AI for Healthcare, Homework 1 - CT Reconstruction

This assignment explores the basic principles of image reconstruction in computed tomography (CT), focusing on filtered back projection and iterative reconstruction. It demonstrates how to convert an image into radon space and create a sinogram using synthetic projections. The assignment uses these synthetic projections to reconstruct the original image.

Step 1: Calculate synthetic projection using Radon Transform

The Radon Transform is a mathematical operation that converts a two-dimensional function ($f(x, y)$) into a one-dimensional function with an angle parameter ($f(\rho, \theta)$). It is used in tomography to reconstruct a two-dimensional image from a set of projections. In this step, we generate synthetic projections using the Radon Transform for the Shepp-Logan head phantom, line, and circle images.

Input:

- Shepp-Logan head phantom, line, and circle images
- Projection angles

Output:

- Sinograms for the Shepp-Logan head phantom, line, and circle images

Functions used:

- **radon()**: This function calculates the Radon Transform for the input image with specified projection angles.

Step 2: Vary the number of projection angles and perform reconstruction

In this step, we reconstruct the original images from their sinograms by varying the number of projection angles. The Inverse Radon Transform is used to perform the reconstruction, which is the inverse of the Radon Transform.

Input:

- Sinograms for the Shepp-Logan head phantom, line, and circle images
- Different numbers of projection angles

Output:

- Reconstructed images for different numbers of projection angles

Functions used:

- **iradon()**: This function calculates the Inverse Radon Transform for the input sinogram with specified projection angles, reconstructing the original image.

Step 3: Demonstrate the difference between back projection and filtered back projection

In this step, we compare the results of unfiltered back projection and filtered back projection. Unfiltered back projection methods are used to reconstruct the original image without any noise reduction. Filtered back projection methods reduce the noise in the reconstructed image.

Input:

- Sinograms for the Shepp-Logan head phantom, line, and circle images
- Projection angles

Output:

- Filtered back projection reconstructed images
- Unfiltered back projection reconstructed images

Functions used:

- **iradon()**: This function calculates the Inverse Radon Transform for the input sinogram with specified projection angles, reconstructing the original image with or without filtering.

Step 4: Apply & describe an algebraic iterative reconstruction technique such as SART

The Simultaneous Algebraic Reconstruction Technique (SART) is an algebraic iterative reconstruction method used primarily in CT to reconstruct an image from a set of projection data. Unlike the back projection methods in the previous steps, SART and other algebraic iterative reconstruction techniques rely on a system of linear equations that describe the relationship between the input image and the projection data. These methods iteratively update the image estimate by minimizing the difference between the estimated projections and the actual projection data. SART, in particular, updates the image estimate pixel by pixel and is known for producing high-quality images with reduced noise and artifacts.

In this step, we apply SART to reconstruct the Shepp-Logan head phantom, line, and circle images from their sinograms.

Input:

- Sinograms for the Shepp-Logan head phantom, line, and circle images
- Projection angles
- Number of iterations

Output:

- SART reconstructed images for different numbers of iterations

Functions used:

- **iradon_sart()**: This function calculates the SART reconstruction for the input sinogram with specified projection angles and a given number of iterations, reconstructing the original image.

Summary:

This assignment demonstrated the process of CT image reconstruction using filtered back projection and iterative reconstruction techniques such as SART. We generated sinograms for the Shepp-Logan head phantom, line, and circle images using the Radon Transform and then reconstructed the original images using filtered back projection and SART methods.

The algebraic iterative reconstruction technique (SART) differs from the filtered back projection method in that it uses a system of linear equations to model the relationship between the input image and the projection data. SART iteratively updates the image estimate to minimize the difference between the estimated projections and the actual projection data. This results in high-quality images with reduced noise and artifacts compared to filtered back projection methods.