№1

1) $x = y''^2 + \ln(y'')$

$$\begin{cases} t = y'' \\ x = t^2 + \ln(t) \end{cases}$$

```
x = y''(x)^2 + ln(y''(x));
x(t) := t^2 + ln(t) :
'x' = x(t);
```

$$x = \left(\frac{d^2}{dx^2}\, y(x)\right)^2 + \ln\left(\frac{d^2}{dx^2}\, y(x)\right)$$

$$x = t^2 + \ln(t)$$

$$dx = \frac{dy'}{t}$$

$\cancel{dy' = t2}$ $\quad dx = \left(2t + \frac{1}{t}\right) dt$

$$dy' = (2t^2 + 1)\,dt$$

$$y' = \frac{2t^3}{3} + t + C_1$$

```
dx := diff(t^2 + ln(t), t) :
diffy := int(t·(dx), t) + _C1;
```

$$diffy := \frac{2}{3} t^3 + t + \_C1$$

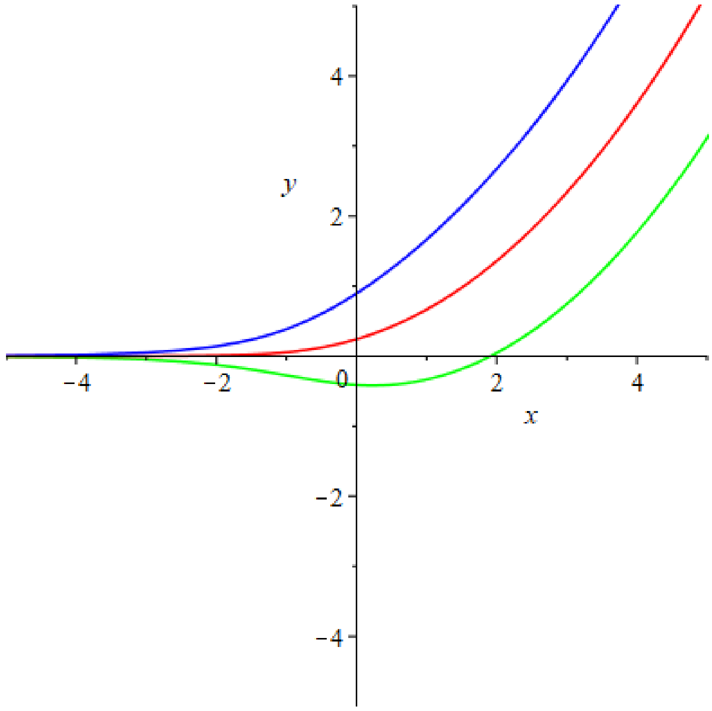
$$\begin{cases} y = \dfrac{t^4}{6} + \dfrac{t^2}{2} + C_1 t + C_2 \\ x = t^2 + \ln(t) \end{cases}$$

```
y(t) := int(diffy, t) + _C2 :
'y' = y(t);
'x' = x(t);
```

$$y = \frac{1}{6}t^4 + \frac{1}{2}t^2 + \_C1\,t + \_C2$$

$$x = t^2 + \ln(t)$$

```
a := array(1 ..3) :
tin := -10 ..10 :
xin := -5 ..5 :
yin := -5 ..5 :
for i from -1 by 1 to 1 do
a[i + 2] := plot([x(t), subs(_C1 = i, _C2 = 0, y(t)), t =-5 ..5], x = xin, y = yin);
end do:
plots[display](a[1], a[2], a[3], color = [green, red, blue]);
```



2) $\operatorname{arctg}(x)(x^2+1)(yy''-y'^2)=yy'$

$$\operatorname{arctg}(x)(x^2+1)=\frac{yy'}{yy''-y'^2}=\frac{1}{\frac{y''}{y'}-\frac{y'}{y}}$$

$$y'=zy,\quad y''=z'y+y'z$$

$$\frac{y'}{y}=z,\quad \frac{y''}{y'}=\frac{z'}{z}+z$$

$$\operatorname{arctg}(x)(x^2+1)=\frac{1}{\frac{z'}{z}+z-z}=\frac{z}{z'}$$

$$\frac{dz}{z}=\frac{dx}{\operatorname{arctg}(x)(x^2+1)}$$

$$\frac{dz}{z}=\frac{d\operatorname{arctg}(x)}{\operatorname{arctg}(x)}$$

$$z=C_1\operatorname{arctg}(x)$$

$$\frac{y'}{y}=\operatorname{arctg}(x)\cdot C_1$$

$$\ln|y|=C_1\int\operatorname{arctg}(x)\,dx$$

$\cancel{\ln|y|}$ $\int\operatorname{arctg}(x)\,dx=x\operatorname{arctg}(x)-\frac{1}{2}\int\frac{dx^2}{1+x^2}=$

