

Review

- Algorithm design
- Testing and debugging

Arrays

Lecture 14-1

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Data Structures So Far

- Various data structures with different characteristics

Collection	Mutable?	Ordered?	Use When...
list	Yes	Yes	You want to keep track of an ordered sequence that you want update
tuple	No	Yes	You want to build an ordered sequence that you know won't change or that you want to use as a key in a dictionary or as a value in a set
set	Yes	No	You want to keep track of values, but order doesn't matter, and you don't want duplicates. The values must be immutable.
dictionary	Yes	Yes	You want to keep a mapping of keys to values. The keys must be immutable.

Data Structures So Far

- Each data structure has its methods for our convenience, which we have used their methods without knowing how they are implemented
 - Append
 - Pop
 - Insert
 - Remove
 - Get
 - Size

From now, let's dive into their implementation and learn more data structures!

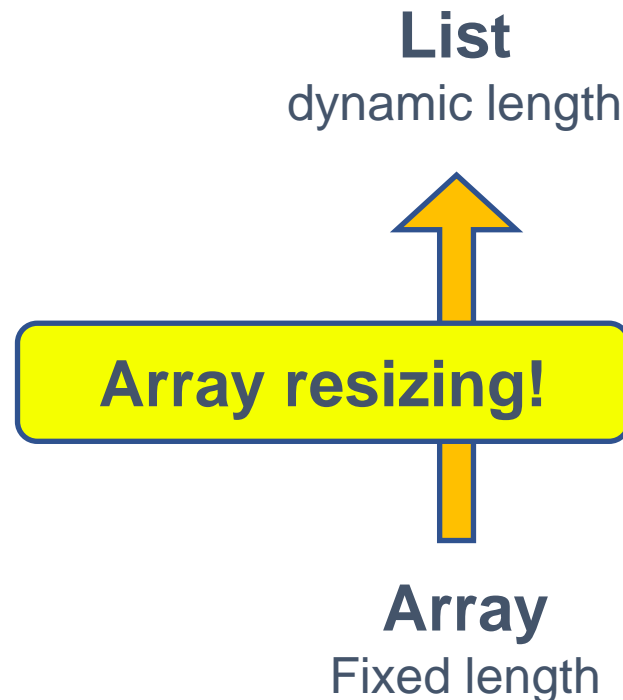


Looking into Lists – Arrays

- We need to declare memory boxes for storing information
 - Ex.) “A = 1” declares a memory box to store an integer object A
- An **array** is an object comprising a **numbered sequence** of memory boxes
 - This is a more fundamental data structure (no method at all) that Python lists are built on
 - This is why we can easily access the i-th element of list A by using **A[i]**
- An array comprises
 - **Fixed** integer length (N) – should be set when initializing it
 - A **sequence** of N memory boxes (numbered 0 through N-1)

Wait... **Fixed** length?

We have inserted, appended, popped, and removed freely using lists! Its length must be **dynamic**!



Array Resizing

- Two problems of an array due to its fixed length
 - Memory wastage: If it contains only $n(\ll N)$ valid elements
 - Memory shortage: If it wants to contain more than N elements
- Array resizing: create another larger array and copy all the elements
 - `L.append(3)` when the current array is **full**



A diagram shows a variable 'L' with an arrow pointing to the first row of a table. The table has two rows and twelve columns. The first row is labeled 'index' and contains values from 0 to 10. The second row is labeled 'values' and contains the corresponding array elements: 5, -2, 0, 100, -6, 7, 4, 9, -7, 50, and 4.

