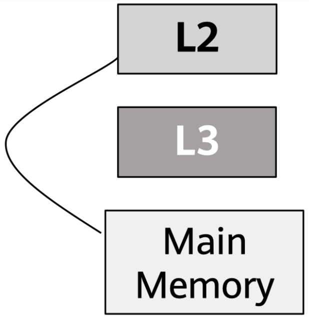
- Partitioned Cache：与Private相同的是，一个进程只能使用对应的部分L3容量；与Private不同的是，L3细分为一个Home的L3和N个Remote的L3，Home的L3类似L4，所以L3和L3之间会通信，由Home的L3来维护多个Partitioned L3之间的一致性



L2 L2 L2 ... L2 L2 L2

Remote L3 Remote L3 ... Home L3

- Non-inclusive L3：支持Non-inclusive模式，Memory和L2间直接数据访问



L2

L3

Main Memory

Cache Performance Metrics

- **Miss Rate**
  - Fraction of memory references not found in cache (misses / accesses) = 1 – hit rate
  - Typical numbers (in percentages):
    - 3-10% for L1
    - can be quite small (e.g., < 1%) for L2, depending on size, etc.
- **Hit Time**
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typical numbers:
    - 4 clock cycle for L1
    - 10 clock cycles for L2
- **Miss Penalty**
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory (Trend: increasing!)

Let’s think about those numbers

- **Huge difference between a hit and a miss**
  - Could be 100x, if just L1 and main memory

- **Would you believe 99% hits is twice as good as 97%?**
  - Consider this simplified example:

    cache hit time of 1 cycle
    miss penalty of 100 cycles

  - Average access time:

    97% hits: 1 cycle + 0.03 x 100 cycles = **4 cycles**

    99% hits: 1 cycle + 0.01 x 100 cycles = **2 cycles**

- **This is why “miss rate” is used instead of “hit rate”**

Writing Cache Friendly Code

- **Make the common case go fast**
  - Focus on the inner loops of the core functions

- **Minimize the misses in the inner loops**
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Today

- Cache organization and operation
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

The Memory Mountain

- **Read throughput (read bandwidth)**
  - Number of bytes read from memory per second (MB/s)

- **Memory mountain: Measured read throughput as a function of spatial and temporal locality.**
  - Compact way to characterize memory system performance.

