Formal verification. Model checking. Formal specifications: LTL, CTL. NuSMV and nuXmv model checkers

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Introduction

Formal verification vs. testing

Specifications

- Functional: input-output relationship (what the system must do)
- Non-functional: qualities of the system, such as safety, security, performance, response time, usability, testability, maintainability, extensibility, scalability...
- Informal and formal (e.g., formulas, standardized diagrams)

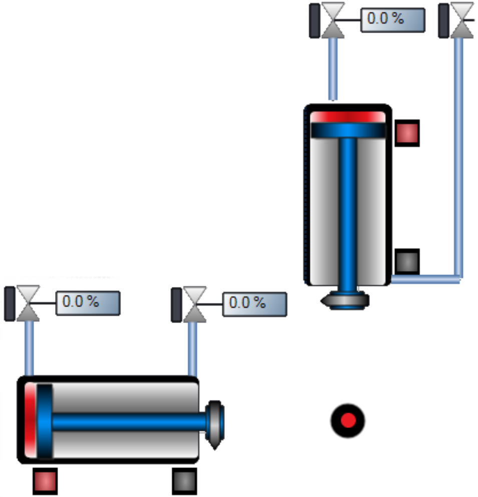
Testing

- Finite, often non-exhaustive number of scenarios to consider
- Test cases can be written manually or generated automatically

Formal verification

- Formal model: unambiguous, often condensed and simplified view on certain aspects of the system to be verified
- Formal specifications are needed
- Often exhaustive checking (under the assumptions of the formal models)

Example of a system: TwoCylinders



- Two cylinders and a tray
- If a workpiece is placed on a tray, it can be pushed away by a cylinder
- The cylinders will collide if they are both extended almost entirely

Formal verification methods

- Static analysis
  - Analyze the system without executing it
  - For example, can detect that the last assignment is unreachable:

```
retracted = position == 0;
extended = position == 10;
if (retracted && extended) { error = True; }
```

- Model checking
  - Mostly for checking functional specifications
  - Prove or disprove that the cylinders cannot collide by examining all possible system behaviors
- Theorem proving
  - Prove a wider range of properties
  - For a particular cylinder collision algorithm, prove that if it works for $i > 1$ cylinders, then it will also work for $i + 1$ cylinders

Model checking

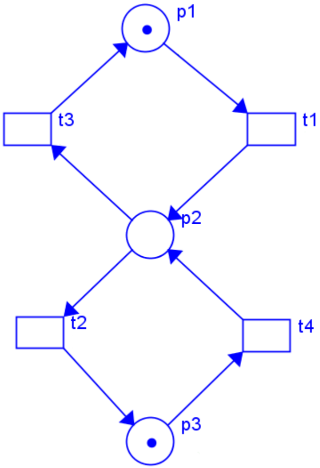
Model checking: overview

- Requires a **formal model** of the behavior of the system
- Mathematically, this model is often represented as some form of a state machine
  - Visual tools to do this, e.g., UPPAAL
- There are formal languages that allow specifying formal models textually
  - E.g., NuSMV, nuXmv, SPIN

- Similarly, **temporal logics** are formal languages that specify properties to be verified
- These properties often examine **infinite model behaviors**

- Due to checking behaviors, the specifications that are checked are often functional
- With carefully created models and temporal formulas, it is also possible to verify response time, safety and security

Model example: Petri nets

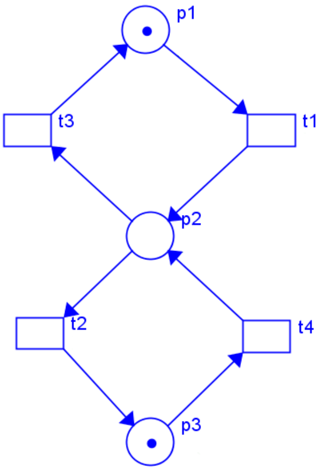


- Locations (circles) can contain tokens (dots)
- Each location can contain 0 or more tokens
- The initial configuration is shown on the slide
- Each transition (box) can move a token from its input location to its output location
- If multiple transitions can execute, then the one to be executed is selected **nondeterministically**