index	0	1	2	3	4	5	6	7	8	9	10
values	5	-2	0	100	-6	7	4	9	-7	50	4

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L →

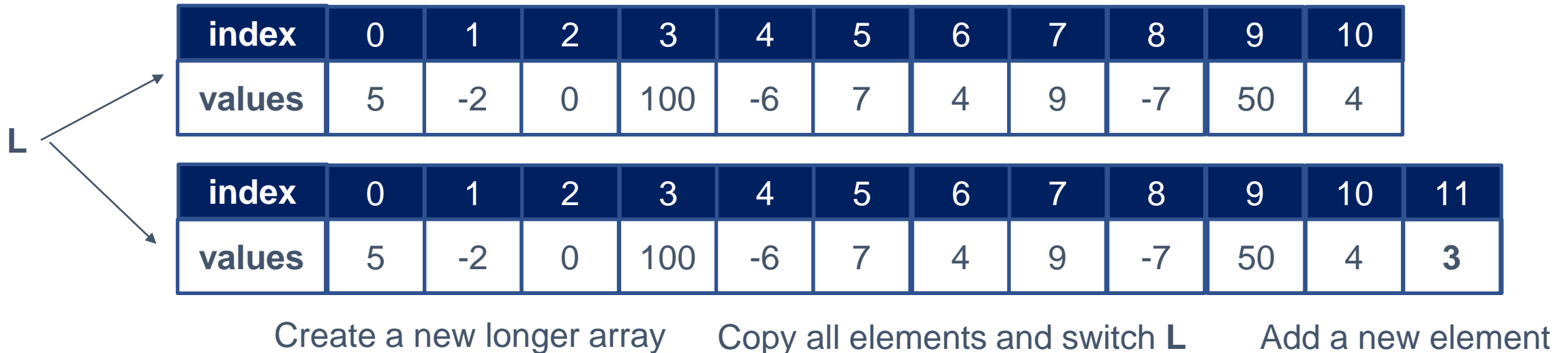
index	0	1	2	3	4	5	6	7	8	9	10
values	5	-2	0	100	-6	7	4	9	-7	50	4

index	0	1	2	3	4	5	6	7	8	9	10	11
values	0	0	0	0	0	0	0	0	0	0	0	0

Create a new longer array

Array Resizing

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Array Resizing

- Array resizing is expensive: new memory boxes and copy operation
 - Increasing size by one every time is not efficient (too many resizing)
 - Increasing size too much at once is not efficient either (memory wastage)
- To resize fewer, Python list size grows as 0, 4, 8, 16, 25, 35, 46, 58, ...
 - Mild over-allocation proportional to the current size
- Anyway... is there another way of organizing a collection of data to support append and pop easily?

Linked Lists

Lecture 14-2

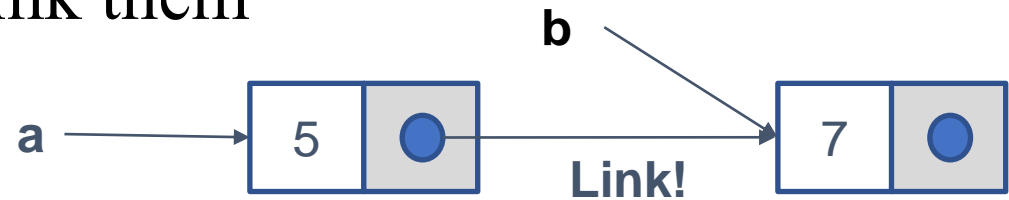
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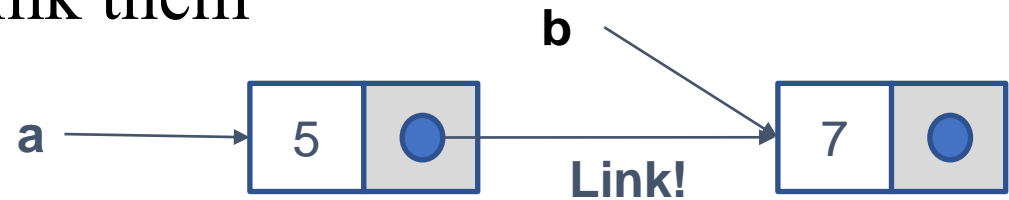
Basis