$=x\operatorname{arctg}(x)-\frac{1}{2}\ln|1+x^2|+C_2$

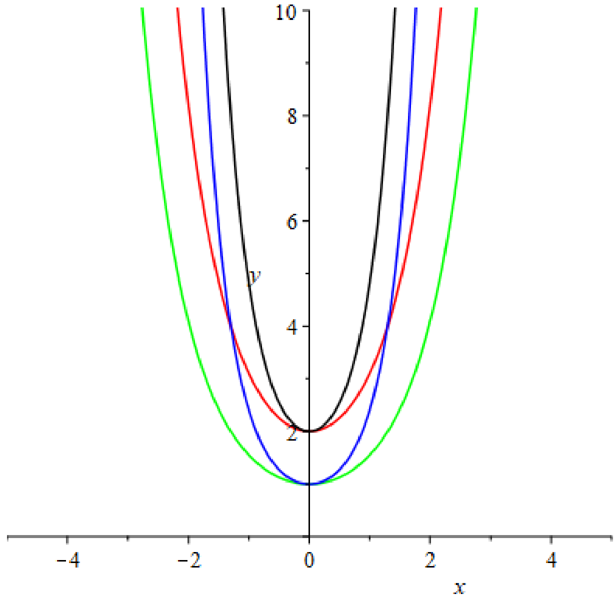
$$\ln|y|=C_1\left(x\operatorname{arctg}(x)-\frac{1}{2}\ln(1+x^2)+C_2\right)$$

$$y=C_2e^{C_1\operatorname{arctg}(x)}(1+x^2)^{-\frac{C_1}{2}}$$

$expr := \arctan(x)\cdot(x^2+1)\cdot\left(y(x)\cdot y''(x) - (y'(x))^2\right) = y(x)\cdot y'(x):$
$dsolve(expr);$
$res := rhs(\%):$

$$y(x) = e^{\_C1\,x\arctan(x)}\left(x^2+1\right)^{-\frac{\_C1}{2}}\_C2$$

$xin := -5..5:$
$yin := -0..10:$
$a = array(1..2):$
for i **from** 1 **by** 1 **to** 2 **do**
for j **from** 1 **by** 1 **to** 2 **do**
$a[2\cdot i + j - 2] := plot(subs(\_C1 = i, \_C2 = j, res), x = xin, y = yin):$
end do
end do:
$plots[display](a[1], a[2], a[3], a[4], color = [green, red, blue, black]);$



