A Solution for Isolating the Source of Vibration Problem

Introduction

Under the continuous action of external periodic force, the vibration of forced vibration generated by the vibration system. There are some examples of application resonance, acoustic resonance of musical instruments, orbital resonances between satellites of some Jupiter planets in the solar system, resonance of basement membrane in animal ears, resonance of electric circuits, etc.
In this project, the forced vibration model is generated by two springs whose coefficient of spring, k, and an external periodic force, $P=P_m\sin(w_f t)$. The details of the model are shown in Figure 1. This project will supply a problem-solving idea to obtain the relationship between the transmissibility T_m of the system, defined as the ratio F_m/P_m of the maximum value F_m of the fluctuating periodic force transmitted to the foundation to the maximum value P_m of the periodic force applied to the machine element, and frequency ratios ω_f/ω_n

Illustration

Analyze the question and determine constraints

1. In the first question, the transmissibility T_m of the system must be solved by changing the frequency ratios ω_f/ω_n from 0 to 5 when the damping factors c/cc equal to 0.2, 0.4, 0.6, 0.8, 1.0 respectively. In this case, the relationship of the transmissibility and the frequency ratios, the damping factors must be found firstly by analyzing the fluctuating periodic force transmitted to the foundation.
2. As for the second question, it can get the damping factors qualified directly by the results of the first question. Filter out the appropriate damping factors to make the transmissibility small or equal to 0.5.

Analyze the forces

1. The differential equation of motion of a damped system with a single degree of freedom under the action of harmonic periodic forces can be deduced by analyzing the forced applied externally according to Newton's second law.
2. The periodic distance is
$$x = x_m \sin(\omega_f - \varphi)$$
Corresponding, the velocity is
$$\dot{x} = x_m \omega_f \cos(w_f t - \varphi)$$
3. For a damped system with a single degree of freedom under the action of

harmonic periodic forces, the maximum distance x_m can be solved by

$$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2(\frac{c}{c_c})(\frac{\omega_f}{\omega_n})\right]^2}}$$

4. Hence, the fluctuating periodic force transmitted to the foundation is

$$F = kx_m \sin(\omega_f t - \varphi) + cx_m\omega_f \cos(w_f t - \varphi)$$

There is

$$F_m = x_m\sqrt{k^2 + (c\omega_f)^2}$$

5. The transmissibility T_m of the system can be obtained by

$$T_m = \frac{F_m}{P_m} = \frac{x_m\sqrt{k^2 + (c\omega_f)^2}}{P_m} = \frac{\sqrt{1+\left(\frac{c\omega_f}{k}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2(\frac{c}{c_c})(\frac{\omega_f}{\omega_n})\right]^2}}$$

6. For the reason that

$$k = m{\omega_n}^2$$

and

$$c_c = 2m\omega_n$$

$\frac{c\omega_f}{k}$ can be reduced to $2\frac{c}{c_c}\frac{\omega_f}{\omega_n}$.

7. Ultimate, the transmissibility T_m of the system is

$$T_m = \frac{\sqrt{1+\left[2(\frac{c}{c_c})(\frac{\omega_f}{\omega_n})\right]^2}}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2(\frac{c}{c_c})(\frac{\omega_f}{\omega_n})\right]^2}}$$

Solute the result

1. The transmissibility changed with frequency ratios and the damping factors are shown in Figure 5.
2. The appropriate damping factors to make the transmissibility small or equal to 0.5 are shown in Figure 6. The corresponding value of transmissibility also shows.
3. As shown in the Figure 6, the damping factors when the values of transmissibility that smaller than 0.5 and frequency ratio equals to 2.5 are C/Cc = 0.4 and C/Cc = 0.2 .

Key Codes

Calculate the values of transmissibility according to the derived formula at first as shown in Figure 2.
Draw the figure of the transmissibility for frequency ratios. (Figure 3)
Calculate the appropriate damping factors to make the transmissibility small or equal to 0.5 and output the results of the values of the damping factors that satisfies the condition. (Figure 4)

Conclusion

By model and solve the problem of a damped system with a single degree of freedom under the action of harmonic periodic forces, the project explores the influence of the frequency ratios and the damping factors on the transmissibility. This model can be used to solve the problem of forced vibration in real life.

Appendix

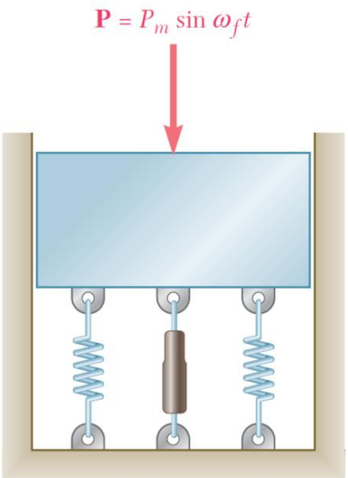


Figure 1 the problem model

```
% frequency ratios (wf/wn) named as wR
% damping factors (c/cc) named as cR
% transmissibility (Tm) named as Tm

wR=0:0.01:5;
cR=[0.2,0.4,0.6,0.8,1.0];

[wRrows,wRcolumns]=size(wR);
[cRrows,cRcolumns]=size(cR);
Tm=[0,0;0,0];
while cRcolumns>0
    cRtmp=cR(cRcolumns);

    wRcoltmp=wRcolumns;
    while wRcoltmp>0
        wRtmp=wR(wRcoltmp);
        Tm(cRcolumns,wRcoltmp)=sqrt((1+(2*wRtmp*cRtmp)^2)/((1-wRtmp^2)^2+(2*wRtmp*cRtmp)^2));
        wRcoltmp=wRcoltmp-1;
    end

    cRcolumns=cRcolumns-1;
end
```

