# Informatics II, Spring 2023, Exercise 12

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## Task 1

Consider the following undirected Graph $G_1$:

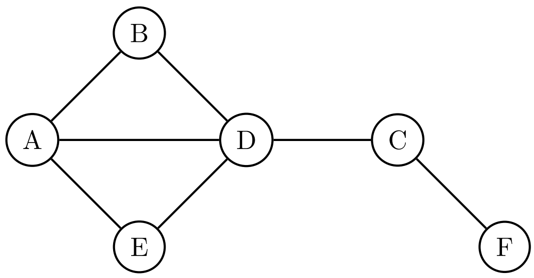



Figure 1: $G_1$

1. Which of the following set of vertices in $G_1$ build up a cycle:

   A. ABDEA

   B. ABDC

   C. ABDBA

2. Which of the following sequences in $G_1$ could be the result of a BFS:

   A. EABDCF

   B. CFDAEB

   C. CDEABF

   D. BADCFE

3. Which of the following sequences in $G_1$ could be the result of a DFS:

A. ABEDCF

B. CDFABE

C. DCFABE

D. EADBCF

4. Which statements about DFS and BFS are correct?

A. The BFS algorithm con be implemented using a Queue, where as DFS relies on a recursive algorithm.

B. By applying the DFS algorithm one can compute the minimal distances from the starting vertex to all Vertices in the Graph by reading its stime (When vertex was first visited).

C. Both algorithms BFS and DFS eventually visit all vertices in a Graph.

D. The BFS algorithm can be applied in a unweighted Graph to find the shortest paths.

## Graphs

Consider a Graph $G_2$ given by the following adjacency matrix.
(To answer the questions, it may help you to draw out the graph $G_2$.)

$$G_2 \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Mark as True or False and justify your answer:

5. The Graph $G_2$ is acyclic.

6. The Graph $G_2$ is a directed Graph.

7. Consider a Graph G with edges of negative weight. To get the shortest path from Vertex s to t one must use the Bellman-Ford Algorithm, since the Dijkstra algorith may return a false result.

## MST and SSSP

In the following tasks, consider the Graph $G_3$ represented below.

8. What is the total weight of the MST?

A. 19

B. 21

C. 23

D. 27

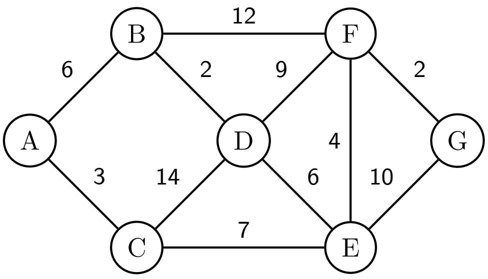



Figure 2: $G_3$

9. What is the shortest path from A to G?

   A. ACEFG

   B. ABDFG

   C. ACEG

   D. ACDFG

10. Select all correct statements:

    A. In the implementation of the Dijkstra Algorithm one can choose to use a min or max Priority Queue.

    B. By applying relaxations the distance of a vertex v from the starting vertex s may be decreased.

    C. The shortest path in a Graph $G = (V, E)$ is always less than $|V|/2$

    D. A shortest path must have no cycle

# Task 2: Finding cycle in undirected Graph

Write a pseudocode algorithm that given a Graph G returns TRUE if the Graph contains a cycle and FALSE if it does not.

Every vertex $v \in V$ has the following properties:

- v.adj: A list with all vertices adjacent to v
- v.col: The coloring of the vertex (White, Grey, Black)
- v.pred: The predecessor of v

You are free to use further properties as it suits your algorithm.

Hint: Try to solve this problem with a DFS approach.

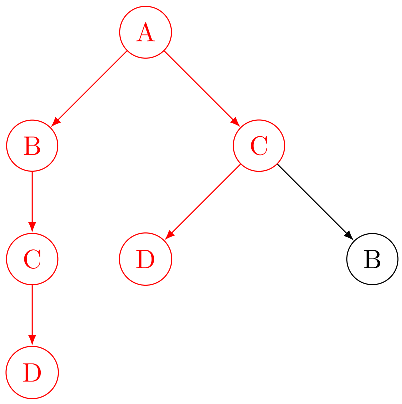
## Task 3: Count all possible Paths between two Vertices

Write a pseudocode algorithm that given the adjacency Matrix M of a Graph, a source vertex s and a destination vertex t returns the number of all possible simple paths between those vertices. Be aware that the Graph may contain cycles and you are not allowed to visit a vertex twice.

Hint: Use a DFS approach to enumerate all possible paths from s to t.

For example, for the Graph given below there are 2 paths from Vertex A to D marked in red. Every branch in the Tree represents a possible simple path from vertex A. Keep track of the visited vertices in the current branch by using a list or similar to avoid visiting a vertex twice which would lead to a cycle.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$




## Task 4: k-hop [20 FS Final Exam]

Consider a directed graph $G = (V, E)$ with nodes $v \in V$ and $u \in V$. Node $u$ is a $k$-**hop neighbor** of node $v$ if there is at least one path from $v$ to $u$ and the minimum distance from $v$ to $u$ is $k$. The distance from $v$ to $u$ is infinite if there is no path from $v$ to $u$.

Let $|V|$ be the number of nodes in $G$. Each node is identified by a unique integer between 1 and $|V|$. A directed edge $(v, u) \in E$ is represented as a pair of nodes and denotes an edge from node $v \in V$ to node $u \in V$.

a. Assume a graph $G = (V, E)$ with vertices $V = \{1, 2, 3, 4, 5, 6\}$ and edges $E = \{(1, 2), (1, 4), (1, 5), (2, 3), (2, 5), (3, 1), (4, 6)\}$. Draw the adjacency matrix of $G$.

b. Let $k = 2$. Determine all 2-hop neighbors of node 1? Explain your approach.

c. Assume a directed graph $G$ with $n$ nodes. An adjacency matrix $a[n, n]$ is used to represent the edges of the graph. Give a pseudocode algorithm that prints all $k$-hop neighbors of node $v$.