3) $y' = xy'' - \sqrt{y''}$

$$\begin{cases} y' = z \\ z = xz' - \sqrt{z'} \end{cases}$$

$$\begin{cases} z' = t \\ z = xt - \sqrt{t} \end{cases}$$

$$dz = x\,dt + t\,dx - \frac{dt}{2\sqrt{t}}$$

$$dz = t\,dx$$

$$t\,dx = x\,dt + t\,dx - \frac{dt}{2\sqrt{t}}$$

$$x = \frac{1}{2\sqrt{t}}\ ; \quad x^2 = \frac{1}{4\cdot t}$$

$$z' = t = \frac{1}{4x^2}\ , \quad z = -\frac{1}{4x}$$

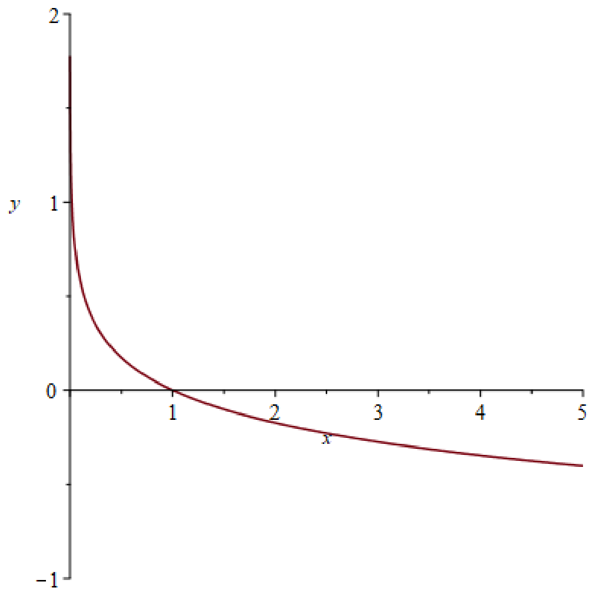
$dsolve\left(z = x \cdot z' - \sqrt{z'}, z(x), parametric\right);$
$i := int(rhs(\%), x) :$

$$z(x) = -\frac{\sqrt{\frac{1}{x^2}}}{4}$$

$$y' = -\frac{1}{4x}\ , \quad y = -\frac{1}{4}\ln|x| + C_1$$

$$y(x) = -\frac{\sqrt{\frac{1}{x^2}}\, x \ln(x)}{4} + \_C1$$

$plot(i, x = 0..5, y = -1..2);$



4) $2y'' = \frac{y'}{x} - \frac{y}{x^2} + \frac{e^{\sqrt{x}}}{\sqrt{x}}$

$\cancel{2}y'' - \frac{y'}{2x} + \frac{y}{2x^2} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

$y'' - \frac{y'}{2x} + \frac{y}{2x^2} = 0$

$y'' = \frac{xy'}{2x^2} - \frac{y}{2x^2}$

$y'' = \frac{1}{2}\,\frac{xy' - y}{x^2}$

$y'' = \left(\frac{y}{2x}\right)' \Rightarrow y' = \frac{y}{2x}$

$\ln|y| = \ln|\sqrt{x}|$

$y_1 = \sqrt{x}$

$W_{y_1 y_2}(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = e^{-\int a_1(x)\,dx}$

```
y1 := sqrt(x) :
W := Matrix([[y1, y1'], [y2, y2']]);
det := LinearAlgebra[Determinant](W);
```

$$W := \begin{bmatrix} \sqrt{x} & \frac{1}{2\sqrt{x}} \\ y2(x) & \frac{d}{dx}\,y2(x) \end{bmatrix}$$

$$det := \frac{2x\left(\frac{d}{dx}\,y2(x)\right) - y2(x)}{2\sqrt{x}}$$

$y_1 y_2' - y_2 y_1' = e^{\int \frac{1}{2x}dx}$

$y_1 y_2' - y_2 y_1' = Ce^{\ln|\sqrt{x}|}$

$y_2' - \frac{1}{2x}y_2 = C$

$y_2 = uv$

$u'v + u\left(v' - \frac{v}{2x}\right) = C$

$$\begin{cases} \frac{\vartheta'}{\vartheta} = \frac{1}{2x} \\ u'\vartheta = c \end{cases} \qquad \begin{cases} \vartheta = \sqrt{x} \\ u' = \frac{c_1}{\sqrt{x}} \end{cases} \qquad \begin{cases} \vartheta = \sqrt{x} \\ u = 2c_1\sqrt{x} + C_2 \end{cases}$$

$$y_2 = (2C_1\sqrt{x} + C_2)\sqrt{x}$$

$$y_2 = C_1 x + C_2\sqrt{x}$$

$$y = y_1 + y_2 = C_1 x + C_2\sqrt{x}$$

$dsolve\left(det = C \cdot e^{\int -\frac{1}{2x}\,dx}, y2(x)\right);$

$y = simplify\left(\sqrt{x} \cdot (rhs(\%))\right);$

$$y2(x) = \sqrt{x}\,\_C1 - 2C$$

$$y = \_C1\,x - 2C\sqrt{x}$$

$dsolve\left(y'' - \frac{y'}{2 \cdot x} + \frac{y}{2 \cdot x^2} = 0, y(x)\right);$

$$y(x) = \sqrt{x}\,\_C1 + \_C2\,x$$

$$\begin{cases} C_1' x + C_2'\sqrt{x} = 0 \\ C_1' + \frac{C_2'}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{\sqrt{x}} \end{cases} \qquad \begin{cases} C_1' x + C_2'\sqrt{x} = 0 \\ C_1' x + \frac{C_2'\sqrt{x}}{2} = \sqrt{x}\,e^{\sqrt{x}} \end{cases}$$

