Лабораторная работа №7

№1

$$\begin{cases} y_1' = 5y_1 + y_2 \\ y_2' = 12y_1 + 9y_2 \end{cases}$$

$$\begin{vmatrix} 5-\lambda & 1 \\ 12 & 9-\lambda \end{vmatrix} = \lambda^2 - 14\lambda + 33 = 0$$

$$\begin{cases} \lambda_1 = 11 \\ \lambda_2 = 3 \end{cases}$$

$$\lambda_1 = 11 \Rightarrow \begin{matrix} y_1(x) = \gamma_1 e^{11x} \\ y_2(x) = \gamma_2 e^{11x} \end{matrix}$$

$$\begin{cases} -6\gamma_{11} + \gamma_{21} = 0 \\ 12\gamma_{11} - 2\gamma_{21} = 0 \end{cases}$$

$$\begin{cases} \gamma_{11} = 1 \\ \gamma_{12} = 6 \end{cases} \Rightarrow \bar{\gamma}_1 = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$\lambda_2 = 3 \Rightarrow \bar{\gamma}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\bar{y} = C_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{11x} + C_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{3x}$$

$$\begin{cases} y_1 = C_1 e^{11x} + C_2 e^{3x} \\ y_2 = 6C_1 e^{11x} - 2C_2 e^{3x} \end{cases} \qquad Y = \begin{bmatrix} e^{11x} & e^{3x} \\ 6e^{11x} & -e^{3x} \end{bmatrix}$$

$$\begin{matrix} \lambda_1 = 11 > 0 \\ \lambda_2 = 3 > 0 \end{matrix} \Rightarrow \text{Неустойчивый узел}$$

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sde := {y1' = 5·y1 + y2, y2' = 12·y1 + 9·y2} :
dsolve(sde);
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$$\{y1(x) = \_C1\, e^{11x} + \_C2\, e^{3x},\ y2(x) = 6\,\_C1\, e^{11x} - 2\,\_C2\, e^{3x}\}$$

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with(DEtools) :
DEplot3d(sde, {y1(x), y2(x)}, x = 0 ..1, [[y1(0) = 0, y2(0) = 0]]);
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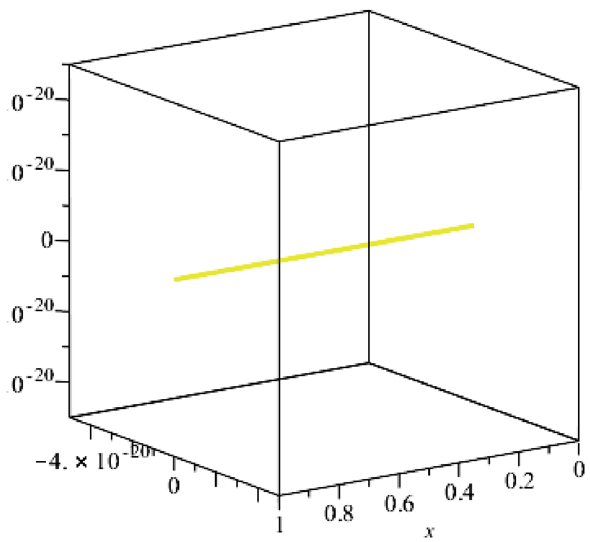
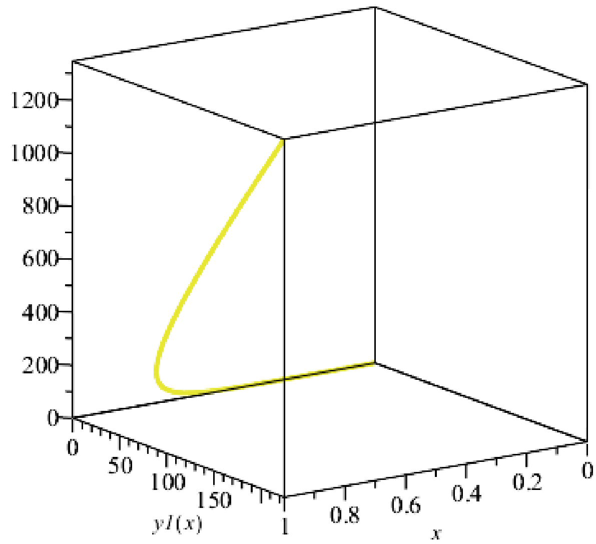
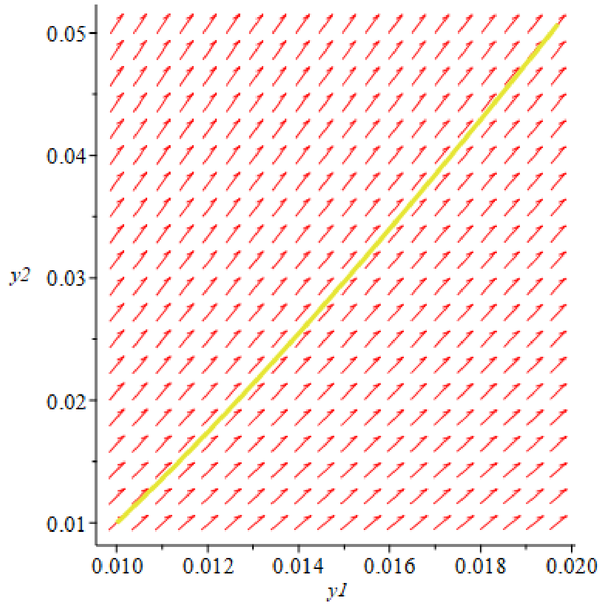