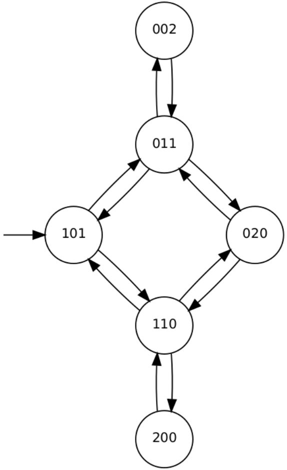
State (reachability) graph of a system

- Nodes: all reachable states of the system
- If the system is modular, then the state of the system consists of the state of all its modules
- Directed edges: one-step evolutions of the state
- Multiple outgoing edges are possible from each state, i.e., **nondeterminism** is common

State graph: example



A Petri net

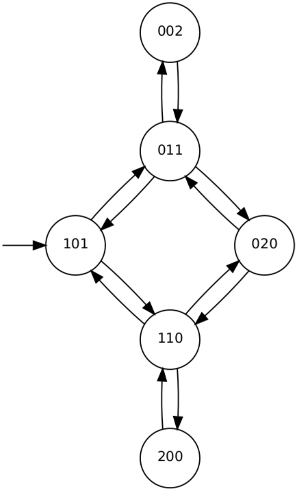


State graph, state $= p_1 p_2 p_3$

Kripke structures

- Formalization of a state graph
- Let AP be a finite set of so-called **atomic propositions**
- Then $M = (S, I, T, L)$ is a **Kripke structure**, where:
- S is a finite set of **states**
- $I \subseteq S$ is a set of **initial states**
- $T \subseteq S \times S$ is a **transition relation**
- $L : S \to 2^{\mathrm{AP}}$ is a **labeling function**
- **No deadlock assumption:** $\forall s \in S \; \exists s' \in S : (s, s') \in T$, i.e., it is possible to proceed to somewhere from any state

State graph interpreted as a Kripke structure



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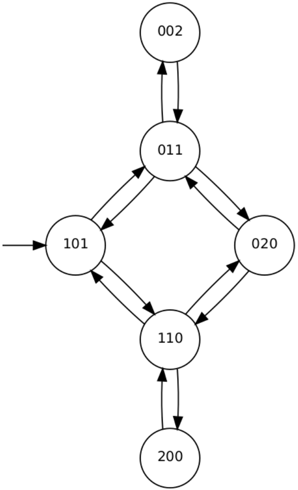

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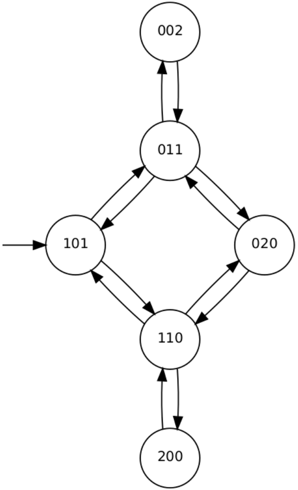
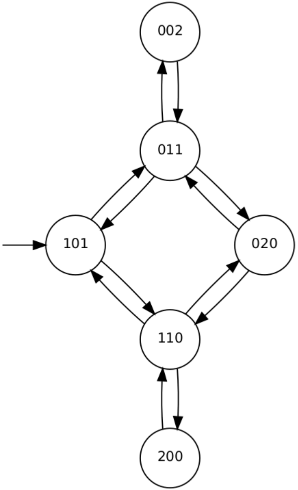


- AP $= \{$"$p_i = j$" $\mid 1 \leq i \leq 3, 0 \leq j \leq 2\}$
- S:

State graph interpreted as a Kripke structure



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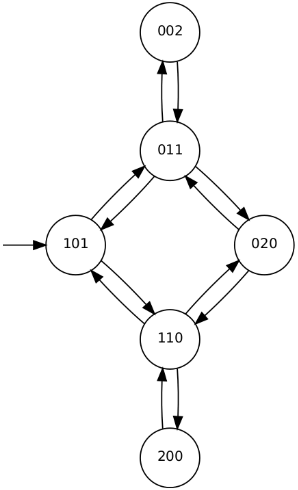