$$\cancel{-\frac{C_2'\sqrt{x}}{2} = \sqrt{x}\,e^{\sqrt{x}}} \qquad \frac{C_2'\sqrt{x}}{2} = -\frac{C_1' x}{2}$$

$$\frac{C_1' x}{2} = \sqrt{x}\,e^{\sqrt{x}}; \quad C_1' = \frac{4e^{\sqrt{x}}}{2\sqrt{x}}$$

$$C_1 = 4e^{\sqrt{x}} + C$$

$$2\sqrt{x}\,e^{\sqrt{x}} = -C_2'\sqrt{x}$$

$$C_2' = -2e^{\sqrt{x}}$$

$$C_2 = -2\int e^{\sqrt{x}}\,dx$$

$$\int e^{\sqrt{x}}\,dx = \int \frac{\sqrt{x}\,e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int \sqrt{x}\,e^{\sqrt{x}}\,d\sqrt{x} =$$

$$= 2\int \sqrt{x}\,d\,e^{\sqrt{x}} = 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} = 2e^{\sqrt{x}}(\sqrt{x} - 1) + C$$

`

$$C_2 = -4e^{\sqrt{x}}(\sqrt{x}-1) + C$$

$$C_2 = e^{\sqrt{x}}(1-\sqrt{x}) + C$$

$$y = x\cdot(4e^{\sqrt{x}} + C_1) + \sqrt{x}(C_1 - 4e^{\sqrt{x}}(\sqrt{x}-1)) =$$

$$= C_2\sqrt{x} - 4e^{\sqrt{x}}(x-\sqrt{x}) + 4xe^{\sqrt{x}} + x\cdot C_1 =$$

$$= C_1 x + 4\sqrt{x}e^{\sqrt{x}} + C_2\sqrt{x}$$

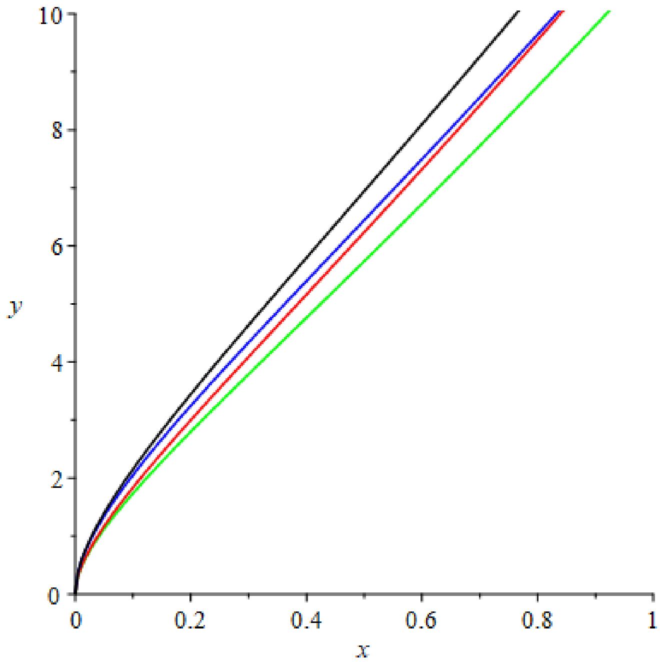
$res := dsolve\left(\left[C1'\cdot\sqrt{x} + C2'\cdot x = 0, \frac{C1'}{2\cdot\sqrt{x}} + C2' = \frac{e^{\sqrt{x}}}{\sqrt{x}}\right], [C1(x), C2(x)]\right);$

$$res := \{C1(x) = -4e^{\sqrt{x}}\sqrt{x} + 4e^{\sqrt{x}} + \_C1, C2(x) = 4e^{\sqrt{x}} + \_C2\}$$