$DEplot3d(sde, \{y1(x), y2(x)\}, x = 0..1, [[y1(0) = 0.01, y2(0) = 0.01]]);$



$phaseportrait(sde, \{y1(x), y2(x)\}, x = 0..0.1, [[y1(0) = 0.01, y2(0) = 0.01]]);$



№2

$$\begin{cases} y_1' = 4y_1 + 13y_2 \\ y_2' = 5y_1 + 3y_2 \end{cases} \qquad \begin{cases} y_1'' = 4y_1' + 13y_2' \\ y_2' = 5y_1 + 3y_2 \end{cases}$$

$$y_1'' = 4y_1' + 65y_1 + 39y_2$$

$$y_1'' = 4y_1' + 65y_1 + 3y_1' - 12y_1$$

$$y_1'' - 7y_1' - 53y_1 = 0$$

$$\lambda^2 - 7\lambda - 53 = 0$$

$$D = \sqrt{49 + 4\cdot 53} = 3\sqrt{29}$$

$$\lambda_{1,2} = \frac{7 \pm 3\sqrt{29}}{2}$$

$$y_1 = C_1 e^{\lambda_1} + C_2 e^{\lambda_2}$$

$$y_2' = 5\cdot C_1 \cdot e^{\lambda_1 x} + 5\cdot C_2 e^{\lambda_2 x} + 3y_2$$

$$y_2' - 3y_2 = 5\cdot C_1 e^{\lambda_1 x} + 5\cdot C_2 e^{\lambda_2 x}$$

$$y_2 = u\vartheta$$

$$u'\vartheta + u(\vartheta' - 3\vartheta) = 5\cdot C_1 e^{\lambda_1 x} + 5\cdot C_2 e^{\lambda_2 x}$$

$$\begin{cases} \vartheta = e^{3x} \\ u' = 5\cdot C_1 e^{(\lambda_1 - 3)x} + 5\cdot C_2 \cdot e^{(\lambda_2 - 3)x} \end{cases}$$

$$u = \frac{5}{\lambda_1 - 3} C_1 e^{(\lambda_1 - 3)x} + \frac{5}{\lambda_2 - 3}\cdot C_2 e^{(\lambda_2 - 3)x}$$

$$y_2 = \frac{5}{\lambda_1 - 3}\cdot C_1 e^{\lambda_1 x} + \frac{5}{\lambda_2 - 3}\cdot C_2 \cdot e^{\lambda_2 x}$$

$$\begin{cases} y_1 = C_1 \cdot e^{\frac{7+3\sqrt{29}}{2}x} + C_2 e^{\frac{7-3\sqrt{29}}{2}x} \\ y_2 = -\frac{1-3\sqrt{29}}{26} C_1 \cdot e^{\frac{7+3\sqrt{29}}{2}x} - \frac{1+3\sqrt{29}}{26} C_2 e^{\frac{7-3\sqrt{29}}{2}x} \end{cases}$$

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dsolve({y1'= 4·y1 + 13·y2, y2'= 5·y1 + 3·y2});
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$$\left\{ y1(x) = \_C1\, e^{\frac{(7+3\sqrt{29})x}{2}} + \_C2\, e^{-\frac{(-7+3\sqrt{29})x}{2}},\ y2(x) = -\frac{3\,\_C2\, e^{-\frac{(-7+3\sqrt{29})x}{2}}\sqrt{29}}{26} + \frac{3\,\_C1\, e^{\frac{(7+3\sqrt{29})x}{2}}\sqrt{29}}{26} - \frac{\_C2\, e^{-\frac{(-7+3\sqrt{29})x}{2}}}{26} - \frac{\_C1\, e^{\frac{(7+3\sqrt{29})x}{2}}}{26} \right\}$$

