Definition of the tuning vector θ

The NMPC configuration is parametrized by the column vector

$$\theta^\top = \left[f,\ \theta_p,\ \theta_m,\ q,\ r^{(u)},\ r^{(\Delta u)} \right].$$

The components satisfy:

$$f \in (0,1),$$
$$\theta_m \in \mathbb{Z}_{\geq 0}, \qquad 0 \leq \theta_m \leq 30,$$
$$\theta_p \in \mathbb{Z}_{\geq 0}, \qquad 0 \leq \theta_p \leq 60,$$
$$q \in [-3,3]^3,$$
$$r^{(u)} \in [-3,3]^3,$$
$$r^{(\Delta u)} \in [-3,3]^3.$$

The weighting matrices are given by

$$Q = \operatorname{diag}(10^q), \quad R_u = \operatorname{diag}(10^{r^{(u)}}), \quad R_{\Delta u} = \operatorname{diag}(10^{r^{(\Delta u)}}).$$

The horizons are

$$m = \theta_m + 1,$$
$$p = \theta_p + m.$$

Surrogate for time

Implementation

```
1    function t_hat = time_model(x, m, p, alfa
         , beta, a)
2    k = 20;
3    softplus = @(z) log1p(exp(k*z)) / k;
4
5    y = a(1) + a(2)*x;
6    t_base = y + softplus(-y);
7
8    scale = (m/10).^alfa .* (p/30).^beta;
9    t_hat = scale .* t_base;
10   end
```

Mathematical form

Let the scalar input be normalised to the interval $[-1,1]$ as

$$x = 2f - 1.$$

The shape term is modelled as an affine function of x,

$$y(x) = a_0 + a_1 x.$$

Define the smooth rectifier

$$\text{softplus}_k(z) = \frac{1}{k} \log\left(1 + e^{kz}\right), \quad k > 0.$$

The base time surrogate is

$$t_{\text{base}}(x) = y(x) + \text{softplus}_{k=20}(-y(x)),$$

which enforces non-negativity while remaining continuously differentiable.

The full surrogate for the computation time **in hours** is

$$\hat{t}(x, m, p) = \left(\frac{m}{10}\right)^{\alpha} \left(\frac{p}{30}\right)^{\beta} t_{\text{base}}(x).$$

Fitting is performed in hours to avoid numerical scaling issues during optimisation.

Fitted parameters

Table 1: Estimated parameters of the surrogate time model.

Category	Parameter	Value
Scaling exponents	α	0.33233435
	β	2.3029582
Affine shape coefficients	a_0	8.843454×10^{-1}
	a_1	1.079230
Softplus parameter	k	20
Fit quality	R^2	0.999417