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3.2

One way to construct the union of the rectangles in O(nlogn) time is to first sort the rectangles by their x-coordinates (or y-coordinates if the rectangles have vertical sides). Next, we can use a data structure such as a balanced binary tree (e.g. a red-black tree) to keep track of the height of each rectangle at any given x-coordinate. As we iterate through the sorted rectangles, we can use the tree to efficiently find the maximum height at the start and end coordinates of each rectangle and update the tree with the new height of the rectangle. This allows us to keep track of the union of all rectangles in a dynamic and efficient manner. Additionally, we can use the same tree to find the maximum height of the union at any given x-coordinate in O(logn) time.

3.5

Upper Bound: The complexity of the union of a set of polygonal pseudodisks with n vertices in total can be upper bounded by 2n-m, where m is the number of original vertices in the union. This is because the union of two pseudodisks with m and n vertices respectively will have at most m+n-1 vertices in the union. This is because when we take the union of the two pseudodisks, we will eliminate the overlapping vertices and keep the non-overlapping ones, resulting in a total of m+n-1 vertices.

By extending this reasoning to more than two pseudodisks, we can see that if we have k pseudodisks with m1, m2, ..., mk vertices respectively, the union of these k pseudodisks will have at most m1+m2+...+mk-k+1 vertices.

As we know that the total number of vertices in all the k pseudodisks is n, we can write the above equation as: m1+m2+...+mk-k+1 <= n

Therefore, the complexity of the union of k pseudodisks is at most 2n-m, where m is the number of original vertices in the union.

Lower Bound: To prove a lower bound of 2n-6, we can construct an example that has this complexity.

Consider two pseudodisks with 3 vertices each. The first pseudodisk can be represented by the points (1,1), (1,2), (2,2) and the second pseudodisk can be represented by the points (2,1), (3,1), (3,2).

The union of these two pseudodisks will have the points (1,1), (1,2), (2,2), (2,1), (3,1), (3,2) resulting in a total of 6 vertices.

As the total number of vertices in both the pseudodisks is 6, we can see that the complexity of the union is 2n-6 which proves that the lower bound of 2n-6 can be achieved.

3.6

Proof:

First, let's consider the case where $P \oplus Q = (P \cup H) \oplus Q$. This means that $Q \oplus \{t\} \subseteq -H$ for all $t \in$ IR2.

To prove this, we will show that $P \cap (Q \oplus \{t\}) = /0$ for all $t \in P \oplus -Q$.

Assume that $P \cap (Q \oplus \{t\}) = /0$ for all $t \in P \oplus -Q$. Then, for any point $t \in P \oplus -Q$, we have that $Q \oplus \{t\} \subseteq -H$.

This is because if $t \in P \oplus -Q$, then $t \in P$ and $t \notin -Q$. Therefore, $t \notin Q \oplus \{t\}$, which means that $Q \oplus \{t\} \subseteq -H$.

Conversely, let's assume that $Q \oplus \{t\} \subseteq -H$ for all $t \in$ IR2. We will show that $P \cap (Q \oplus \{t\}) = /0$ for all $t \in P \oplus -Q$.

Assume that $t \in P \oplus -Q$. Then, $t \in P$ and $t \notin -Q$. Since $Q \oplus \{t\} \subseteq -H$, we have that $t \notin Q \oplus \{t\}$. Therefore, $P \cap (Q \oplus \{t\}) = /0$.

Therefore, we have proved that $P \oplus Q = (P \cup H) \oplus Q$ iff $\exists t \in$ IR2, such that $Q \oplus \{t\} \subseteq -H$.

Now, let's consider the case where $P \oplus Q \neq (P \cup H) \oplus Q$. This means that there exists a $t \in$ IR2 such that $Q \oplus \{t\} \nsubseteq -H$.

To prove this, we will show that $P \cap (Q \oplus \{t\}) \neq /0$ for some $t \in P \oplus -Q$.

Assume that there exists a $t \in IR2$ such that $Q\bigoplus\{t\} \nsubseteq -H$. Then, there exists a point $x \in Q\bigoplus \{t\}$ such that $x \notin -H$.

Since $t \in IR2$, it follows that $t \in P\bigoplus -Q$. Therefore, $P\cap(Q\bigoplus \{t\}) = /0$.

Therefore, we have proved that $P\bigoplus Q \neq (P\cup H)\bigoplus Q$ iff $\exists t \in IR2$, such that $Q\bigoplus\{t\} \nsubseteq -H$.

Thus, we have proved the theorem that $P\bigoplus Q$ 6= $(P\cup H)\bigoplus Q$ iff $\exists t \in IR2$, such that $Q\bigoplus\{t\} \subseteq -H$.