Computational Models – Exercise 5

Due Wednesday, 10 May 2023

Each student must solve the problems on their own. If you encounter difficulties, you may ask a classmate for a hint or the general idea. However, detailed discussion, note-taking, or sharing of written solutions is not allowed. Do not write down your answers while communicating with other people or show the answers for feedback.

Our grading app has severe limitations, such as no zoom tool. To make sure we can grade your work, please follow these technical guidelines:

Submit a **single PDF file** through Moodle.

The file size is limited to **10 MB**. If necessary, google *reduce PDF file size*.

Fill in your answers **on this form*** in the allocated spaces. The space provided gives you an indication of the expected length and level of detail of the answer. You may add a little more space if you need.

Include everything from this form in your submission. In particular, **include the problem statements**. Do not delete any text or omit pages, just add your answers.

Ensure your answers are **legible** (easy to read) at zoom 100% on a standard computer screen. Your text should be **large**, **sharp**, and in **high contrast** with the background.

Do not squeeze scanned solutions to fit in the space, as the text will become small.

Verify that pages are properly **ordered** and **oriented**.

The page size must be **A4**. Before submitting your file, check its page size using Acrobat Reader: go to File > Properties > Description and confirm that Page Size is around 21 × 29 cm. Note that scanning A4 pages does not guarantee the resulting page size will be A4, due to scaling. If necessary, google *resize PDF to A4*.

Do not add your answers as PDF comments. If you can drag them in Acrobat Reader, they are comments. If necessary, google *flatten PDF*.

A **5-point bonus** will be given to solutions typed in a word processor. Hand-sketched illustrations or diagrams will not deny you this bonus.

If there are technical issues with your submission, you may receive a fine. In extreme cases, your submission may not be graded at all.

If you need help or have questions, please use the course forum at Piazza.

*The only exception is in case you use LaTeX or a similar typesetting system. In that case, copy-paste everything from this file, except for illustrations or other hard-to-reproduce graphical elements. No need to fix corrupted formulas.

Worked with Jemma Diamond - 806839

Problem 1

For each of the following languages prove whether it's regular or not. If it is not regular, use the pumping lemma for regular languages and if it is regular prove it. No need to prove formally that your DFA/NFA/Regex recognizes the language.

[10 pt] **1.** $L = \{w \in \{0,1\}^* \mid Every\ substring\ of\ length\ 10\ appears\ only\ once\ in\ w\}$.

L is not a regular language. ATC that L is a regular language, therefore the pumping lemma holds for L.

Let n be the number promised by the lemma. Choose the word w= $1^n 0^n$

We have that w ∈L and |w| >=n, therefore by the lemma w can be divided into 3 parts, w=xyz such that the 3 conditions for the lemma hold.
Since |xy| <= n and |y| > 0, then we know that x and y consist only of 1's.
Denote x=1^s and y=1^t where t>0 and s+t<=n.

By the lemma, it must hold that for each i >= 0, w'=xy^iz= $1^{(n+(i-1)t)}0^n$ is part of L.

Therefore to get a contradiction we must choose an i such that (n+(i-1)t)=20

(n+(i-1)t)=20 ∴(i-1)t= 20-n ∴(i-1)= (20-n)/t ∴i= (20-n)/t +1.

And Since t<=n we have (20-n)/t is a whole number (as required).

Therefore w'=xy^iz= $1^{(n+((20-n)/t+1-1)t)}0^n$= $1^{20}0^n$. Therefore w' is not part of L as a substring of length 10 of only 1's appeares more than once. Therefore L is not a regular language.

[10 pt] **2.** $L = \{a^{k^2} \mid k \geq 0\}$.

L is not a regular language. ATC that L is a regular language and therefore the pumping lemma holds for L. Let n be the number promised by the lemma. We choose the word w= a^{n^2}

We have that w ∈L and |w| >=n, therefore by the pumping lemma, w can be divided into 3 parts, w=xyz such that the 3 conditions for lemma hold.

Denote x=$a^s$and y=a^t where t>0 and s+t<=n.

Consider the word w'= xy^2z = a^{n^2+t}.

n^2 +t <= n^2+n < n^2+2n +1= $(n+1)^2$

and also n^2+t > n^2 since t > 0. Therefore n^2 < n^2 +t < $(n+1)^2$. Therefore w' is not part of L . Therefore L is not a regular language.

[10 pt] **3.** $L = \{w \in \{a,b\}^* | \, w = w^R\}$.

L is not a regular language. ATC that L is a regular language and therefore the pumping lemma holds for L. Let n be the number promised by the lemma. Choose the word w= a^nbba^n.

We have that w ∈L and |w| >=n, therefore by the pumping lemma, w can be divided into 3 parts, w=xyz such that the 3 conditions for lemma hold. Since |xy| <= n and |y| > 0, then we know that x and y consist only of a's.