Figure 2 keycodes of calculation of the transmissibility

```
% plot the figure
% Tm--(wf/wn) for different (c/cc)
figure(1);
fig1=plot(wR,Tm(1,:),wR,Tm(2,:),wR,Tm(3,:),wR,Tm(4,:),wR,Tm(5,:));
hold on
legend([fig1(1),fig1(2),fig1(3),fig1(4),fig1(5)],'C/Cc = 0.2','C/Cc = 0.4','C/Cc = 0.6','C/Cc = 0.8','C/Cc = 1.0');
title('Transmissibility (Tm) for frequency ratios (wf/wn)');
xlabel('wf/wn');
ylabel('Tm');
```

Figure 3 plot the transmissibility for frequency ratios

```
% frequency ratios (wf/wn) named as wR
% damping factors (c/cc) named as cR
% transmissibility (Tm) named as Tm

wR=2.5;
cR=[0.2,0.4,0.6,0.8,1.0];
[cRrows,cRcolumns]=size(cR);
Tm=[0,0];

fileID1 = fopen('Question2Results.txt','w');
while cRcolumns>0
    cRtmp=cR(cRcolumns);
    Tm(cRcolumns)=sqrt((1+(2*wR*cRtmp)^2)/((1-wR^2)^2+(2*wR*cRtmp)^2));
    if Tm(cRcolumns)<=0.5
        disp("Tm = "+Tm(cRcolumns));
        disp("C/Cc = "+cRtmp);
        fprintf(fileID1,'Tm = %f -> C/Cc = %.1f\n',Tm(cRcolumns),cRtmp);
    end
    cRcolumns=cRcolumns-1;
end
fclose(fileID1);
```

Figure 4 filter out the appropriate damping factors

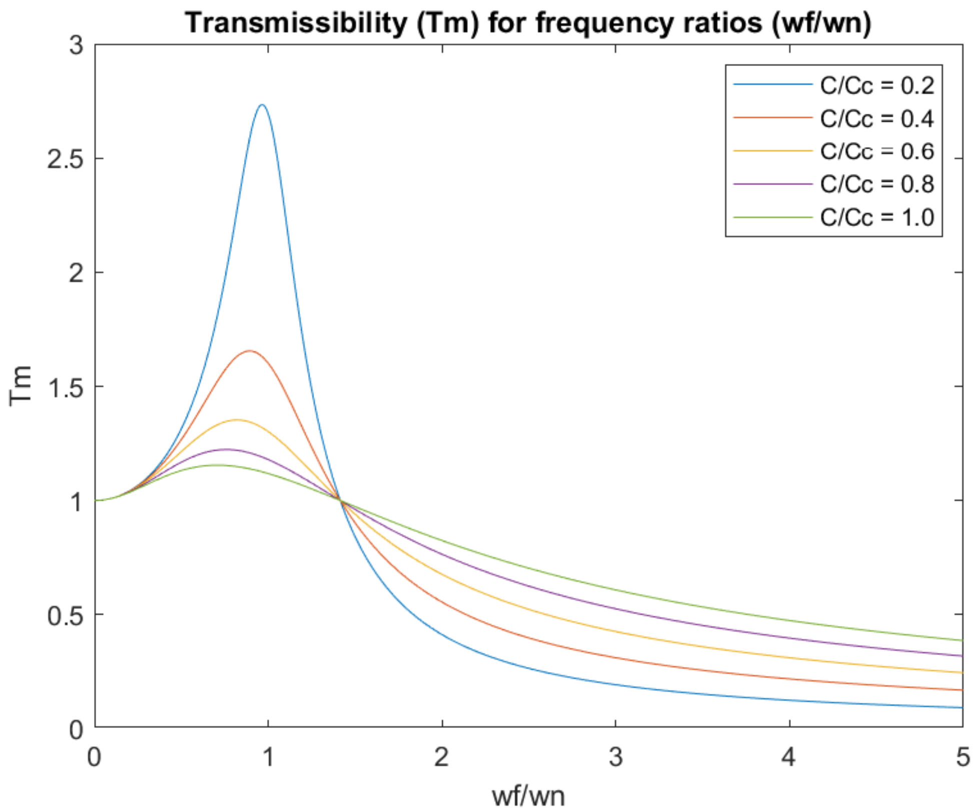
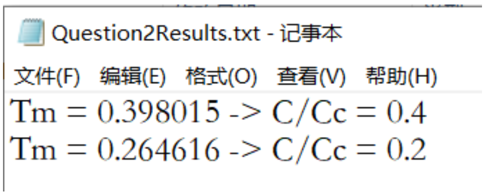


Figure 5 results for question 1


Question2Results.txt - 记事本

文件(F) 编辑(E) 格式(O) 查看(V) 帮助(H)

Tm = 0.398015 -> C/Cc = 0.4

Tm = 0.264616 -> C/Cc = 0.2

Figure 6 results for question 2