Memory Mountain Test Function

```
long data[MAXELEMS];  /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *        array "data" with stride of "stride‶,
 *        using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

Call `test()` with many combinations of `elems` and `stride`.

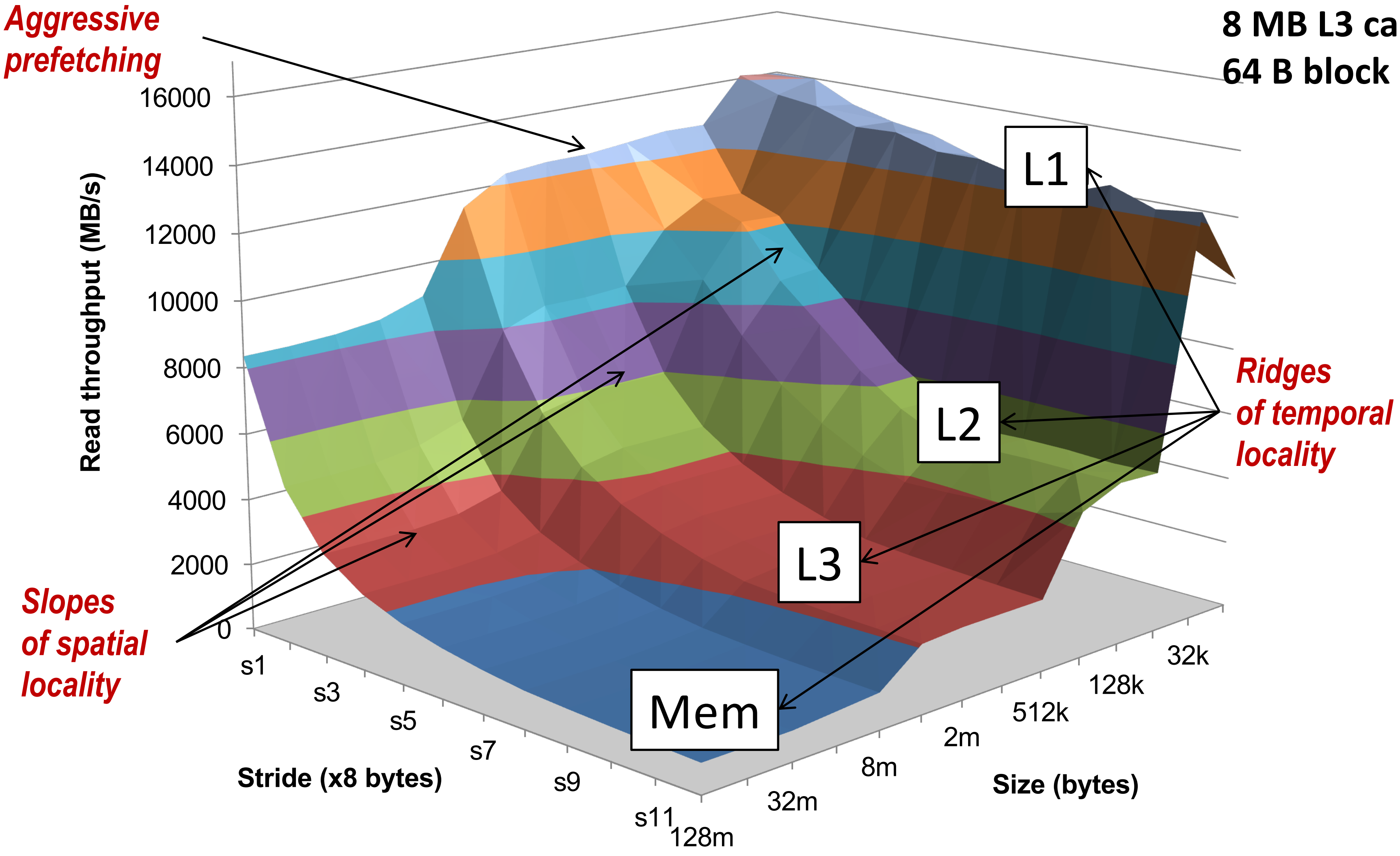
For each `elems` and `stride`:

1. Call `test()` once to warm up the caches.

2. Call `test()` again and measure the read throughput(MB/s)

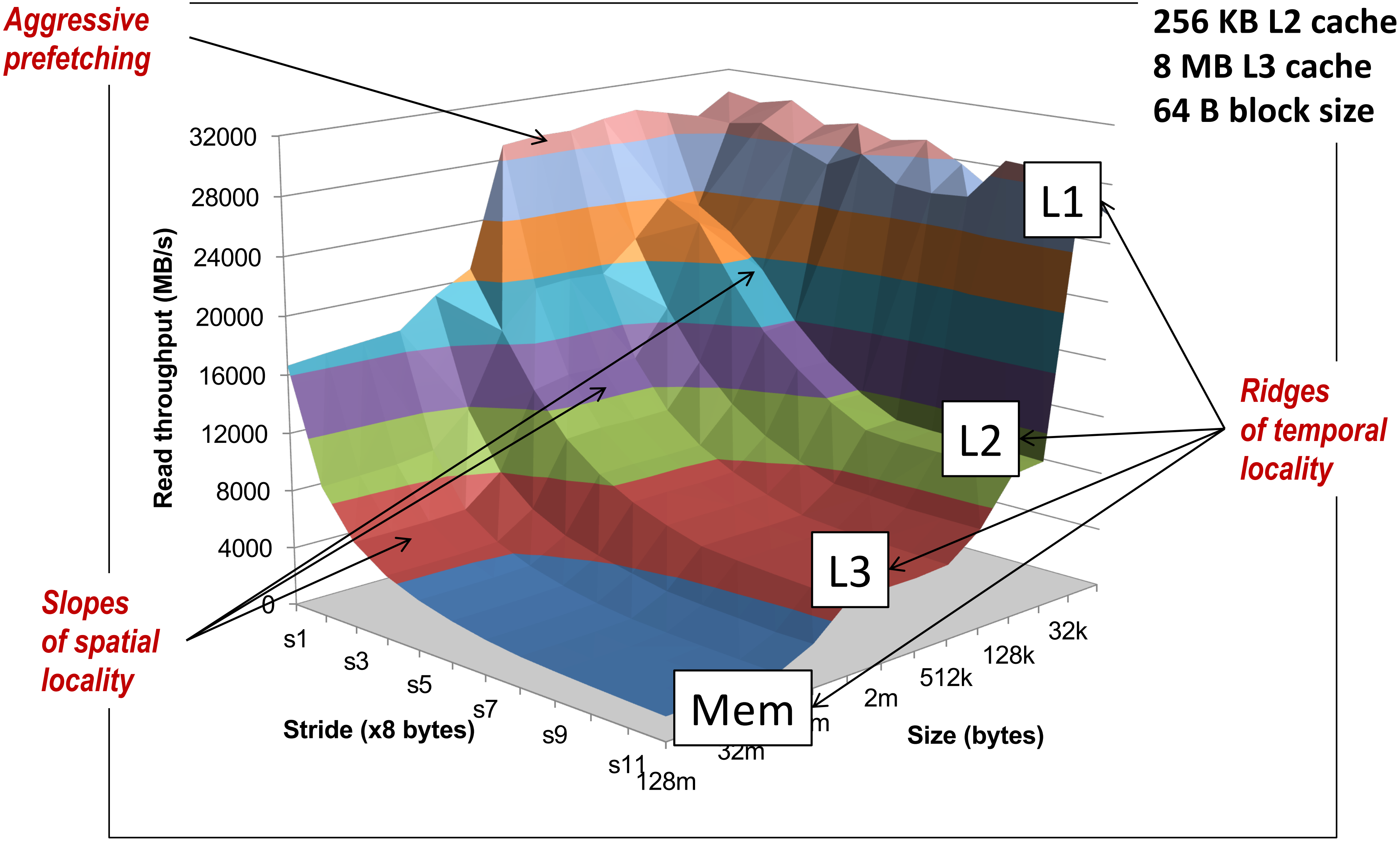
The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



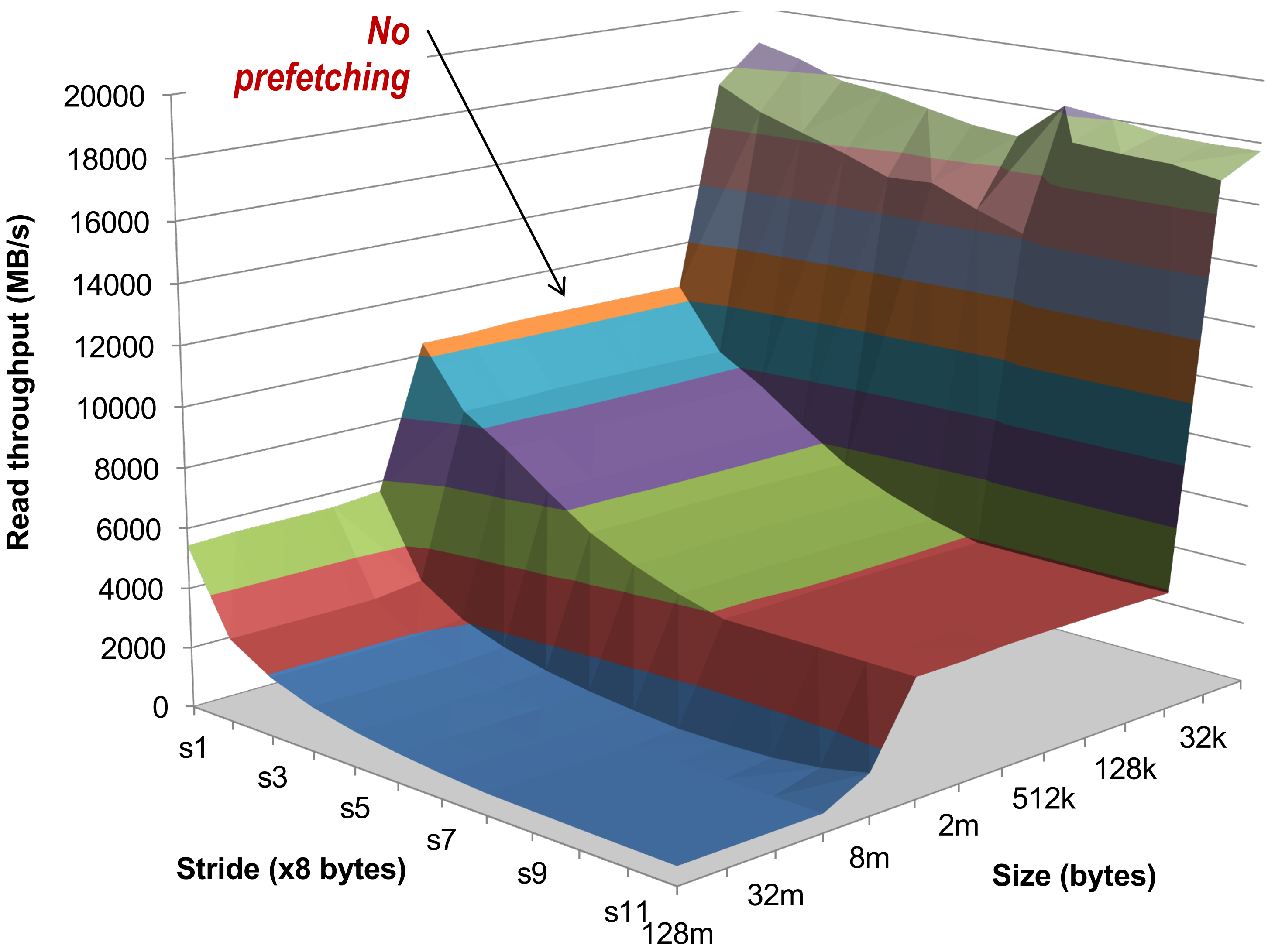
The Memory Mountain

Core i5 Haswell
3.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