Метод Лагранжа:

$$\begin{cases} \dot{x} = -x - 2y + 1 \\ \dot{y} = -\frac{3}{2}x + y \end{cases} \qquad \begin{matrix} x(0) = 1 \\ y(0) = 0 \end{matrix}$$

$$\begin{vmatrix} -1-\lambda & -2 \\ -\frac{3}{2} & 1-\lambda \end{vmatrix} = (\lambda-1)(\lambda+1) - 3 = 0$$

$$\lambda^2 - 4 = 0$$

$$\begin{cases} \lambda_1 = 2 \\ \lambda_2 = -2 \end{cases}$$

$\lambda = 2$:

$$\gamma_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$\lambda = -2$:

$$\gamma_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{cases} x = 2 \cdot C_1 e^{2t} + 2C_2 e^{-2t} \\ y = -3C_1 e^{2t} + C_2 e^{-2t} \end{cases}$$

$$\begin{cases} C_1 = C_1(t) \\ C_2 = C_2(t) \end{cases}$$

$$\begin{cases} 2C_1'(t) e^{2t} + 2C_2'(t) e^{-2t} = 1 \\ -3C_1'(t) e^{2t} + C_2'(t) e^{-2t} = 0 \end{cases}$$

$$8C_1'(t) e^{2t} = 1$$

$$\begin{cases} C_1'(t) = \frac{e^{-2t}}{8} \\ C_2'(t) = \frac{3e^{2t}}{8} \end{cases} \qquad \begin{cases} C_1(t) = -\frac{e^{-2t}}{16} + C_1 \\ C_2(t) = \frac{3e^{2t}}{16} + C_2 \end{cases}$$

$$\begin{cases} x = 2\left(C_1 - \frac{e^{-2t}}{16}\right) e^{2t} + 2\left(\frac{3e^{2t}}{16} + C_2\right) e^{-2t} \\ y = -3\left(C_1 - \frac{e^{-2t}}{16}\right) e^{2t} + \left(\frac{3e^{2t}}{16} + C_2\right) e^{-2t} \end{cases}$$

$$\begin{cases} x = 2C_1 e^{2t} + 2C_2 e^{-2t} + \frac{1}{4} \\ y = -3C_1 e^{2t} + C_2 e^{-2t} + \frac{3}{8} \end{cases}$$

dsolve({*sde*});

$$\left\{x(t) = e^{-2t}\_C2 + e^{2t}\_C1 + \frac{1}{4}, y(t) = \frac{e^{-2t}\_C2}{2} - \frac{3e^{2t}\_C1}{2} + \frac{3}{8}\right\}$$

Задача Коши:

$$\begin{cases} 1 = 2C_1 + 2C_2 + \frac{1}{4} \\ 0 = -6C_1 + 2C_2 + \frac{3}{4} \end{cases}$$

$$\begin{cases} C_1 = \frac{3}{16} \\ C_2 = \frac{3}{16} \end{cases} \qquad \begin{cases} x = \frac{3}{8}e^{2t} + \frac{3}{8}e^{-2t} + \frac{1}{4} \\ y = -\frac{9}{16}e^{2t} + \frac{3}{16}e^{-2t} + \frac{3}{8} \end{cases}$$

dsolve({*sde*, $x(0) = 1, y(0) = 0$});

$$\left\{x(t) = \frac{3e^{-2t}}{8} + \frac{3e^{2t}}{8} + \frac{1}{4}, y(t) = \frac{3e^{-2t}}{16} - \frac{9e^{2t}}{16} + \frac{3}{8}\right\}$$

Метод Д'Аламбера:

$$(x+\lambda y)' = \left(-1 - \frac{3}{2}\lambda\right)\left(x + \frac{-2+\lambda}{-1-\frac{3}{2}\lambda}y\right) + 1$$

$$(x+\lambda y)' = \left(-1 - \frac{3}{2}\lambda\right)\left(x + \frac{-2+\lambda}{-1-\frac{3}{2}\lambda}y\right) + 1$$

$$\frac{4-2\lambda}{2+3\lambda} = \lambda$$

$$\begin{cases} \lambda_1 = -2 \\ \lambda_2 = \frac{2}{3} \end{cases}$$

$\lambda_1 = -2$:

$$z = (x + \lambda y)$$

$$z' = z(-1+3) + 1$$

$$z' = 2z + 1$$

$$z' - 2z = 1$$

$$z = uv$$

$$u'v + u(v' - 2v) = 1$$

$$z = u \cdot v = \left(\frac{e^{-2t}}{-2} + C_1\right)e^{2t} = C_1 e^{2t} - \frac{1}{2}$$

