A Solution for A Double-Car Freight Train Problem

Introduction

The double-car freight train problem aims to explore the relationship between spring performance and roughness of the road, which can guide the selection of materials of the spring in the production of vehicles. Such kind of practical problem can be modeled into the ideal model of a double-car freight train problem. Through researching the relationship between the stretch in the spring and the friction coefficient, the state in the motion of two cars which are connected with each other by a spring when they sliding down a slope can be able to maintain at a fixed reasonable interval by controlling its acceleration. In order to simplify the model while retaining the university of the result, the model regards rolling friction as sliding friction indicated as μ_k. Car B has brake while Car A does not. They are at rest on the slope and both were relatively stationary finally. The motion model is shown in Figure 1. This project will supply a problem-solving idea for the model.

Illustration

Analyze the question and determine constraints

1. The first question attempts to determine the stretch of the spring when both of cars move without friction force.
2. In the second question, car B is affected by friction. Compared to question 1, its force analysis will increase complexity which is reflected in the number of the forces car B and the system have. The stretch of the spring changes dynamically when the cars slide down.
3. In question 3, the friction coefficient and the stretch of the spring have constrained condition. The former must be less than 0.6 while another should not be more than 1.2 meter. The motion of the cars will be restricted more.

Analyze the force

1. Car A and car B are under corresponding weight, normal fore and the elastic force. The elastic force is numerical equal but in opposite direction. In this case, car A and car B will run in the same state in the whole motion process because their force analysis is identical. The accelerations of them keep still all of the time. The detailed analysis process is shown in Figure 2.
2. Compared to question 1, cars in the second question have a different running

process and the stretch of the spring changes in the motion before they are relatively stationary. When the system reaches a steady state, the acceleration of cars and the stretch of the spring will remain as constants. The detailed analysis process is shown in Figure 3.

3. Through the constraints on friction coefficient and the stretch of the spring, cars' motion must balance the influence of these two coefficients. The detailed analysis process is shown in Figure 4.

Solute the result

1. No compression of spring.
2. The deformation of the spring is 0.955 meter in the stretch direction.
3. The minimum of the acceleration of cars is affected by friction coefficient and the stretch of the spring.
   $a_{min}=7.848+0.75x-5.886(\mu_k)_B$ ($x<=1.2m$, $0<(\mu_k)_B<=0.6$, $(\mu_k)_B=0.4187x$)
   Solve by matlab to get $a_{min}=5.7906m/s^2$ when x=1.2 meter and $(\mu_k)_B=0.5024$.

Key Codes

Define a function for the acceleration equation.

```
fun = @(x)7.848+0.75*x(1) - 5.886*x(2);
```

Use Matlab's built-in functions fmincon() to find the minimum value of the function and get the stretch of the spring and the friction coefficient.

```
[x,fval,exitflag,output] = fmincon(fun,x0,A,b,Aeq,beq,lb,ub)
```

The restrictions is assigned for the function parameters which are A,b,Aeq,beq,lb and ub.

The details are shown in Figure 5 and the output of the program is shown in Figure 6.

Discussion

If more cars are present in the system, the spring connecting the brake cars and the cars without brake will be stretched by a largest length than others. All of the case of the problem can analyze in that spring. The brake cars can be seemed as a system and the others can be seemed as another system to simply the model. By this way, the acceleration equation $a_{min}=g\sin\theta-kx/m_A$, where m_A is the mass of the system of the cars without brake. The interval of the stretch of the spring x will change with the weights of the two system as $x=[m_Am_Bg\cos\theta/k(m_A+m_B)](\mu_k)_B$, where $(\mu_k)_B$ is less than 0.6.

Conclusion

By model and solve the double-car freight train problem, the project explores the relationship between spring performance and roughness of the road, which can guide the selection of materials of the spring in the production of vehicles. This ideal model ignores spring vibration affection. In practical production, this approach is not reasonable and tolerated. Thence the project is expected to optimize urgently.