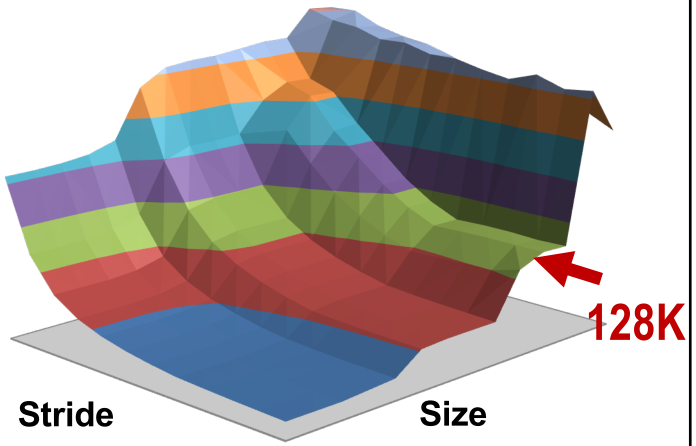


2008 Memory Mountain

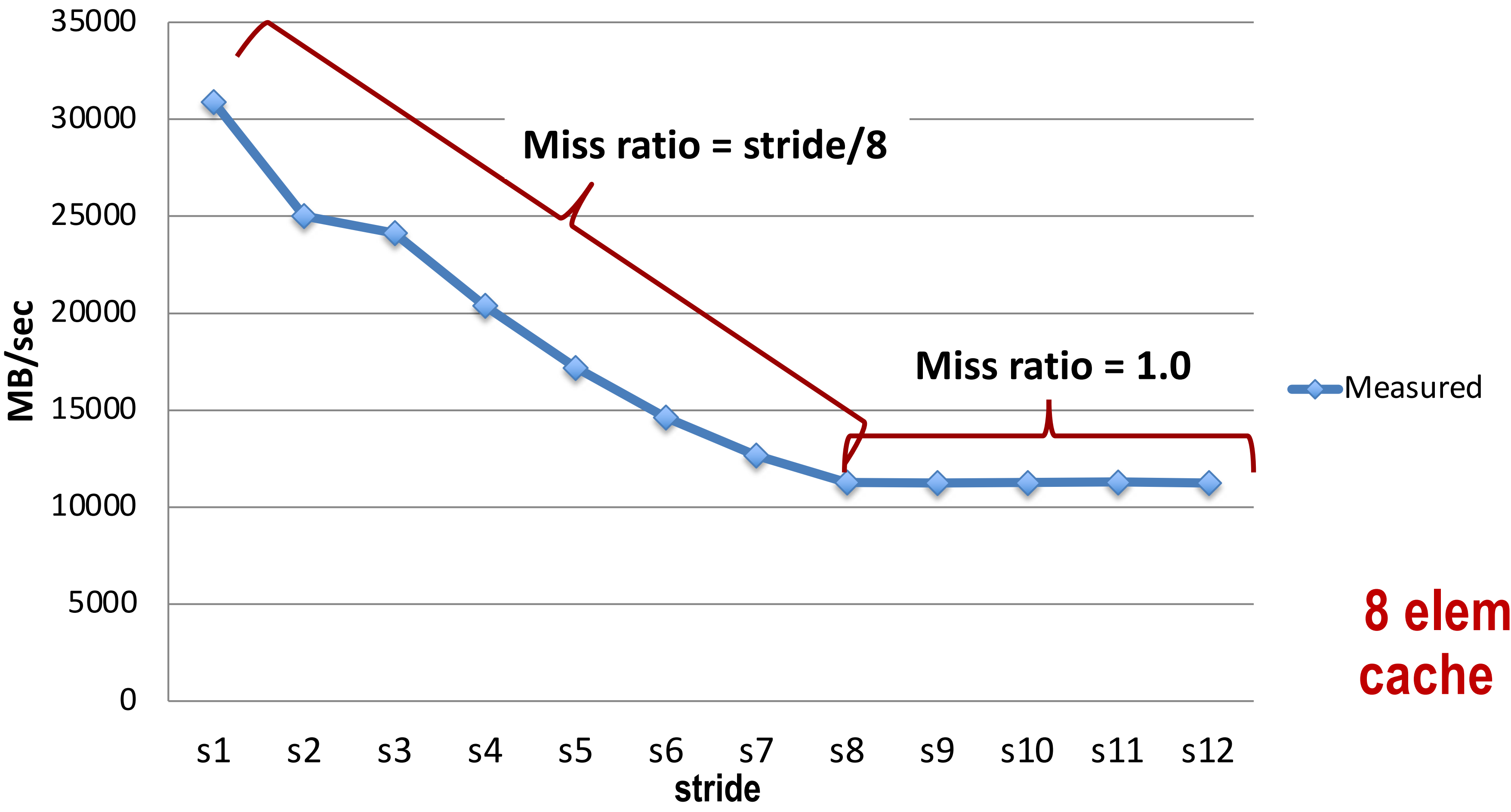
Core 2 Duo
2.4 GHz
32 KB L1 d-cache
6MB L2 cache
64 B block size



Closer Look at Stride Effects



Throughput for size = 128K



8 elems per cache block

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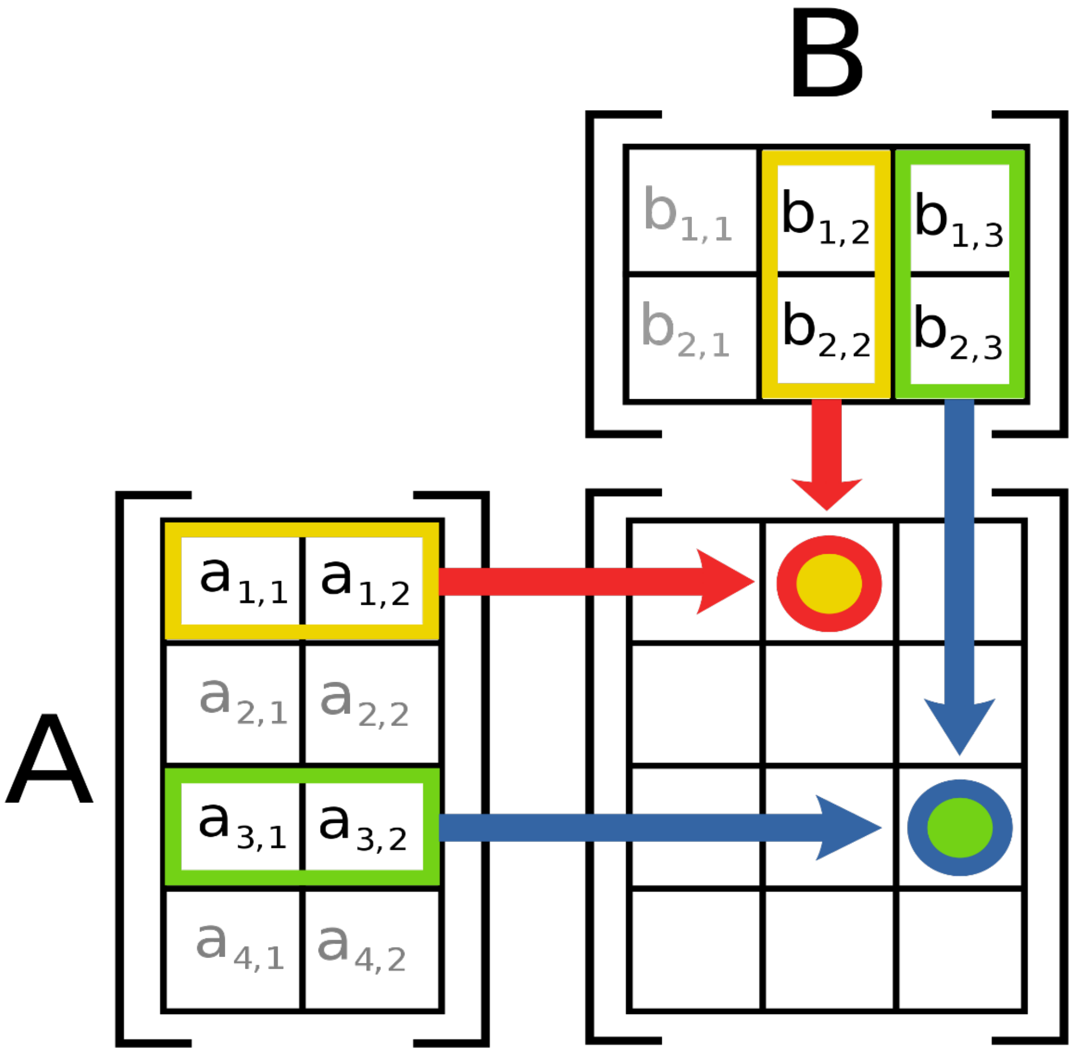
Matrix multiplication



