Complex Systems — PROBLEM SHEET 3

Complex networks, 2022/2023

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1. First we check the robustness of networks in case of errors: we randomly select and remove an f fraction of nodes and measure the relative size of the giant component $P_\infty(f)/P_\infty(0)$ as a function of f for several networks (listed below) of size $N = 10^4$ (take $N = 10^3$ if you are not able to make it for larger system). Use an ensemble average over L samples, which means that for each type of a graph and for each set of parameters you should create L networks (check different values of L to see what L is sufficient). For each type of network plot $P_\infty(f)/P_\infty(0)$. What do you see? Please think for a while and create all networks with the same average degree $< k >$. How $< k >$ influences results? Think of a figure, which illustrates the role of the network structure and the role of parameters in the most informative way.

   (a) Random graph $G(N, p)$ for $k = 0.5, 1, 2, 4$.

   (b) The Barabasi-Albert graph for $k = 2, 4$.

   (c) The Watts-Strogatz graph $WS(N, k, \beta)$ with $\beta = 0.01$ for two values of $k = 2$ and $k = 4$.

   Please remember to label axis properly and provide a legend for N, as well as the title or the caption for a graph. Can you draw any conclusions from the obtained results?

2. Now we check the robustness of networks in case of attacks: we first remove the highest degree node, followed by the node with the next highest degree and so on until an f fraction of nodes is removed. Then, analogously like in the previous task, we measure the relative size of the giant component $P_\infty(f)/P_\infty(0)$ as a function of f.

3. In the previous task attacks are based on the simplest measure of centrality, namely degree centrality. However, it could be done also based on the others. Please check robustness analogously as in the previous task based on the closeness and the betweenness centralities.

The tasks should be finished till 29.12.2022