- Let's define a class that contains a single integer value as below:
 - `class ListNode():`
 - `def __init__(self, x):`
 - `self.val = x`
 - `self.next = None` # A special variable for **linking** to another node
- Let's create two ListNode objects and link them
 - `a = ListNode(5)`
 - `b = ListNode(7)`
 - `a.next = b`



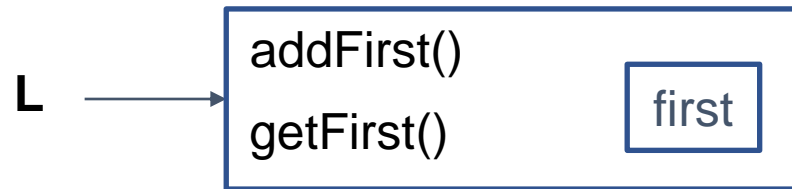
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- Let's define a class that contains a single integer value as below:
 - `class ListNode():`
 - `def __init__(self, x):`
 - `self.val = x`
 - `self.next = None` # A special variable for **linking** to another node
- Let's create two ListNode and link them
 - `a = ListNode(5)`
 - `b = ListNode(7)`
 - `a.next = b`
- Now we can access ListNode **b** through ListNode **a** because they are **linked**!
 - `b.val`
 - `a.next.val`



Single Linked Lists

- A linked list whose node has a single link as we've just seen
 - Every node can be access through the **first** node
- An example of a SLList consisting of two basic methods and one variable
 - `L = SLList()`



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 - `L = SLList()`
 - `L.addFirst(5)`



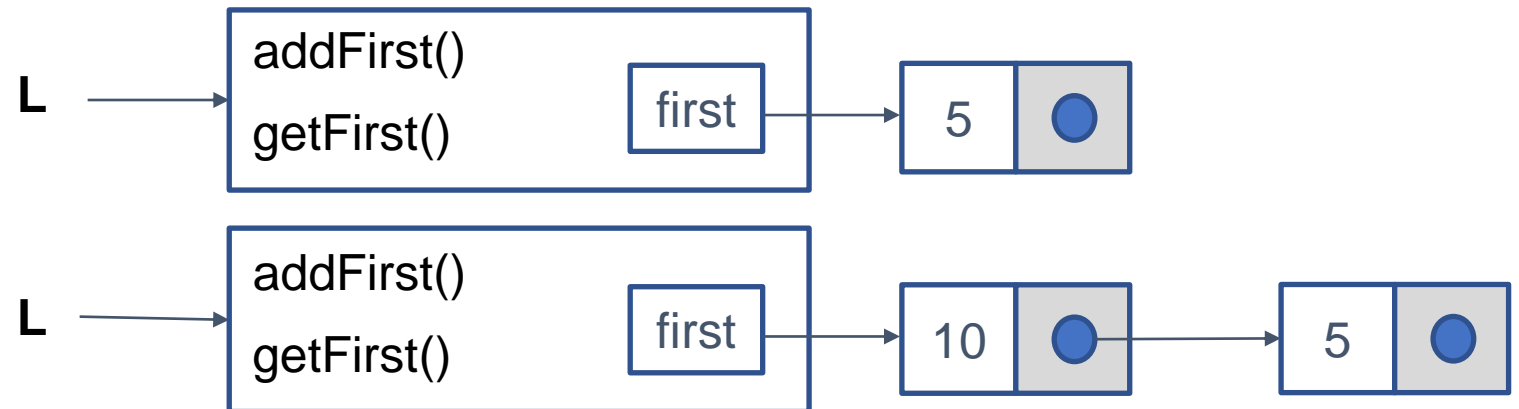
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 - `L = SLList()`
 - `L.addFirst(5)`
 - `L.getFirst()`
 - 5



