Computational Models – Exercise 4

Due Thursday, 4 May 2023

Each student must solve the problems on their own. If you encounter difficulties, you may ask a classmate for a hint or the general idea. However, detailed discussion, note-taking, or sharing of written solutions is not allowed. Do not write down your answers while communicating with other people or show the answers for feedback.

Our grading app has severe limitations, such as no zoom tool. To make sure we can grade your work, please follow these technical guidelines:

Submit a **single PDF file** through Moodle.

The file size is limited to **10 MB**. If necessary, google *reduce PDF file size*.

Fill in your answers **on this form*** in the allocated spaces. The space provided gives you an indication of the expected length and level of detail of the answer. You may add a little more space if you need.

Include everything from this form in your submission. In particular, **include the problem statements**. Do not delete any text or omit pages, just add your answers.

Ensure your answers are **legible** (easy to read) at zoom 100% on a standard computer screen. Your text should be **large**, **sharp**, and in **high contrast** with the background.

Do not squeeze scanned solutions to fit in the space, as the text will become small.

Verify that pages are properly **ordered** and **oriented**.

The page size must be **A4**. Before submitting your file, check its page size using Acrobat Reader: go to File > Properties > Description and confirm that Page Size is around 21 × 29 cm. Note that scanning A4 pages does not guarantee the resulting page size will be A4, due to scaling. If necessary, google *resize PDF to A4*.

Do not add your answers as PDF comments. If you can drag them in Acrobat Reader, they are comments. If necessary, google *flatten PDF*.

A **5-point bonus** will be given to solutions typed in a word processor. Hand-sketched illustrations or diagrams will not deny you this bonus.

If there are technical issues with your submission, you may receive a fine. In extreme cases, your submission may not be graded at all.

If you need help or have questions, please use the course forum at Piazza.

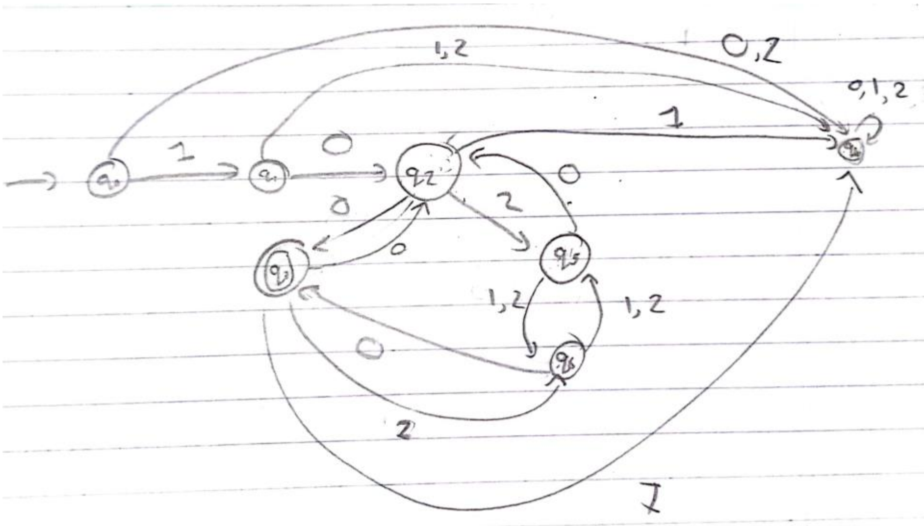
*The only exception is in case you use LaTeX or a similar typesetting system. In that case, copy-paste everything from this file, except for illustrations or other hard-to-reproduce graphical elements. No need to fix corrupted formulas.

Worked with Jemma Diamond - 806839

Problem 1

Build a minimal DFA for the following languages.

[5 pt] **1.** $L = \left\{ w \in \{0,1,2\}^* \,\middle|\, \begin{array}{c} w \text{ starts with } 10, \text{ends with } 0, \\ \text{does not have substring } 01 \text{ and } |w| \text{ is odd} \end{array} \right\}$.

