

Chap. 6 (Part 2 : 6.4~6.6)

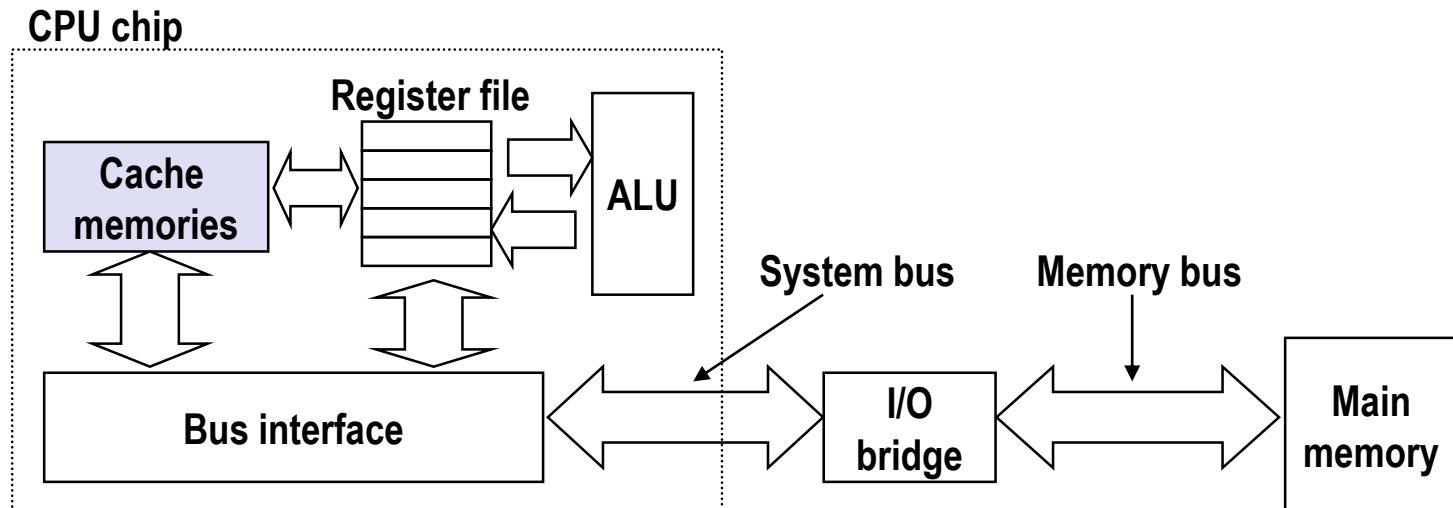
The Memory Hierarchy

Today

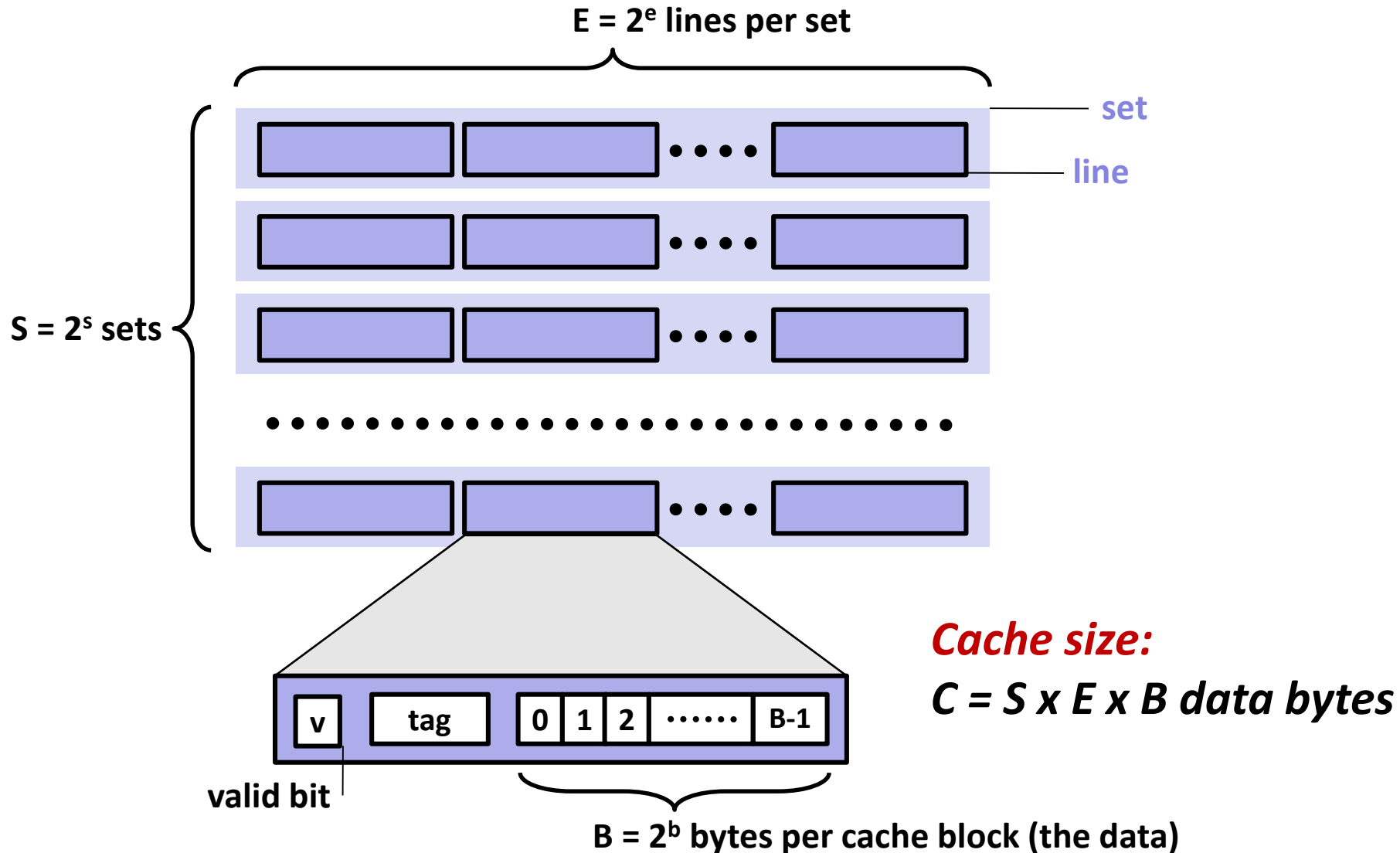
- **Cache memory organization and operation**
- **Performance impact of caches**
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

Cache Memories

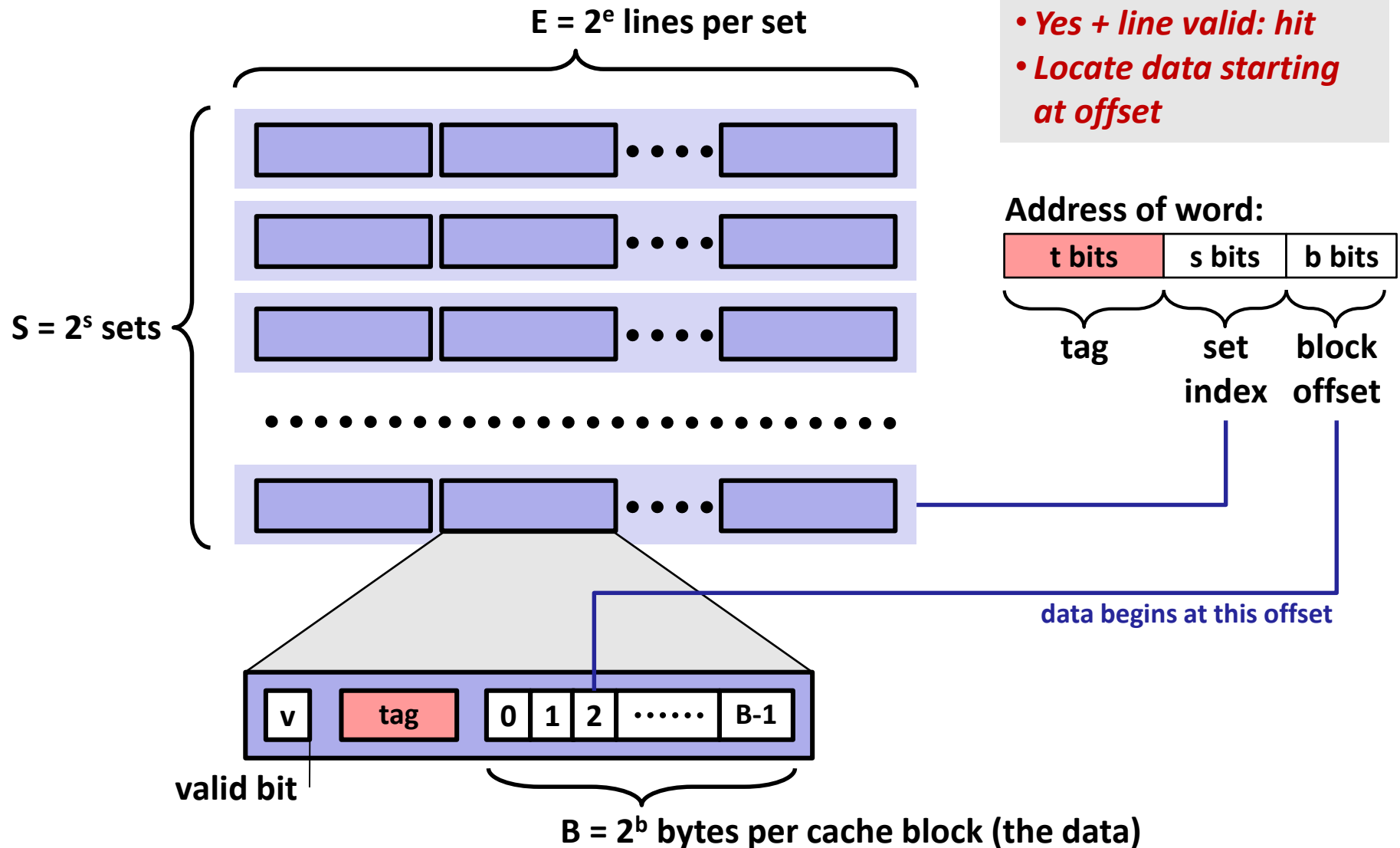
- **Cache memories** are small, fast SRAM-based memories managed automatically in hardware.
 - Hold frequently accessed blocks of main memory
- CPU looks first for data in caches (e.g., L1, L2, and L3), then in main memory.
- Typical system structure:



General Cache Organization (S, E, B)



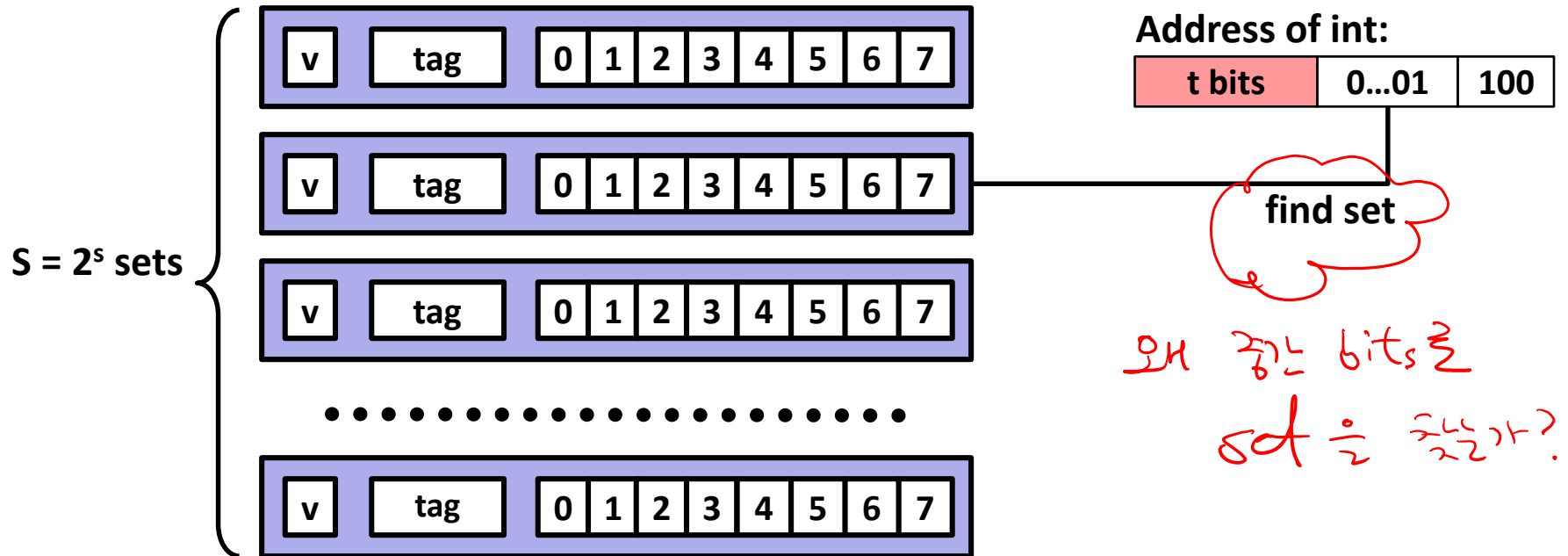
Cache Read



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

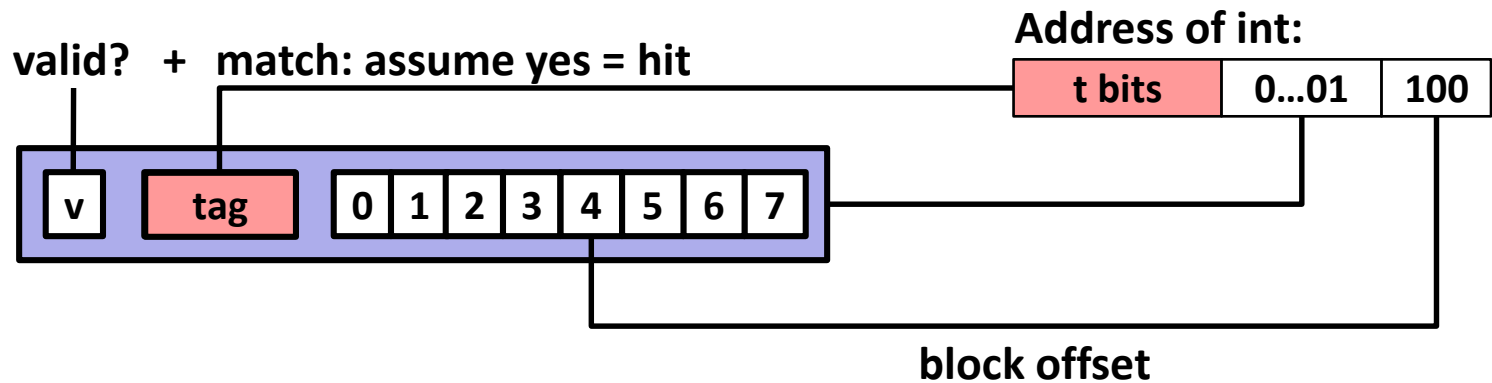
Assume: cache block size 8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

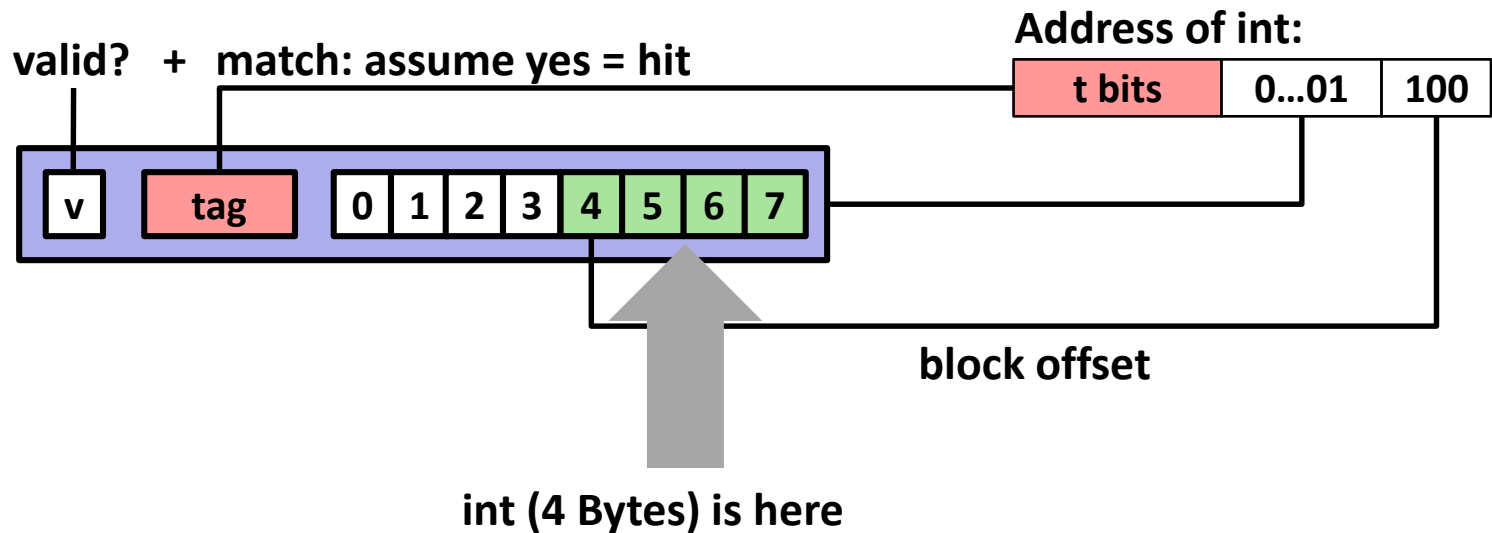
Assume: cache block size 8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

Assume: cache block size 8 bytes



No match: old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 byte addresses, B=2 bytes/block,
S=4 sets, E=1 Blocks/set

Address trace (reads, one byte per read):

0	[<u>0000</u> ₂],	miss
1	[<u>0001</u> ₂],	hit
7	[<u>0111</u> ₂],	miss
8	[<u>1000</u> ₂],	miss
0	[<u>0000</u> ₂]	miss

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1			
Set 2			
Set 3	1	0	M[6-7]

