Ejercicios de Funciones Indeterminadas Aplicando la regla de l'Hopital

a) $\lim_{x \to 1} \frac{\ln(2x^2-1)}{\tan(x-1)}$

$$\lim_{x \to 1} \left(\frac{\frac{d}{dx}\left(\ln(2x^2-1)\right)}{\frac{d}{dx}\left(\tan(x-1)\right)} \right)$$

$$\lim_{x \to 1} \left(\frac{\frac{4x}{2x^2-1}}{\frac{d}{dx}\left(\tan(x+1)\right)} \right)$$

$$\lim_{x \to 1} \left(\frac{\frac{4x}{2x^2-1}}{\sec(x-1)^2} \right)$$

$$\lim_{x \to 1} \left(\frac{4x}{(2x^2-1)\sec(x-1)^2} \right)$$

$$\lim \frac{4(1)}{2(1)^2-1\sec(1-1)^2} \quad R// 4$$

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b) $\lim_{x \to 1} \frac{x - sen(x)}{x^3}$

$$\lim_{x \to 0} \left(\frac{\frac{d}{dx}\left(x - sen(x)\right)}{\frac{d}{dx}(x^3)} \right)$$

$$\lim_{x \to 0} \left(\frac{1-\cos(x)}{\frac{d}{dx}(x^3)} \right) = \lim_{x \to 0} \frac{1-\cos(x)}{3x^2}$$

$$\lim_{x \to 0} \left(\frac{\frac{d}{dx}(1-\cos(x))}{\frac{d}{dx}(3x^2)} \right) = \left(\frac{sin(x)}{\frac{d}{dx}(3x^2)} \right)$$

$$\lim_{x\to 0}\left(\frac{\sin(x)}{6x}\right)=\left(\frac{\frac{d}{dx}(\sin(x))}{\frac{d}{dx}(6x)}\right)$$

$$\lim_{x\to 0}\left(\frac{\cos(x)}{\frac{d}{dx}(6x)}\right)=\lim_{x\to 0}\left(\frac{\cos(x)}{6}\right)$$

$$\lim_{x\to 0}\frac{\cos 0}{6}=\frac{1}{6}\quad R//$$

c) $$\lim_{x\to 1}\frac{1-\cos(x-1)}{(\ln(x))^2}$$

$$\lim_{x\to 1}\left(\frac{\frac{d}{dx}(1-\cos(x-1))}{\frac{d}{dx}(\ln(x)^2)}\right)=\lim_{x\to 1}\left(\frac{\sin(x-1)}{\frac{d}{dx}(\ln(x)^2)}\right)$$

$$\lim_{x\to 1}\left(\frac{\sin(x-1)}{\frac{2\ln(x)}{x}}\right)=\lim_{x\to 1}\left(\frac{\sin(x-1)x}{2\ln(x)}\right)$$

$$\lim_{x\to 1}\left(\frac{\frac{d}{dx}(\sin(x-1)x)}{\frac{d}{dx}(2\ln(x))}\right)$$

$$\lim_{x\to 1}\left(\frac{\cos(x-1)x+\sin(x-1)}{\frac{d}{dx}(2\ln(x))}\right)=\frac{\cos(x-1)x+\sin(x-1)}{\frac{2}{x}}$$

$$\lim_{x\to 1}\left(\frac{(\cos(x-1)x+\sin(x-1))x}{2}\right)=\left(\frac{\cos(x-1)x^2+\sin(x-1)x}{2}\right)$$

$$\lim_{x\to 1}\frac{\cos(1-1)(1)^2+\sin(1-1)1}{2}=\frac{1}{2}\quad R//$$

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d: $\lim_{x \to 0} \frac{\cos^2(x) - 1}{x^2}$

$\lim_{x \to 0} \left(\frac{\frac{d}{dx}(\cos(x)^2 - 1)}{\frac{d}{dx}(x^2)} \right)$

$\lim_{x \to 0} \left(\frac{\sin(2x)}{\frac{d}{dx}(x^2)} \right)$

$\lim_{x \to 0} \left(\frac{-\sin(2x)}{2x} \right)$

$\lim_{x \to 0} \left(\frac{-\sin(\cancel{2x})}{\cancel{2x}} \right) = 1$ R//

e: $\lim_{x \to \infty} \frac{3x^2 - 5x}{2x^2 + 1}$

$\lim_{x \to \infty} \left(\frac{\frac{d}{dx}(3x^2 - 5x)}{\frac{d}{dx}(2x^2) + 1} \right) = \left(\frac{6x - 5}{\frac{d}{dx}(2x^2 + 1)} \right)$

$\lim_{x \to \infty} \left(\frac{6x - 5}{4x} \right) = \left(\frac{\frac{d}{dx}(6x - 5)}{\frac{d}{dx}(4x)} \right)$

$\lim_{x \to \infty} \left(\frac{6}{\frac{d}{dx}(4x)} \right) = \left(\frac{6}{4} \right) = \frac{3}{2}$ R//

