Computational Models – Exercise 8

Due Saturday, 17 June 2023

Each student must solve the problems on their own. If you encounter difficulties, you may ask a classmate for a hint or the general idea. However, detailed discussion, note-taking, or sharing of written solutions is not allowed. Do not write down your answers while communicating with other people or show the answers for feedback.

Our grading app has severe limitations, such as no zoom tool. To make sure we can grade your work, please follow these technical guidelines:

Submit a **single PDF file** through Moodle.

The file size is limited to **10 MB**. If necessary, google *reduce PDF file size*.

Fill in your answers **on this form*** in the allocated spaces. The space provided gives you an indication of the expected length and level of detail of the answer. You may add a little more space if you need.

Include everything from this form in your submission. In particular, **include the problem statements**. Do not delete any text or omit pages, just add your answers.

Ensure your answers are **legible** (easy to read) at zoom 100% on a standard computer screen. Your text should be **large**, **sharp**, and in **high contrast** with the background.

Do not squeeze scanned solutions to fit in the space, as the text will become small.

Verify that pages are properly **ordered** and **oriented**.

The page size must be **A4**. Before submitting your file, check its page size using Acrobat Reader: go to File > Properties > Description and confirm that Page Size is around 21 × 29 cm. Note that scanning A4 pages does not guarantee the resulting page size will be A4, due to scaling. If necessary, google *resize PDF to A4*.

Do not add your answers as PDF comments. If you can drag them in Acrobat Reader, they are comments. If necessary, google *flatten PDF*.

A **5-point bonus** will be given to solutions typed in a word processor. Hand-sketched illustrations or diagrams will not deny you this bonus.

If there are technical issues with your submission, you may receive a fine. In extreme cases, your submission may not be graded at all.

If you need help or have questions, please use the course forum at Piazza.

*The only exception is in case you use LaTeX or a similar typesetting system. In that case, copy-paste everything from this file, except for illustrations or other hard-to-reproduce graphical elements. No need to fix corrupted formulas.

