Dynamics and Control presentation

Group 27

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1 Maths for 1D Motion

PID Controller

$$
\begin{aligned}
k_c &= m_{eq}\omega_c^2\sqrt{\alpha}, \qquad & \tau_z &= \frac{\sqrt{\alpha}}{\omega_c} \\
\tau_i &= \beta\tau_z, & \tau_p &= \alpha\tau_z
\end{aligned} \tag{1}
$$

A skew-sine reference profile has a maximum jerk at:

$$
\dddot{r}_{max} = \frac{4\pi^2 h_m}{t_m^3} \tag{2}
$$

$$
\begin{aligned}
\omega_c &= \left(\frac{\dddot{r}_{max}\beta}{\alpha e_{LF}}\right)^{\frac{1}{3}} && (3) \\
&= \left(\frac{4\pi^2 h_m\beta}{\alpha e_{LF}}\right)^{\frac{1}{3}} \frac{1}{t_m} && (4)
\end{aligned}
$$

Our values.
Based off: `https://new.abb.com/products/robotics/industrial-robots/irb-360`. See IRB 360-1/1600 model.
Cycle from 0 mm$\rightarrow$ d_m $\rightarrow$ 0 mm

$$
\begin{aligned}
m_{eq} &= 1\ kg \\
e_{max} &= 0.1\ mm \\
d_m &= 280\ mm \\
h_m &= arctan\left(\frac{d_m}{L_u + L_l}\right) \\
&= 0.1391\ rad\ = 8^\circ \\
t_m &= 0.35\ s
\end{aligned}
$$

2 Maths for 2D motion

Position

$$\Phi = \begin{bmatrix} M_x \mp \frac{D}{2} - U_{j,x} - \frac{L_u}{2}cos(\theta_{U,j}) \\ M_y - U_{j,y} - \frac{L_u}{2}sin(\theta_{U,j}) \\ U_{j,x} - \frac{L_u}{2}cos(\theta_{U,j}) - L_{j,x} - \frac{L_l}{2}cos(\theta_{L,j}) \\ U_{j,y} - \frac{L_u}{2}sin(\theta_{U,j}) - L_{j,x} - \frac{L_l}{2}sin(\theta_{L,j}) \\ L_{j,x} - \frac{L_l}{2}cos(\theta_{L,j}) - P_x \pm \frac{L_e}{2} \\ L_{j,y} - \frac{L_l}{2}sin(\theta_{L,j}) - P_y \\ \vdots \\ \vdots \end{bmatrix} \tag{5}$$

$$\Phi^{driving} = \begin{bmatrix} \theta_{U,1} - \omega_1 t - (\theta_{U,1})_0 \\ \theta_{U,2} - \omega_2 t - (\theta_{U,2})_0 \end{bmatrix} \tag{6}$$

Independent and Dependent Co-ordinates

$$q_i = \begin{bmatrix} \theta_{U,1} \\ \theta_{U,2} \end{bmatrix} \tag{7}$$

$$q_d = \begin{bmatrix} \vdots \end{bmatrix} \tag{8}$$

$$J = \Phi_{q_d}^{-1}\Phi_{q_i} \tag{9}$$

$$\dot{q}_d = -J\dot{q}_i \tag{10}$$

$$\hat{M} = M_{ii} + J^T M_{dd} J \tag{11}$$

$$\hat{Q} = -J^T Q_{A,d} + J^T M_{dd}\Phi_{q_d}^{-1}\gamma \tag{12}$$

Inverse Kinematics

$$\Phi^{IK} = \begin{bmatrix} (\mp\frac{D}{2} - L_u cos(\theta_{U,j}) \pm \frac{L_e}{2} - P_x)^2 + (P_y + L_u sin(\theta_{U,j}))^2 - L_l^2 \\ \vdots \end{bmatrix} \tag{13}$$

$$\ddot{q}_i = -(\Phi_{q_i}^{IK})^{-1}(\Phi_{PP}^{IK}\dot{P} + 2\Phi_{q_i P}^{IK}\dot{q}_i\dot{P} + \Phi_{q_i q_i}^{IK}\dot{q}_i + \Phi_{P}^{IK}\ddot{P}) \tag{14}$$