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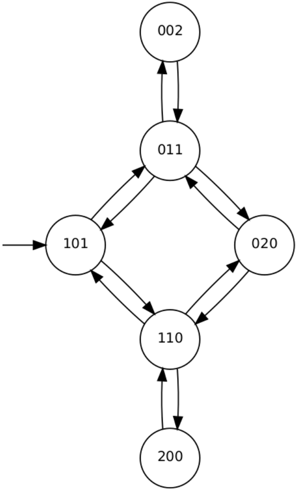

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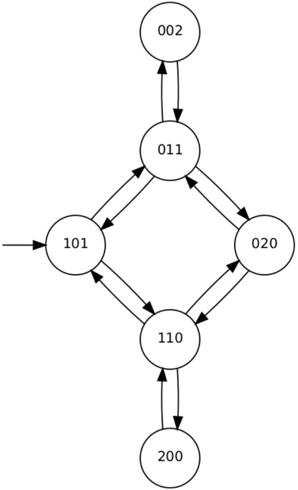

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- $L : S \to 2^{\mathrm{AP}}$:

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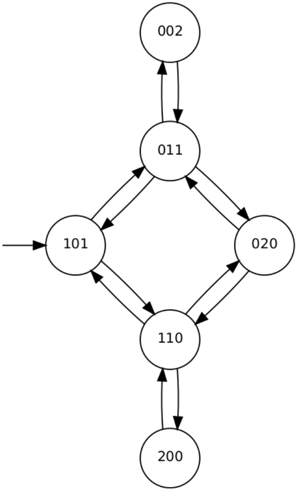

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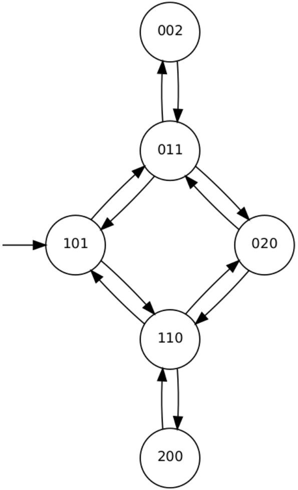

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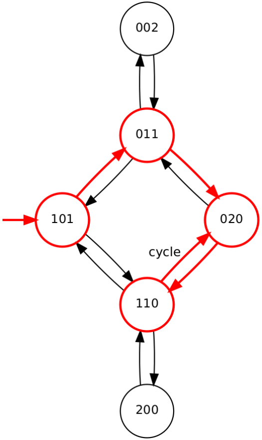
- **Specifications** can be interpreted as predicates over Kripke structures

System behaviors are paths in Kripke structures



Infinite paths are common in formal verification. This is the reason why deadlocks are undesirable

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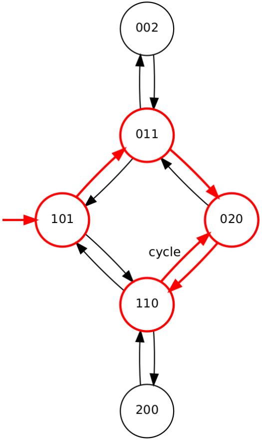

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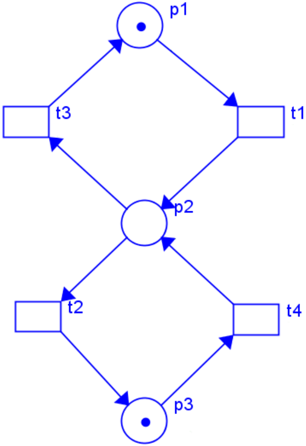

What happens in terms of the original model?

