A Solution for Three-Bar-Linkage Mechanism Problem

Introduction

The plane linkage mechanism is widely used in various machines and instruments, such as in internal combustion engines, mixers, conveyors, robots and so on. The simplest plane linkage is composed of four components, called plane four-bar mechanism. It has a wide range of applications and is the basis of a multi-bar mechanism. In this project, we are expected to simplify the plane multi-bar mechanism, to analyze the motion law of the three-bar-linkage mechanism model. Bars are fixed on two points, point O and point C. The bar 1 has an initial angular velocity and a constant angular acceleration. The details of the model are shown in Figure 1. This project will supply a problem-solving idea to obtain the linkage change relationship of their motion state by researching these three bars' angular velocity and their angular acceleration.

Illustration

Analyze the question and determine constraints

1. With the change of the motion state of the bar 1, bar 2 and bar 3 will change their motion state relatively. Therefore, the question is expected to determine the relationship of the angular velocity and angular acceleration of three bars.
2. One end of bar 1 is fixed at point O. Meanwhile, one end of bar 3 if fixed at point C. The positions of the Point O and Point C are known. In this case, the space Cartesian coordinates can be created and all of bars can be digitized with the determined coordinate values. The details analysis of the points' position is shown in Figure 2.

Analyze the angular velocity and the angular acceleration

1. Point A and Point B are all in bar2. The angular velocities of these two points have a relationship that the angular velocity of Point B equals to the angular velocity of Point A plus the rotational velocity of Point B based on Point A. The details analysis process is shown in Figure 3.
2. Similarly, the angular accelerations of Point A and Point B have a relationship that the angular acceleration of Point B equals to the angular acceleration of Point A plus the rotational acceleration of Point B based on Point A. The detailed analysis process is shown in Figure 4.

Solute the result

1. The angular velocity of bars changed with time and theta are shown in Figure 10 and Figure 11.
2. The angular acceleration of bars changed with time and theta are shown in Figure 12 and Figure 13.
3. As shown in the figures, the motion of bar 1 is simpler than others. The motion of bar 2 and bar 3 are similar.

Key Codes

The positions of points are determined and calculated at fist as shown in Figure 7. With the help of the built-in function of Matlab, solve(), the complex equations with multiple unknowns can be solved. Then the angular velocity equation and the angular acceleration equation can be defined as shown as Figure 8 and Figure 9.
Define the independent variables theta and time. And then used them to calculate the results by calling the functions with the help of the while loop statement. (Figure 5)
Finally, visualize the results by Matlab's built-in functions plot(). (Figure 6)

Conclusion

By model and solve the three-bar-linkage mechanism problem, the project explores the relationship of the motion state among these three bars, which can guide the analysis of the plane multi-bar mechanism motion. This model ignores the mass of the bars and the friction effect of the joints. In practical production, this approach is not reasonable and cannot be tolerated. Thence the project is expected to optimize urgently.

Appendix

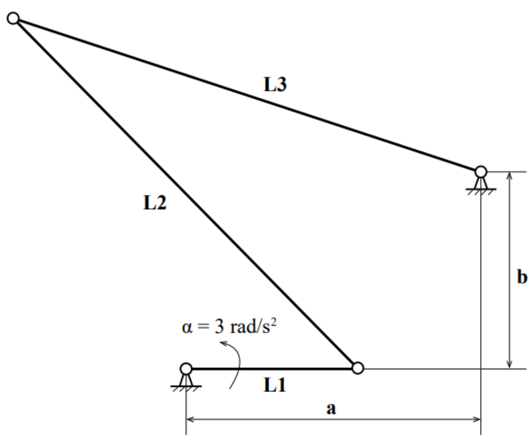


Figure 1 the three-bar-linkage mechanism model

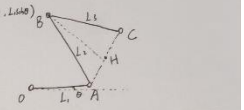
Solu:

$\theta = \frac{1}{2}\alpha t^2 \quad \theta \in [0, 2\pi] \rightarrow t \in [0, 2.05] \quad \alpha = 3 rad/s^2$

$L_1 = 0.35m \quad L_2 = L_3 = 1m \quad a = 0.6m \quad b = 0.4m$

$O(0,0,0) \quad A(L_1\cos\theta, L_1\sin\theta) \quad \vec{OA}(L_1\cos\theta, L_1\sin\theta)$

~~Bt~~ $\quad C(a, b, 0)$



For point B:

