# Data Structures – Assignment 1
# IDC, Spring 2022

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**Submission day:** 11.3.22 – (you can use an extension and submit by 15.3.22).

**Honor code:**

- Do not copy the answers from any source.
- You may work in small groups but write your own solution - write whom you worked with.
- Cheating students will face Committee on Discipline (COD).
- Do not forget – you are here to learn!

**Submission:**

- Submit your solution as a PDF file only. Other formats will not be graded.
- Typed submissions will get a bonus of 3 points.
- If you choose not to type your solution, make sure the scan looks good. We will deduct points for hard-to-read submissions.
- In order to ask question regarding the assignment on the piazza the title of the question must be of the form 'assignment x, question x', and should be written in English only.

## Question A

Answer the following questions. You are allowed to use known geometric and algebraic formulas. Write clearly the formulas you use.

1. Consider the following sequence: $a_0 = 10$, $a_{i+1} = 5 \cdot a_i$. Prove by induction that $a_n = 10 \cdot 5^n$.
2. Prove that for all $n \geq 1$: $\sum_{i=1}^{n} i \cdot 2^i = (n-1) \cdot 2^{n+1} + 2$.
3. Prove that for every integer $n \geq 1$, all integers in the range $[1, 2^n - 1]$ can be written as a sum of distinct elements from the set $\{2^0, 2^1, ..., 2^{n-1}\}$. For example $6 = 2^2 + 2^1$

## Question B

Which of the following proofs are correct? For an incorrect proof – explain what went wrong.

1. **Claim1:** For every non-negative integers $a$ and $n$, $a^n = 1$.
   **Proof:** We prove that the claim holds for all integers $n \geq 0$, using strong induction.
   **Base case:** For $n = 0$, $a^0 = 1$, hence the claim holds.
   **Induction Hypothesis:** Assume that for all integers $k \leq n$ $a^k = 1$.
   **Induction step:** We will show that for $n+1$, $a^{n+1} = 1$
   Write $n + 1 = n + n - (n - 1)$ It follows that $a^{n+1} = \frac{a^n \cdot a^n}{a^{n-1}}$.
   By the induction hypothesis, $a^n = 1$ and $a^{n-1} = 1$ since $n, n-1 \leq n$.
   It follows that $a^{n+1} = \frac{1 \cdot 1}{1} = 1$.

2. **Claim 2:** Given $n \geq 2$ points in the plane they all lie on the same line.

   **Proof:** We prove that the claim holds for all integers $n \geq 2$.
   **Base case:** For $n = 2$ the claim holds.
   **Induction Hypothesis:** Assume the claim holds for $n$.
   **Induction step:** We will show that the claim holds for $n + 1$ .
   Let $p_1, ..., p_{n+1}$ be any set of points in the plane. Applying the induction assumption on the set $p_1, ..., p_n$, we know that $p_1, ..., p_n$ lie on a line $\ell_1$.

   Applying the induction assumption again on $p_2, ..., p_{n+1}$ we get that they on lie on a line $\ell_2$. Since $\ell_1$ and $\ell_2$ share the points $p_2, ..., p_n$ we conclude that $\ell_1 = \ell_2$ and so $p_1, ..., p_{n+1}$ lie on $\ell_1$.

3. **Claim 3:** Given the following recursive bound on a function $T$: $T(2) = 4$ and $T(n) \leq 2T(n/2) + 10n$ for $n = 2^k$. It follows that $T(n) \leq 10n \log n$

   **Proof:** We prove the claim by induction on $k$.
   **Base case:** For $k = 1$ we have $T(2) = 4 \leq 10 \cdot 2 \log 2 = 20$.

   **Induction Hypothesis:** Assume that for $2^{k-1} = \frac{n}{2}$ the claim holds, that is,

   $$T(\frac{n}{2}) \leq 10\frac{n}{2}\log(\frac{n}{2}).$$

   **Induction step:** We will show that the claim holds for $n$.

   (i) $T(n) \leq 2T(\frac{n}{2}) + 10n$ (given recursive bound on $T$)

   (ii) $T(\frac{n}{2}) \leq 10\frac{n}{2}\log(\frac{n}{2})$ (by the induction hypothesis)

   (iii) $T(n) \leq 2 \cdot 10 \cdot \frac{n}{2}\log(\frac{n}{2}) + 10n$ (putting (i) and (ii) together)

   (iv) $T(n) \leq 10n(\log(n/2) + 1) = 10n(\log(\frac{n}{2}) + \log 2) = 10n \log n$ ( algebra + (iii))

## Question C

Consider the following program:

**Algorithm 1:** func(n)

```
x ← 1
s ← 2
r ← n
for i=1 to n do
    x ← 2 · x
    s ← s + i · x
end
while x > 1 do
    x ← x/2
    r ← r + 1
end
```

What is the value of $x$, $s$ and $r$ at the end of the run as a function of $n$? Give a short explanation.

## Question D

Let $\ell_1$ and $\ell_2$ be two *sorted* linked lists (in increasing order).

1. Write a pseudocode that merges the two linked lists into a sorted linked list (that contains the union of their elements).

2. Provide pseudocode to deal with the following situation: the first element of $\ell_2$ is not the minimal one in the list (all other elements are in increasing order).

## Question E

Recall the implementation of linked lists using two arrays learned in class. Describe an implementation of a *double linked* list using an array of size $n \times 3$, and two variables. In this implementation a pointer is an index, and '-1' will represent NIL. Answer the following questions.

1. Describe the idea and the role of each of the array dimensions (the content of the array).

2. Provide a pseudocode for InsertLast(L,k), which inserts a key $k$ to the tail of the list $L$. Double the size of the array, if there is an overflow, and update the list accordingly.

3. Provide a pseudocode for DeletePrevKey(L,k), which receives a value $k$ and deletes the previous key from the list $L$ (if it exists).