Linear temporal logic (LTL)

Single behavior view

- Assume that now we have only two atomic propositions: p and q
- All possible behaviors are infinite sequences over $2^{\{p,q\}}$
- Example: $\{p, q\}, \{p\}, \{\}, \text{cycle}(\{q\}, \{p, q\})$

Single behavior view

- Assume that now we have only two atomic propositions: p and q
- All possible behaviors are infinite sequences over $2^{\{p,q\}}$
- Example: $\{p, q\}, \{p\}, \{\}, \text{cycle}(\{q\}, \{p, q\})$
- Boolean logic is able to characterize single elements of such sequences
- Can we somehow introduce predicates over infinite sequences of atomic propositions?
- For example, to formulate a specification: each p is followed by $\neg p$ on the next step (which is false for this example)

Linear temporal logic (LTL)

- Formal language that extends the usual propositional Boolean logic
- Variables: atomic propositions, e.g., p and q
- Usual Boolean operators are allowed, e.g., $p \wedge \neg q$ is an LTL formula, but it refers to the **first element** of an infinite sequence

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  - **U**: **u**ntil (binary operator), e.g., $p\mathbf{U}q$ means "q must happen at some step, and the sequence must satisfy p until (non-inclusive) q happens"

Examples of LTL formulas

- Path 1: $\{p, q\}, \{p\}, \{\}, \text{cycle}(\{q\}, \{p, q\})$
- Path 2: $\text{cycle}(\{p, q\})$
- Path 3: $\{\}, \text{cycle}(\{p\}, \{p, q\}, \{q\})$

- $f_1 = \mathbf{G}p$

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- $f_4 = \mathbf{XXXX}p$ ("on the fifth step") – paths 1, 2, 3
- $f_5 = \mathbf{FG}(p \wedge q)$ ("globally from some point") – path 2
- $f_6 = \mathbf{GF}(p \wedge q)$

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- $f_7 = \mathbf{G}(p \to \mathbf{X}q)$ ("p is always followed by q") – paths 2, 3

LTL: some simplification and equivalence rules

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- $\neg\mathbf{G}(f) = \mathbf{F}(\neg f)$
- $\neg\mathbf{F}(f) = \mathbf{G}(\neg f)$

- Also remember that Boolean rules can be applied to Boolean subformulas, e.g., $\mathbf{G}(f \to g)$ is equivalent to $\mathbf{G}(\neg f \lor g)$

LTL model checking: definition

- Kripke structure M satisfies LTL formula f (written: $M \models f$), if **all paths** in M which **start in M's initial states** satisfy f

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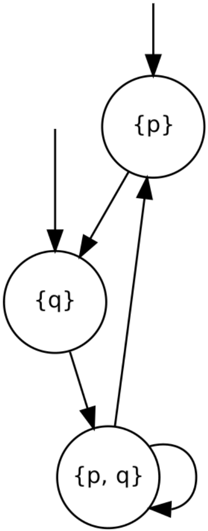

Quiz: which of these LTL formulas are satisfied by the KS on the right? Why?

- $f_1 = \mathbf{G}p$
- $f_2 = \mathbf{F}(\neg p \wedge \neg q)$
- $f_3 = p\mathbf{U}(\neg p \wedge \neg q)$
- $f_4 = \mathbf{XXXX}p$
- $f_5 = \mathbf{FG}(p \wedge q)$
- $f_6 = \mathbf{GF}(p \wedge q)$
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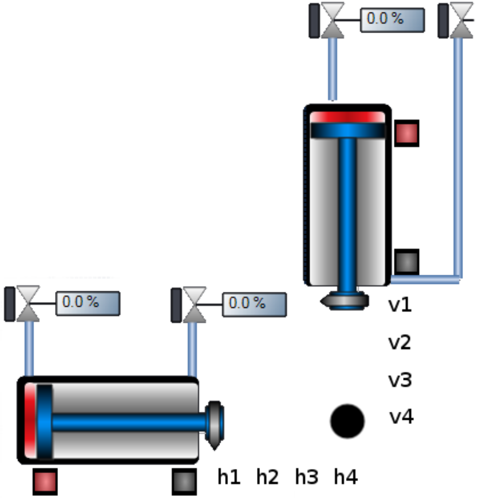
Quiz answers

Quiz answers

- Only $f_6 = \mathbf{GF}(p \wedge q)$

Formal model of the TwoCylinders system



- The extension of each cylinder is discretized into four intervals
- When both cylinders share interval 4, they collide
- A workpiece can be placed into the shared interval
- If a cylinder reaches interval 4 and there is a workpiece, it is pushed