$y := simplify(rhs(res[1])\cdot\sqrt{x} + rhs(res[2])\cdot x);$

$$y := 4e^{\sqrt{x}}\sqrt{x} + \sqrt{x}\,\_C1 + \_C2\,x$$

a := *array*(1 ..4) :
for *i* **from** 0 **by** 1 **to** 1 **do**
for *j* **from** 0 **by** 1 **to** 1 **do**
a[2·*i* + *j* + 1] := *plot*(*subs*(_C1 = *i*, _C2 = *j*, *y*(*x*)), *x* = 0 ..1, '*y*' = 0 ..10) :
end do
end do:
plots[*display*](*a*[1], *a*[2], *a*[3], *a*[4], *color* = [*green*, *red*, *blue*, *black*]);



№2

$$2xy''' = y''$$

$$y'' = z$$

$$2xz' = z$$

$$\frac{2x\,dz}{dx} = z; \qquad \frac{dx}{2x} = \frac{dz}{z}$$

$$C_1\sqrt{x} = z$$

$$y'' = C_1\sqrt{x}$$

$$y' = C_1 x^{\frac{3}{2}} + C_2$$

$$y = C_1 x^{\frac{5}{2}} + C_2 x + C_3$$

```
#task2
 restart;
 dsolve(2·x·y'''= y'', y(x));
```

$$y(x) = \_C1 + \_C2 x^{5/2} + \_C3\, x$$

№3

$y' = z$

$z' + 2z = -2e^{x}(\sin(x) + \cos(x))$

$z' + 2z = 0 \Rightarrow \frac{dz}{z} = -2dx \Rightarrow z = Ce^{-2x}$

$z = C(x)e^{-2x}$

$z' = C'(x)e^{-2x} - 2C(x)e^{-2x}$

$C'(x)e^{-2x} - 2C(x)e^{-2x} + 2C(x)e^{-2x} = -2e^{x}(\sin(x) + \cos(x))$

$C'(x) = -2e^{3x}(\sin(x) + \cos(x))$

$C(x) = -2\int e^{3x}(\sin(x) + \cos(x)) = -2\int e^{3x}\sin(x)dx -$

$-2\int e^{3x}\cos(x)dx$

$\int e^{3x}\sin(x)dx = \frac{1}{3}\int \sin(x)\,de^{3x} = \frac{1}{3}e^{3x}\sin(x) -$

$-\frac{1}{3}\int e^{3x}\cos(x)dx$

$C(x) = -\frac{2}{3}e^{3x}\sin(x) - \frac{4}{3}\int e^{3x}\cos(x)dx$

$\int e^{3x}\cos(x)dx = \frac{1}{3}\int \cos(x)\,de^{3x} = \frac{1}{3}e^{3x}\cos(x) +$

$+\frac{1}{3}\int e^{3x}\sin(x)dx = \frac{1}{3}e^{3x}\cos(x) + \frac{1}{9}\int \sin(x)\,de^{3x} =$

$= \frac{1}{3}e^{3x}\cos(x) + \frac{1}{9}e^{3x}\sin(x) - \frac{1}{9}\int e^{3x}\cos(x)dx$

$\frac{10}{9}\int e^{3x}\cos(x)dx = \frac{1}{3}e^{3x}\cos(x) + \frac{1}{9}e^{3x}\sin(x)$

$10\int e^{3x}\cos(x)dx = 3e^{3x}\cos(x) + e^{3x}\sin(x)$

$\int e^{3x}\cos(x)dx = \frac{1}{10}e^{3x}(\sin(x) + \cos(x))$

$C(x) = -\frac{2}{3}e^{3x}\sin(x) - \frac{4}{3}\int e^{3x}\cos(x)dx =$

$= -\frac{2}{3}\left(e^{3x}\sin(x) + 2\int e^{3x}\cos(x)dx\right) = -\frac{2}{3}\left(e^{3x}\sin(x) + \frac{1}{5}e^{3x}\sin(x) +\right.$

$\left.+\frac{3}{5}e^{3x}\cos(x)\right) = -\frac{2}{15}\left(5e^{3x}\sin(x) + e^{3x}\sin(x) + 3e^{3x}\cos(x)\right) =$

