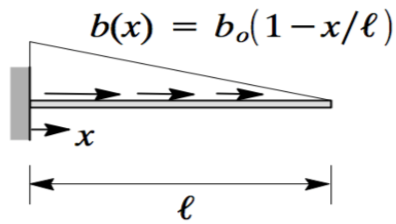
Homework 2

(due on Thursday, Oct 19, 11:59 PM)

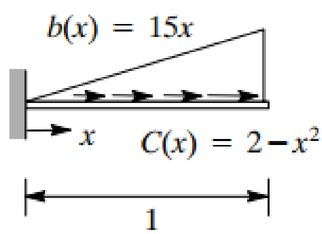
Problem 1. (25pts) Consider the uniaxial rod shown below, fixed at $x = 0$, free at $x = 1$ and subjected to the linearly varying body force indicated. The rod is made from a composite material with a variable elastic modulus $C(x) = C_0(2 - x/l)$, making it twice as stiff at $x = 0$, as it is at $x = 1$.The governing differential equation for a rod with variable modulus is

$$(C(x)u')' + b(x) = 0$$

where a prime indicates differentiation with respect to x. **Find the exact (classical) solution to the problem by directly integrating the governing equations.**



Problem 2. (25pts) Consider the unit length and modulus *C(x)* that varies as shown in the sketch. The rod is fixed at the left end, is free at the right end, and is subjected to a linear varying body force *b(x)* as shown. Consider the following displacement map: $u(x) = a(x^3 + 2x^2 - 3x)$ where a is some constant. **Does the displacement map a solution to the given problem? Why or why not?**



Problem 3. (25pts). The principle of virtual work for a certain boundary value problem can be stated as

$$G(u, \bar{u}) = \int_0^l [A(x)u''\bar{u}'' + B(x)u\bar{u} - b(x)\bar{u}]dx = 0 \quad \text{for} \quad \bar{u}(x) \in \mathcal{F}(0, L)$$

where *A*, *B*, and *b* are known functions of *x*, *u(x)* is the unknown field, and a prime denotes derivative with respect to *x*. **What is the classical differential equation that is equivalent to this variational statement?**

Problem 4. (25pts) The classical (4th order) differential equation and boundary conditions for a certain boundary value problem are

$$Au'''' + Bu'' + Cu = b \quad \text{for all} \quad x \in [0, l]$$
$$u(0) = 0, \quad u(l) = 0, \quad Au''(0) = 0, \quad Au''(l) = 0$$

where A, B, C, and b are known constants, $u(x)$ is the unknown field, and a prime denotes derivative with respect to x. **Find an expression for the virtual work functional associated with the classical differential equation.** In other words, find the functional G that has the property that the statement "$G(u,\bar{u})=0$ for all $\bar{u} \in F_e$" is equivalent to the classical differential equation and the highest derivative that appears in G is second order. Describe and restrictions that must be placed on F_e.