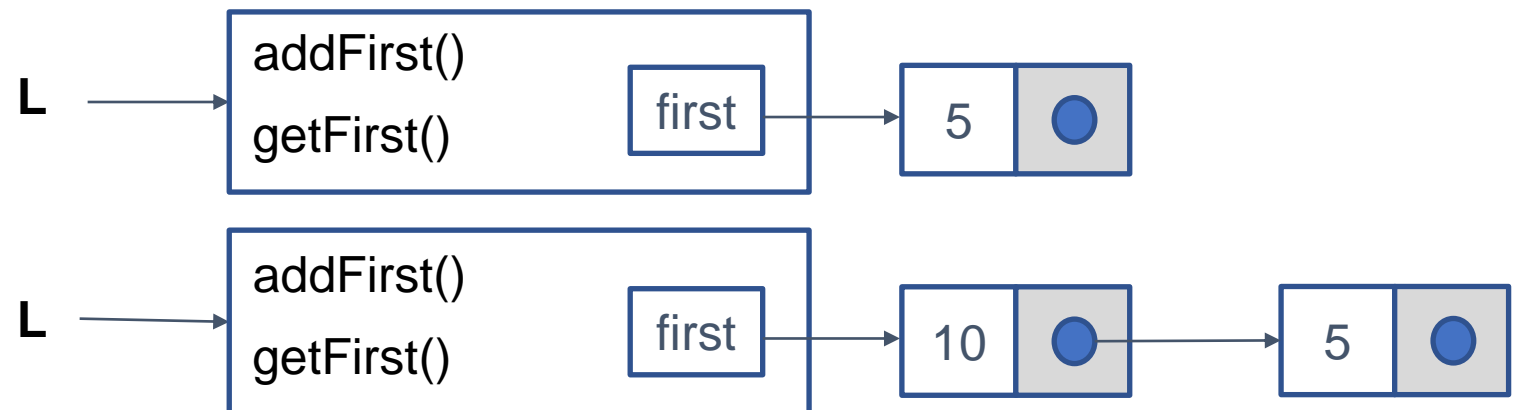
Single Linked Lists

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- An example of a SLList consisting of two basic methods and one variable
 - `L = SLList()`
 - `L.addFirst(5)`
 - `L.getFirst()`
 - 5
 - `L.addFirst(10)`



Single Linked Lists

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 - Every node can be access through the **first** node
- An example of a SLList consisting of two basic methods and one variable
 - `L = SLList()`
 - `L.addFirst(5)`
 - `L.getFirst()`
 - 5
 - `L.addFirst(10)`
 - `L.getFirst()`
 - 10



Single Linked Lists – addFirst and getFirst

- `class SLList():`
- `def __init__(self) -> None:`
- `self.first = None`
- `def addFirst(self, x: int) -> None:`
- `newFirst = ListNode(x)`
- `newFirst.next = self.first`
- `self.first = newFirst`
- `def getFirst(self) -> int:`
- `if self.first:`
- `return self.first.val`
- `return None`

addFirst()

getFirst()

first

Let's add more functionality to make our linked list useful!

Single Linked Lists – size

- `class SLList():`
- `def __init__(self) -> None:`
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`addFirst()`

`getFirst()`

`getSize()`

first

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- `return None`

`addFirst()`

`getFirst()`

`getSize()`

first

- `def getSize(self) -> int:`
- `curNode = self.first`
- `size = 0`
- `while curNode != None: #Navigate the whole list`
- `size += 1`
- `curNode = curNode.next`
- `return size`

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`addFirst()`

`getFirst()`

`getSize()`

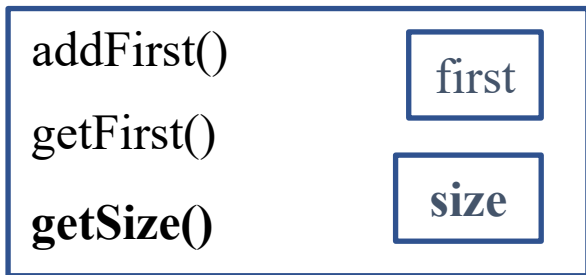
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- `curNode = self.first`
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- `while curNode != None: #Navigate the whole list`
- `size += 1`
- `curNode = curNode.next`
- `return size`

But it takes $O(N)$ time...
How to reduce the time cost?