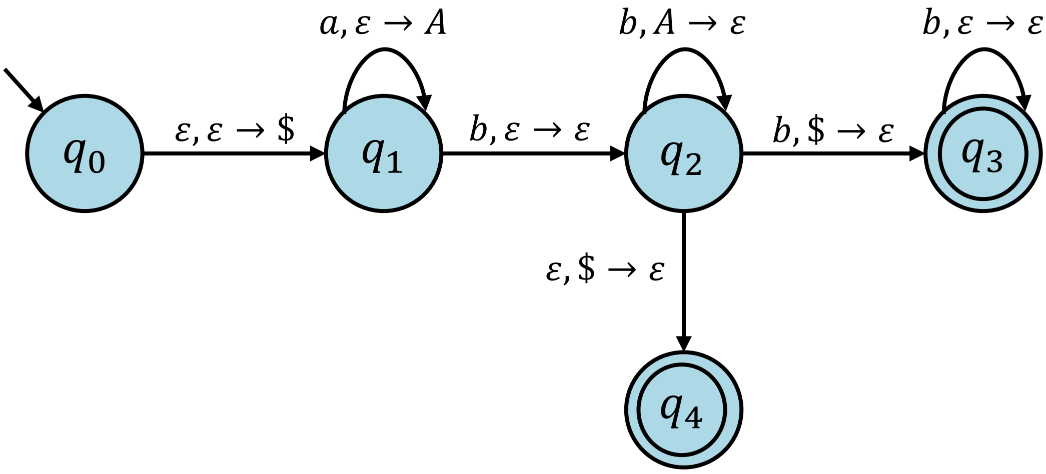
Worked with Junil Lee - 000805387

Problem 1

Who is the man in the picture? Only wrong answers :-) ______Elvis presley_______

Problem 2

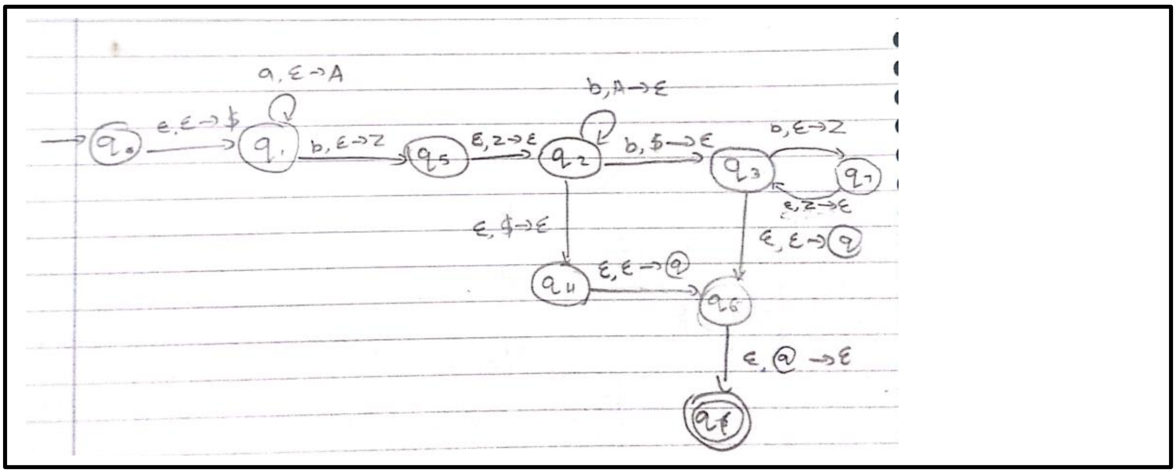
Let $P = (Q, \Sigma_P, \Gamma, \delta, q_0, F)$ be the following PDA:



[2 pt] **1.** What is $L(P)$?

The language of all the words of the form $a^m b^n$ where n > m, and m >= 0, n > 0

[3 pt] **2.** Simplify P.



[6 pt] **3.** Construct a context-free grammar $G = (V, \Sigma_G, S, R)$ such that $L(G) = L(P)$ using the algorithm learned in class.

$V = \{\mathrm{A}_{pq} \mid p, q \in \{0,1,2,3,4,5,6,7,f\}\}$____.

$\Sigma_G = \Sigma_\mathrm{P}$ = {a,b}____.

$S = \mathrm{A}_0\mathrm{A}_f$____.

$R =$

$\mathrm{A}_{00} = \mathrm{A}_{00}\mathrm{A}_{00}$
$\mathrm{A}_{00} = \mathrm{A}_{01}\mathrm{A}_{10}$
$\mathrm{A}_{00} = \mathrm{A}_{02}\mathrm{A}_{20}$
$\mathrm{A}_{00} = \mathrm{A}_{03}\mathrm{A}_{04}$
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.
.
$\mathrm{A}_{01} = \mathrm{A}_{00}\mathrm{A}_{01}$
$\mathrm{A}_{01} = \mathrm{A}_{01}\mathrm{A}_{11}$
.
.
.
$\mathrm{A}_{ff} = \mathrm{A}_{f6}\mathrm{A}_{6f}$
$\mathrm{A}_{ff} = \mathrm{A}_{f7}\mathrm{A}_{7f}$
$\mathrm{A}_{ff} = \mathrm{A}_{ff}\mathrm{A}_{ff}$

$\mathrm{A}_{00} = \varepsilon$
$\mathrm{A}_{11} = \varepsilon$
$\mathrm{A}_{22} = \varepsilon$
$\mathrm{A}_{33} = \varepsilon$
$\mathrm{A}_{44} = \varepsilon$
$\mathrm{A}_{55} = \varepsilon$
$\mathrm{A}_{66} = \varepsilon$
$\mathrm{A}_{77} = \varepsilon$
$\mathrm{A}_{ff} = \varepsilon$

$\mathrm{A}_{12} = \mathrm{bA}_{55}$
$\mathrm{A}_{12} = \mathrm{aA}_{12}b$
$\mathrm{A}_{03} = \mathrm{A}_{12}$
$\mathrm{A}_{04} = \mathrm{A}_{12}$
$\mathrm{A}_{4f} = \mathrm{A}_{66}$
$\mathrm{A}_{3f} = \mathrm{A}_{66}$

Problem 3

Let M be a classic TM (as defined in first lecture/recitation) with $Q = \{q_0, q_1, q_{acc}, q_{rej}\}$, $\Sigma = \{0,1\}$ and $\Gamma = \{0,1,\sqcup\}$.