F: $\lim_{x \to 0} \frac{\sin x}{x}$

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$\lim_{x \to 0} \left(\frac{\frac{d}{dx}(\sin(x))}{\frac{d}{dx}(x)} \right) = \left(\frac{\cos(x)}{\frac{d}{dx}(x)} \right)$

$\lim_{x \to 0} \left(\frac{\cos(x)}{1} \right) = \lim_{x \to 0} (\cos(x))$

$\lim_{x \to 0} \cos(0) = 1$ R//

g: $\lim_{x \to 0} \frac{\text{sen}(5x)}{\text{sen}(2x)}$

$\lim_{x \to 0} \left(\frac{\frac{d}{dx}(\text{sen}(5x))}{\frac{d}{dx}(\sin(2x))} \right) = \left(\frac{5\cos(5x)}{\frac{d}{dx}(\sin(2x))} \right)$

$\lim_{x \to 0} \left(\frac{5\cos(5x)}{2\cos(2x)} \right) = \frac{5\cos(5 \times 0)}{2\cos(2 \times 0}$

$\lim_{x \to 0} \frac{5}{2}$ R//.

h: $\lim_{x \to 2} \frac{\text{Tang}^2(x-2)}{x^2-4}$

$\lim_{x \to 2} \left(\frac{\frac{d}{dx}(\text{Tang}^2(x-2))}{\frac{d}{dx}(x^2-4)} \right) = \left(\frac{\frac{2\,\text{sen}(x-2)}{\cos(x-2)^3}}{\frac{d}{dx}(x^2-4)} \right)$

$\lim_{x \to 2} \left(\frac{\frac{2\sin(x-2)}{\cos(x-2)^3}}{2x} \right) = \left(\frac{\sin(x-2)}{\cos(x-2)^3\, x} \right)$

$\lim_{x \to 2} \frac{\sin(2-2)}{\cos(2-2)^3\, 2} = 0$ R//

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I: $\lim_{x \to 0} \frac{1\cos(7x)}{4x^2}$

$\lim_{x \to 0} \left(\frac{\frac{d}{dx}(1-\cos(7x))}{\frac{d}{dx}(4x^2)} \right) = \left(\frac{7\sin(7x)}{\frac{d}{dx}(4x^2)} \right)$

$\lim_{x \to 0} \left(\frac{7\sin(7x)}{8x} \right) = \left(\frac{\frac{d}{dx}(7\sin(7x))}{\frac{d}{dx}(8x)} \right)$

$\lim_{x \to 0} \left(\frac{49\cos(7x)}{\frac{d}{dx}(8x)} \right) = \lim_{x \to 0} \frac{49\cos(7 \times 0)}{8} = \frac{49}{8}$ R//

$$J: \lim_{x \to 0} \frac{e^{x} - e^{x}}{\sin x}$$

$$\lim_{x \to 0} \left(\frac{\frac{d}{dx} (e^{x} - e^{x})}{\frac{d}{dx} (\sin(x))} \right) = \left(\frac{\frac{e^{2x}+1}{e^{x}}}{\frac{d}{dx} (\sin(x))} \right)$$

$$\lim_{x \to 0} \left(\frac{\frac{e^{2x}+1}{e^{x}}}{\cos(x)} \right) = \left(\frac{e^{2x}+1}{(e^{x}) \cos(x)} \right)$$

$$\lim_{x \to 0} \frac{e^{2x0}+1}{e^{0} \times \cos(0)} = 2 \text{ R//}$$

$$K: \lim_{n \to 2} \frac{\sqrt{x^2+5} - 3}{x^2 - 2x}$$

$$\lim_{n \to 2} \frac{\sqrt{x^2+5} - 3}{x^2 - 2x}$$

El limite de una constante es igual a la consta nte.

$$L: \lim_{x \to 0} \frac{1 - \sqrt{1-x^2}}{x^2}$$

$$\lim_{x \to 0} \left(\frac{\frac{d}{dx} (1 - \sqrt{1-x^2})}{\frac{d}{dx} (x^2)} \right) = \left(\frac{\frac{x}{\sqrt{1-x^2}}}{\frac{d}{dx} (x^2)} \right) = \left(\frac{\frac{x}{\sqrt{1-x^2}}}{2x} \right)$$

$$\lim_{x \to 0} \left(\frac{1}{2\sqrt{1-x^2}} \right) = \frac{1}{2\sqrt{1-0^2}} = \frac{1}{2} \text{ R//}$$

$$M: \lim_{x \to 3} \left(\frac{1}{x-3} - \frac{5}{x^2-x-6} \right)$$

$$\lim_{x \to 3} \left(\frac{1}{x-3} - \frac{5}{x^2-x-6} \right) = \frac{1}{3}$$

$$\lim_{x \to 3} \left(\frac{1}{x-3} - \frac{5}{x^2-x-6} \right) = \frac{1}{5}$$

Los limites son iguales $\frac{1}{5}$ R//

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