Single Linked Lists – size and size variable

- `class SLList():`
- `def __init__(self) -> None:`
- `self.first = None`
- `def addFirst(self, x: int) -> None:`
- `newFirst = LinkedNode(x)`
- `newFirst.next = self.first`
- `self.first = newFirst`
- `def getFirst(self) -> int:`
- `if self.first:`
- `return self.first.val`
- `return None`



A special variable that caches the size information!
Then `getSize()` implementation becomes very simple

Single Linked Lists – size and size variable

- `class SLList():`
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- `if self.first:`
- `return self.first.val`
- `return None`

<code>addFirst()</code>	<code>first</code>
<code>getFirst()</code>	
<code>getSize()</code>	<code>size</code>

A special variable that caches the size information!
Then `getSize()` implementation becomes very simple

- `def getSize(self) -> int:`
- `return self.size # O(1)!`

Now we need to manage the size variable properly.
+1 operation in each add function call

Single Linked Lists – size and size variable

- `class SLList():`
- `def __init__(self) -> None:`
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- `def addFirst(self, x: int) -> None:`
- `newFirst = ListNode(x)`
- `newFirst.next = self.first`
- `self.first = newFirst`
- `self.size += 1`
- `def getFirst(self) -> int:`
- `if self.first:`
- `return self.first.val`
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<code>addFirst()</code>	<code>first</code>
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A special variable that caches the size information!
Then `getSize()` implementation becomes very simple

- `def getSize(self) -> int:`
- `return self.size # O(1)!`

Now we need to manage the size variable properly.
+1 operation in each add function call

Single Linked Lists – append

```
• class SLList():  
•     def __init__(self) -> None:  
•         self.first = None  
•         self.size = 0  
•     def addFirst(self, x: int) -> None:  
•         newFirst = ListNode(x)  
•         newFirst.next = self.first  
•         self.first = newFirst  
•         self.size += 1  
•     def getFirst(self) -> int:  
•         if self.first:  
•             return self.first.val  
•         return None  
•     def getSize(self) -> int:  
•         return self.size
```

addFirst()	append()	first
getFirst()		
getSize()		size

Now we want to **append** a new node
at the **end** of a linked list

Single Linked Lists – append

- `class SLList():`
- `def __init__(self) -> None:`
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- `def addFirst(self, x: int) -> None:`
- `newFirst = ListNode(x)`
- `newFirst.next = self.first`
- `self.first = newFirst`
- `self.size += 1`
- `def getFirst(self) -> int:`
- `if self.first:`
- `return self.first.val`
- `return None`
- `def getSize(self) -> int:`
- `return self.size`

<code>addFirst()</code>	<code>append()</code>	<code>first</code>
<code>getFirst()</code>		
<code>getSize()</code>		<code>size</code>

Now we want to **append** a new node at the **end** of a linked list

- `def append(self, x: int) -> None:`
- `self.size += 1`
- `curNode = self.first`
- `while(curNode.next != None):`
- `curNode = curNode.next`
- `curNode.next = ListNode(x)`

Single Linked Lists – append

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- `self.first = newFirst`
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- `def getFirst(self) -> int:`
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- `return self.first.val`
- `return None`
- `def getSize(self) -> int:`
- `return self.size`

<code>addFirst()</code>	<code>append()</code>	<code>first</code>
<code>getFirst()</code>		
<code>getSize()</code>		<code>size</code>

Now we want to **append** a new node at the **end** of a linked list

Is anything **wrong** with it?

What if SLList is **empty** (first = None)?