For each of the following pairs of configurations, C_i and C_{i+1}, determine whether it is possible that C_i will yield C_{i+1}. (i.e., whether that's possible under some definition of δ). If possible, define the specific transition that will cause M to go from C_i to C_{i+1}. If not, explain why.

[2 pts] **1.** $C_i = 001q_{rej}1$
$C_{i+1} = 0011q_1$

Not possible. In configuration C_i, the head is currently on a rejecting state, which means that the run/computation immediately ends, and therefore it cannot make another transition to a new configuration.

[2 pts] **2.** $C_i = 1q_110$
$C_{i+1} = 10q_20$

Possible with the transition function $\delta(q_1, 1) = (q_2, 0, R)$

[2 pts] **3.** $C_i = 0q_1100$
$C_{i+1} = 01q_000$

Possible with the transition function $\delta(q_1, 1) = (q_0, 1, R)$

[2 pts] **4.** $C_i = 0q_0010$
$C_{i+1} = q_01010$

Not possible. In config. C_i the tape head is on the second symbol in the tape. Any transition function could therefore only write over that one symbol for it's next transition. However in config. C_{i+1} , the first symbol of the tape was changed/overwritten

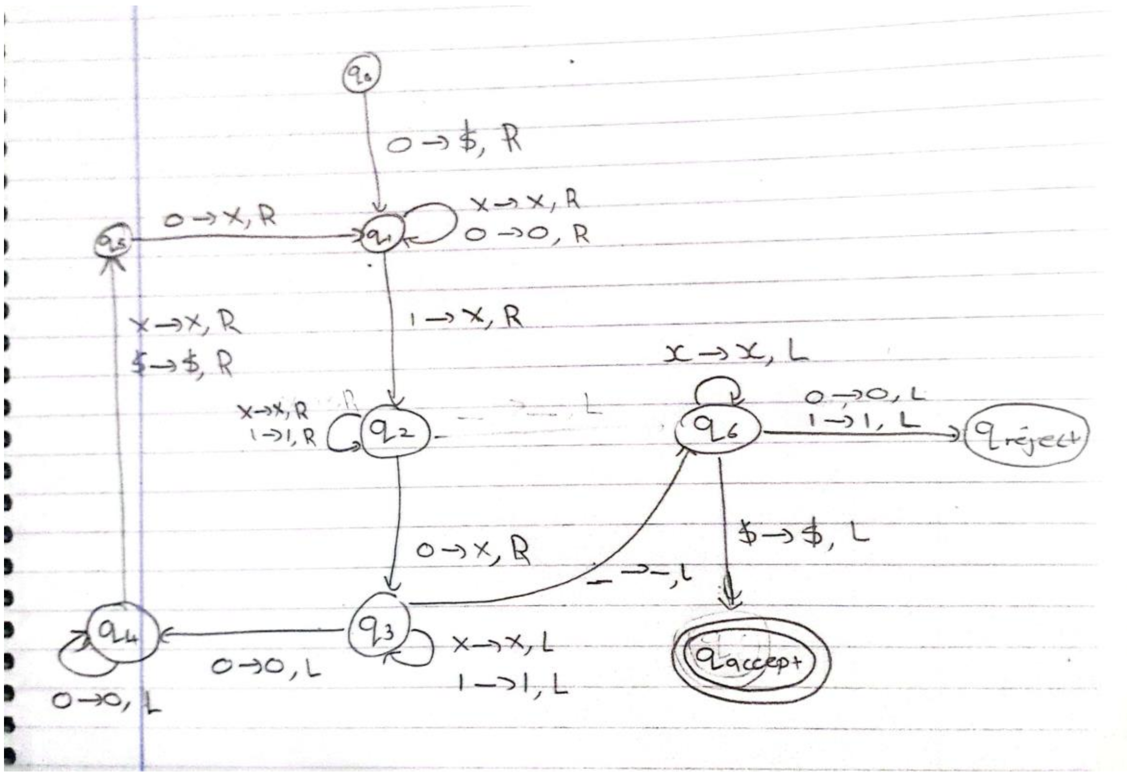
[2 pts] **5.** $C_i = q_001$
$C_{i+1} = q_101$