Appendix

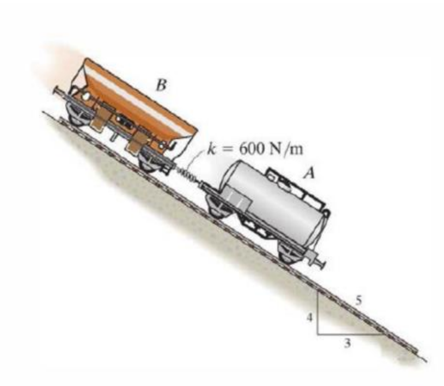


Figure 1 the motion model

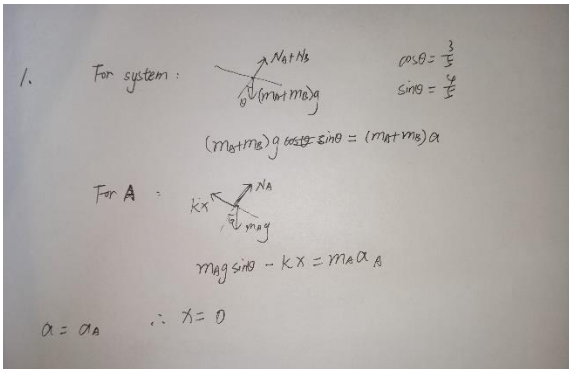


Figure 2 question 1 analysis

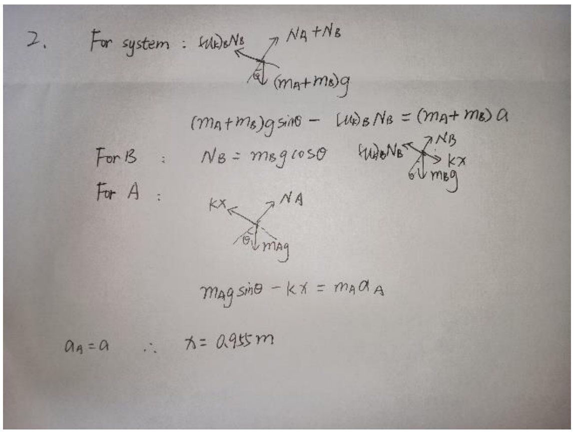
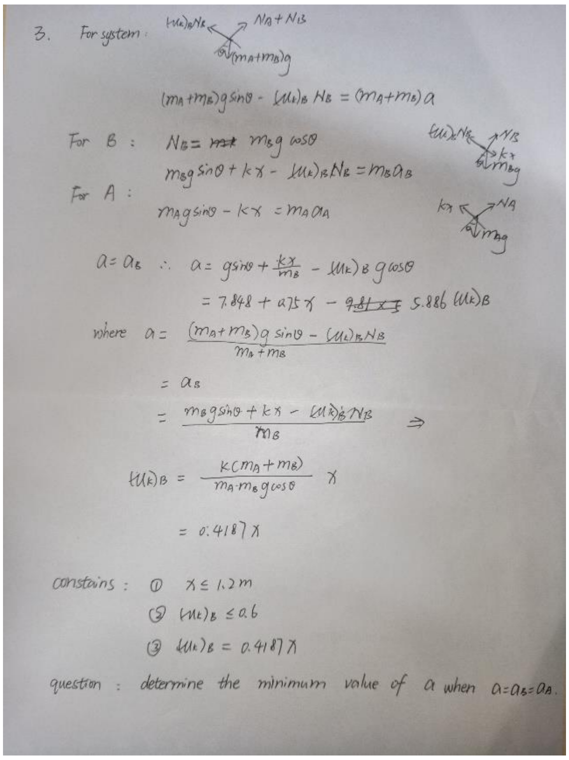


Figure 3 question 2 analysis



3. For system:

$$(m_A+m_B)g\sin\theta - (\mu_k)_B N_B = (m_A+m_B)a$$

For B: $N_B = \cancel{m_A} \ m_B g\cos\theta$

$$m_B g\sin\theta + kx - (\mu_k)_B N_B = m_B a_B$$

For A:

$$m_A g\sin\theta - kx = m_A a_A$$

$a = a_B$ $\therefore$ $a = g\sin\theta + \frac{kx}{m_B} - (\mu_k)_B g\cos\theta$

$$= 7.848 + 0.75x - \cancel{9.81\times\frac{3}{5}}\ 5.886(\mu_k)_B$$

where $a = \frac{(m_A+m_B)g\sin\theta - (\mu_k)_B N_B}{m_A+m_B}$

$$= a_B$$

$$= \frac{m_B g\sin\theta + kx - (\mu_k)_B N_B}{m_B} \Rightarrow$$

$$(\mu_k)_B = \frac{k(m_A+m_B)}{m_A \cdot m_B g\cos\theta} x$$

$$= 0.4187x$$

constains: ① $x \le 1.2m$

② $(\mu_k)_B \le 0.6$

③ $(\mu_k)_B = 0.4187x$

question: determine the minimum value of a when $a = a_B = a_A$.

Figure 4 question 3 analysis

```
fun = @(x)7.848+0.75*x(1) - 5.886*x(2);

A = [];
b = [];
Aeq = [0.4187,-1];
beq = 0;
lb = [0,0];
ub = [1.2,0.6];
x0 = [1,0.5];
[x,fval,exitflag,output] = fmincon(fun,x0,A,b,Aeq,beq,lb,ub)
```

Figure 5 key codes

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
x = 1×2
      1.2000    0.5024

fval = 5.7906
exitflag = 1
output = struct with fields:
         iterations: 5
          funcCount: 18
    constrviolation: 0
           stepsize: 1.9160e-06
          algorithm: 'interior-point'
      firstorderopt: 2.0000e-06
       cgiterations: 0
            message: '↵Local minimum found that satisfies the constraints.↵↵Optimization completed because the
```

Figure 6 output of the program