Ex 3.c

Lin search $\Theta(n)$, Bin Search $\Theta(\log(n))$

Task 2

a) Comparison: n

additions:

$$\sum_{i=1}^{n} \sum_{kj=i}^{n} 1$$

$i + k + k \ldots$

bis n

$$i + xk \leq n \Rightarrow x = \frac{n-i}{k}$$

$$\Rightarrow \sum_{i=1}^{n} \frac{n-i}{k} = \sum_{i=1}^{n} \frac{n}{k} - \sum_{i=1}^{n} \frac{i}{k}$$

$$= \frac{n}{k} \sum_{i=1}^{n} 1 - \frac{1}{k} \sum_{i=1}^{n} i$$

$$= \frac{n^2}{k} - \frac{1}{k} \cdot \frac{n(n+1)}{2}$$

$$= \frac{n^2}{k} - \frac{n^2+n}{k \cdot 2}$$

$$= \frac{2n^2 - n^2 + n}{k \cdot 2}$$

$$= \frac{n^2+n}{2 \cdot k}$$

both operations

$$\frac{n^2+n}{2k} + n$$

for assignements worst case

$$\frac{n^2+n}{2k} + n$$

$\Rightarrow$ Algo $\in O(n^2)$

Task 3

fastest $\longrightarrow f_8(n) = 1440$, constant time

$$\wedge|$$

$$f_7(n) = \log^2(n) + 50\sqrt{n} + \log(n) =: \text{(I)}$$

speculation this goes to 0

$$\frac{\log^2(n) + 50\sqrt{n} + \log(n)}{12\sqrt{n} + 10^{223} + \underbrace{\log 5^n}_{\overset{\shortparallel}{n \log 5}}} \cdot 1 \quad 1 = \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \frac{\frac{\log^2(n)}{n} + \frac{50\sqrt{n}}{n} + \frac{\log(n)}{n}}{\frac{12\sqrt{n}}{n} + \frac{10^{223}}{n} + \frac{n\log(5)}{n}} =: \text{(II)}$$

$$\text{(III)} := \frac{\log^2(n)}{n} = \frac{e^{\ln(\log^2(n))}}{e^{\ln(n)}}$$

$$= e^{\ln(\log^2(n)) - \ln(n)}$$

$$= e^{2 \cdot \ln(\log(n)) - \ln(n)}$$

$$\downarrow -\infty$$

$\Rightarrow$ (III) $\to 0$

$\Rightarrow$ (II) $\to 0$

$$\Rightarrow \quad \underset{\mathrm{12\sqrt{n} + 10^{223} + \log 5^n = (IV)}}{\underset{\wedge|}{(I)}}$$

$$\log n^{2n+1} = 2n \log(n) + \log(n)$$

speculation this goes to 0

$$\frac{2n \log(n) + \log(n)}{12\sqrt{n} + 10^{223} + n \log(5)} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \frac{2\log(n) + \frac{\log(n)}{n}}{\frac{12\sqrt{n}}{n} + \frac{10^{223}}{n} + \log(5)} \to \infty$$

$\Rightarrow 2n\log(n) + \log(n) \leq$ (IV)

thus

$$f_8 \leq \underbrace{f_4 \leq}_{\text{Proof}} f_7 \leq f_6$$

$$\frac{12\sqrt{n} + c + n\log 5}{\log^2(n) + 50\sqrt{n} + \log(n)} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$\downarrow$$

c equally fast

speculation: goes to 0

$$\frac{2n\log(n) + \log(n)}{f_5} \longrightarrow 0$$

$$\Rightarrow f_6 \leq f_5$$

$$\frac{10^{1020}n^4 + 8^{227}n^3 + 20^{231}n^2 + 128n\log n}{4^{\log_2 n}} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}$$

spec! goes to 0

$$\frac{4^{\log_2 n}}{n^4} = \frac{e^{\ln(4^{\log_2(n)})}}{e^{4\ln(n)}}$$

$$= e^{\log_2(n)\ln(4) - 4\ln(n)} \longrightarrow 0$$

$$\Rightarrow \quad f_3 \leq f_5 \quad , f_6 \leq f_3$$

$$\frac{2n\log(n) + \log(n)}{4^{\log_2(n)}}$$

$$\frac{e^{\ln(2n\log(n))} + e^{\ln(\log(n))}}{e^{\log_2(n)\ln(4)}} \to 0$$

$$f_3 \leq f_2 \quad , f_2 \leq f_5$$

easy to show

Lastly

show $f_5 \leq f_7$

$$\frac{C_1 n^4 + C_2 n^3 + C_4 n^2 + C_5 \log(n)}{(2n+3)!}$$

$$= \frac{C_1 n^4}{(2n+3)!} + \frac{C_2 n^3}{(2n+3)!} + \frac{C_4 n^2}{(2n+3)!} + \frac{C_5 \log(n)}{(2n+3)!}$$

if $\frac{c_1 n^4}{(2n+3)!} \longrightarrow 0$ then the other sums also go to 0

$$\shortparallel$$

$$\frac{c_1 n \cdot n \cdot n \cdot n}{(2n+3)(2n+2)(2n+1)(2n)\cdot(2n-1)}$$

$$\leq \frac{c_1}{(2n-1)!} \longrightarrow 0$$

hence

$$f_8 \leq f_7 \leq f_6 \leq f_3 \leq f_2 \leq f_5 \leq f_1$$

Task 7

a) Pre condition

array A[1...n] with n distinct integers and $k \leq n$

Post

A[1...k] is sorted in de-creasing order

sum is the sum of all elements 1...k

b) i) both loops are up.

ii) the left subarray

- 1 ... i-1 is alwas decreasing sorted (inner-loop)

- maxi is the largest element in the sub-array A[i ... j] (outer-loop)

c) best/worst-case (comparisons)

$\Theta(n^2)$

best / Worst case (assignements)

$4k$ $k^2 + 4k$

c) - Array empty

- correct output

d)

comparisons :

$$\sum_{i=1}^{k} \sum_{j=i}^{n} 1 = \sum_{i=1}^{k} n - i + 1$$

$$= k \cdot n - \sum_{i=1}^{k} i + k$$

$$= n \cdot k + k - \frac{k(k+1)}{2}$$

$$\in O(n^2)$$

exam task

false

$f_1(n) + f_2(n) + f_3(n)$

$= O(1) + O(N^2) + O(N \log N)$

$= O(1 + N^2 + N \log N)$

$= O(N^2)$