Possible with the transition function $\delta(q_0, 0) = (q_1, 0, L)$

Problem 4

Let $L = \{0^n 1^n 0^n \mid n \geq 0\}$.

[10 pts] **1**. Draw the state diagram of a classic TM (as defined in first lecture/recitation) M that accepts every $w \in L$ and rejects every $w \notin L$. You are allowed not specify the transitions to the reject state, but this must be explicitly stated in your solution.



[5 pts] **2**. Explain in words each component of the construction.

The left component of the diagram (q0 through q5) is repeating the same process: replace the first occurrence of a 0 in the first group of zeroes with an x, and then skipping to the first occurrence of 1 and replacing it with an x, and then moving to the group of zeroes after the 1's and replacing the first 0 of that group with an x. It repeats this process until the end of the input word (i.e when all the symbols become x's), or if at any point in the repeating process an unexpected symbol appears, then the word is rejected and the TM goes to a rejecting state (the transitions to the rejecting state were not drawn – anytime a state reads an input that has no corresponding transition defined, it should go to the rejecting state). One Exception is that the very first 0 of the word, i.e the first symbol, is actually replaced with a \$. This is for the second component of the graph to know that it has reached the beginning of the word (the left most symbol of the tape). The second component of the graph (states q6, the accepting state and the rejecting state) only becomes active once the TM has reached the end of the word. If the word is a valid word, then all the symbols on the tape should've been replaced with an x (with exception of the first symbol, which is a \$ sign). This component checks that all the symbols on the tape have been altered to an x, if it finds a non x symbol, it goes to a reject state and if it reaches the \$ sign, then it knows that its at the start of the word and that all the other symbols have been replaced with an x and so it goes to an accepting state.

For each of the input strings below give the configuration sequence of M on the corresponding inputs.

[2 pts] **1.** 001100.

$q_1$001100 -> \$$q_1$01100 -> \$0$q_1$1100 -> \$0x$q_1$100 -> \$0x1$q_1$00 -> \$0x1x$q_1$0 -> \$0x1x0q_1 ->* \$$q_1 0x1x0$ (skipped configs where the head just gets moved) -> \$x$q_1 x1x0$ -> \$xx$q_1$1x0 -> \$xxx$q_1 x$0 -> \$xxxx$q_1$0 -> \$xxxxxq_1->* q_1\$xxxxx

[no pts] **2.** 0011.

[2 pts] **3.** 0101.

Problem 5

Construct a TM that decides the language: $L = \{x\$y|\ x, y \in \{0,1\}^*\ and\ y = x + 1\}$, here x, y can be interpreted as the binary representation of non-negative integers. For example: $w_1 = 010\$011 \in L$ while $w_2 = 101\$100 \notin L$ since $x + 1 = 101 + 1 = 110 \neq 100 = y$.

You may use a multiple-tape TM with left, right and stay-put (denoted by S) moves.

[5 pts] **1.** describe the transition function δ in words (how many tapes do you use, how is your TM going to work [what is the algorithm], etc.).

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[15 pts] **2**. Write formally the definition of the 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$. You are allowed not to specify the transitions to the reject state, but this must be explicitly stated in your solution. You may assume that the input is of the form $x\$y$ ($x, y \in \{0,1\}^*$) and there is no need to check this.