```
Out[i, j] =
   dot product(A[i, ..], B[..,j])
 = sum (
          a[i, 0] * b[0, j],
          a[I, 1] * b[1, j]
                 )
```

Matrix Multiplication Example

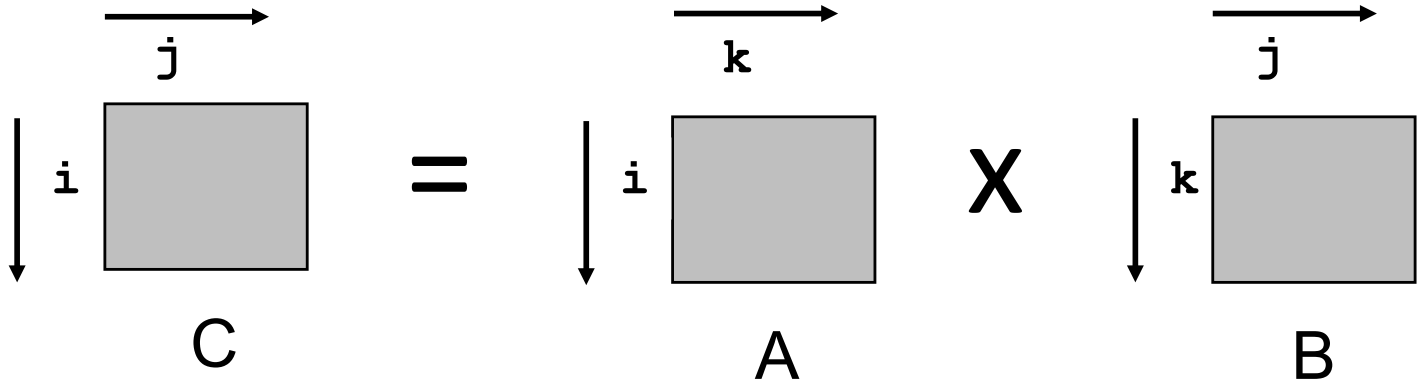
- **Description:**
  - Multiply *N* x *N* matrices
  - Matrix elements are doubles (8 bytes)
  - O(N^3) total operations
  - *N* reads per source element
  - *N* values summed per destination
    - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                          matmult/mm.c
```