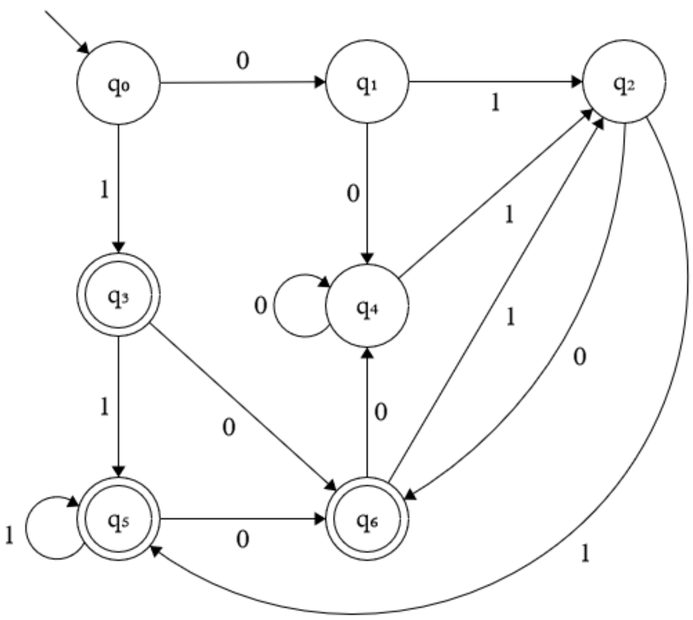


[5 pt] **2.** $L = \{w \in \{x, y\}^* | w \text{ contains all possible combinations of consequtive pair}\}$.
I.e., w contains xx, xy, yy and yx.

I don't know

Problem 2

Let A be a DFA as described in the figure below.



[10 pt] **1**. Find the equivalent classes of A using the equivalent algorithm we saw in class. show intermediate steps.

$E_0 = \{\{q_5, q_6, q_3\}, \{q_0, q_1, q_2, q_4\}\}$

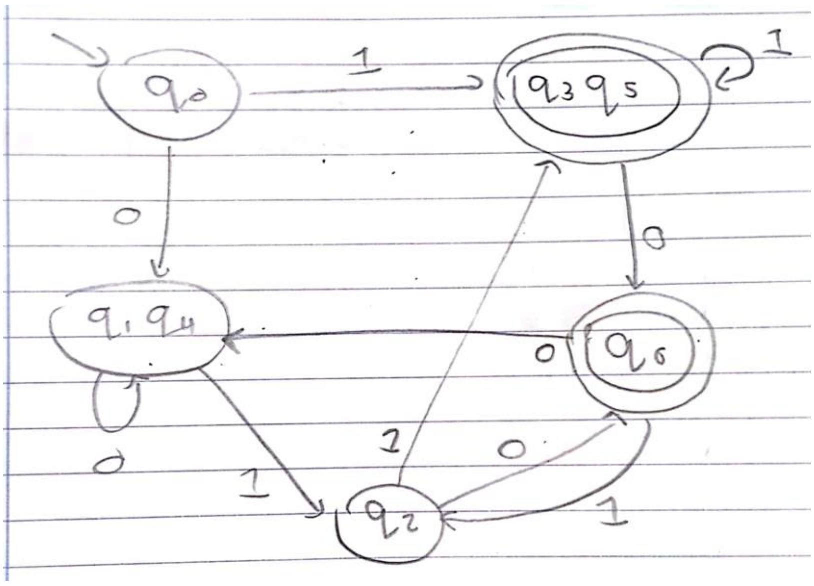
	0	1
q_0	q_1	q_3
q_1	q_4	q_2
q_2	q_6	q_5
q_3	q_6	q_5
q_4	q_4	q_2
q_5	q_6	q_5
q_6	q_4	q_2

