The rotation matrix for a rotation of 45° around the y-axis is:

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} .707 & 0 & .707 \\ 0 & 1 & 0 \\ -.707 & 0 & .707 \end{bmatrix}.$$

Rotating the unit vector along the x-axis by 45° around the $y$axis gives:

$$\begin{bmatrix} .707 & 0 & .707 \\ 0 & 1 & 0 \\ -.707 & 0 & .707 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} .707 \\ 0 \\ -.707 \end{bmatrix}.$$

The quaternion for a rotation of 45° around the y-axis is:

$$\cos\frac{\theta}{2} + j\sin\frac{\theta}{2} = \cos 22.5° + j\sin 22.5° = .924 + .383j\,.$$

The norm is one as required for a quaternion representing a rotation: $\sqrt{.924^2 + .383^2} = 1$.
Rotating the unit vector along the x-axis by 45° around the $y$axis gives:

$$\begin{aligned} q_R v q_R^* &= (.924 + 0i + .383j + 0k)(0 + 1i + 0j + 0k)(.924 - 0i - .383j - 0k) \\ &= 0 + (.853 - .147)i + 0j + (-.354 - .354)k \\ &\approx (.707i - .707k)\,, \end{aligned}$$

which is what we got from the Euler angle rotation matrix.
This can also be computed by transforming the rotation quaternion $q_0 + q_1 i + q_2 j + q_3 k$ into a rotation matrix:

$$M = 2 \cdot \begin{bmatrix} q_0^2 + q_1^2 - 0.5 & q_1q_2 - q_0q_3 & q_0q_2 + q_1q_3 \\ q_0q_3 + q_1q_2 & q_0^2 + q_2^2 - 0.5 & q_2q_3 - q_0q_1 \\ q_1q_3 - q_0q_2 & q_0q_1 + q_2q_3 & q_0^2 + q_3^2 - 0.5 \end{bmatrix}.$$

For $.924 + 0i + .383j + 0k$:

$$M = 2 \cdot \begin{bmatrix} .924^2 - .5 & 0 & .924 \cdot .383 \\ 0 & .924^2 + .383^2 - .5 & 0 \\ -.924 \cdot .383 & 0 & .924^2 - .5 \end{bmatrix} = 2 \cdot \begin{bmatrix} .354 & 0 & .354 \\ 0 & .5 & 0 \\ -.354 & 0 & .354 \end{bmatrix}.$$

Rotating the unit vector along the x-axis:

$$2 \cdot \begin{bmatrix} .354 & 0 & .354 \\ 0 & .5 & 0 \\ -.354 & 0 & .354 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} .707 \\ 0 \\ .-707 \end{bmatrix}$$

gives the same result.