- `def append(self, x: int) -> None:`
- `self.size += 1`
- `curNode = self.first`
- `while(curNode.next != None):`
- `curNode = curNode.next`
- `curNode.next = ListNode(x)`

Linked Lists with Sentinel

Lecture 14-3

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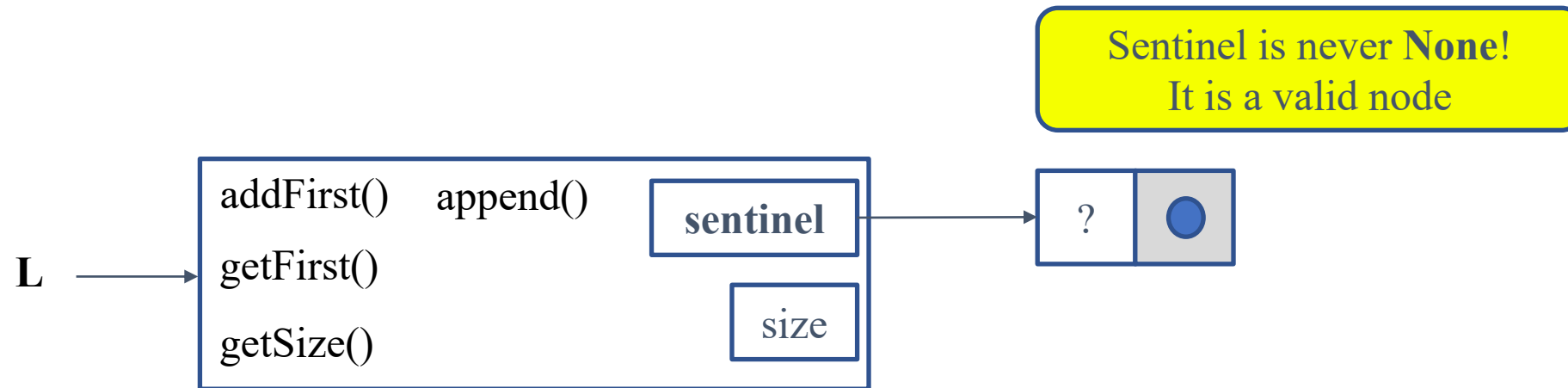
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Single Linked Lists – Sentinel Node