$E_1 = \{\{q_5, q_3\}, \{q_6\}, \{q_1, q_4\}, \{q_2\}, \{q_0\}\}$

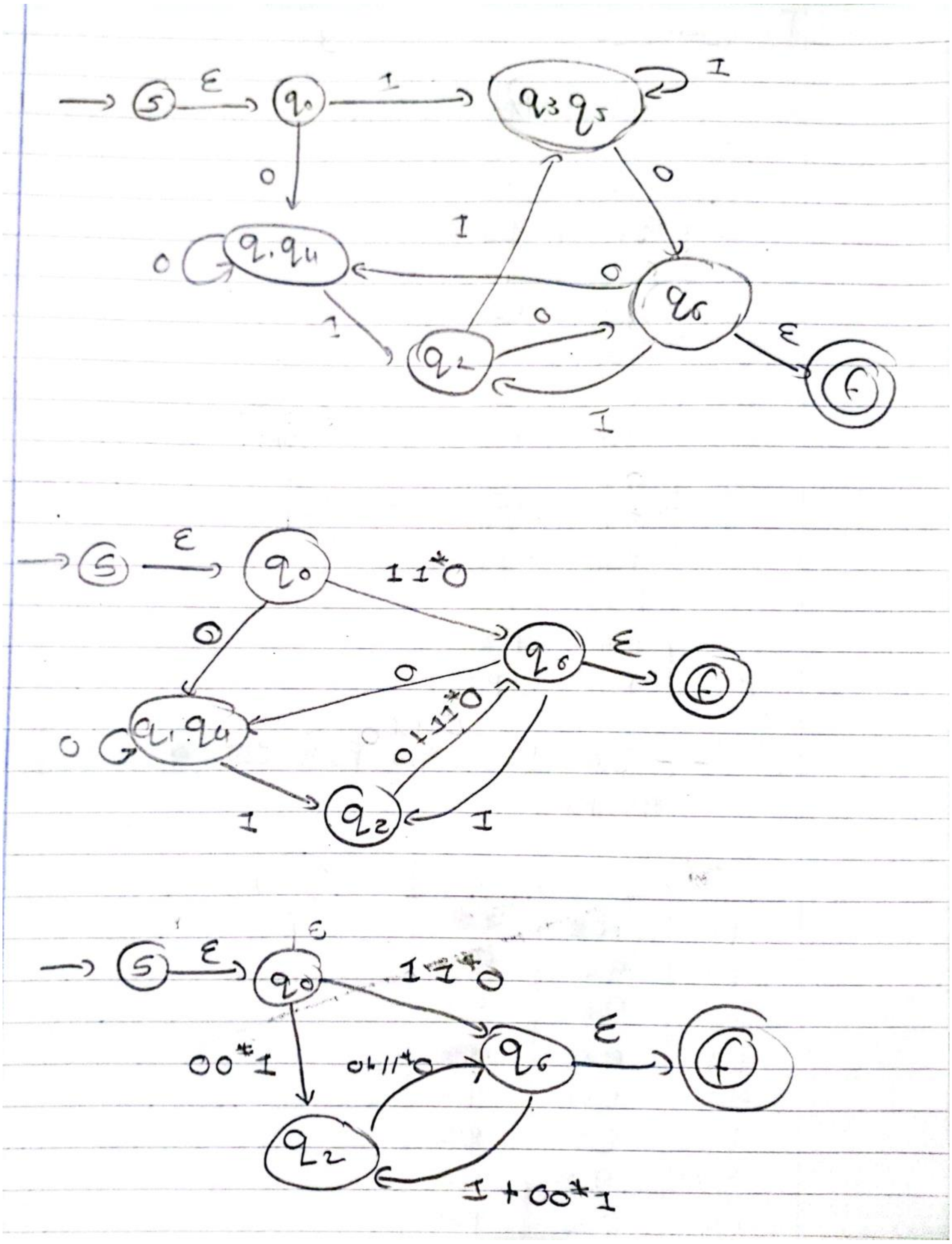
	0	1
q_0	q_1	q_3
q_1	q_4	q_2
q_2	q_6	q_5
q_3	q_6	q_5
q_4	q_4	q_2
q_5	q_6	q_5
q_6	q_4	q_2