A Higher Level Example

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (i = 0; i < 16; i++)
        for (j = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

```
int sum_array_cols(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (j = 0; i < 16; i++)
        for (i = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

Ignore the variables sum, i, j

assume: cold (empty) cache,
a[0][0] goes here



작업
84바이트!

32 B = 4 doubles

blackboard

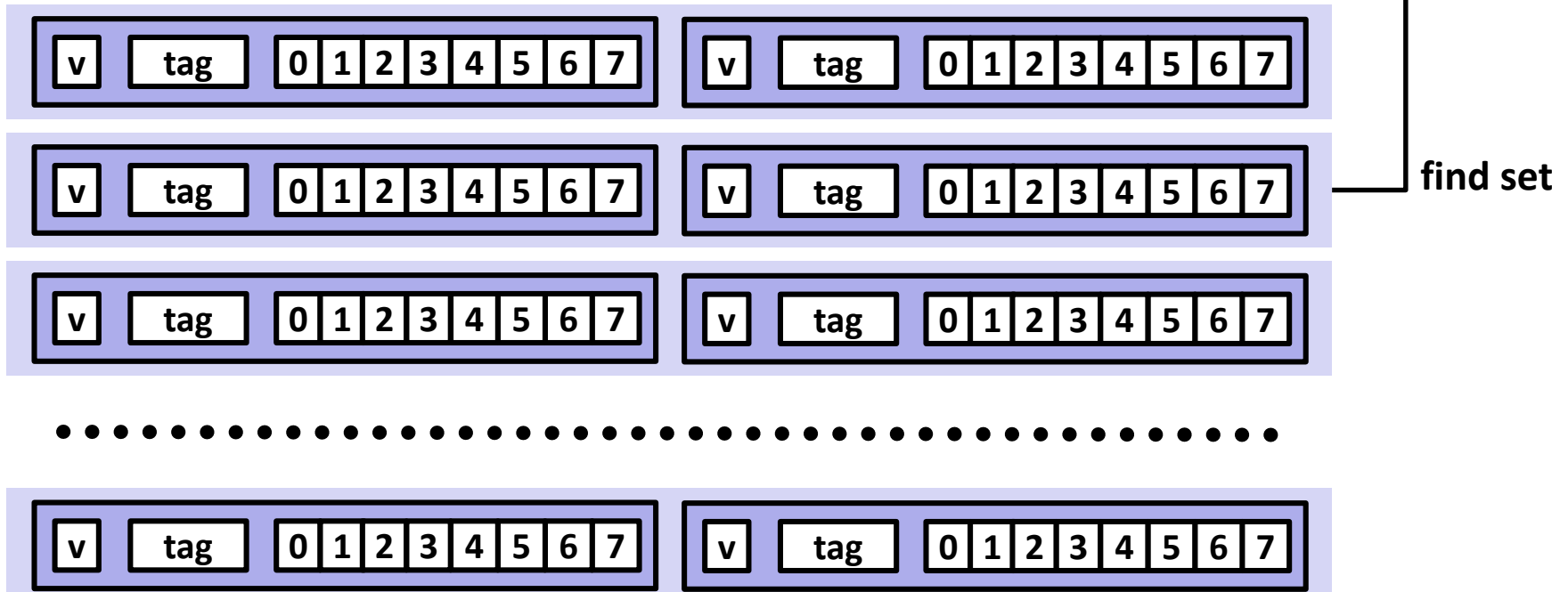
E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes

Address of short int:

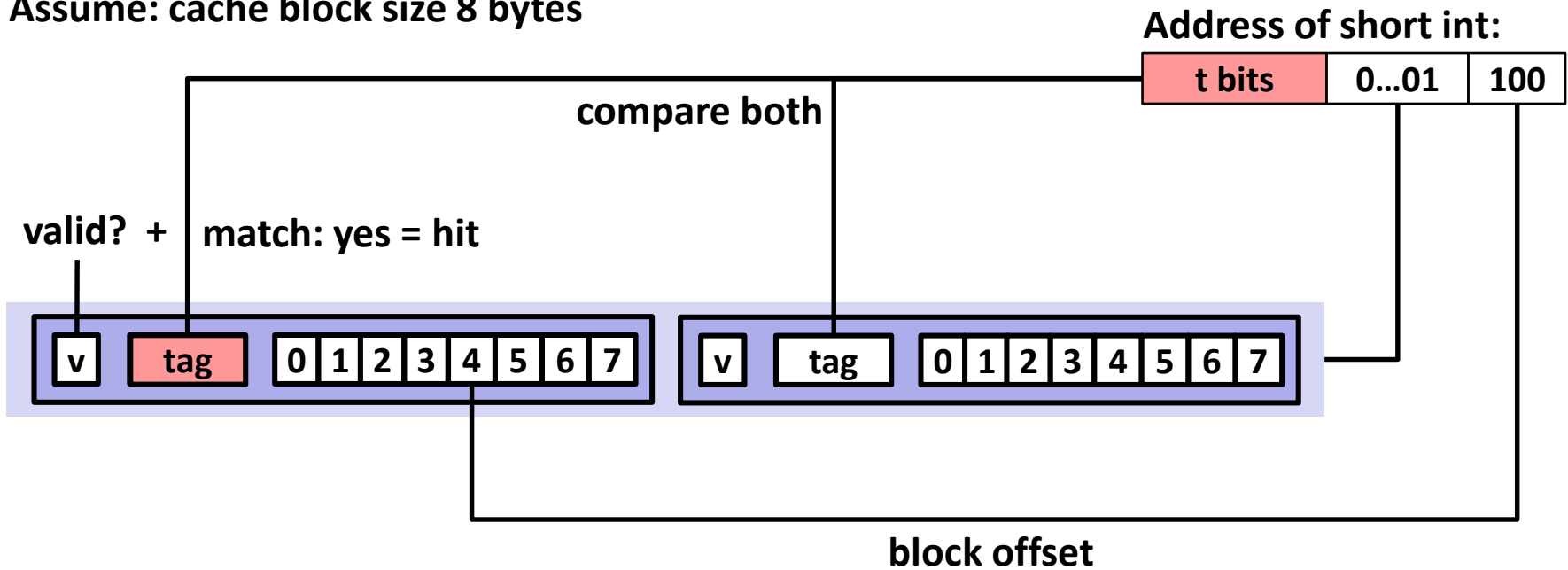
t bits	0...01	100
--------	--------	-----



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

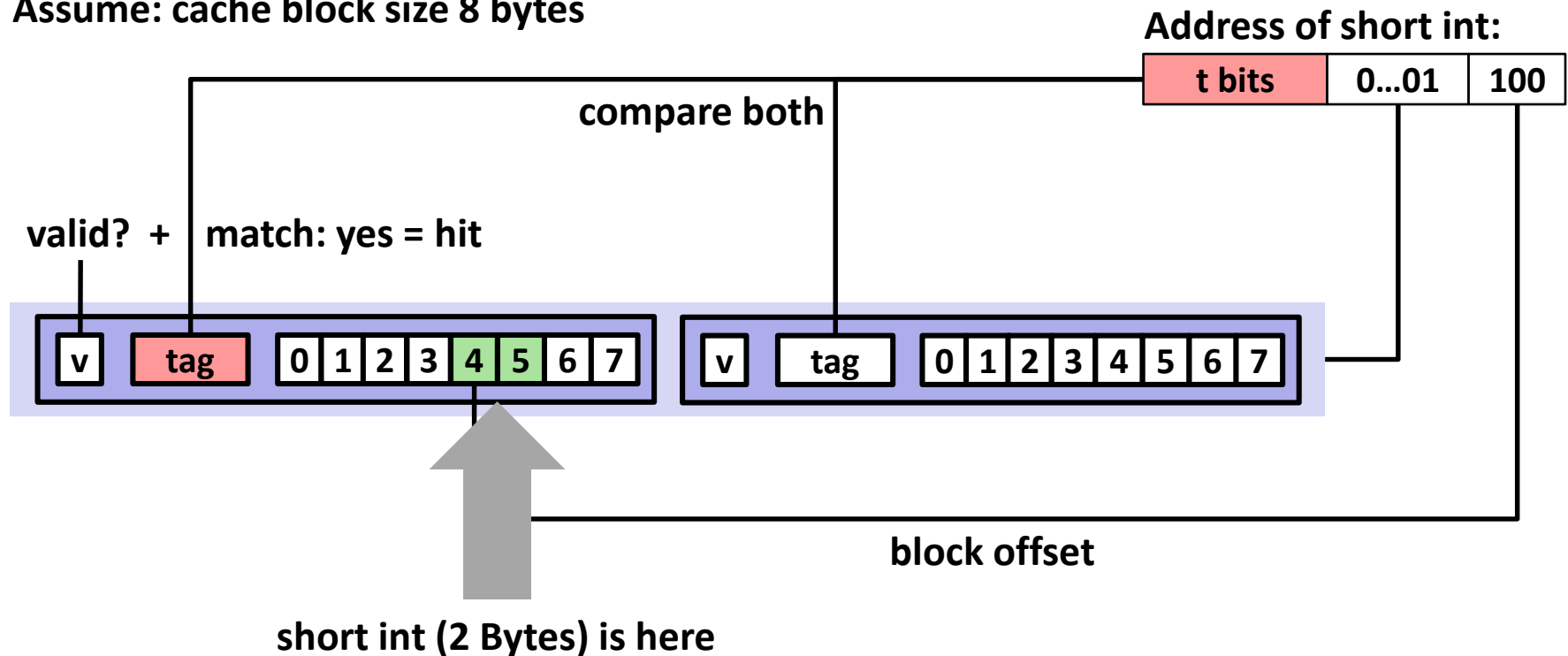
Assume: cache block size 8 bytes



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes



No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0	[0000 ₂],	miss
1	[0001 ₂],	hit
7	[0111 ₂],	miss
8	[1000 ₂],	miss
0	[0000 ₂]	hit

	v	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

A Higher Level Example

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

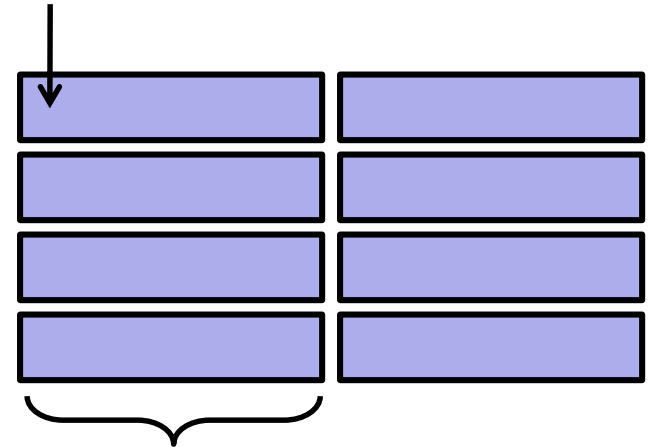
    for (i = 0; i < 16; i++)
        for (j = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (j = 0; i < 16; i++)
        for (i = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

Ignore the variables sum, i, j

assume: cold (empty) cache,
a[0][0] goes here



32 B = 4 doubles

blackboard

What about writes?

■ Multiple copies of data exist:

- L1, L2, Main Memory, Disk

■ What to do on a write-hit?

- **Write-through** (write immediately to memory)
- **Write-back** (defer write to memory until replacement of line)
 - Need a dirty bit (line different from memory or not)

■ What to do on a write-miss?

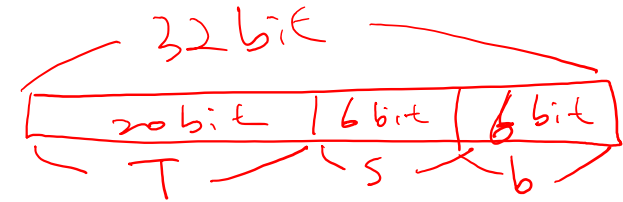
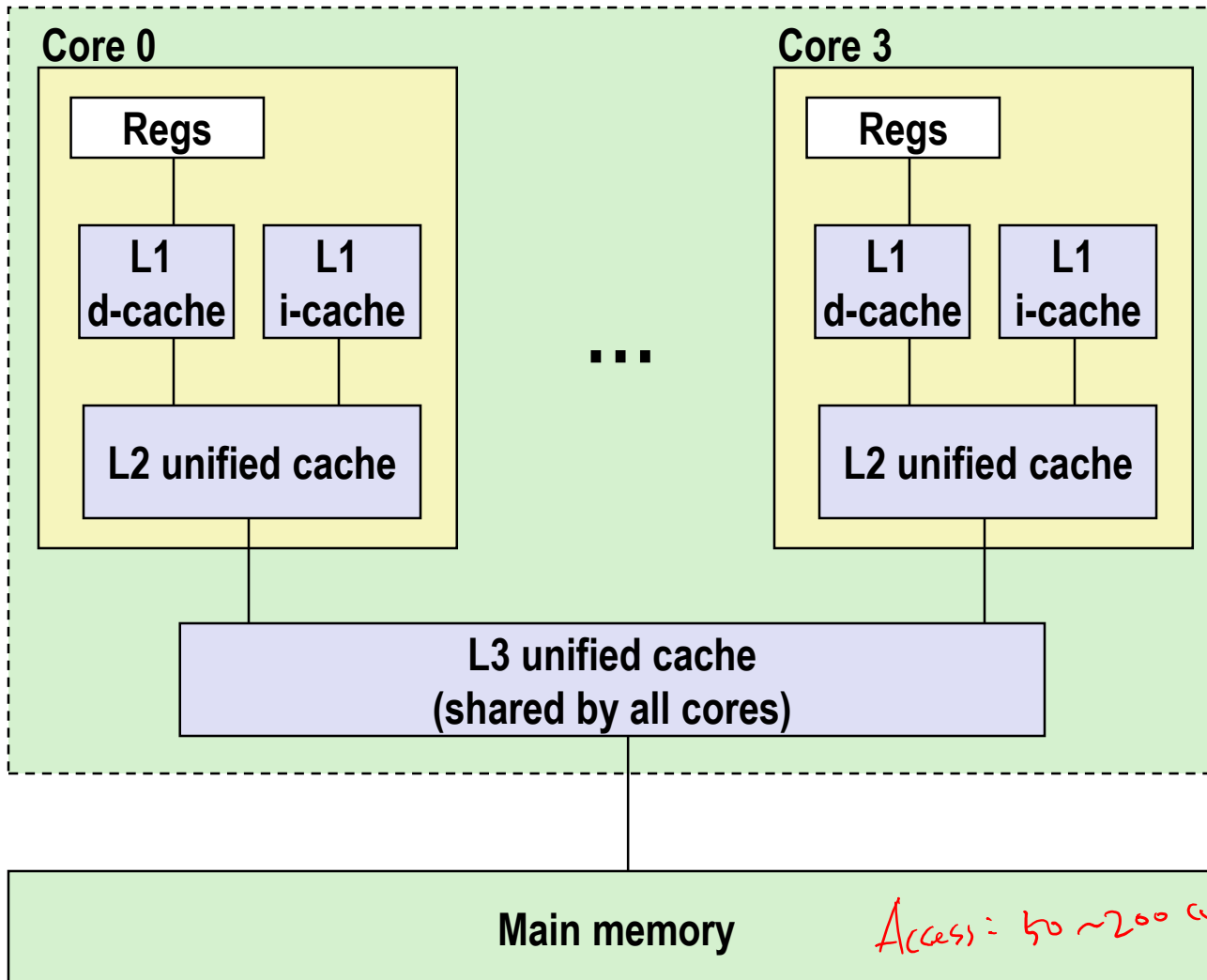
- **Write-allocate** (load into cache, update line in cache)
 - Good if more writes to the location follow
- **No-write-allocate** (writes immediately to memory)

■ Typical

- Write-through + No-write-allocate
- **Write-back + Write-allocate**

Intel Core i7 Cache Hierarchy

Processor package



L1 i-cache and d-cache:

32 KB, 8-way, 64 sets