- We now replace **first** with **sentinel**, which is a **dummy node**

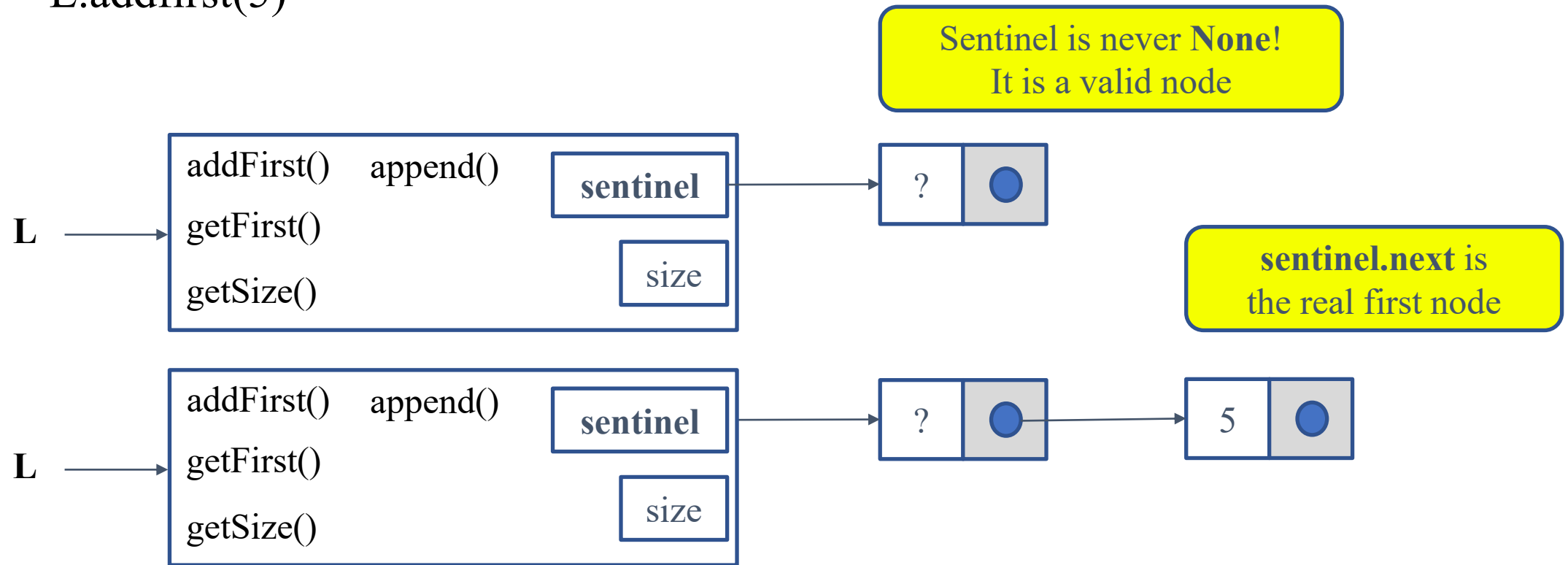
Single Linked Lists – Sentinel Node

- We now replace **first** with **sentinel**, which is a **dummy node**
 - $L = \text{SLList}()$



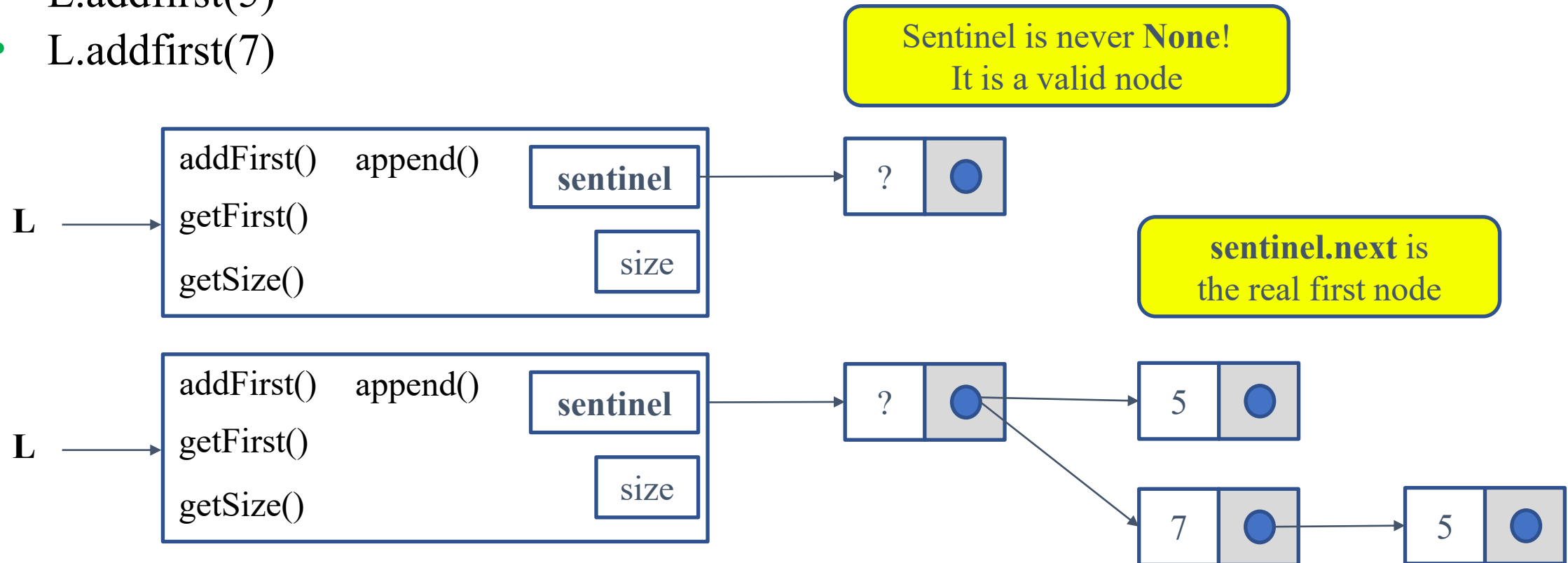
Single Linked Lists – Sentinel Node

- We now replace **first** with **sentinel**, which is a **dummy node**
 - `L = SLList()`
 - `L.addfirst(5)`