LTL specification for the two cylinders system: plant model

- Atomic propositions: h_1, h_2, h_3, h_4 (displacements of the horizontal cylinder), v_1, v_2, v_3, v_4 (displacements of the vertical cylinder), w (workpiece is present)
- Cylinder has a position:

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  $\mathbf{G}(\neg(h_1 \wedge h_2) \wedge \neg(h_1 \wedge h_3) \wedge \neg(h_1 \wedge h_4) \wedge \neg(h_2 \wedge h_3) \wedge \neg(h_2 \wedge h_4) \wedge \neg(h_3 \wedge h_4))$,
  $\mathbf{G}(\neg(v_1 \wedge v_2) \wedge \neg(v_1 \wedge v_3) \wedge \neg(v_1 \wedge v_4) \wedge \neg(v_2 \wedge v_3) \wedge \neg(v_2 \wedge v_4) \wedge \neg(v_3 \wedge v_4))$
- If a cylinder is fully extended, then there is no workpiece:

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- Cylinder can't have more than one position:
  $\mathbf{G}(\neg(h_1 \wedge h_2) \wedge \neg(h_1 \wedge h_3) \wedge \neg(h_1 \wedge h_4) \wedge \neg(h_2 \wedge h_3) \wedge \neg(h_2 \wedge h_4) \wedge \neg(h_3 \wedge h_4))$,
  $\mathbf{G}(\neg(v_1 \wedge v_2) \wedge \neg(v_1 \wedge v_3) \wedge \neg(v_1 \wedge v_4) \wedge \neg(v_2 \wedge v_3) \wedge \neg(v_2 \wedge v_4) \wedge \neg(v_3 \wedge v_4))$
- If a cylinder is fully extended, then there is no workpiece: $\mathbf{G}(h_4 \vee v_4 \to \neg w)$
- ...

- Such specifications can help "debug" the plant model

LTL specification for the two cylinders system: requirements for the controller

- We won't use any additional atomic propositions: all we need can be specified in terms of the plant!

LTL specification for the two cylinders system: requirements for the controller

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- Cylinders iterate (each new workpiece is pushed by a different cylinder): $\mathbf{G}((h_4 \wedge (\mathbf{X}\neg h_4) \wedge \mathbf{F}w) \to \mathbf{X}((\neg w \wedge \neg v_4 \wedge \neg h_4)\mathbf{U}(w \wedge (w\mathbf{U}(v_4 \wedge \neg h_4)))))$, and the same for the other cylinder

Computation tree logic (CTL)

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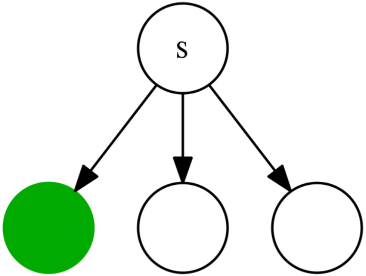
- In LTL, there is always an implicit quantification over all paths starting in initial states
- In **CTL**, all temporal operators are annotated with quantifiers

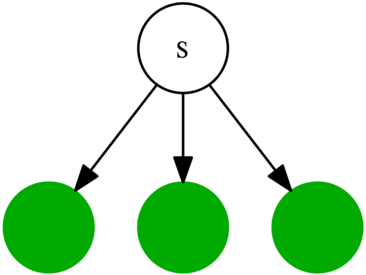
- CTL formulas characterize not infinite sequences, but rather states of the Kripke structure
- A Kripke structure satisfies a CTL formula, if **all its initial states** satisfy this formula
- Let s be a state of the KS, then $s \models f$ means s satisfies f

CTL: temporal operator **EX**



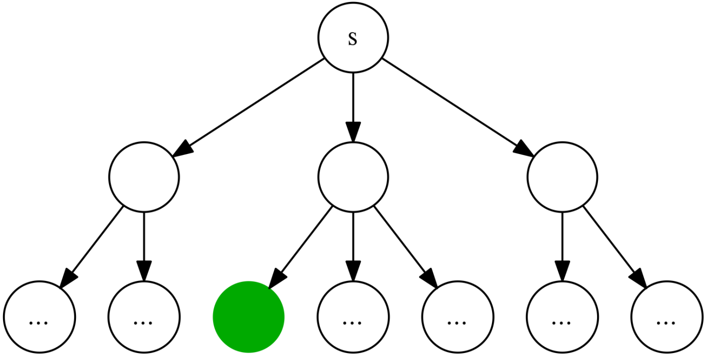
- $s \models \mathbf{EX}(f)$ ("exists next"): in some successor of s, f holds