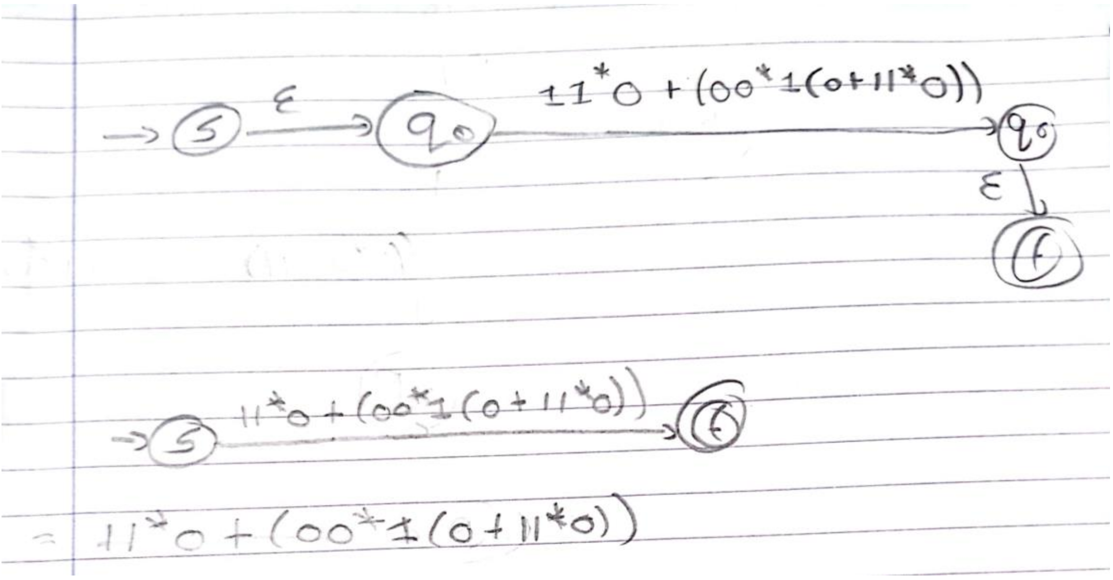
$E_2 = \{\{q_5, q_3\}, \{q_6\}, \{q_1, q_4\}, \{q_2\}, \{q_0\}\} = E_1$

[10 pt] **2**. Build a minimal DFA for $L(A)$.



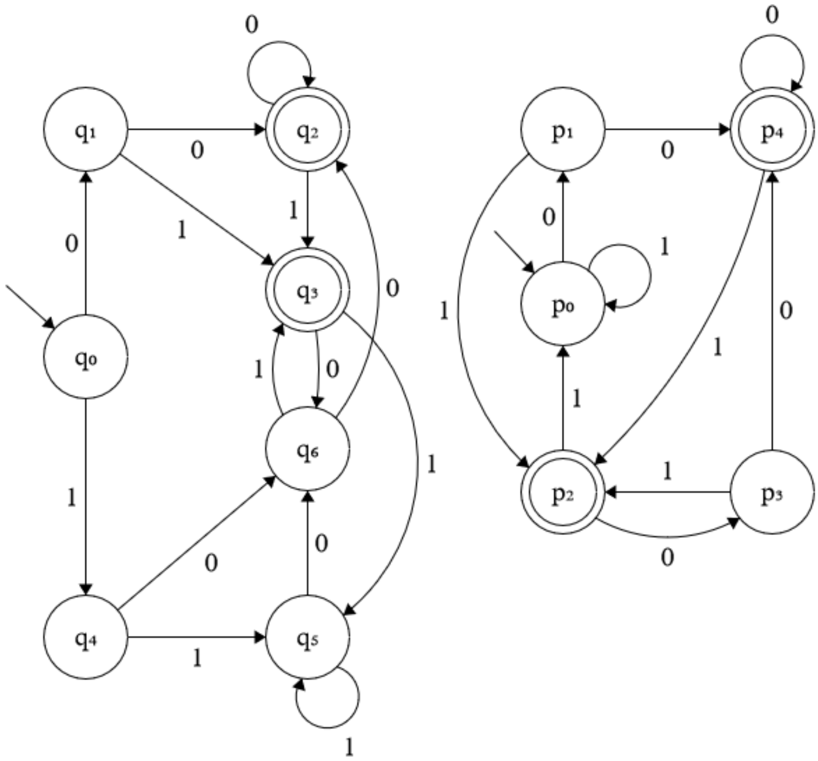
[10 pt] **3**. Using the minimal DFA you built, build a regular expression for $L(A)$. Use the algorithm taught in class to convert a DFA into a regular expression. Show your intermediate steps.





