Assignment no. 2 due: Jan. 3rd, 2022

You may work on and submit this assignment **in pairs**.

2.1. Describe the gift wrapping algorithm in 3D space shown in class in detail. In particular, break down the functions exist(e), processed(e), and mark_processed(e) into operations applied to the current stack Q and convex hull H, where the latter is represented by a half-edge data structure, so that the overall time complexity remains $\Theta(n^2)$ in the worst case.

2.2. A graph is simple if it has no loops or parallel edges.

1. Prove that every planar map has either a vertex with degree at most 3 or a face with degree at most 3.
2. Prove that every simple planar graph has a vertex with degree less than 6.

2.3. Given a simple polygon with n vertices in the plane and a positive rational number $r = w^2$, describe an efficient algorithm that computes the orientation of the polygon that minimizes its height, such that its width does not exceed w; see the figure below for examples. Imagine that the polygon is about to be printed on a "two-dimensional" 3D printer with a base of width w.

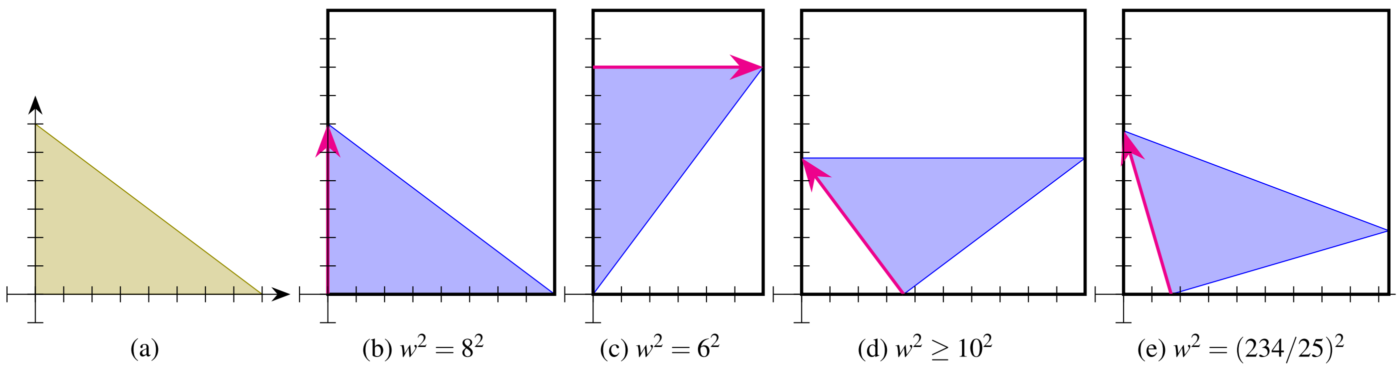

Figure 1: (a) A polygon. (b) direction $(0,1)$. (c) direction $(1,0)$. (d) direction $(-9,12)$. (e) direction $(-7,24)$.

Implement the algorithm. The resulting orientation should be represented by the (not necessarily normalized) direction of the up vector (0,1) rotated together with the polygon so that it ends up at a minimal-height position. You may use inexact arithmetic.

(optional, bonus) Implement the algorithm in a robust manner (following the EGC paradigm).

I suggest that you produce graphic results as well as alphanumerical, using *ipe* for instance.

2.4. In each of the following settings, describe a construction (a polygon and guard placements) where the specified vertex guards do not fully cover the polygon in the art gallery sense, and such that your construction could be generalized to any number of vertices, as specified in the setting:

1. A simple polygon with $2k$ vertices, for every $k > 2$, and a specific assignment of guards placed at every other vertex along the boundary of the polygon. Namely, guards placed at the vertices $v_i, v_{i+2}, v_{i+4}, \ldots$, do not fully cover the polygon.
2. Similarly, a simple polygon with $3k$ vertices, for every $k > 2$, and a specific assignment of guards placed at every third vertex along the boundary of the polygon.
3. A simple polygon with n vertices, for every $n > 5$, and guards placed only at *convex vertices*. A vertex is convex if its interior angle is less than π.