Computational Models – Exercise 6

Due Friday, 26 May 2023

Each student must solve the problems on their own. If you encounter difficulties, you may ask a classmate for a hint or the general idea. However, detailed discussion, note-taking, or sharing of written solutions is not allowed. Do not write down your answers while communicating with other people or show the answers for feedback.

Our grading app has severe limitations, such as no zoom tool. To make sure we can grade your work, please follow these technical guidelines:

Submit a **single PDF file** through Moodle.

The file size is limited to **10 MB**. If necessary, google *reduce PDF file size*.

Fill in your answers **on this form*** in the allocated spaces. The space provided gives you an indication of the expected length and level of detail of the answer. You may add a little more space if you need.

Include everything from this form in your submission. In particular, **include the problem statements**. Do not delete any text or omit pages, just add your answers.

Ensure your answers are **legible** (easy to read) at zoom 100% on a standard computer screen. Your text should be **large**, **sharp**, and in **high contrast** with the background.

Do not squeeze scanned solutions to fit in the space, as the text will become small.

Verify that pages are properly **ordered** and **oriented**.

The page size must be **A4**. Before submitting your file, check its page size using Acrobat Reader: go to File > Properties > Description and confirm that Page Size is around 21 × 29 cm. Note that scanning A4 pages does not guarantee the resulting page size will be A4, due to scaling. If necessary, google *resize PDF to A4*.

Do not add your answers as PDF comments. If you can drag them in Acrobat Reader, they are comments. If necessary, google *flatten PDF*.

A **5-point bonus** will be given to solutions typed in a word processor. Hand-sketched illustrations or diagrams will not deny you this bonus.

If there are technical issues with your submission, you may receive a fine. In extreme cases, your submission may not be graded at all.

If you need help or have questions, please use the course forum at Piazza.

*The only exception is in case you use LaTeX or a similar typesetting system. In that case, copy-paste everything from this file, except for illustrations or other hard-to-reproduce graphical elements. No need to fix corrupted formulas.

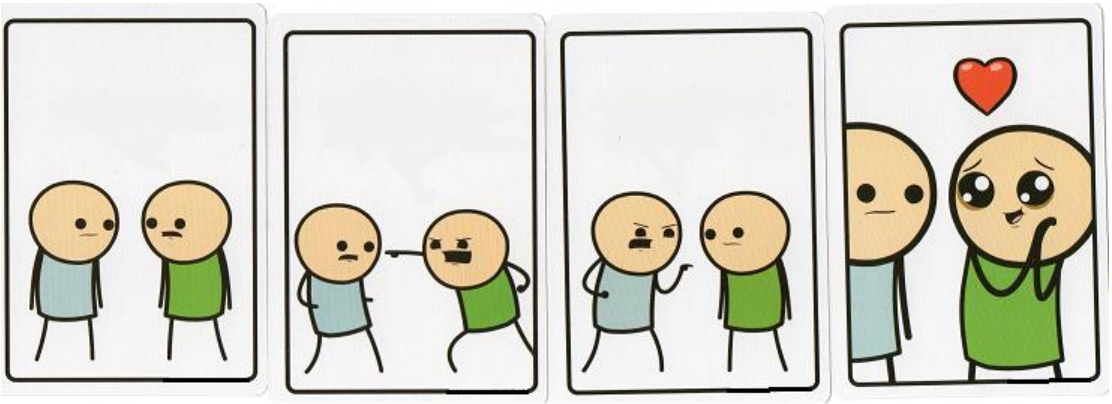
Worked with Jemma Diamond - 806839

Problem 1

[1 pt] **1.** Before Mount Everest was discovered, what was the highest mountain in the world?

Kangchenjunga

[1 pt] **2.** Edit the following comic and add meaning to it.



Frame 1: are you… not wearing any pants? (said by green guy)

Frame 2: OMG LOOK AT THIS GUY HE'S COMPLETELY PANTSLESS!! (green guy)

Frame 3: HEY YOU'RE ALSO NOT WEARING ANY PANTS!! (blue guy)

Frame 4: Maybe this means we're meant for each other! (green guy)

Problem 2

Construct a context-free grammar for each of the following languages. Unless explicitly stated otherwise, assume $\Sigma = \{a, b, c\}$.

[7 pt] **1.** $L = \{a^i b^k c^i \mid (i + k) \bmod 3 = 1\}$.

G = (V, Σ, S, R) with:

V= {S, D} , Σ= {a, b, c}

R: S-> aaaSccc | aabDbcc | aDc | bD

D-> bbbD | ε

[7 pt] **2.** $L = \{a^n b^{2m} c^{n-m} \mid n \geq m\}$.

G = (V, Σ , S, R) with:

V= {S, A, D}, Σ= {a, b, c}

R: S-> aSc |A

A-> aAbb | ε

[7 pt] **3.** $L = \left\{uv \,\middle|\, \begin{matrix} u, v \in \{a, b\}^*, |u| = |v|, \\ \text{there is only a singe } k \text{ where } u(k) \neq v^R(k) \end{matrix}\right\}$.