$\vec{AC}(a - L_1\cos\theta, b - L_1\sin\theta, 0)$

$\vec{AH} = \frac{1}{2}\vec{AC} = \left(\frac{a-L_1\cos\theta}{2}, \frac{b-L_1\sin\theta}{2}, 0\right) \quad |\vec{AH}| = \sqrt{\left(\frac{a-L_1\cos\theta}{2}\right)^2 + \left(\frac{b-L_1\sin\theta}{2}\right)^2}$

$\vec{HB} = \frac{|BH|}{|\vec{AH}|} \times \left(-\frac{b-L_1\sin\theta}{2}, \frac{a-L_1\cos\theta}{2}, 0\right)$ where $|BH| = \sqrt{L_2^2 - |\vec{AH}|^2}$

$\therefore \vec{AB} = \vec{AH} + \vec{HB}$

$\vec{CB} = \vec{AB} - \vec{AC}$

Figure 2 position analysis

For angular velocity w_1, w_2, w_3: $\vec{w}_1 = (0,0,w_1)$, $\vec{w}_2 = (0,0,w_2)$, $\vec{w}_3 = (0,0,w_3)$

① $w_1 = \alpha t$

$\vec{V}_B = \vec{V}_A + \vec{w}_2 \times \vec{AB}$

$\vec{V}_B = \vec{w}_3 \times \vec{CB} + \cancel{\vec{w}_2 \times \vec{AB}}$

$\vec{V}_A = \vec{w}_1 \times \vec{OA}$

$\therefore$ we can get the results of w_2 ②, w_3 ③.

Figure 3 angular velocity analysis

For angular acceleration $\alpha_1, \alpha_2, \alpha_3$: $\vec{\alpha}_1 = (0,0,\alpha_1)$, $\vec{\alpha}_2 = (0,0,\alpha_2)$, $\vec{\alpha}_3 = (0,0,\alpha_3)$

① $\alpha_1 = \alpha = 3 rad/s^2$

$\vec{a}_B = \vec{a}_A + \vec{\alpha}_2 \times \vec{AB} - w_2^2 \times \vec{AB}$

$\vec{a}_A = \vec{\alpha}_1 \times \vec{OA} - w_1^2 \cdot \vec{OA}$

$\vec{a}_B = \vec{\alpha}_3 \times \vec{CB} - w_3^2 \cdot \vec{CB}$

$\therefore$ we can get the results of α_2 ② α_3 ③.

Figure 4 angular acceleration analysis

```
t=0:0.05:2.05; %time
theta=0.5*3*(t.*t); % angle theta
[rows,columns]=size(theta);
while columns>0
    % get the angular velocity
    [a1,a2,a3]=velocitySolution(theta(columns));
    b0(columns)=theta(columns);
    b1(columns)=a1;
    b2(columns)=a2;
    b3(columns)=a3;
    %get the angular acceleration
    [c1,c2,c3]=accelerationSolution(theta(columns),a1,a2,a3);
    d1(columns)=c1;
    d2(columns)=c2;
    d3(columns)=c3;

    columns=columns-1;
end
```

Figure 5 calculate results by calling the functions

```
% plot the figure
% angular velocity -- time
figure(1);
fig1=plot(t,b1,t,b2,t,b3);
hold on
legend([fig1(1),fig1(2),fig1(3)],'w1','w2','w3');
title('Angular Velocity of L1, L2 and L3');
xlabel('time (s)');
ylabel('angular velocity (rad/s)');
```

Figure 6 visualize the results

```
%O, A, B, C 的坐标值
point0=[0, 0, 0];
pointA=[L1*cos(theta), L1*sin(theta), 0];
OA=pointA-point0;
pointC=[a, b, 0];
  %pointB->
AC=pointC-pointA;
AH=AC/2;
lengthHB=sqrt(L2^2-(norm(AH))^2);
HB=[L1*sin(theta)-b, a-L1*cos(theta), 0]*((lengthHB)/(norm(AH))); %
AB=AH+HB; %即rAB的长度
CB=AB-AC;
```

Figure 7 define the position of points

```
%angular velocity
arufa1=[0, 0, 3];
w1=[0, 0, (sqrt(2*arufa1(3)*theta))];
syms w2unknown w3unknown
w2=[0, 0, w2unknown];
w3=[0, 0, w3unknown];
vA=cross(w1, OA);
vB=[-w3(3)*CB(2), w3(3)*CB(1), 0]; %=cross(w3, CB);
vRotate=[-w2(3)*AB(2), w2(3)*AB(1), 0]; %=cross(w2, AB)
f=vB-vA-vRotate; % =[0, 0, 0];
fresult=solve(f==0);
w1K=w1(3);
w2K=fresult.w2unknown;
w3K=fresult.w3unknown;
```