Access: 4 cycles

L2 unified cache:

256 KB, 8-way, 512 sets
Access: 11 cycles

L3 unified cache:

8 MB, 16-way, 8192 sets
Access: 30-40 cycles

Block size: 64 bytes for all caches.

Cache Performance Metrics

■ Miss Rate

- Fraction of memory references not found in cache (misses / accesses)
= $1 - \text{hit rate}$
- Typical numbers (in percentages):
 - 3-10% for L1
 - can be quite small (e.g., $< 1\%$) for L2, depending on size, etc.

■ Hit Time

- Time to deliver a line in the cache to the processor
 - includes time to determine whether the line is in the cache
- Typical numbers:
 - 1-2 clock cycle for L1
 - 5-20 clock cycles for L2

■ Miss Penalty

- Additional time required because of a miss
 - typically 50-200 cycles for main memory (Trend: increasing!)

Lets think about those numbers

■ Huge difference between a hit and a miss

- Could be 100x, if just L1 and main memory

■ Would you believe 99% hits is twice as good as 97%?

- Consider:
cache hit time of 1 cycle
miss penalty of 100 cycles

- Average access time:

97% hits: 1 cycle + 0.03 * 100 cycles = **4 cycles**

99% hits: 1 cycle + 0.01 * 100 cycles = **2 cycles**

hit 이면 Miss 이면 100배 먼저 cache를 뒤져봐야
있을것라!

■ This is why “miss rate” is used instead of “hit rate”

Writing Cache Friendly Code

- **Make the common case go fast**
 - Focus on the inner loops of the core functions
- **Minimize the misses in the inner loops**
 - Repeated references to variables are good (**temporal locality**)
 - Stride-1 reference patterns are good (**spatial locality**)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

Today

- Cache organization and operation
- **Performance impact of caches**
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

The Memory Mountain

- **Read throughput** (read bandwidth)
 - Number of bytes read from memory per second (MB/s)
- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
 - Compact way to characterize memory system performance.

Memory Mountain Test Function

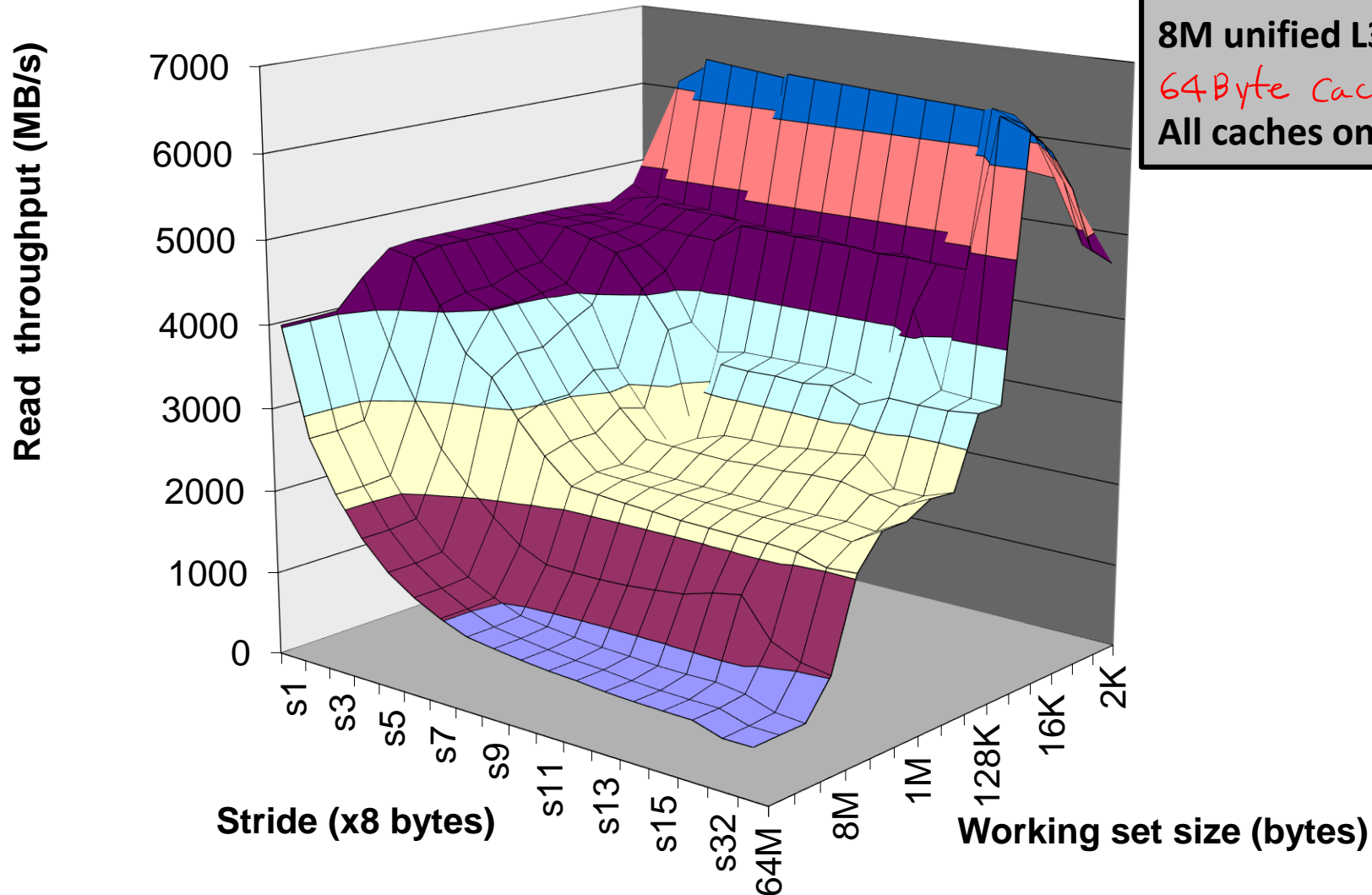
```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride); /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0); /* call test(elems, stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