$\lambda = \frac{2}{3}$:

$$z' + 2z = 1$$

$z = uv$

$$u'v + u(v' + 2v) = 1$$

$$z = u \cdot v = \left(\frac{e^{2t}}{2} + C_2\right)e^{-2t} = C_2 e^{-2t} + \frac{1}{2}$$

$$\begin{cases} x - 2y = C_1 e^{2t} - \frac{1}{2} \\ x + \frac{2}{3}y = C_2 e^{-2t} + \frac{1}{2} \end{cases}$$

$$\frac{8}{3}y = C_2 e^{-2t} + 1 - C_1 e^{2t}$$

$$y = -\frac{3}{8} C_1 e^{2t} + \frac{3}{8} C_2 e^{-2t} + \frac{3}{8}$$

$$x + \frac{3}{4} C_1 e^{2t} - \frac{3}{4} C_2 e^{-2t} - \frac{3}{4} = C_1 e^{2t} - \frac{1}{2}$$

$$\begin{cases} x = \frac{1}{4} C_1 e^{2t} + \frac{3}{4} C_2 e^{2t} + \frac{1}{4} \\ y = -\frac{3}{8} C_1 e^{2t} + \frac{3}{8} C_2 e^{-2t} + \frac{3}{8} \end{cases}$$

dsolve({*sde*});

$$\left\{x(t) = e^{-2t}\_C2 + e^{2t}\_C1 + \frac{1}{4}, y(t) = \frac{e^{-2t}\_C2}{2} - \frac{3e^{2t}\_C1}{2} + \frac{3}{8}\right\}$$

Задача Коши:

$$\begin{cases} 1 = \frac{1}{4} C_1 + \frac{3}{4} C_2 + \frac{1}{4} \\ 0 = -\frac{3}{8} C_1 + \frac{3}{8} C_2 + \frac{3}{8} \end{cases}$$

...

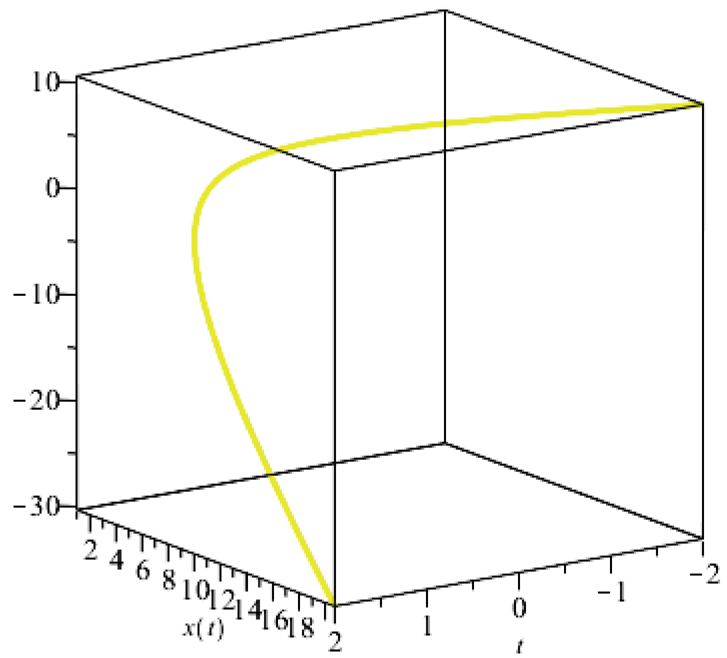
$$\begin{cases} C_1 = \frac{3}{2} \\ C_2 = \frac{1}{2} \end{cases}$$

$$\begin{cases} x = \frac{3}{8} e^{2t} + \frac{3}{8} e^{-2t} + \frac{1}{4} \\ y = -\frac{9}{16} e^{2t} + \frac{3}{16} e^{-2t} + \frac{3}{8} \end{cases}$$

dsolve({*sde*, $x(0) = 1$, $y(0) = 0$});

$$\left\{x(t) = \frac{3e^{-2t}}{8} + \frac{3e^{2t}}{8} + \frac{1}{4}, y(t) = \frac{3e^{-2t}}{16} - \frac{9e^{2t}}{16} + \frac{3}{8}\right\}$$

$DEplot3d(\{sde\}, [x(t), y(t)], t=-2..2, [[x(0)=1, y(0)=0]]);$



$phaseportrait(\{sde\}, [x(t), y(t)], t=-2..2, [[x(0)=1, y(0)=0]])$