[10 pt] **<u>Problem 3</u>**

Decide whether the following two DFAs are equivalent or not using the algorithm we saw in class. Show your intermediate steps.



Problem 3.

$E_0 = \{\{q_2, q_3, p_2, p_4\}, \{q_0, q_1, q_4, q_5, q_6, p_0, p_1, p_3\}\}$

	0	1
q_0	q_1	q_4
q_1	q_2	q_3
q_2	q_2	q_3
q_3	q_6	q_5
q_4	q_6	q_5
q_5	q_6	q_5
q_6	q_2	q_3
p_0	p_1	p_0
p_1	p_4	p_2
p_2	p_3	p_0
p_3	p_4	p_2
p_4	p_4	p_2

$E_1 = \{\{q_2, p_4\}, \{q_3, p_2\}, \{q_1, q_6, p_1, p_3\}, \{q_0, q_4, q_5, p_0\}\}$

	0	1
q_0	q_1	q_4
q_1	q_2	q_3
q_2	q_2	q_3
q_3	q_6	q_5
q_4	q_6	q_5
q_5	q_6	q_5
q_6	q_2	q_3
p_0	p_1	p_0
p_1	p_4	p_2
p_2	p_3	p_0
p_3	p_4	p_2
p_4	p_4	p_2

$E_2 = \{\{q_2, p_4\}, \{q_3, p_2\}, \{q_1, q_6, P_1, P_3\}, \{q_0, q_4, q_5, P_0\}\}$
$= E_1$

$\therefore$ Done Equivelance Alg.

$\therefore$ Since q_0 and P_0 are in the same class, then $q_0 \equiv P_0$

$\therefore$ The two DFA's are equivelant

Problem 4

Denote by $Rank(L)$ the number of states in the minimal DFA recognizing the regular language L.

For each one of the following claims decide whether it is correct or incorrect. Justify your answer.

[7 pt] **1.** $Rank(L) = Rank(\overline{L})$, L is regular language.

True. ATC that $Rank(L) \neq Rank(\overline{L})$.

If Rank(L) < Rank($\overline{L}$): Let M be the minimal DFA that recognizes L and let the number of states in M be n. Convert all of M's accepting states to rejecting states and all its rejecting states to accepting states. This new DFA recognizes $\overline{L}$. Therefore Rank($\overline{L}$) is at most n. However n is equal to rank(L). Contradiction to Rank(L) < Rank($\overline{L}$).

If Rank(L) > Rank($\overline{L}$): Let M be the minimal DFA that recognizes $\overline{L}$ and let the number of states in M be n. Convert all of M's accepting states to rejecting states and all its rejecting states to accepting states. This new DFA recognizes L. Therefore Rank(L) is at most n. However n is equal to rank($\overline{L}$). Contradiction to Rank(L) > Rank($\overline{L}$).

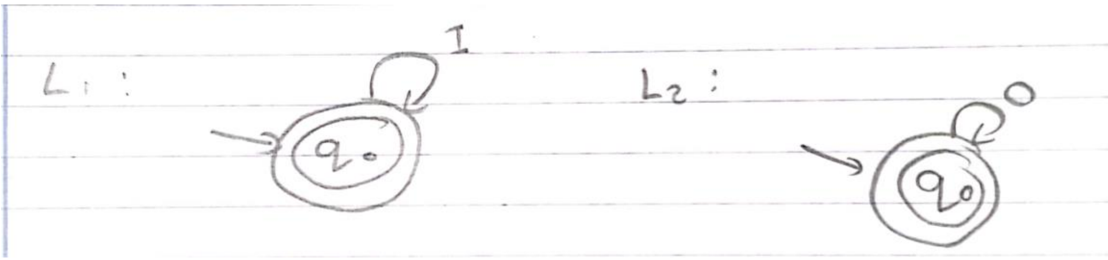
[7 pt] **2.** $Rank(L_1 \cap L_2) \leq Rank(L_1) \cdot Rank(L_2)$, L_1 and L_2 are regular languages.