Figure 8 define the angular velocity of three bars

```
%angular acceleration of L1,  L2,  L3
arufa1=[0, 0, 3];
w2=[0, 0, w2K];
syms arufa2Kunknown arufa3Kunknown
arufa2=[0, 0, arufa2Kunknown];
arufa3=[0, 0, arufa3Kunknown];
aA=(cross(arufa1, OA))+((-(w1K^2))*OA);
aB=[-arufa3(3)*CB(2), arufa3(3)*CB(1), 0]+((-(w3K^2))*CB); %=cross(arufa3, CB)
aRotate_t=[-arufa2(3)*AB(2), arufa2(3)*AB(1), 0]; %=cross(arufa2, AB)
aRotate_n=(-w2(3)*w2(3))*AB;
f=aB-aA-aRotate_t-aRotate_n; % =[0, 0, 0];
fresult=solve(f==0);
arufa1K=arufa1(3);
arufa2K=fresult.arufa2Kunknown;
arufa3K=fresult.arufa3Kunknown;
```

Figure 9 define the angular acceleration of three bars

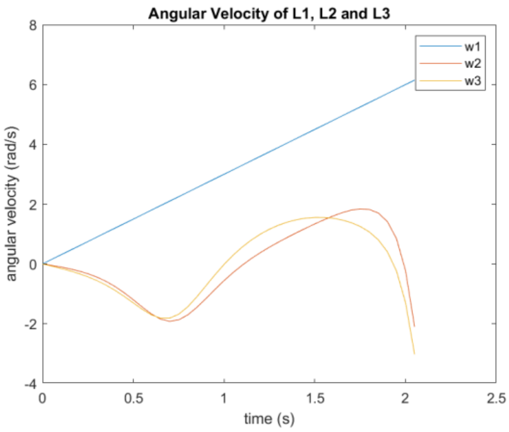


Figure 10 angular velocity / time

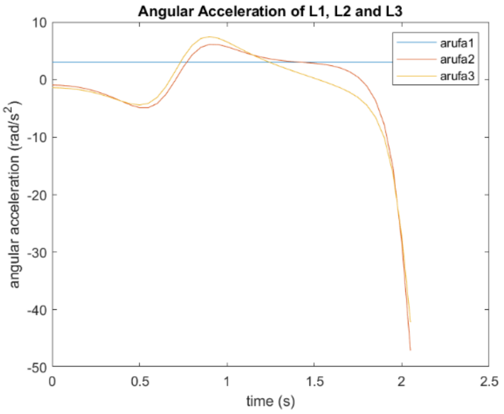


Figure 11 angular velocity / theta

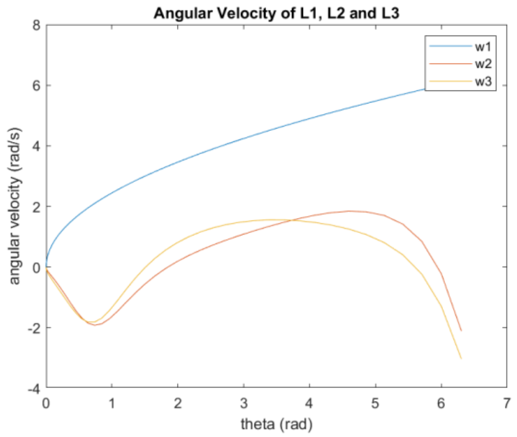


Figure 12 angular acceleration / time

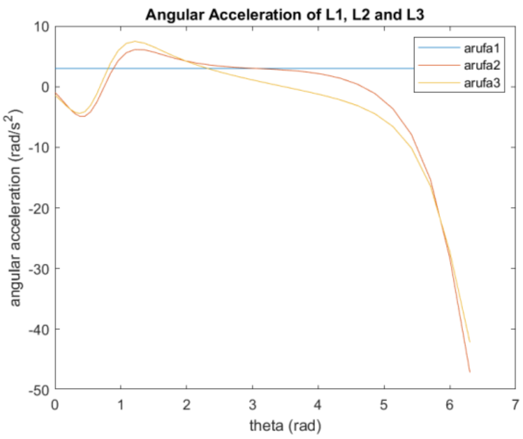


Figure 13 angular acceleration / theta