$= -\frac{2}{15}e^{3x}(6\sin(x) + 3\cos(x)) = -\frac{2}{5}e^{3x}(2\sin(x) + \cos(x))$

$$C(x) = -\frac{2}{5} e^{3x}(2\sin(x) + \cos(x)) + C_1$$

$$z = -\frac{2}{5} e^{x}(2\sin(x) + \cos(x)) + C_1 e^{-2x}$$

$$z = y'$$

$$y' = -\frac{4}{5} e^{x}\sin(x) - \frac{2}{5} e^{x}\cos(x) + C_1 e^{-2x}$$

$$y = -\frac{4}{5}\int e^{x}\sin(x)dx - \frac{2}{5}\int e^{x}\cos(x)dx + C_1\int e^{-2x}dx + C_2$$

$$\int e^{x}\sin(x)dx = \int \sin(x)de^{x} = e^{x}\sin(x) - \int e^{x}\cos(x)dx$$

$$y = -\frac{4}{5} e^{x}\sin(x) + \frac{2}{5}\int e^{x}\cos(x)dx + C_1\int e^{-2x}dx + C_2$$

$$\int e^{x}\cos(x)dx = \int \cos(x)de^{x} = e^{x}\cos(x) + \int e^{x}\sin(x)dx =$$

$$= e^{x}\cos(x) + \int \sin(x)de^{x} = e^{x}(\cos(x) + \sin(x)) - \int e^{x}\cos(x)dx$$

$$\int e^{x}\cos(x) = \frac{1}{2} e^{x}(\cos(x) + \sin(x))$$

$$\int e^{-2x}dx = -\frac{e^{-2x}}{2}$$

$$y = -\frac{4}{5} e^{x}\sin(x) + \frac{1}{5} e^{x}\cos(x) + \frac{1}{5} e^{x}\sin(x) - C_1 e^{-2x} + C_2$$

$$y = C_2 + e^{x}\left(-\frac{3}{5}\sin(x) + \frac{1}{5}\cos(x)\right) - C_1 e^{-2x}$$

$$y = C_2 + \frac{1}{5} e^{x}(\cos(x) - 3\sin(x)) - C_1 e^{-2x}$$

№3

$$y'' + 2y' = -2e^{x}(\sin(x) + \cos(x))$$

$$y'' + 2y' = 0$$

$$\lambda^2 + 2\lambda = 0$$

$$\begin{cases}\lambda_1 = 0\\ \lambda_2 = -2\end{cases} \qquad \begin{cases}y_1 = 1\\ y_2 = e^{-2x}\end{cases}$$

$$y_{oo} = C_1 + C_2 e^{-2x}$$

$$y_{чн} = -2e^{x}(A\sin(x) + B\cos(x))$$

$$y'_{чн} = -2e^{x}(A\sin(x) + B\cos(x)) - 2e^{x}(A\cos(x) - B\sin(x))$$

$$y'_{чн} = -2e^{x}((A-B)\sin(x) + (A+B)\cos(x))$$

$$y''_{чн} = -2e^{x}((A-B)\sin(x) + (A+B)\cos(x)) - 2e^{x}((A-B)\cdot\cos(x) - (A+B)\sin(x))$$

$$y''_{чн} = -2e^{x}(2A\cos(x) - 2B\sin(x))$$

$$-2e^{x}(A\cos(x) - B\sin(x)) - 4e^{x}((A-B)\sin(x) + (A+B)\cos(x)) = -2e^{x}(\sin(x) + \cos(x))$$

$$e^{x}((2A-4B)\sin(x) + (4A+2B)\cos(x)) = e^{x}(\sin(x) + \cos(x))$$

$$\begin{cases}2A - 4B = 1\\ 4A + 2B = 1\end{cases} \qquad \begin{cases}2A - 4B = 1\\ 8A + 4B = 2\end{cases}$$

$$\begin{cases}10A = 3\\ B = \frac{2A-1}{4}\end{cases} \qquad \begin{cases}A = \frac{3}{10}\\ B = -\frac{1}{10}\end{cases}$$

$$y_{чн} = \frac{1}{5}e^{x}(\cos(x) - 3\sin(x))$$

$$y = \frac{1}{5}e^{x}(\cos(x) - 3\sin(x)) + C_1 + C_2 e^{-2x}$$

```
#task3
restart;
dsolve(y'' + 2·y' = -2·exp(x)·(sin(x) + cos(x)), y(x));
```

$$y(x) = \frac{e^{x}\cos(x)}{5} - \frac{3\,e^{x}\sin(x)}{5} - \frac{\_C1}{2\,(e^{x})^2} + \_C2$$