Denote x=$a^s$and y=a^t where t>0 and s+t<=n.

Consider the word w'=xy^2z = a^{n+1}bba^n

Since t>0 we have that w' has more a's to begin with than to end with so w' ≠ w'^R. Therefore w' is not part of L . Therefore L is not a regular language.

[10 pt] **4.** $L = \{a^s b^t | \, s \neq t^2 + 3\}$.

L is not a regular language. ATC that L is a regular language and therefore the pumping lemma holds for L. Let n be the number promised by the lemma. Choose the word w= $a^n b^n$.

We have that w ∈L and |w| >=n, therefore by the pumping lemma, w can be divided into 3 parts, w=xyz such that the 3 conditions for lemma hold. Since |xy| <= n and |y| > 0, then we know that x and y consist only of a's.

Denote x=$a^s$and y=a^t where t>0 and s+t<=n.

By the lemma, it must hold that for each i >= 0, w'=xy^iz= $a^{(n+(i-1)t)} b^n$ is part of L.

Therefore to get a contradiction we must choose an i such that (n+(i-1)t)= n^2+3

(n+(i-1)t)= n^2+3 ∴ (i-1)t= n^2 -n +3 ∴(i-1)= (n^2 -n +3)/t

∴ i= (n^2 -n +3)/t +1.

And Since t<=n we have (n^2 -n +3)/t +1 is a whole number (as required).

Therefore after substituting out choice for i we get that w'=xy^iz= $a^{(n^2+3)} b^n$. Therefore w' is not part of L. Therefore L is not a regular language.

[10 pt] **5**. $L = \{w \in \{a,b,c\}^* |\ w = (abc)^n a(bca)^n b(cab)^n\ and\ n \geq 0\}$.

L is a regular language as it can be expressed as a regular expression:

L = (abcabcabc)*ab

[10 pt] **6**. $L = L_1 \cdot L_2$ where $L_1 = \{0^n | n \geq 0\}$ and $L_2 = \{1^n | n \geq 0\}$.

L is a regular language as it can be expressed as a regular expression:

L = 0*1*

Problem 2

Prove or disprove.

[7 pt] **1**. Given two languages, L_1 and L_2. If L_1 and $L_1 \cdot L_2$ are regular languages, then L_2 is regular language.

False. Counterexample:

Let L_1={a^m| $m \geq 0$} and let L_2= {$a^n b^n$|n >= 0}

L_1 is a regular language and L_2 is not a regular language. However $L_1 \cdot L_2$ = {$a^m a^n b^n$ | m,n >=0} is a regular language since it can be seen as the language L= {a*b*} which is regular.

[7 pt] **2**. Given two languages, L_1 and L_2. If $L_1 \cup L_2$ and L_2 are regular languages, then L_1 is regular language.

False. Counterexample:

Let L_1= {$a^n b^n$|n >= 0} and let L_2= {a,b}*

L_1 is not a regular language and L_2 is a regular language. However $L_1 \cup L_2$= {a,b}* which is a regular.

[7 pt] **3**. Given language L over $\Sigma = \{0,1\}$ such that $|L| > 70$. If the minimal DFA that recognizes L has 6 states, then L is an infinite language.

True. Proof:

If L has 6 states, then there are a finite number of distinct strings that can be accepted by the DFA, as each state of the DFA corresponds to a different letter of the string. Therefore, there are at most $2^6 = 64$ such distinct strings. However, since $| L | > 70$, that means that there are more than 70 distinct strings in the language. Therefore, some of the words in L must contain loops. Therefore, some words can be pumped and will still go on to be accepted by L. Therefore L is an infinite language.

[7 pt] **4**. Given two non-regular languages, L_1 and L_2. If $L_1 \subseteq L \subseteq L_2$, then L must be non-regular language.

False. Counterexample:

Let $L_1 = 0^{k^2}$and let L = 0^*. L_1 was proven to be a regula rlangiage in Q1.2 and L is clearly a regular language as it can be described by a RegEx. Clearly, $L_1 \subseteq$ L

Let L_2= $L_1 \cup$ L This is not a regular language as it is the union of a regular and non regular language. It's clear that L$\subseteq L_2$. Therefore all the conditions hold, but L is regular.

[12 pt] **Problem 3**

Let $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i \neq 1\} \cup \{ab^j c^j \mid j \geq 0\}$.

In the recitation we proved that L satisfies the pumping lemma for regular languages.

Prove that L is not a regular language by using regular closures and the pumping lemma for regular languages.

Guidance:

1. Prove that $L' = \{ab^j c^k \mid j, k \geq 0\}$ is regular language.
2. Define the language $L'' = L \cap L'$.
3. Show that L'' is not regular language using the pumping lemma for regular languages.
4. Explain why the proof you wrote proves that L is not a regular language.

I don't know.