

1. For each given regular expression P , construct a DFA (Deterministic Finite Automaton), and find the number of accepted word of length at most 5, i.e. the size of the set $\mathcal{L}' = \{w \in \mathcal{L}(P) \mid |w| \leq 5\}$. For “any” ($.$) and “negative” ($[\wedge .]$) matches, assume that the alphabet is $\Sigma = \{a, b, c, d\}$.

- (a) $P_1 = ab^*$ (c) $P_3 = [\wedge cd]^+ c\{3\}$ (e) $P_5 = d(a|bc)^*$
(b) $P_2 = a+b?c$ (d) $P_4 = [\wedge a](. | ddd)?$ (f) $P_6 = ((a|ab)[cd])\{2\}$

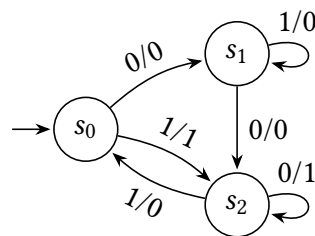
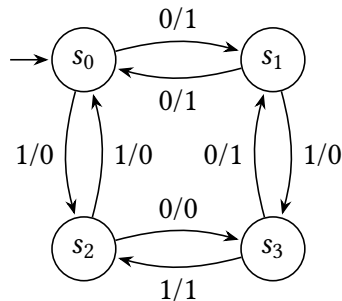
2. Describe the set of strings defined by each of these sets of productions in EBNF (extended Backus-Naur form).

- (a) $\langle string \rangle ::= \langle L \rangle + \langle D \rangle ? \langle L \rangle +$
 $\langle L \rangle ::= a \mid b \mid c$
 $\langle D \rangle ::= 0 \mid 1$
(b) $\langle string \rangle ::= \langle sign \rangle ? \langle N \rangle$
 $\langle sign \rangle ::= '+' \mid '-'$
 $\langle N \rangle ::= \langle D \rangle (\langle D \rangle \mid 0)^*$
 $\langle D \rangle ::= 1 \mid \dots \mid 9$
(c) $\langle string \rangle ::= \langle L \rangle^* (\langle D \rangle +)^? \langle L \rangle^*$
 $\langle L \rangle ::= x \mid y$
 $\langle D \rangle ::= 0 \mid 1$
(d) $\langle string \rangle ::= \langle C \rangle \langle R \rangle^*$
 $\langle C \rangle ::= a \mid \dots \mid z \mid A \mid \dots \mid Z$
 $\langle D \rangle ::= 0 \mid \dots \mid 9$
 $\langle R \rangle ::= \langle C \rangle \mid \langle D \rangle \mid '-'$

3. Let $\mathcal{G} = \langle V, T, S, P \rangle$ be the phrase-structure grammar with vocabulary $V = \{A, S\}$, terminal symbols $T = \{0, 1\}$, start symbol $S = S$, and set of productions $P: S \rightarrow 1S, S \rightarrow 00A, A \rightarrow 0A, A \rightarrow 0$.

- (a) Show that 111000 belongs to the language generated by \mathcal{G} .
(b) Show that 11001 does not belong to the language generated by \mathcal{G} .
(c) What is the language generated by \mathcal{G} ?

4. Find the output generated from the input string 01110 for each of the following Mealy machines.



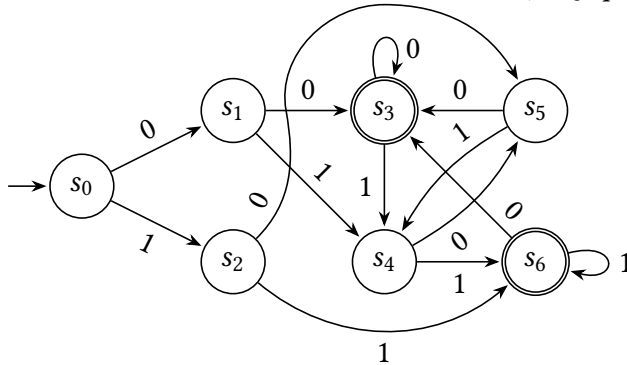
5. Construct a Moore machine for each of the following descriptions.

- (a) Determine the residue modulo 3 of the input treated as a binary number. For example, for input ε (which corresponds to “value” 0) the residue is 0; 101 (5 in decimal) has residue 2; and 1010 (value 10) has residue 1.
(b) Output the residue modulo 5 of the input from $\{0, 1, 2\}^*$ treated as a ternary (base 3) number.
(c) Output A if the binary input ends with 101; output B if it ends with 110; otherwise output C .