Variable ***sum*** *held in register*

Miss Rate Analysis for Matrix Multiply

- **Assume:**
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- **Analysis Method:**
  - Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**
  - each row in contiguous memory locations
  - a[i][j] = a[i*N + j] where N is the number of columns
- **Stepping through columns in one row:**

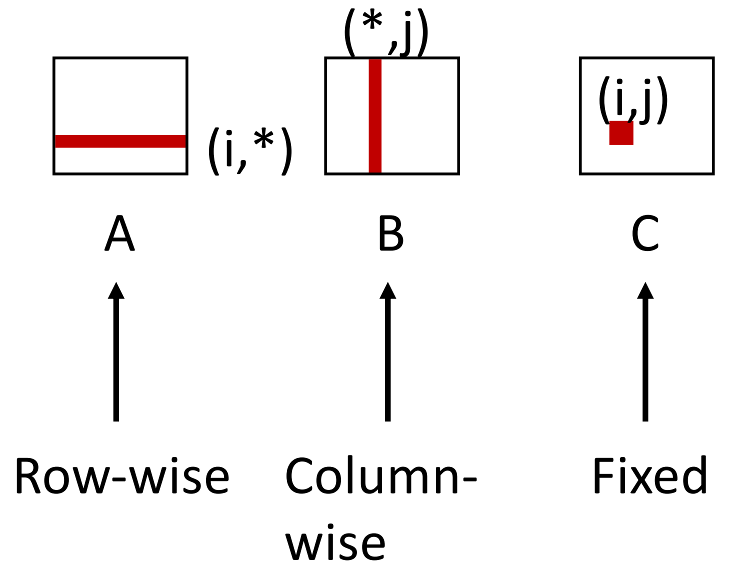
  -
    ```
    for (i = 0; i < N; i++)
     sum += a[0][i];
    ```
  - accesses successive elements
  - if block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B
- **Stepping through rows in one column:**

  -
    ```
    for (i = 0; i < n; i++)
     sum += a[i][0];
    ```
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                          matmult/mm.c
```

Inner loop:



Miss rate for inner loop iterations:

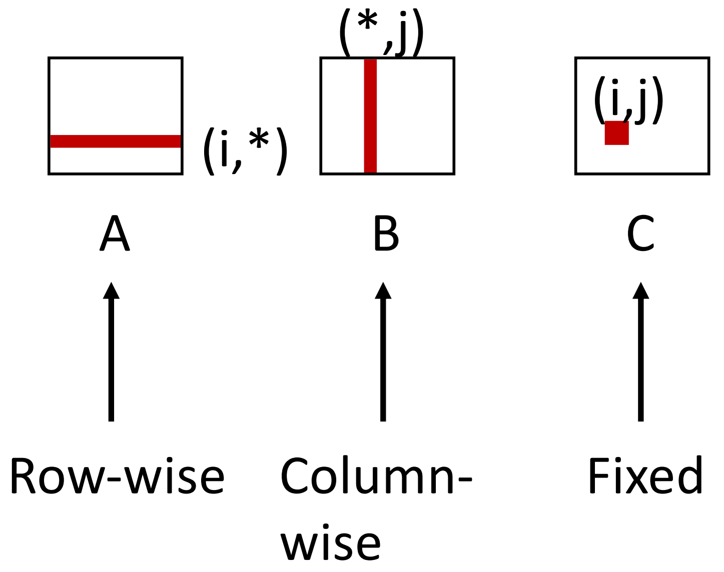
A B C

Block size = 32B (four doubles)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                           matmult/mm.c
```

Inner loop:



Miss rate for inner loop iterations:

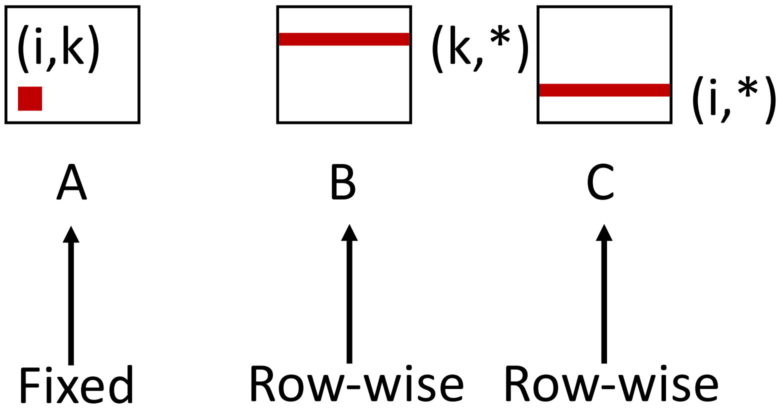
A	B	C
0.25	1.0	0.0

Block size = 32B (four doubles)

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}                         matmult/mm.c
```

Inner loop:



Miss rate for inner loop iterations:

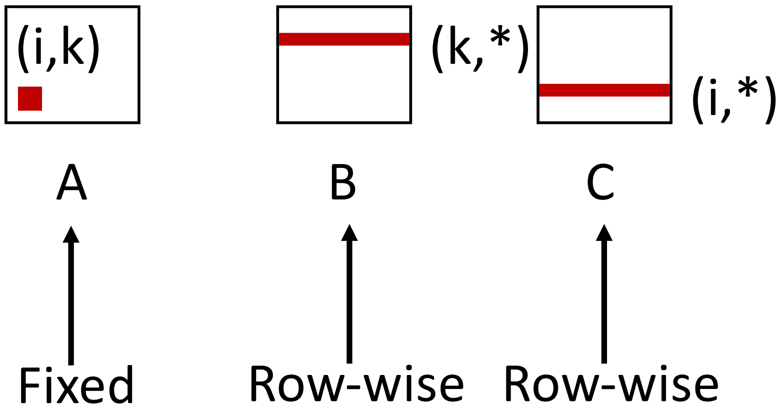
A B C

Block size = 32B (four doubles)

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}                        matmult/mm.c
```

Inner loop:



Miss rate for inner loop iterations:

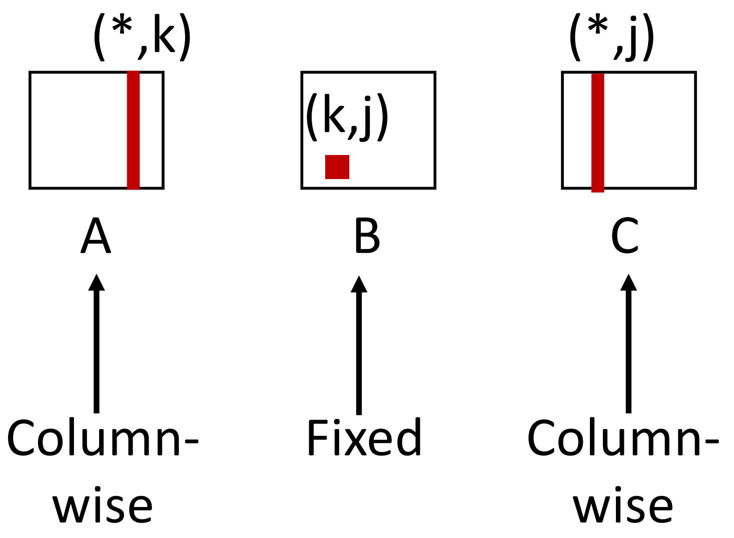
A	B	C
0.0	0.25	0.25

Block size = 32B (four doubles)

