Homework 1

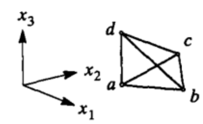
(due on Thursday, Oct 5, 11:59 PM)

Problem 1. (20 pts) Compute the values of the following expressions

a) δ_{ii}
b) $\delta_{ij}\delta_{ij}$
c) $C_{ij}\delta_{ik}\delta_{jk}$
d) $\delta_{ab}\delta_{bc}\delta_{cd}\dots\delta_{xy}\delta_{yz}$ (enough terms to exhaust all 26 letters of the alphabet)

Problem 2. (20 pts) The vertices of a triangle are given by the position vectors **a**, **b**, and **c**. The components of these vectors in a particular basis are **a** = (0,0,0), **b** = (1,4,3), and **c** = (2,3, l). Using a vector approach, compute the area of the triangle. Find the area of the triangle projected onto the plane with normal vector **n** = (0, 0,1). Find the unit normal vector to the triangle.

Problem 3. (20 pts) Let the coordinates of four points **a**, **b**, **c,** and **d** be given by the following position vectors **a** = (l, 1,1), **b** = (2,1,1), **c** = (l, 2, 2), and **d** = (l, 1,3) in the coordinate system shown. Find vectors normal to planes **abc** and **bcd**. Find the angle between those vectors. Find the area of the triangle **abc**. Find the volume of the tetrahedron **abcd**.



Problem 4. (20 pts) Use the observation that $||\mathbf{u}-\mathbf{v}||^2 = (\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})$ along with the distributive law for the dot product to show that

$$\mathbf{u}\cdot\mathbf{v} \equiv \frac{1}{2}(||\mathbf{u}||^2 + ||\mathbf{v}||^2 - ||\mathbf{u}-\mathbf{v}||^2)$$

Problem 5) (20 pts) Consider a tensor field **T** defined on a tetrahedral region bounded by the coordinate planes $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, and the oblique plane $6x_1 + 3x_2 + 2x_3 = 6$, as shown in the sketch. The tensor field has the particular expression $\mathbf{T} = \mathbf{b} \otimes \mathbf{x}$, where **b** is a constant vector and **x** is the position vector $\mathbf{x} = x_i e_i$. Compute the integral of div(**T**) over the volume and the integral of **Tn** over the surface of the tetrahedron (and thereby show that they give the same result, as promised by the divergence theorem). Note that the volume of the tetrahedron of the given dimensions is one.