6. Show that regular languages are *closed* under the following operations.

- (a) Union, that is, if L_1 and L_2 are regular languages, then $L_1 \cup L_2$ is also regular.
(b) Concatenation, that is, if L_1 and L_2 are regular languages, then $L_1 \cdot L_2$ is also regular.
(c) Kleene star, that is, if L is a regular language, then L^* is also regular.
(d) Complement, that is, if L is a regular language, then $\bar{L} = \Sigma^* - L$ is also regular.
(e) Intersection, that is, if L_1 and L_2 are regular languages, then $L_1 \cap L_2$ is also regular.

7. Determine whether the following languages are regular or not. For non-regular languages, use Pumping lemma to prove that they are not regular. For each regular language, provide a regular expression and construct an ε -NFA.
- (a) $L_1 = \{w \in \{0, 1\}^* \mid \text{length of } w \text{ is odd}\}$
 - (b) $L_2 = \{0^n 1^n \mid n \in \mathbb{N}\}$
 - (c) $L_3 = \{w \in \{0, 1\}^* \mid w \text{ contains an even number of 1s}\}$
 - (d) $L_4 = \{1^{n^2} \mid n \in \mathbb{N}\}$
8. Consider a finite-state automaton $M = (\Sigma, Q, q_0, F, \delta)$ and a non-negative integer k . Let R_k be the relation on the set of states of M such that $s R_k t$ if and only if for every input string $w \in \Sigma^*$ with $|w| \leq k$, $\delta(s, w)$ and $\delta(t, w)$ are both final states or both not final states. Furthermore, let R^* be the relation on the set of states of M such that $s R^* t$ if and only if for every input string $w \in \Sigma^*$, regardless of length, $\delta(s, w)$ and $\delta(t, w)$ are both final states or both not final states.
- (a) Show that for every nonnegative integer k , R_k is an equivalence relation on S .
Two states s and t are called k -equivalent if $s R_k t$.
 - (b) Show that R^* is an equivalence relation on S .
Two states s and t are called $*$ -equivalent if $s R^* t$.
 - (c) Show that if two states s and t are k -equivalent ($k > 0$), then they are also $(k - 1)$ -equivalent.
 - (d) Show that the equivalence classes of R_k are a *refinement* of the equivalence classes of R_{k-1} .
 - (e) Show that if two states s and t are k -equivalent for every non-negative integer k , then they are $*$ -equivalent.
 - (f) Show that all states in a given R^* -equivalence class are final or all are not final.
 - (g) Show that if two states s and t are $*$ -equivalent, then $\delta(s, a)$ and $\delta(t, a)$ are also $*$ -equivalent for all $a \in \Sigma$.
9. Consider the finite-state automaton $M = (\Sigma, Q, q_0, F, \delta)$ depicted below.



- (a) Find the k -equivalence classes of M for $k = 0, 1, 2, 3$.
- (b) Find the $*$ -equivalence classes of M .
- (c) Construct the quotient automaton \overline{M} of M .
 ▶ The quotient automaton \overline{M} of the deterministic finite-state automaton $M = (\Sigma, S, s_0, F, \delta)$ is the finite state automaton $\overline{M} = (\Sigma, \overline{S}, [s_0]_{R^*}, \overline{F}, \overline{\delta})$, where the set of states \overline{S} is the set of R^* -equivalence classes of S ; the transition function $\overline{\delta}$ is defined by $\overline{\delta}([s]_{R^*}, a) = [\delta(s, a)]_{R^*}$ for all states $[s]_{R^*}$ of \overline{M} and input symbols $a \in \Sigma$; and \overline{F} is the set consisting of R^* -equivalence classes of final states of M .

10. Solve the following regex crosswords¹. Fill each cell with a single ASCII character (an uppercase letter, a digit, a punctuation mark, or a space). Each row/column, when read left to right or top to bottom must match the regular expression(s) given for that row/column.

[^SPEAK]+ EP|IP|EF
HE|LL|O+
[PLEASE]+

(Y|F)(.)\2[DAF]\1
(U|O|I)*T[FRO]+
[KANE]*[GIN]*

[^NRU](NO|ON)
[RUNT]*
O.*[HAT]
(.)*DO\1

[^MCI]+ (TM|BF)
(CAT|A-T)+
[MA\-\sE]+

[^KI\sP]+
(M|APS|EA)*

(.)\1(.)\2
[PU\sH]+
[C\sOU]+
[LUH]+
(P|K)[^U]+
.*C+[TIF]
(NO|ONE|ION)*

.*L+
[PUF\s]*
[TIC]*
[NOI\sE]+

[AI][E\s]
[A\~Z]+
[\sT\~M]+
(EM|FE)(IT|IP)
[^P]I(IT|ME)
.*E.*
(EP|ST)*
T[A-Z]*
.M.T
.*P.[S-X]+

[PIF]+
.*[OWE]*
(TN|LF|TF)*

¹ Credits: <https://regexcrossword.com>