G = (V, Σ,S,R) with:

V= {S, D} , Σ= {a, b}

R: S-> aSa | bSb | aDb |bDa

D-> aDa | bDb | ε

[7 pt] **4.** $L = \{w \in \{a, b\}^* \mid w \neq w^R\}$.

G = (V, Σ,S,R) with:

V= {S, A}

Σ= {a, b}

R: S -> aAb | bAa | aSa | bSb

A -> a | b | ε

Problem 2

The following languages are defined over $\Sigma = \{0,1\}$. For each of that following languages determine whether the language is a context-free but not regular language or a regular language. Prove this by writing a context-free grammar or a regular grammar.

[5 pt] **1.** $L = \{\sigma_1 u \sigma_2 v \sigma_3 | \sigma_1, \sigma_2, \sigma_2 \in \Sigma, u, v \in \Sigma^*, |u| = |v| \text{ and } \sigma_1 = \sigma_2 = \sigma_3\}$ that is, the first, the middle and the last letters are identical.

This is not a regular language but it is a context free language as it can be described by a grammar but it can't be described in the regular grammar format.

G = (V, Σ,S,R) with:

V= {S,D,B,A }

Σ= {0,1}

R: S -> 1D1 | 0B0

D -> ADA | 1

B -> ABA | 0

A -> 0 | 1

[5 pt] **2.** $L = \{\sigma_1 u \sigma_2 v \sigma_3 | \sigma_1, \sigma_2, \sigma_2 \in \Sigma, u, v \in \Sigma^*, |u| = |v| \text{ and } \sigma_1 = \sigma_2 \text{ or } \sigma_2 = \sigma_3 \text{ but not both}\}$ that is, the middle letter is identical to the first or the last letter but not both.

This is a regular language as it can be described by a regular grammar as such:

G = (V, Σ,S,R) with:

V= {S,A,B,D}

Σ= {0,1}

R: S -> 0A1 | 1B0

A -> DAD | 0 | 1

B -> DBD | 0 | 1

D -> 0 | 1

Problem 3

Prove the following claims using substitution closures.

[7 pt] **1.** Let L be a regular language over $\Sigma = \{a, b\}$.

Define, $mid_a(L) = \{uav | uv \in L \; and \; |u| = |v|\}$.

Prove that $mid_a(L)$ is context-free language.

Define the regular substitution g over Σ as:

g(@)={@,#@).

Define L*= g(L) ∪ {#}, which is a regular language (closed under substitution and intersection).

Define L**= L* ∩ Σ* ·{#} ·Σ* which is regular by closure under intersection.

Define L'= {u#v | u,v ∈ Σ* and |u| = |v|} which is a CFL as it has a CFG such as:

R: S -> DSD|#

D-> 0|1.

Define L*** = L** ∩ L' = {u#v | u,v ∈ L and |u| = |v|} which is a CFL as it is an intersection of a regular language and a CFL.

Define the substitution, h over Σ ∪ {#} as:

h(#) = {a} , h(@≠#) = {@}

h(L***) is a CFL by closure under context free substitution and h(L***) = $mid_a(L)$ = {uav | uv ∈ L and |u| = |v|}.

[7 pt] **2**. Let L be a regular language over some Σ such that $\# \notin \Sigma$.

Define, $trim-mid(L) = \{u\#w | \exists v \in \Sigma^* \ s.t \ uvw \in L\}$.

Prove that $trim-mid(L)$ is regular language.

I don't know.

[7 pt] **3**. Let L be a regular language over some Σ such that $\# \notin \Sigma$.

Define, $trim-rev(L) = \{u\#w | \exists v \in \Sigma^* \ uvw \in L \ and \ u = w^R\}$.

Prove that $trim-rev(L)$ is context-free language.

I don't know.

Problem 4

Prove or disprove.

[5 pt] **1**. If L_1 is context-free language and L_2 is regular language, then $L_1 \setminus L_2$ is context-free language.

I don't know.

[5 pt] **2**. If L_1 is regular language and L_2 is context-free language, then $L_1 \setminus L_2$ is regular language.

I don't know.

[5 pt] **3**. If L_1 a finite but not empty language and L_2 is not regular language, then $L_1 \cdot L_2$ is not regular language.

I don't know.

Problem 5

For each of the following languages, build a pushdown automaton using a state diagram.

[8 pt] **1.** $L = \{x\#y | x, y \in \{a,b\}^* \text{ and } x \neq y\}$.

I don't know.

[8 pt] **2.** $L = \{xy | x, y \in \{a,b\}^*, |x| = |y| \text{ and } x \neq y\}$.

I don't know.

[8 pt] **3**. $L = \{0^i 1^j 0^k | j \geq \min(i, k)\}$.

I don't know.