CTL: temporal operator **AX**



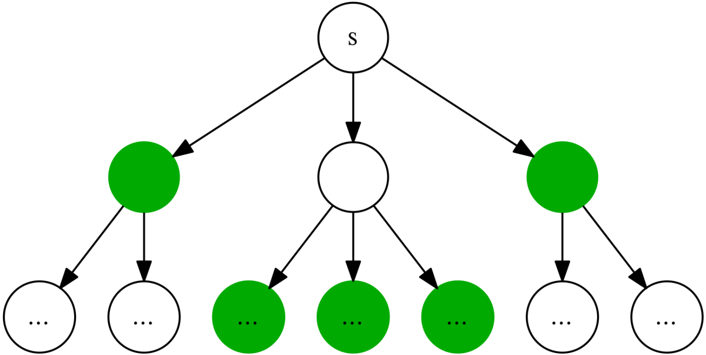
- $s \models \mathbf{AX}(f)$ ("for all next"): in all successors of s, f holds

CTL: temporal operator **EF**

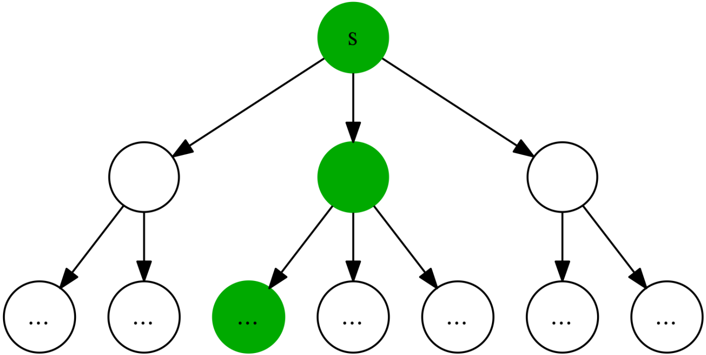


- $s \models \mathbf{EF}(f)$ ("exists in the future"): there exists a path starting in s such that f becomes valid at some point of this path

CTL: temporal operator **AF**

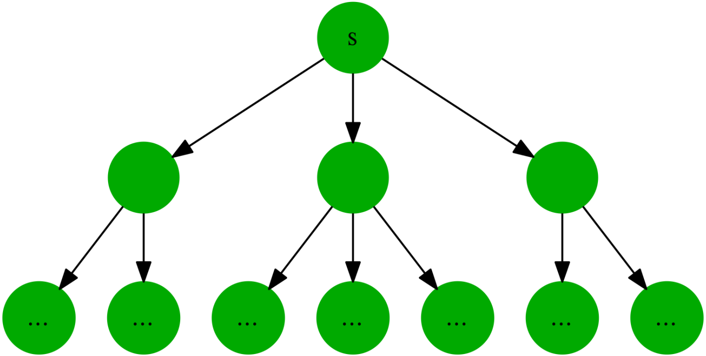


- $s \models \mathbf{AF}(f)$ ("for all in the future"): for all possible paths starting in s, f becomes true at some point



- $s \models \mathbf{EG}(f)$ ("exists globally"): there exists a path starting in s such that f holds at every state along this path

CTL: temporal operator **AG**



- $s \models \mathbf{AG}(f)$ ("for all globally"): for all possible paths starting in s, f is always true

CTL: temporal operators **EU** and **AU**

- $s \models f\,\mathbf{EU}\,g$ ("exists until"): there exists a path starting in s such that f holds until (non-inclusive) g, and g eventually happens
- $s \models f\,\mathbf{AU}\,g$ ("for all until"): for all possible paths starting in s, f holds until (non-inclusive) g, and g eventually happens