The Memory Mountain



Intel Core i7

32 KB L1 i-cache

32 KB L1 d-cache

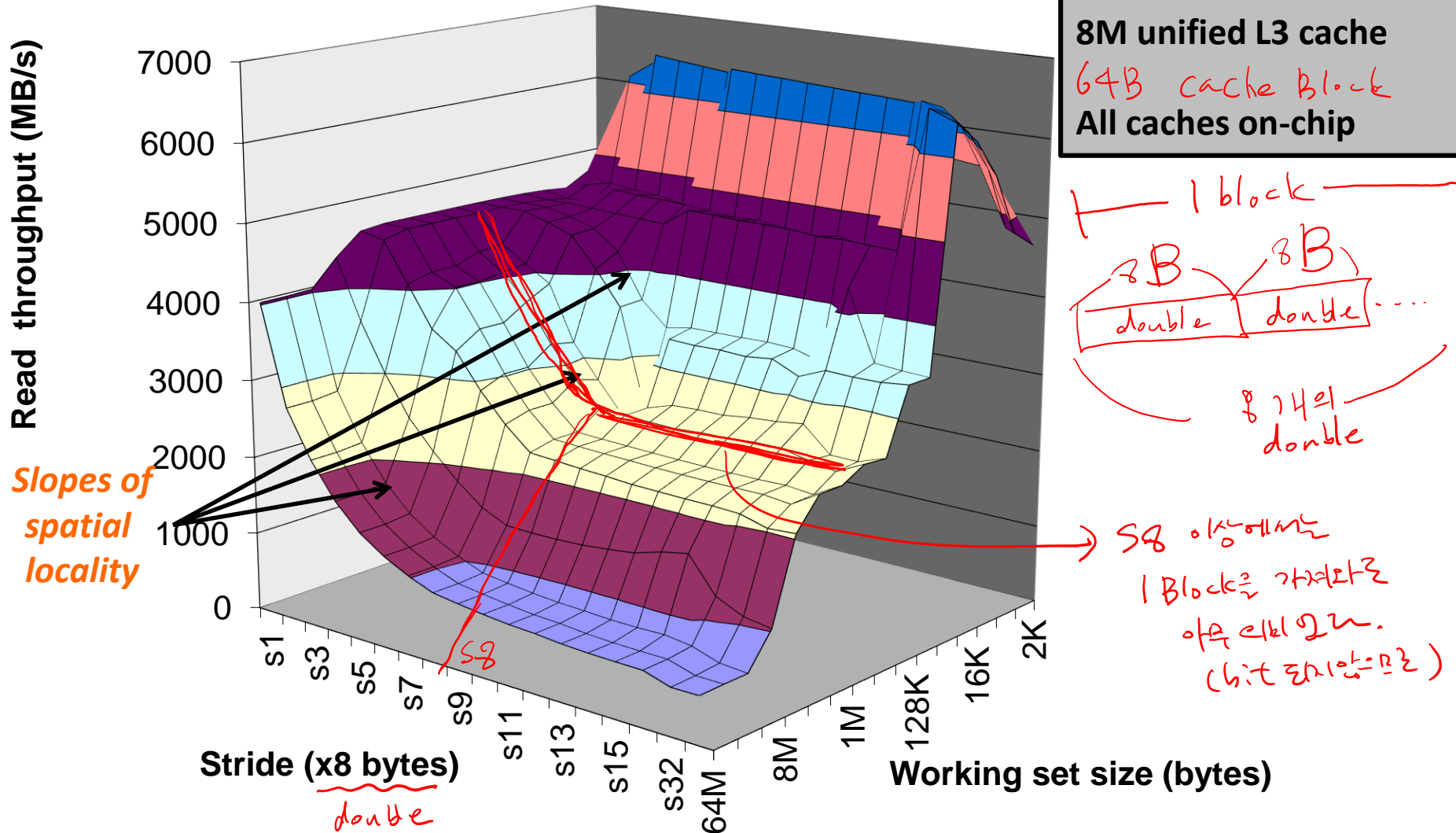
256 KB unified L2 cache

8M unified L3 cache

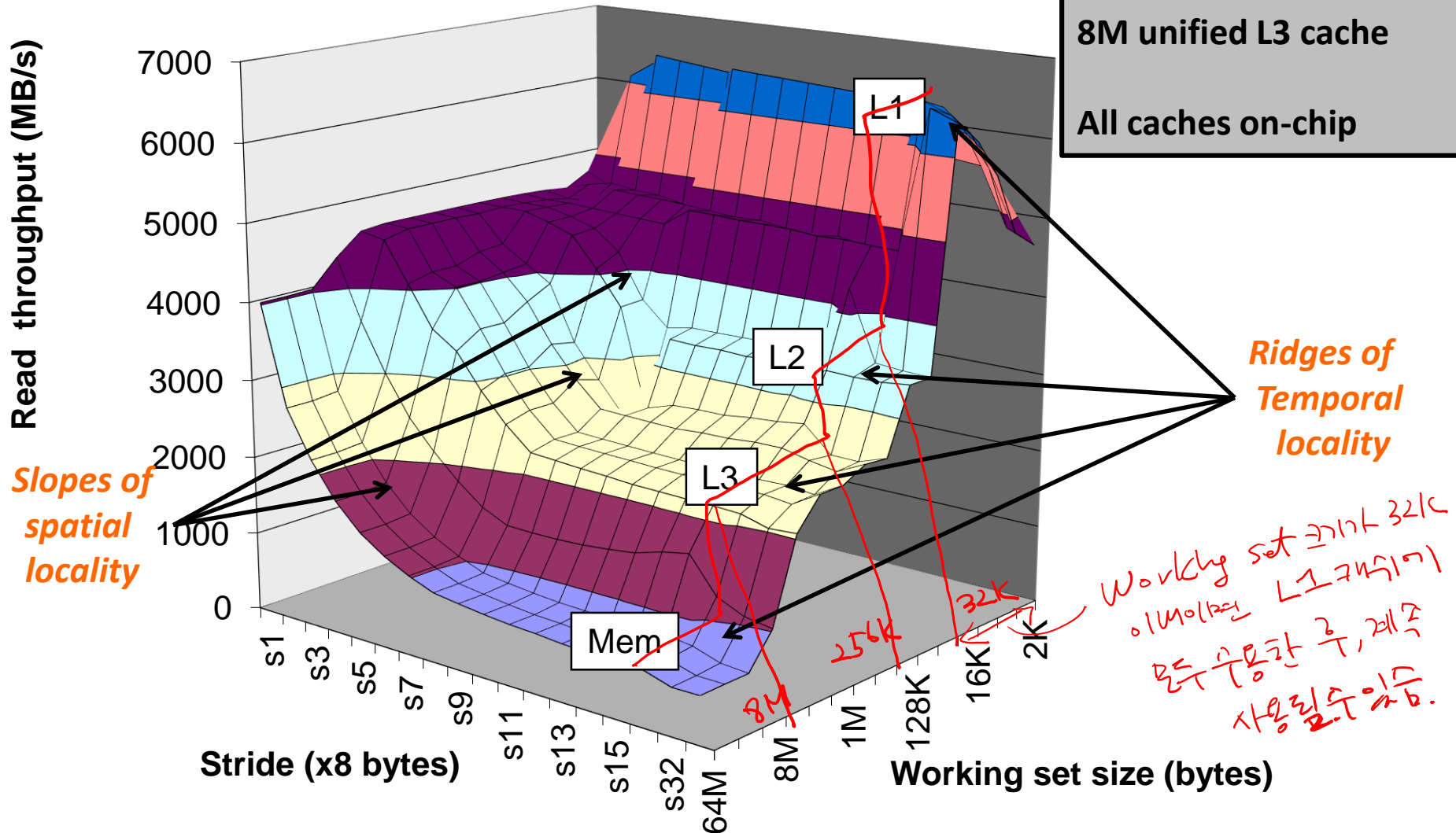
64 Byte Cache Block

All caches on-chip

The Memory Mountain



The Memory Mountain



Today

- Cache organization and operation
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 - Using blocking to improve temporal locality

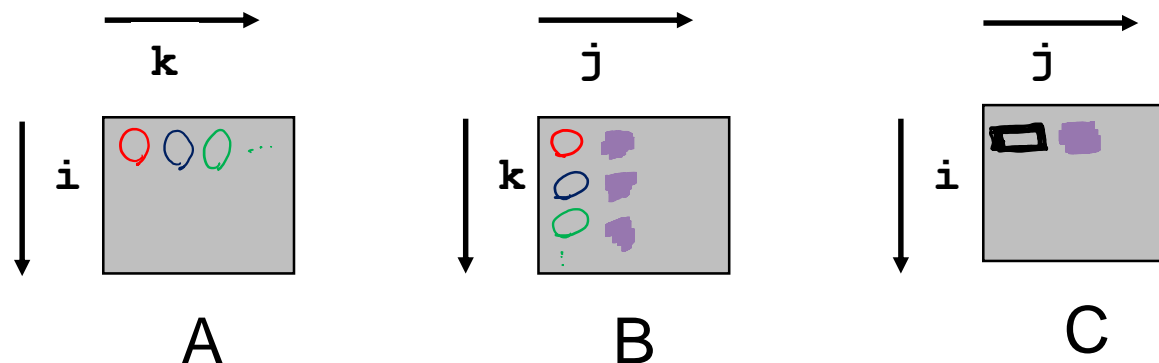
Miss Rate Analysis for Matrix Multiply

■ Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
 - Approximate $1/N$ as 0.0
- Cache is not even big enough to hold multiple rows

■ Analysis Method:

- Look at access pattern of inner loop



8B
double 4개 저장 공간

Matrix Multiplication Example

■ Description:

- Multiply $N \times N$ matrices
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination
 - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*Variable **sum**
held in register*

Layout of C Arrays in Memory (review)

■ C arrays allocated in row-major order

- each row in contiguous memory locations

■ Stepping through columns in one row:

- `for (i = 0; i < N; i++)`
`sum += a[0][i];`
- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
 - compulsory miss rate = $4 \text{ bytes} / B$

■ Stepping through rows in one column:

- `for (i = 0; i < n; i++)`
`sum += a[i][0];`
- accesses distant elements
- no spatial locality!
 - compulsory miss rate = 1 (i.e. 100%)