3 Maths for 3D motion

Position

$$\Phi = \begin{bmatrix} M + R_{A,j}\begin{bmatrix}\frac{D}{2}\\0\\0\end{bmatrix} - U_j - R_{U,j}\begin{bmatrix}0\\0\\\frac{L_u}{2}\end{bmatrix} \\ U_j + R_{U,j}\begin{bmatrix}0\\0\\-\frac{L_u}{2}\end{bmatrix} - L_j - R_{L,j}\begin{bmatrix}0\\0\\\frac{L_l}{2}\end{bmatrix} \\ L_j + R_{L,j}\begin{bmatrix}0\\0\\\frac{-L_l}{2}\end{bmatrix} - P - R_{A,j}\begin{bmatrix}\frac{L_e}{2}\\0\\0\end{bmatrix} \\ \begin{bmatrix}0 & 1 & 0\end{bmatrix} R_{L,j}^T (R_{L,j})_0 \begin{bmatrix}1\\0\\0\end{bmatrix} \\ \vdots \end{bmatrix} \tag{15}$$

$$\Phi^{driving} = \begin{bmatrix} (\theta_{U,j})_x + sin((\theta_{a,j})_0)\omega_j t - 0 \\ (\theta_{U,j})_y - cos((\theta_{a,j})_0)\omega_j t - (\theta_{a,j})_0 \\ (\theta_{U,j})_z - (\theta_{U,j})_{z,0} \\ \vdots \end{bmatrix} \tag{16}$$

$$\Phi_q \dot{q} = \begin{bmatrix} I & -I & -R_{U,j}\tilde{r}_U^{U,A}R_{U,j}^T & 0 & 0 & ... \\ 0 & I & +R_{U,j}\tilde{r}_U^{U,B}R_{U,j}^T & -I & -R_{L,j}\tilde{r}_L^{L,B}R_{L,j}^T & ... \\ 0 & 0 & 0 & I & +R_{L,j}\tilde{r}_L^{L,C}R_{L,j}^T & ... \\ 0 & 0 & 0 & 0 & e_y^0 R_{L,j}^T \tilde{e}_x^U & ... \\ & & & \vdots & & \end{bmatrix} \begin{bmatrix} \dot{M} \\ \dot{U}_j \\ \omega_{U,j} \\ \dot{L}_J \\ \omega_{L,j} \\ \vdots \end{bmatrix} \tag{17}$$

Independent and Dependent Co-ordinates

$$q_i = \begin{bmatrix} (\theta_{U,1})_x \\ (\theta_{U,1})_y \\ (\theta_{U,1})_z \\ \vdots \end{bmatrix} \tag{18}$$

$$q_a = \begin{bmatrix} \theta_{a,1} \\ \theta_{a,2} \\ \theta_{a,3} \end{bmatrix} \tag{19}$$

$$
\begin{aligned}
\omega_a^{0,U} &= R_U^0 \omega_a^{U,U} \\
&= \begin{bmatrix} cos(\theta_a)cos(\psi_0) & -sin(\psi_0) & cos(\psi_0)sin(\theta_a) \\ cos(\theta_a)sin(\psi_0) & cos(\psi_0) & sin(\psi_0)sin(\theta_a) \\ -sin(\theta_a) & 0 & cos(\theta_a) \end{bmatrix} \begin{bmatrix} 0 \\ \omega_a \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} -sin(\psi_0)\omega_a \\ cos(\psi_0)\omega_a \\ 0 \end{bmatrix}
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
\int \omega_a^{0,U} &= \int R_U^0 \omega_a^{U,U} \\
\theta_U &= \begin{bmatrix} -sin(\psi_0)\theta_a - \theta_{a,0} \\ cos(\psi_0)\theta_a \\ \psi_0 \end{bmatrix}
\end{aligned} \tag{21}
$$

Inverse kinematics

$$
\begin{aligned}
r = R_{A,j} \begin{bmatrix} \frac{D-L_e}{2} \\ 0 \\ 0 \end{bmatrix} - R_{U,j} \begin{bmatrix} 0 \\ 0 \\ L_u \end{bmatrix} - P \quad &= R_{L,j} \begin{bmatrix} 0 \\ 0 \\ L_l \end{bmatrix} \\
r^T r = r^T R^T R r &
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
&\left(\left(\frac{D-L_e}{2} - L_u sin(\theta_{a,j}) \right) cos(\psi_0) - P_x \right)^2 + \\
&\left(\left(\frac{D-L_e}{2} - L_u sin(\theta_{a,j}) \right) sin(\psi_0) - P_y \right)^2 + \\
&\qquad\qquad\qquad (-L_u cos(\theta_{a,j}) - P_z)^2 = L_l^2
\end{aligned} \tag{23}
$$

$$
\ddot{q}_a = -(\Phi_{q_a}^{IK})^{-1}(\Phi_{PP}^{IK}\dot{P} + 2\Phi_{q_aP}^{IK}\dot{q}_a\dot{P} + \Phi_{q_aq_a}^{IK}\dot{q}_a + \Phi_P^{IK}\ddot{P}) \tag{24}
$$