$Q =$ ____.

$\Sigma =$ ____.

$\Gamma =$ ____.

$\delta =$

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Problem 6

A $\boldsymbol{5R-4L-TM}$ is similar to a standard TM with the change in which the head can only move either 4 cells to the left or 5 cells to the right (those are the only possible moves). That is, the transition function of a $5R-4L-TM$ is defined as $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{5R, 4L\}$ where Q is the set of the states of the $5R-4L-TM$ and Γ is the tape alphabet of the $5R-4L-TM$.

E.g., suppose that in the transition function there is the transition $\delta(q_0, 1) = (q_1, 0{,}5R)$ and the current configuration is $q_0 111111$ then the yield configuration will be $01111 q_1 1$.

In this problem you will prove equivalence between $5R-4L-TM$ and standard TMs with stay-put **(one tape and the head can move single cell right, to left only or stay-put)**.

Recall: to prove that two models of TMs are equivalent, we need to show:

- For every TM M_1 in the first model there is a TM M_2 in the second model s.t $L(M_1) = L(M_2)$.
- For every TM M_2 in the second model there is a TM M_1 in the first model s.t $L(M_1) = L(M_2)$.

Recall: to show that $L(M_1) = L(M_2)$ we need to show that for every string w it holds that M_1 accepts w iff M_2 accepts w.

To save you some trouble we only ask you to prove one direction of the equivalence. Prove formally that for every standard TM with stay-put M_1, there is a $5R-4L-TM$ M_2, s.t $L(M_1) = L(M_2)$.

Hints: note that $1 \cdot (-4) + 1 \cdot 5 = 1$, $4 \cdot (-4) + 3 \cdot 5 = -1$ and $5 \cdot (-4) + 4 \cdot 5 = 0$.

[5 pts] **1**. Describe the idea of your solution. A reader with the appropriate background (such as the grader) should be able to understand your solution from this description.

[25 pts] **2**. Full proof:

Please use low level descriptions of the TMs in your answer (describe the 7-tuple, as you saw in class, you may write the transition function or draw it as a state diagram, in case there are no changes in one or more of the elements of the 7-tuple then it's enough to note that there is no change). You need to write all the inductive arguments (the entire inductive proof). To save you time, we will ask you to give a full proof only for the case that M_1 takes a step to the left or for the case that M_1 takes a stay step, choose one of the options, just indicate which option you chose.

Problem 7

Give an implementation-level description of TM's that compute the following functions or decide the following languages. There's no need to draw the state-diagram or define the 7-tuple.

Note: You may use multi-tape TM's (in this case, the output of the TM is defined to be the sequence of symbols which is to the left of the head of the last tape). You may use the TM's we saw in class. There's no need to explain the construction from class, but you must specify which TM's you are using.

[7 pts] **1**. Given a string $w = w_1\#w_2$, $w_1, w_2 \in \{0,1\}^*$. The TM accepts if $w_1 \neq w_2$ and rejects otherwise.

[7 pts] **2**. Given a string $w = w_1\#w_2$, $w_1, w_2 \in \{0,1\}^*$. The TM outputs the number of times (in binary) that w_1 appears as a substring in w_2. Note that the occurrences of the substrings may overlap, for example $w_1 = 010$ and $w_2 = 01010$ (in this case w_1 appears as a substring in w_2 both in the first three characters and in the last three characters when there is an overlap in the middle 0 of w_2).

[no pts] **Problem 8**

Show how to simulate a TM with two-sided infinite tape using a standard TM. The tape is initially filled with blanks except for the portion that contains the input. The head is initially over the first input symbol. Computation is defined as usual except that the head never encounters the end of the tape as it moves left. Show the simulation directly without relying on the fact that a two-tape TM is equivalent to a standard TM.

[no pts] **Problem 9**

Show that a TM that can move its head only in two-cell jumps (i.e., two cells left or two cells right) **isn't** equivalent to a standard TM.