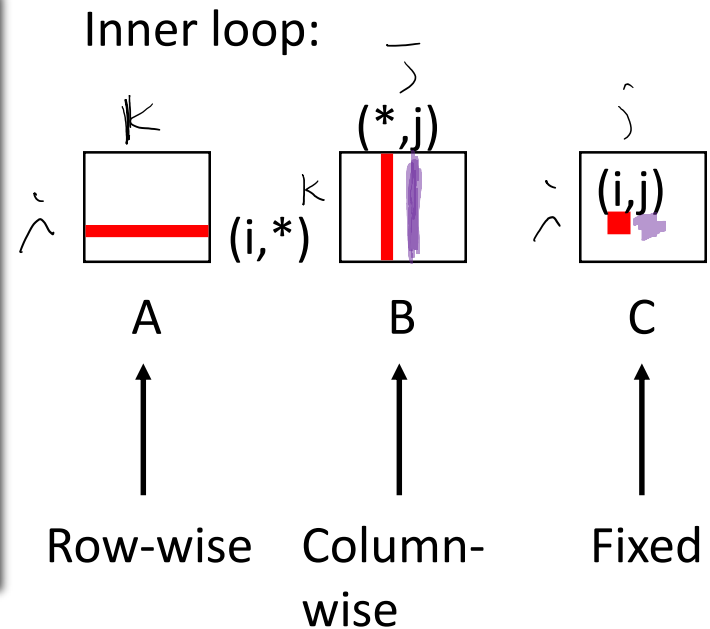
4B이면 원소 1개 크기
 4B가 가짐.
 ex) block 크기가 16B이면
 4개 원소 4개 블록.
 ⇒ 12-15번 원소부터 1 block가 가짐
 miss가 1번 발생하네
 3번은 hit!

Matrix Multiplication (ijk)

```

/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}

```



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

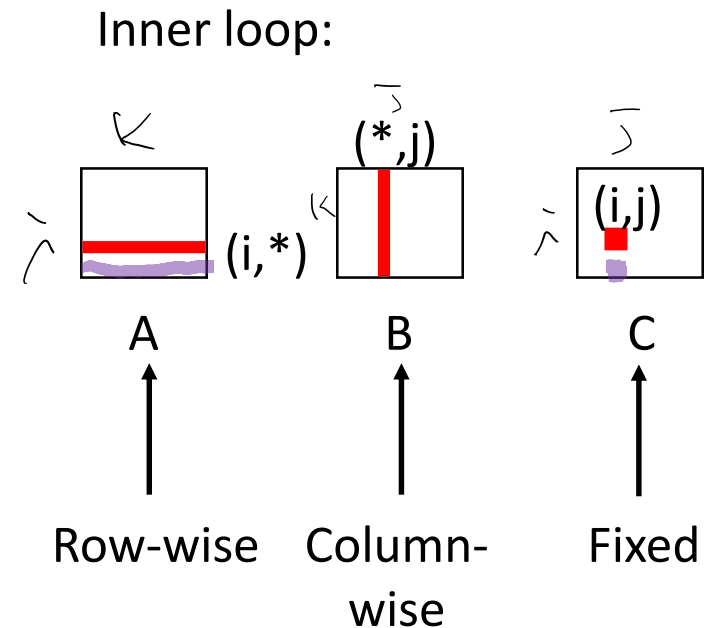
cache 한 번 읽기: $32B$
 비버려진 공간 크기: $8B$

Matrix Multiplication (jik)

```

/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}

```



Misses per inner loop iteration:

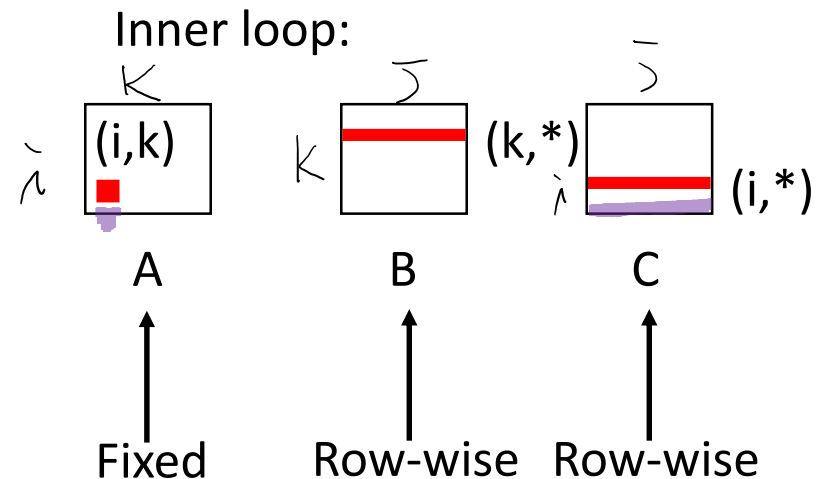
<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (kij)

```

/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}

```



Misses per inner loop iteration:

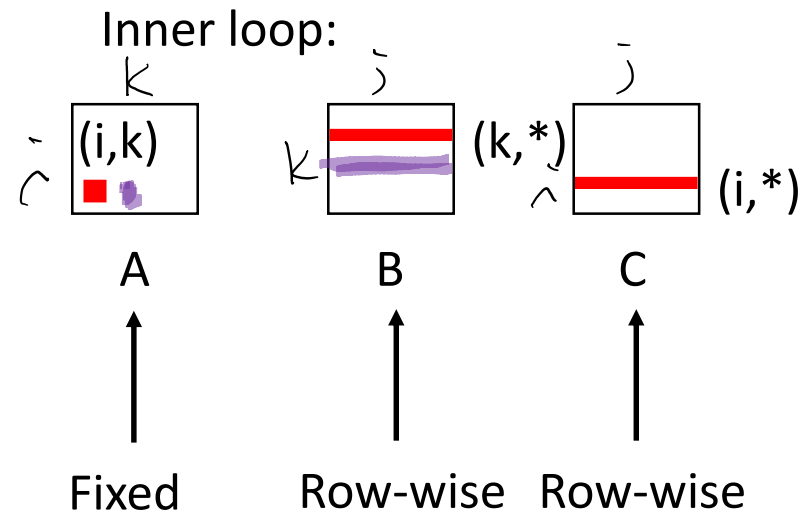
<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (ikj)

```

/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}

```



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

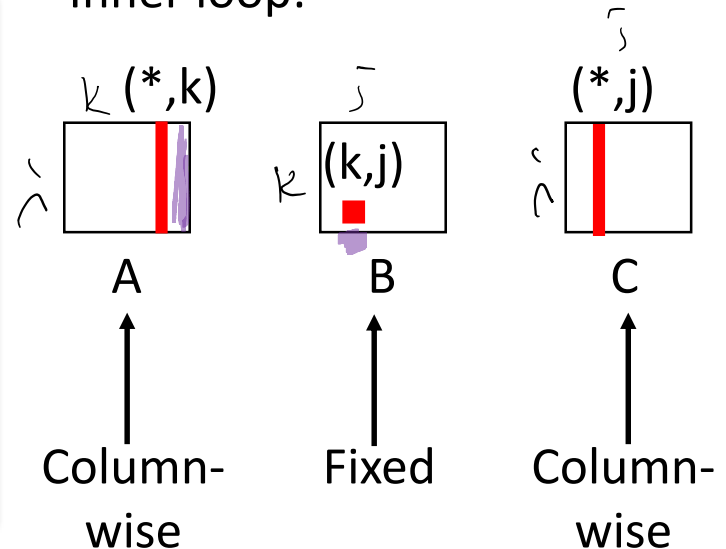
Matrix Multiplication (jki)

```

/* jki */
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}

```

Inner loop:



Misses per inner loop iteration:

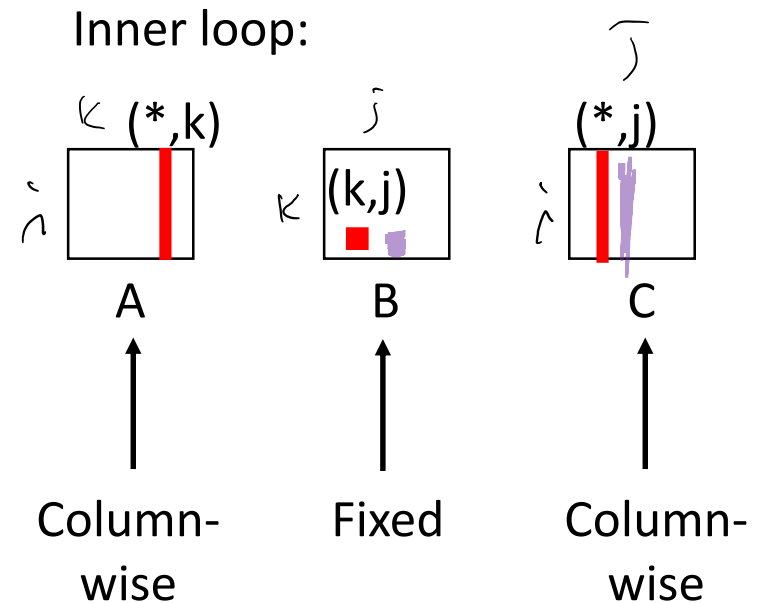
<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

Matrix Multiplication (kji)

```

/* kji */
for (k=0; k<n; k++) {
    for (j=0; j<n; j++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}

```



Misses per inner loop iteration:

A
1.0

B
0.0

C
1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = **1.25** $\frac{2}{1.6}$

```
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

kij (& ikj):

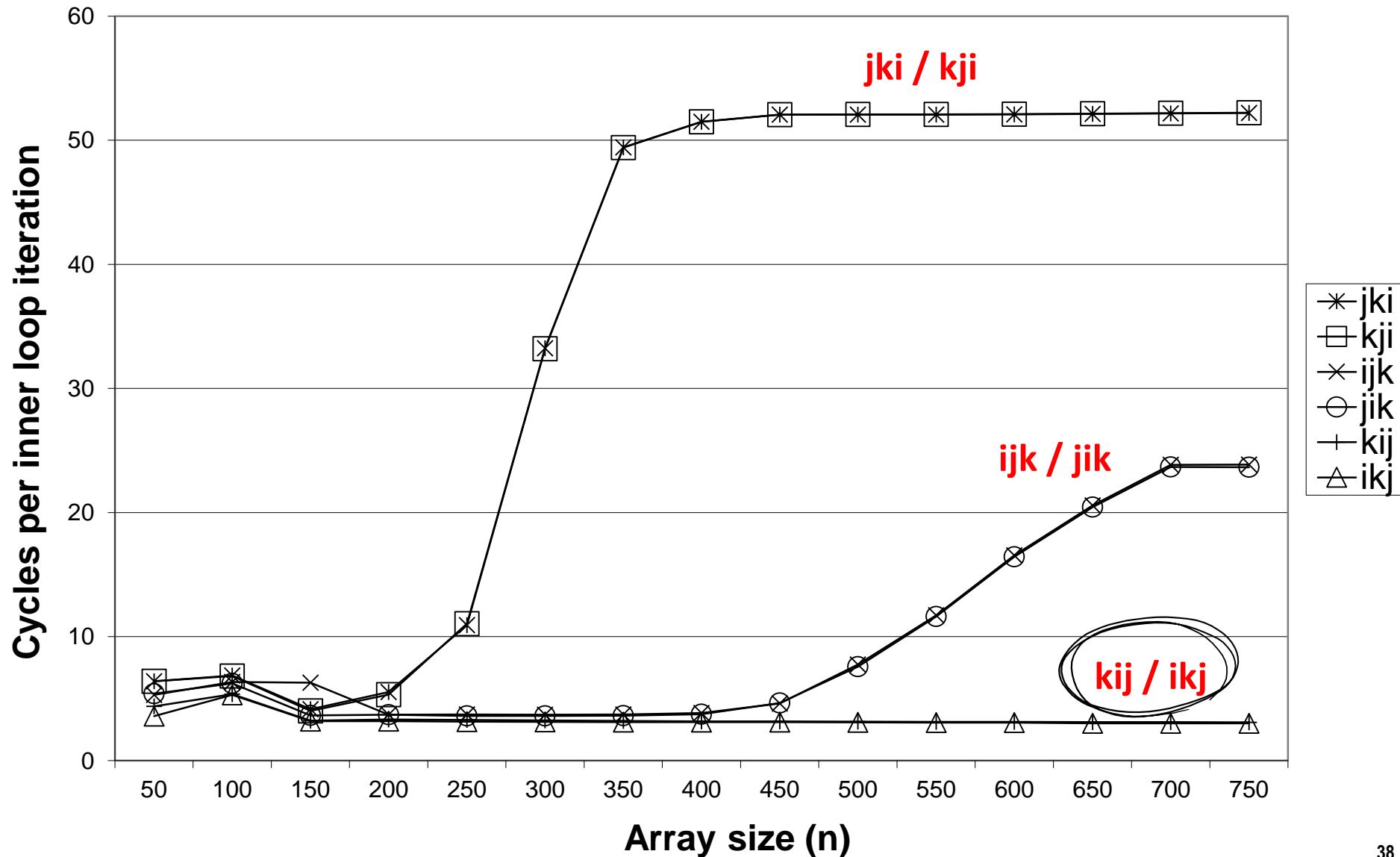
- 2 loads, 1 store
- misses/iter = **0.5** $\frac{1}{2}$

```
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

jki (& kji):

- 2 loads, 1 store
- misses/iter = **2.0** $\frac{2}{1}$

Core i7 Matrix Multiply Performance



Today

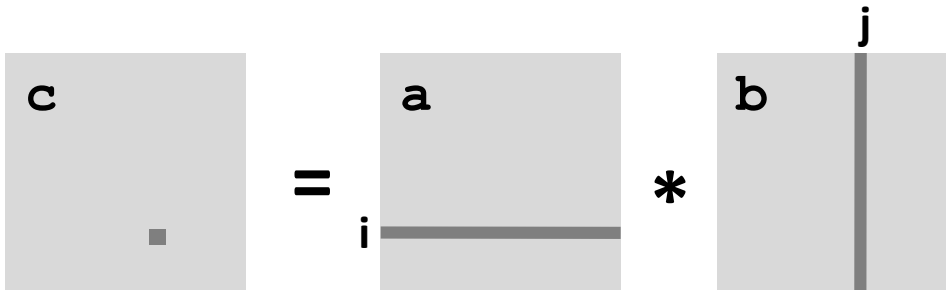
- Cache organization and operation
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행복하세요.

Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n+j] += a[i*n + k]*b[k*n + j];
}
```



Cache Miss Analysis

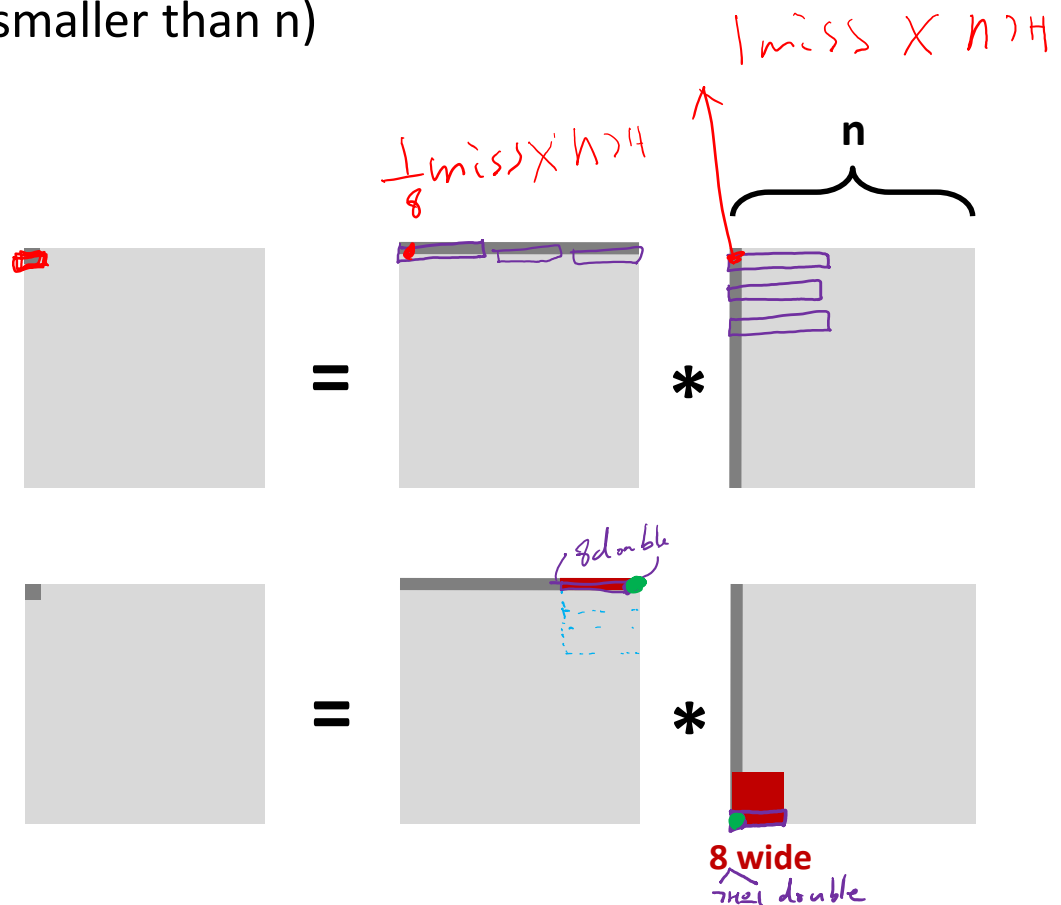
■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)

■ First iteration:

- $n/8 + n = 9n/8$ misses

- Afterwards **in cache:**
(schematic)



Cache Miss Analysis

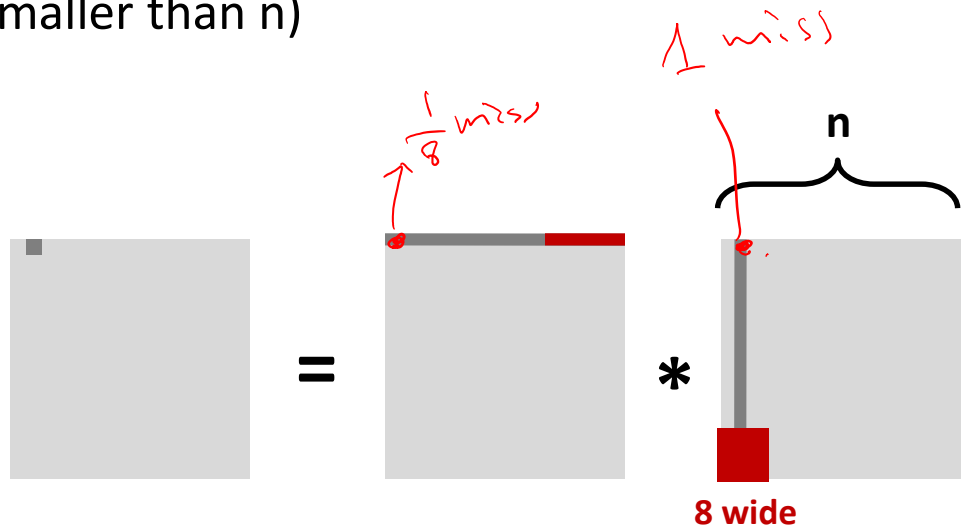
■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)

■ Second iteration:

- Again:

$$n/8 + n = 9n/8 \text{ misses}$$



■ Total misses:

- $$9n/8 * n^2 = (9/8) * n^3$$

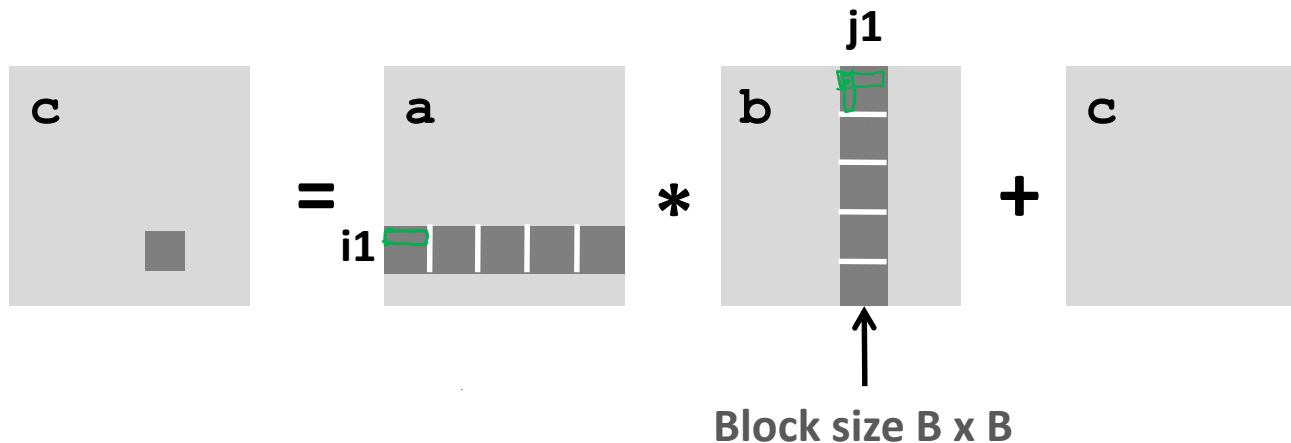
Blocked Matrix Multiplication

