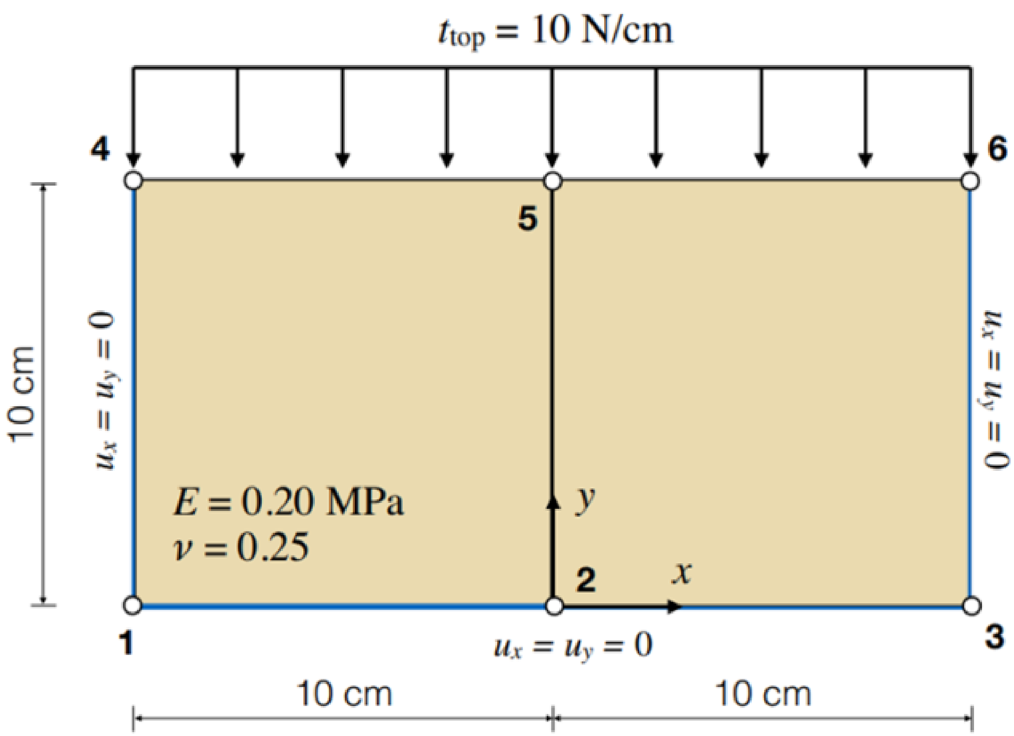
Homework 5

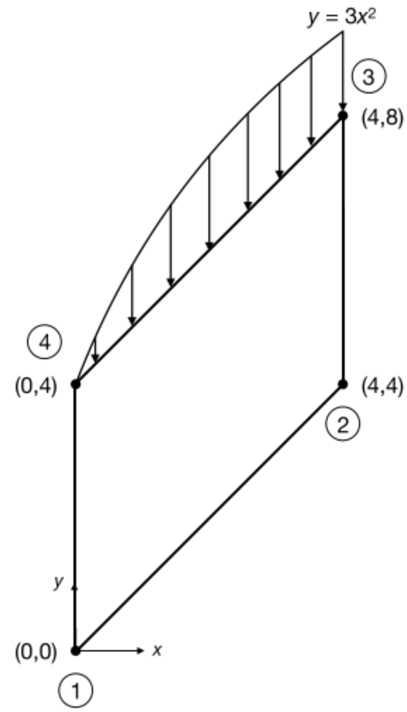
(due on Monday, Dec. 11, 11:59 PM)

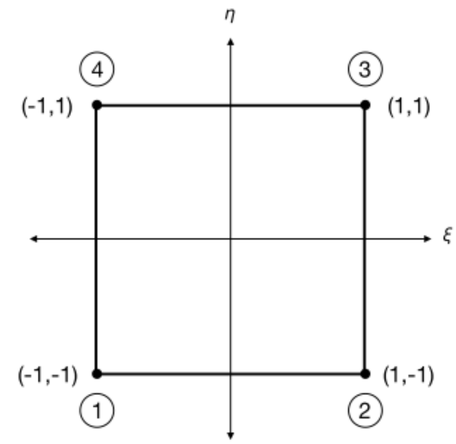
Problem 1. Consider the two-dimensional (plane-strain) problem shown below that is discretized with two four-node bilinear finite elements.



(a) Compute the finite element stiffness matrix for both elements (use exact integration). **Do not compute all components of the 8x8 element matrices**; instead, compute only the terms you will need to obtain the nodal displacement vector for this problem. Use symmetry arguments as needed.
(b) Compute the equivalent nodal force vectors for both elements (use exact integration). Again, **do not compute all components of the 8x1 element vectors**; instead, compute only the terms you will need to obtain the nodal displacement vector for this problem.
(c) Obtain the full nodal displacement vector.
(d) Compute the corresponding stress vector.

Problem 2. Consider the four-node quadrilateral finite element shown below:





(a) Compute the area of this element by evaluating the following integral (*hint: Use isoparametric mapping and transform the integral to the natural coordinates shown on the right*):

$$A_W = \int_W dA$$

(b) Assume that the nodal DOFs are ordered as follows:

$$\mathbf{d} = [d_1, d_2, ..., d_7, d_8]^T$$

Compute the finite element matrix component K_{17} using a one-point Gaussian integration rule. For Lamé constants, $E = 30$ and $\nu = 0.25$.

(c) Compute the equivalent nodal load vector due to the distributed load between nodes 4 and 3. Carry out the integration either exactly, or numerically by using an exact Gaussian quadrature integration rule.

(d) Assume that the nodal displacements for this element are computed as follows:

$$d = [4,0,0,-2,0,0,0,10]^T$$

Compute the displacement at the centroid of the element.