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- $p \wedge \neg q$

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- Which of these CTL formulas are satisfied by the KS on the right? Why?
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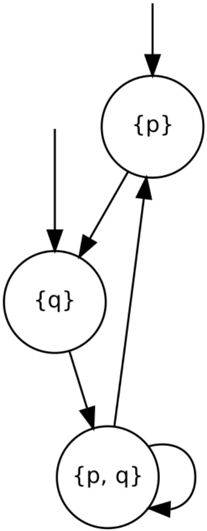
- $f_2 = \mathbf{AG}(p \vee q)$
- $f_3 = \mathbf{AF}(p \wedge q)$
- $f_4 = \mathbf{EF}(\neg p \wedge \neg q)$
- $f_5 = \mathbf{AXAXAXAX}p$
- $f_6 = \mathbf{EFEG}(p \wedge q)$
- $f_7 = \mathbf{EGEF}(p \wedge q)$
- $f_8 = \mathbf{AG}(p \to \mathbf{AX}q)$



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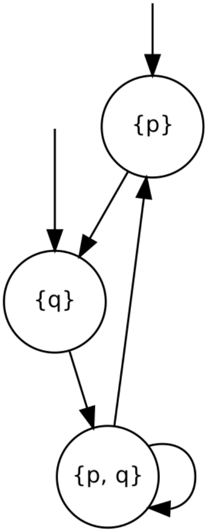
- Answer:



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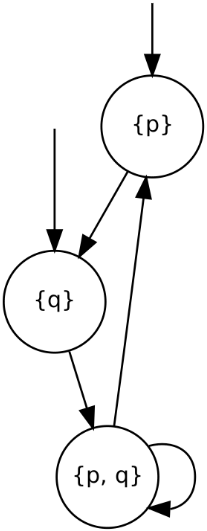
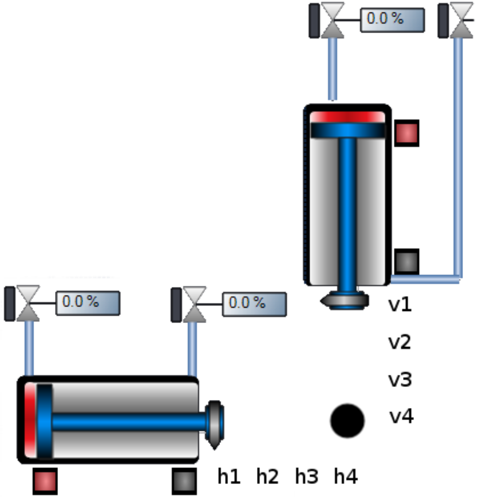
- Answer: f_2, f_3, f_6, f_7



CTL model checking: two cylinders



- Atomic propositions: $h_1..h_4, v_1..v_4, w$
- **Quiz:** specify the following properties in CTL:
  - If a cylinder is fully extended, then there is no workpiece
  - Cylinders do not collide
  - When a workpiece appears, it must be eventually pushed away
  - Cylinders iterate

Quiz answers

Quiz answers

- If a cylinder is fully extended, then there is no workpiece: $\mathbf{AG}(h_4 \lor v_4 \to \neg w)$
- Cylinders do not collide: $\mathbf{AG}\neg(h_4 \land v_4)$
- When a workpiece appears, it must be eventually pushed away: $\mathbf{AG}(w \to \mathbf{AF}\neg w)$
- Cylinders iterate: ¯_(ツ)_/¯

Common specification types ("patterns")

Name	LTL	CTL
Generality / Invariance	$\mathbf{G}f$	$\mathbf{AG}f$
Bounded response	$\mathbf{G}(p \rightarrow \mathbf{X}^n q)$	$\mathbf{AG}(p \rightarrow (\mathbf{AX})^n q)$
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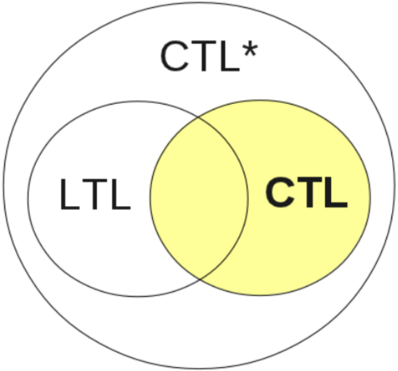
- These are lucky cases, but unfortunately, transforming one logic to the other cannot be always done by appending/removing **A** or **E**
- Sometimes such a transformation is impossible – see the next slide