```

c = (double *) calloc(sizeof(double), n*n);


/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}

```



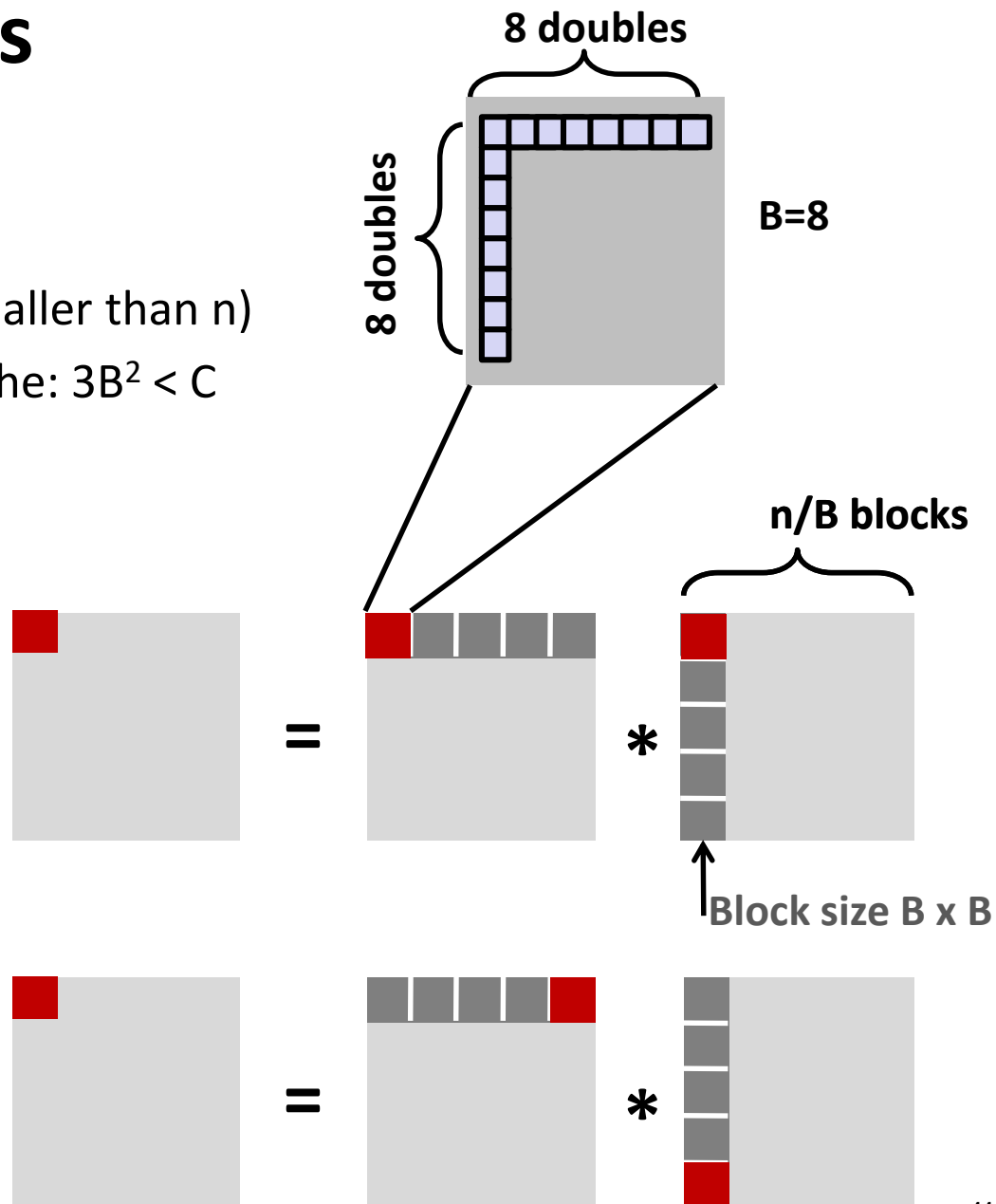
Cache Miss Analysis

■ Assume:

- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)
- Three blocks  fit into cache: $3B^2 < C$

■ First (block) iteration:


- $B^2/8$ misses for each block
- $2n/B * B^2/8 = nB/4$
(omitting matrix c)



- Afterwards in cache
(schematic)

Cache Miss Analysis

■ Assume:

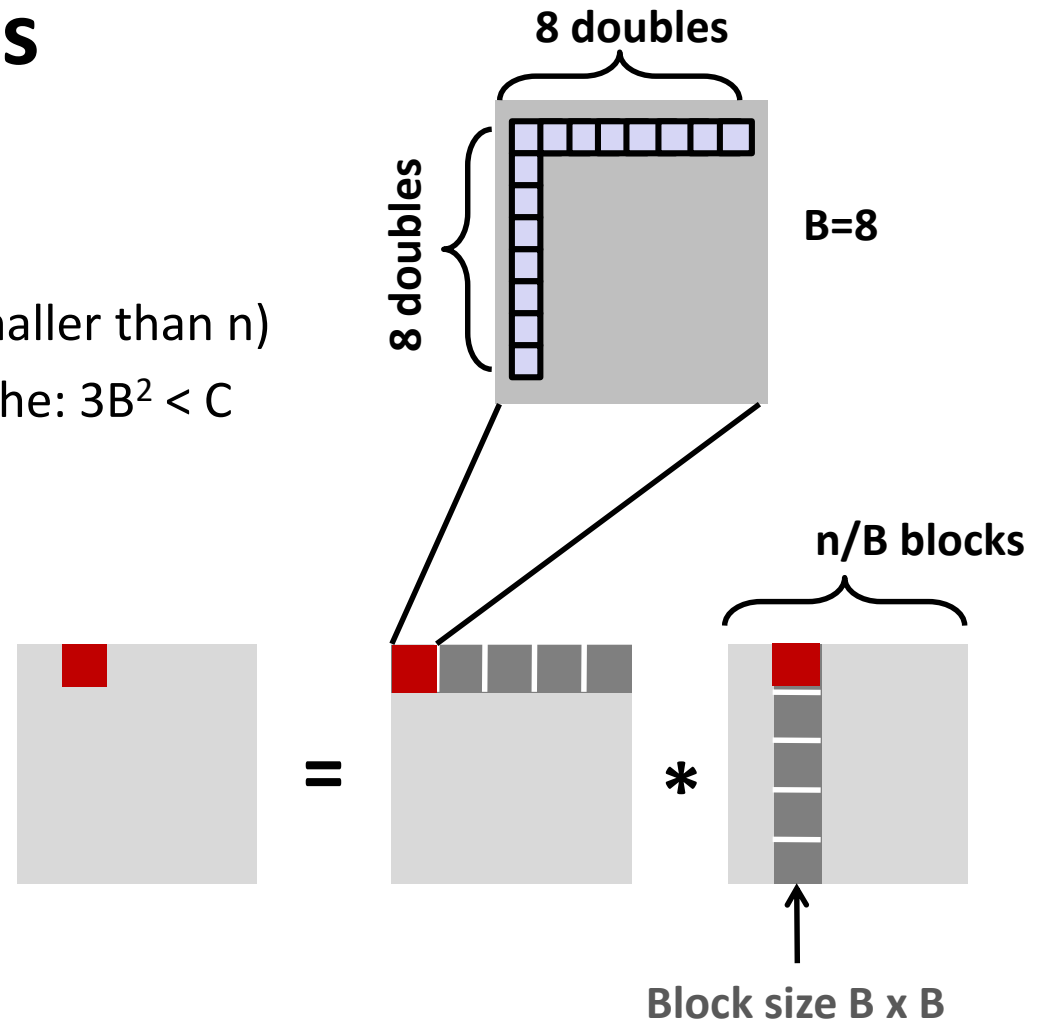
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)
- Three blocks  fit into cache: $3B^2 < C$

■ Second (block) iteration:

- Same as first iteration
- $2n/B * B^2/8 = nB/4$

■ Total misses:

- $nB/4 * (n/B)^2 = n^3/(4B)$



Summary

- No blocking: $(9/8) * n^3$
- Blocking: $1/(4B) * n^3$

- Suggest largest possible block size B , but limit $3B^2 < C!$

- Reason for dramatic difference:
 - Matrix multiplication has inherent temporal locality:
 - Input data: $3n^2$, computation $2n^3$
 - Every array elements used $O(n)$ times!
 - But program has to be written properly

Concluding Observations

- **Programmer can optimize for cache performance**
 - How data structures are organized
 - How data are accessed
 - Nested loop structure
 - Blocking is a general technique
- **All systems favor “cache friendly code”**
 - Getting absolute optimum performance is very platform specific
 - Cache sizes, line sizes, associativities, etc.
 - Can get most of the advantage with generic code
 - Keep working set reasonably small (temporal locality)
 - Use small strides (spatial locality)