Single Linked Lists – Sentinel Node

- We now replace **first** with **sentinel**, which is a **dummy node**
 - `L = SLList()`
 - `L.addfirst(5)`
 - `L.addfirst(7)`



Single Linked Lists – Modification with Sentinel

- `class SLList():`
- `def __init__(self) -> None:`
- `self.sentinel = ListNode(0)`
- `self.size = 0`
- `def addFirst(self, x: int) -> None:`
- `newFirst = ListNode(x)`
- `newFirst.next = self.sentinel.next`
- `self.sentinel.next = newFirst`
- `self.size += 1`
- `def getFirst(self) -> int:`
- `if self.sentinel.next:`
- `return self.sentinel.next.val`
- `return None`
- `def getSize(self) -> int:`
- `return self.size`

<code>addFirst()</code>	<code>append()</code>	<code>sentinel</code>
<code>getFirst()</code>		
<code>getSize()</code>		<code>size</code>

Single Linked Lists – Append with Sentinel

- `class SLList():`
- `def __init__(self) -> None:`
- `self.sentinel = ListNode(0)`
- `self.size = 0`
- `def addFirst(self, x: int) -> None:`
- `newFirst = ListNode(x)`
- `newFirst.next = self.sentinel.next`
- `self.sentinel.next = newFirst`
- `self.size += 1`
- `def getFirst(self) -> int:`
- `if self.sentinel.next:`
- `return self.sentinel.next.val`
- `return None`
- `def getSize(self) -> int:`
- `return self.size`

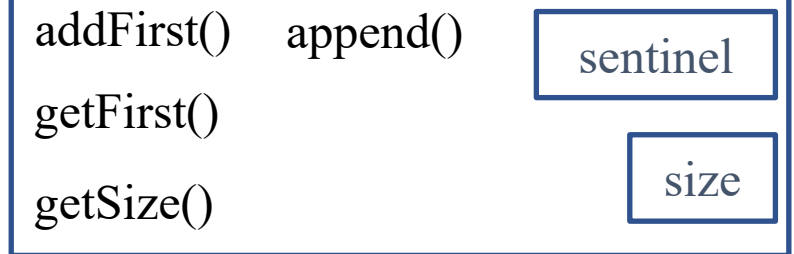
<code>addFirst()</code>	<code>append()</code>	<code>sentinel</code>
<code>getFirst()</code>		
<code>getSize()</code>		<code>size</code>

- `def append(self, x: int) -> None:`
- `self.size += 1`
- `curNode = self.sentinel`
- `while curNode.next != None:`
- `curNode = curNode.next`
- `curNode.next = ListNode(x)`

Now we don't have any special case 😊

Single Linked Lists – Summary

- class **SLList**():
- def **__init__**(self) -> None:
- self.sentinel = ListNode(0)
- self.size = 0
- def **addFirst**(self, x: int) -> None:
- newFirst = ListNode(x)
- newFirst.next = self.sentinel.next
- self.sentinel.next = newFirst
- self.size += 1
- def **getFirst**(self) -> int:
- if self.sentinel.next:
- return self.sentinel.next.val
- return None
- def **getSize**(self) -> int: *# Improved with caching!*
- return self.size
- def **append**(self, x: int) -> None: *# Improved with sentinel!*
- self.size += 1
- curNode = self.sentinel
- while curNode.next != None:
- curNode = curNode.next
- curNode.next = ListNode(x)



Looking Forward ...

- **Problem:** `append()` is still much lower than `addFirst()`
- **Solution:** Doubly linked list (DLList)
 - Add **another sentinel** at the back
 - Each node has not only **next** but also **prev** (pointing at the previous node)

Don't **panic!**
This is out of scope of this course 😊

Thanks!