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}                                  matmult/mm.c
```

Inner loop:



Miss rate for inner loop iterations:

A B C

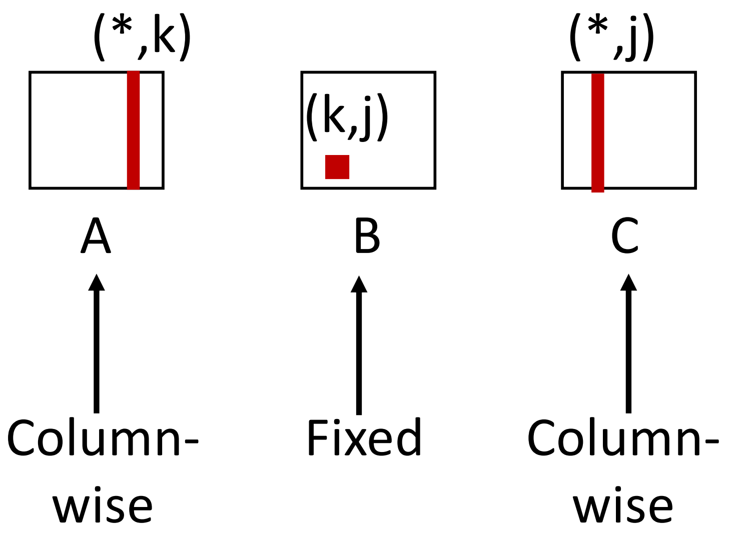
Block size = 32B (four doubles)

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

matmult/mm.c

Inner loop:



Miss rate for inner loop iterations:

A	B	C
1.0	0.0	1.0

Block size = 32B (four doubles)

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

`ijk` (& `jik`):

- 2 loads, 0 stores
- avg misses/iter = **1.25**

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

`kij` (& `ikj`):

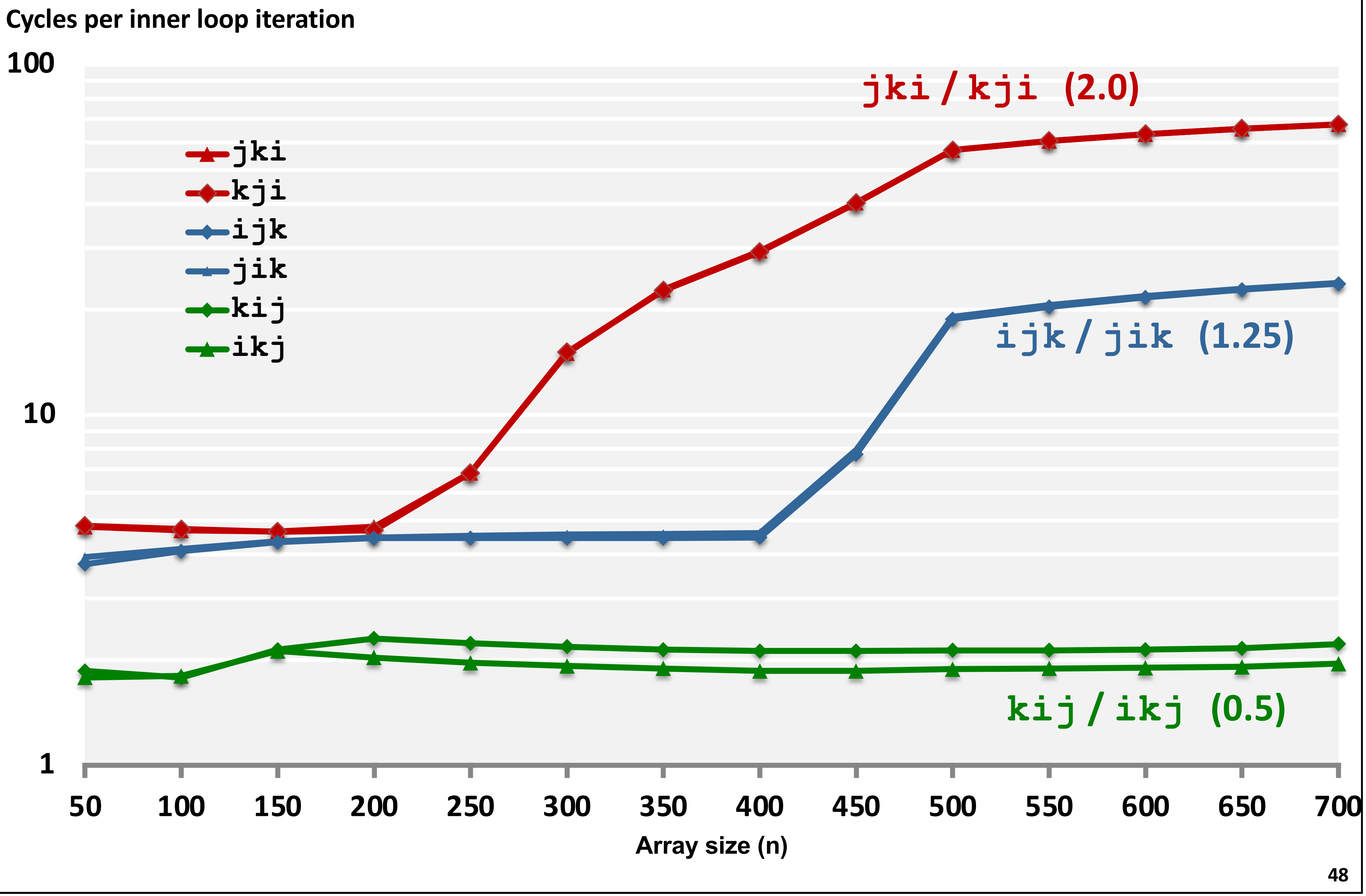
- 2 loads, 1 store
- avg misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

`jki` (& `kji`):

- 2 loads, 1 store
- avg misses/iter = **2.0**

Core i7 Matrix Multiply Performance



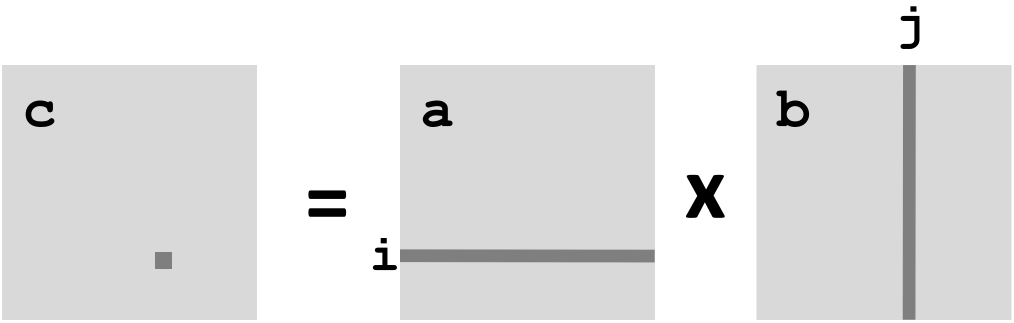
Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Example: Matrix Multiplication

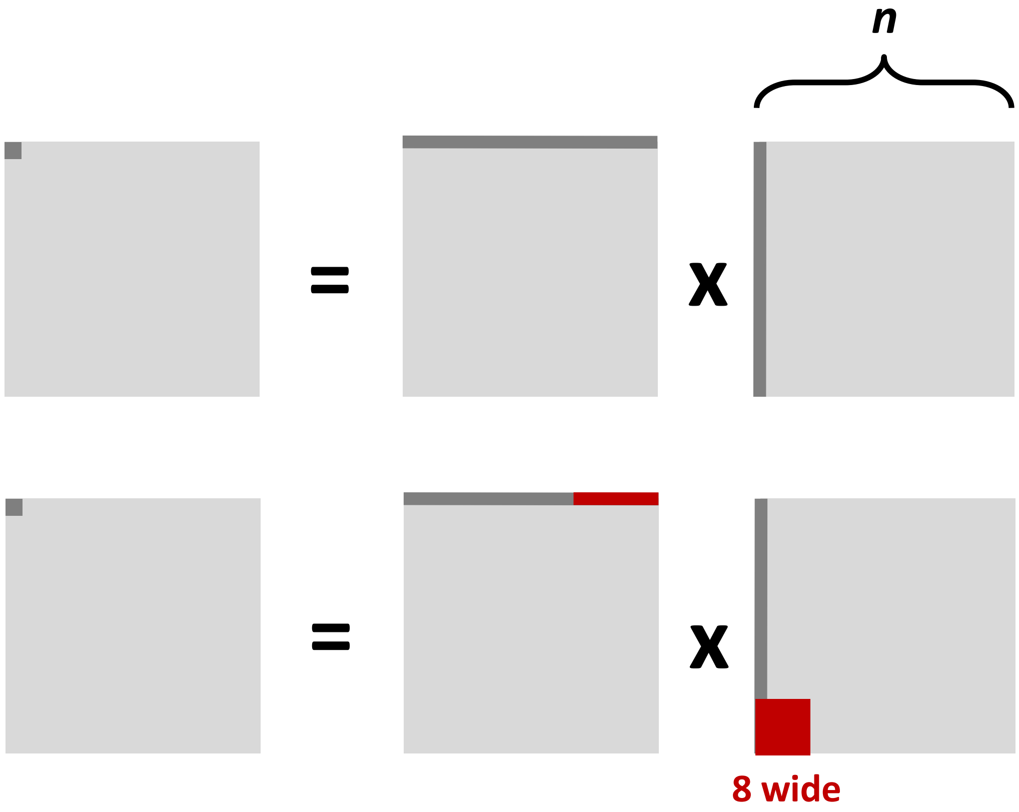
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
     int i, j, k;
     for (i = 0; i < n; i++)
         for (j = 0; j < n; j++)
                for (k = 0; k < n; k++)
                    c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



Cache Miss Analysis

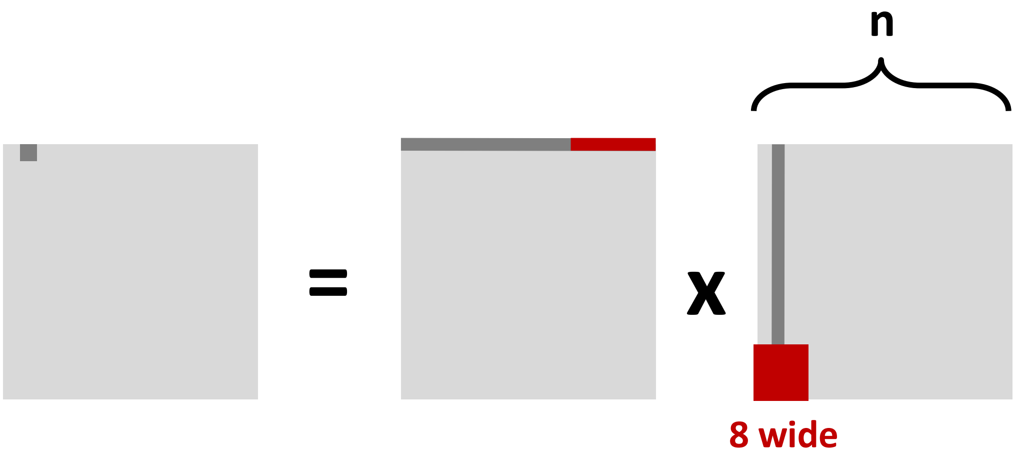
- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than *n*)
- **First iteration:**
  - *n*/8 + n = 9*n*/8 misses
  - Afterwards in cache: (schematic)



Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than *n*)

- **Second iteration:**
  - Again:
    $n/8 + n = 9n/8$ misses



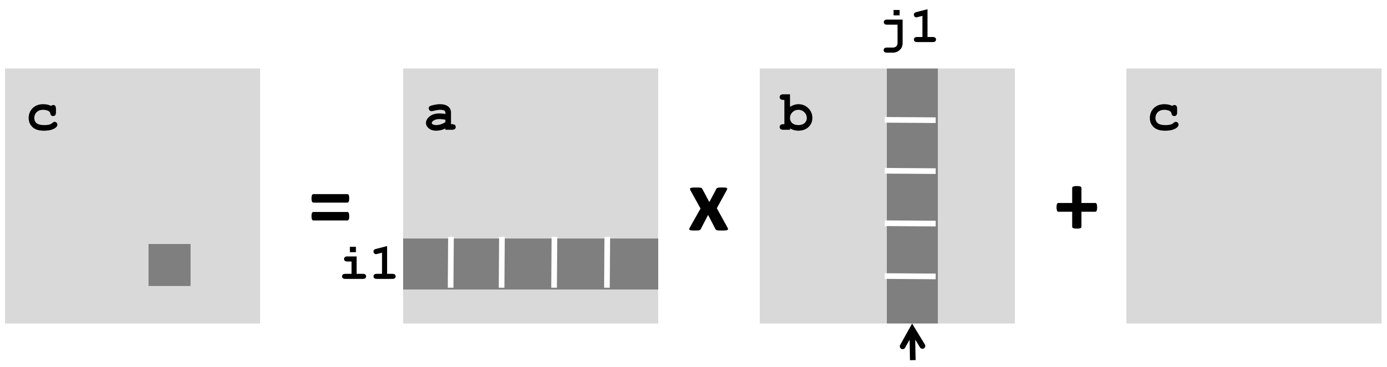
- **Total misses:**
  - $9n/8\ n^2 = (9/8)\ n^3$

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
             for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                  for (i1 = i; i1 < i+B; i1++)
                      for (j1 = j; j1 < j+B; j1++)
                          for (k1 = k; k1 < k+B; k1++)
                             c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

matmult/bmm.c

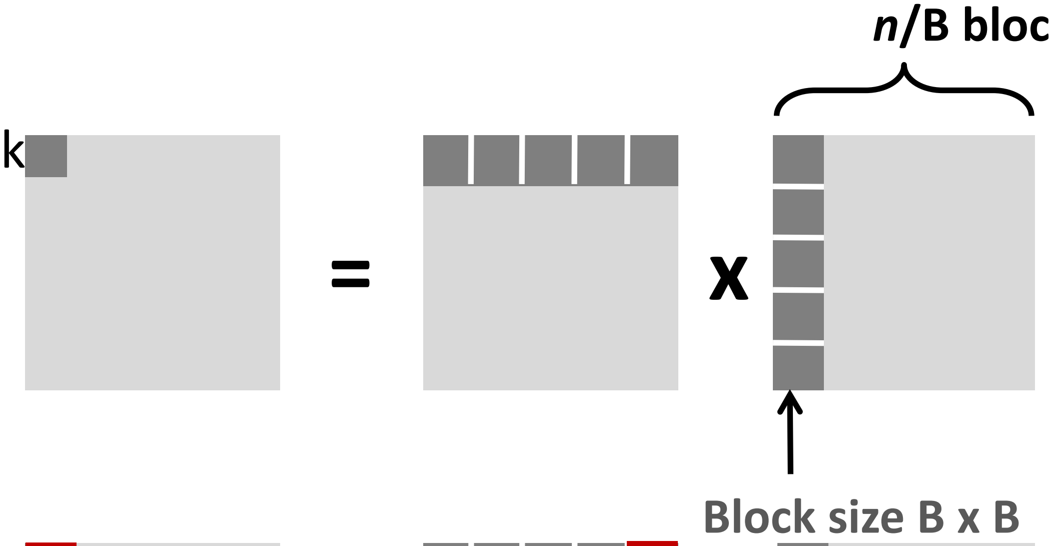


Block size B x B

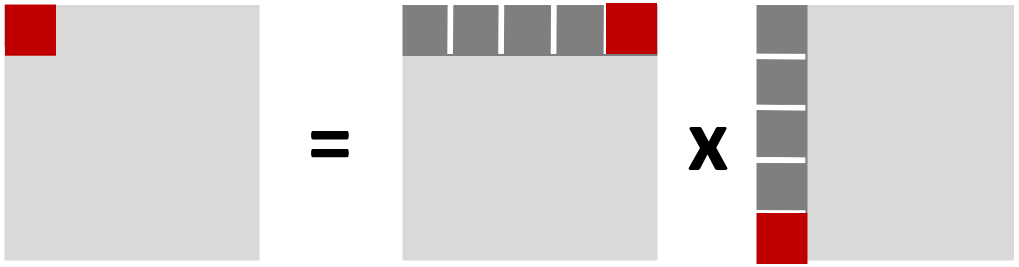
Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$

- **First (block) iteration:**
  - B*B/8 misses for each block
  - $2n/B \times B^2/8 = nB/4$
    (omitting matrix c)

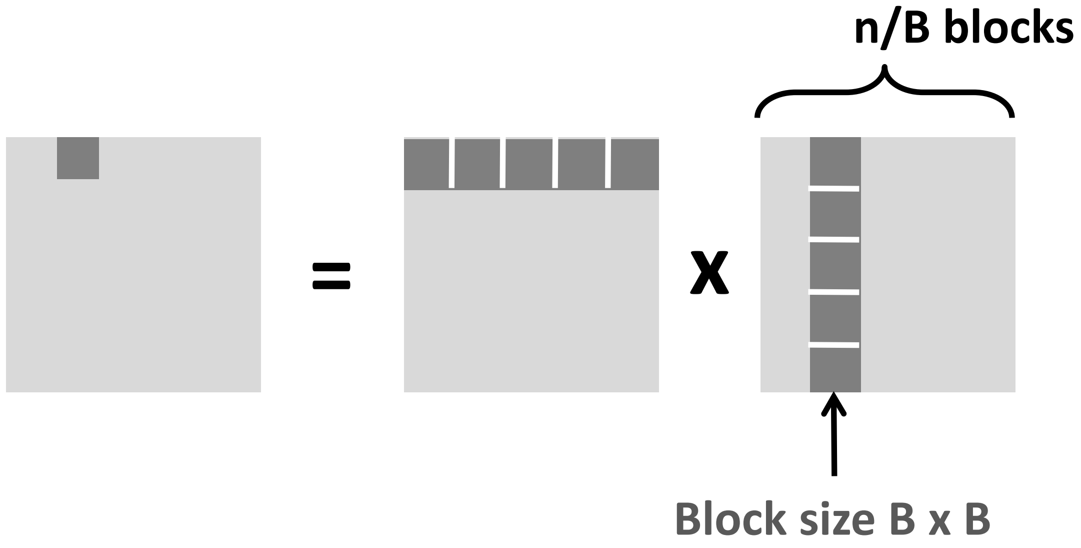


  - Afterwards in cache
    (schematic)



Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$
- **Second (block) iteration:**
  - Same as first iteration
  - $2n/B \times B^2/8 = nB/4$



- **Total misses:**
  - $nB/4 * (n/B)^2 = n^3/(4B)$

Blocking Summary

- **No blocking: (9/8) n^3 misses**
- **Blocking: (1/(4B)) n^3 misses**

- **Use largest block size B, such that B satisfies $3B^2 < C$**
  - Fit three blocks in cache! Two input, one output.

- **Reason for dramatic difference:**
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

Cache Summary

- **Cache memories can have significant performance impact**

- **You can write your programs to exploit this!**
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it’s read from memory.