True. Let M1 be the minimal DFA that recognizes L_1 and let M2 be the minimal DFA that recognizes L_2. The product automaton, P, of L_1 and L_2 has $Rank(L_1) \cdot Rank(L_2)$ number of states (by definition of product automaton). The product automaton recognizes the language $L_1 \cap L_2$ (when we set the accepting states as follows: F = F1 × F2). And therefore P recognizes $L_1 \cap L_2$ and has $Rank(L_1) \cdot Rank(L_2)$. Therefore the minimal DFA of $L_1 \cap L_2$ is at most size $Rank(L_1) \cdot Rank(L_2)$.

[7 pt] **3.** $Rank(L_1 \cup L_2) \leq Rank(L_1) + Rank(L_2)$, L_1 and L_2 are regular languages.

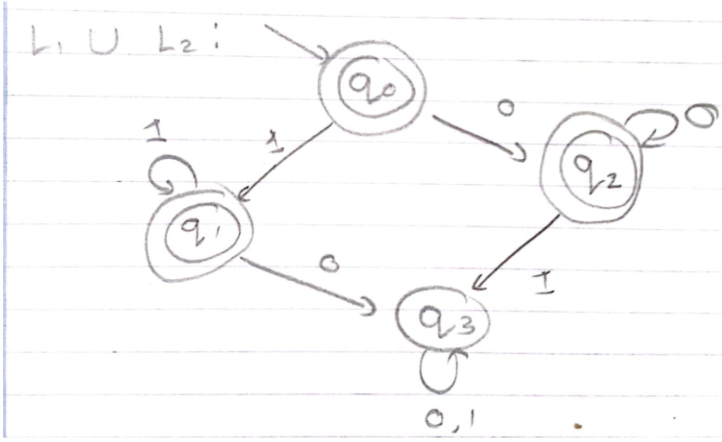
False. Counterexample:

Let $L_1 = \{w \mid w \in 1^*\}$ and $L_2 = \{w \mid w \in 0^*\}$. The DFA for L_1 and L_2 are as follows:



$\text{Rank}(L_1) = \text{Rank}(L_2) = 1$. Therefore $Rank(L_1) + Rank(L_2) = 2$

The DFA of $L_1 \cup L_2$ is:

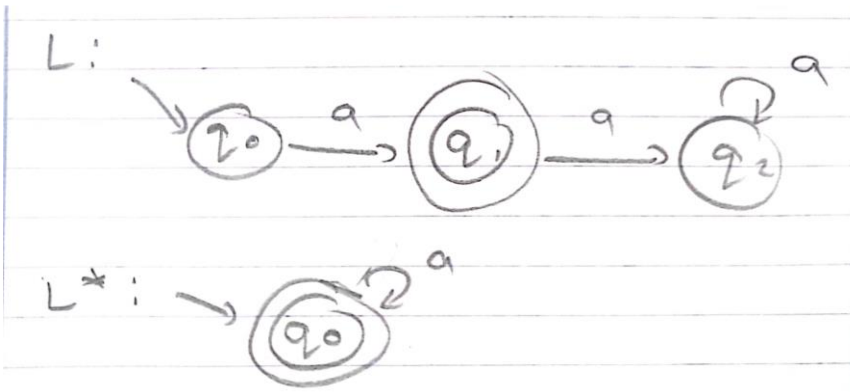


$\text{Rank}(L_1 \cup L_2) = 4 \neq 2 = Rank(L_1) + Rank(L_2)$

[7 pt] **4.** $Rank(L) = Rank(L^*)$, L is regular language.

False. Counterexample:

Let $L_1 = \{1\}$ and $L_2 = \{1^*\}$. The DFA for L_1 and L_2 are as follows:



$\text{Rank}(L_1) = 3 \neq 1 = \text{Rank}(L_2)$

Problem 5

Prove by construction and/or regular closures:

[11 pt] **1**. For languages L_1 and L_2 we define: $L_1 \otimes L_2 = \{xyz \mid xz \in L_1,\ y \notin L_2\}$. Prove that if L_1 *and* L_2 are regular languages then $L_1 \otimes L_2$ is regular language.

I don't know

[11 pt] **2**. For a language L over $\Sigma = \{0,1\}$ define $L' = \{w \mid \overline{w}w \in L\}$ where $\overline{w}$ denotes the bit-flip of w. Prove that if L is regular language, then L' is regular language.

I don't know