Definition of the tuning vector θ

The NMPC configuration is parametrized by the column vector

$$\theta^\top = \left[f,\ \theta_p,\ \theta_m,\ q,\ r^{(u)},\ r^{(\Delta u)}\right].$$

The components satisfy:

$$\begin{aligned}
f &\in (0,1), \\
\theta_m &\in \mathbb{Z}_{\geq 0}, \qquad 0 \leq \theta_m \leq 30, \\
\theta_p &\in \mathbb{Z}_{\geq 0}, \qquad 0 \leq \theta_p \leq 60, \\
q &\in [-3,3]^3, \\
r^{(u)} &\in [-3,3]^3, \\
r^{(\Delta u)} &\in [-3,3]^3.
\end{aligned}$$

The weighting matrices are given by

$$Q = \mathrm{diag}(10^{q}), \quad R_u = \mathrm{diag}(10^{r^{(u)}}), \quad R_{\Delta u} = \mathrm{diag}(10^{r^{(\Delta u)}}).$$

The horizons are

$$\begin{aligned}
m &= \theta_m + 1, \\
p &= \theta_p + m.
\end{aligned}$$

Surrogate for time

Implementation

```
function t_hat = time_model(x, m, p, alfa, beta, cT)
k = 20;
softplus = @(z) log1p(exp(k*z)) / k;

y = Cheb3(x, cT);
t_base = y + softplus(-y);

scale = (m/10).^alfa .* (p/30).^beta;
t_hat = scale .* t_base;
end

function y = Cheb3(x, c)
c = c(:);
if numel(c) ~= 4
error('Cheb3 expects 4 coefficients (c0..c3).');
end
```

```
            T0 = ones(size(x));
            T1 = x;
            T2 = 2*x.^2 - 1;
            T3 = 4*x.^3 - 3*x;

            y = c(1)*T0 + c(2)*T1 + c(3)*T2 + c(4)*T3;
            end
```

Mathematical form

Let the scalar input be normalised to the Chebyshev domain as

$$x = 2f - 1 \in [-1, 1].$$

The third-order Chebyshev polynomials of the first kind are

$$\begin{aligned} T_0(x) &= 1, \\ T_1(x) &= x, \\ T_2(x) &= 2x^2 - 1, \\ T_3(x) &= 4x^3 - 3x. \end{aligned}$$

The Chebyshev surrogate is then

$$y(x) = c_0 T_0(x) + c_1 T_1(x) + c_2 T_2(x) + c_3 T_3(x).$$

Define the smooth rectifier

$$\mathrm{softplus}_k(z) = \frac{1}{k} \log\Big(1 + e^{kz}\Big), \quad k > 0.$$

The base time surrogate is

$$t_{\mathrm{base}}(x) = y + \mathrm{softplus}_{k=20}(-y),$$

which enforces non-negativity while remaining differentiable.

The full surrogate for the time **in hours** is then

$$\hat{t}(x, m, p) = \left(\frac{m}{10}\right)^{\alpha} \left(\frac{p}{30}\right)^{\beta} t_{\mathrm{base}}(x).$$

It is **extremely important** to fit using hours to prevent **numerical blowup**.

Fitted parameters

Table 1: Estimated parameters of the surrogate time model.

Category	Parameter	Value
Scaling exponents	α	1.80346
	β	-0.136052
Chebyshev coefficients	c_0	1.003973
	c_1	1.187784
	c_2	1.389282×10^{-1}
	c_3	9.068274×10^{-3}
Softplus parameter	k	20