There are properties which cannot be expressed in both LTL and CTL

- $\mathbf{G}p/\mathbf{AG}p$, $\mathbf{F}p/\mathbf{AF}p$, $\mathbf{GF}p/\mathbf{AGAF}p$ – both LTL and CTL
- $\mathbf{EF}p$ – only CTL, but there is a workaround to check it in LTL!
- $\mathbf{FG}p$ – only LTL
- $\mathbf{AGEF}p$ – only CTL
- CTL* is a larger logic which allows combining quantified and unquantified temporal operators



NuSMV and nuXmv model checkers

NuSMV

- Open-source symbolic model checker
- Supports LTL and CTL
- Can be downloaded here: `http://nusmv.fbk.eu/`
- Command-line tool, models are specified in text files
- If an LTL specification is false, the corresponding counterexample can be visualized with the tool
  `https://github.com/igor-buzhinsky/nusmv_counterexample_visualizer`

NuSMV: example

```
MODULE main
VAR
    p: boolean;
    c: 0..10;
    e: {value_a, value_b, value_c};
DEFINE
    c_plus_1 := c + 1;
ASSIGN
    init(c) := 0;
    next(c) := c_plus_1 mod 10;
    init(p) := FALSE;
    next(p) := next(c = 5) | p;

CTLSPEC AG(c != 10)
LTLSPEC G(p -> X(p))
```

- `VAR`: variable declarations (Boolean, integer, enumeration)

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- `next` assignments can refer to next values of other variables and current values of all variables

NuSMV: cylinder

```
MODULE CYLINDER(fwd, back)
VAR
    pos: 0..5;
ASSIGN
    init(pos) := 0;
    next(pos) := fwd ? next_pos : back ? prev_pos : pos;
DEFINE
    next_pos := pos < 5 ? (pos + 1) : pos;
    prev_pos := pos > 0 ? (pos - 1) : pos;
    home := pos = 0;
    end := pos = 5;
```

- Modules can have inputs (in the declaration), and their variables and definitions can be interpreted as outputs
- C-style choice operator ?:

NuSMV: controller

```
MODULE CONTROLLER(home, end)
VAR
    state: {moving_fwd, moving_back};
ASSIGN
    init(state) := moving_fwd;
    next(state) := case
        home: moving_fwd;
        end: moving_back;
        TRUE: state;
    esac;
DEFINE
    fwd := state = moving_fwd;
    back := state = moving_back;
```

- Example of explicit state machine modeling

NuSMV: closed-loop composition

```
MODULE main
VAR
    -- this is the way to write comments, by the way
    cyl: CYLINDER(ctr.fwd, ctr.back);
    ctr: CONTROLLER(cyl.home, cyl.end);

LTLSPEC G F cyl.end -- TRUE
LTLSPEC G F cyl.home -- TRUE
```

- **Synchronous**: all the modules make a step together!

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- **Synchronous**: all the modules make a step together!
- How to model asynchronous interaction? – introduce execution permissions; no permission = keep the state of the module unchanged

nuXmv model checker

- Successor of NuSMV
- Mostly backward compatible with NuSMV (i.e., you can run it on the same models)
- Can be downloaded here: `https://es-static.fbk.eu/tools/nuxmv/`
- New features
  - Infinite-state integer and real variables
  - Verification of real-time models

Literature

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- Schneider, K. (2013). Verification of reactive systems: formal methods and algorithms. Springer Science & Business Media
- Cimatti, A., et al. NuSMV 2: An opensource tool for symbolic model checking. International Conference on Computer Aided Verification. 2002
- Cimatti, A., et al. Extending nuXmv with timed transition systems and timed temporal properties. International Conference on Computer Aided Verification. 2019
- NuSMV tutorial: `http://nusmv.fbk.eu/NuSMV/tutorial/`