```
#task1
 restart;
fourier :=
  proc ( f, x1, x2, t := x2 − x1)
  global a0; global an; global bn;
```

$$a0 := simplify\left(\frac{2}{t} \cdot int(f(x), x = x1..x2) \right);$$

$$an := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t} \right), x = x1..x2 \right) \right) \text{ assuming } n :: posint;$$

$$bn := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t} \right), x = x1..x2 \right) \right) \text{ assuming } n :: posint;$$

$$(x, k) \rightarrow \left(\frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t} \right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t} \right), n = 1..k \right) \right):$$

end proc:

$$f := x \rightarrow piecewise\left(-\pi \leq x < 0, \frac{\pi + x}{2}, 0 \leq x < \pi, -\frac{\pi}{2} \right):$$

$$ff := fourier(f, -\pi, \pi, 2 \cdot \pi):$$

```
 a0;
 an;
 bn;
```

$$-\frac{\pi}{4}$$

$$\frac{(-1)^{1+n} + 1}{2\pi n^2}$$

$$\frac{(-1)^n - 2}{2n}$$

(1)

$$r := -3 \cdot \pi..3 \cdot \pi:$$
$$p1 := plot(ff(x, 1000), x = r, color = red, thickness = 4):$$
$$p2 := plot(ff(x, 1), x = r, color = navy):$$
$$p3 := plot(ff(x, 3), x = r, color = cyan):$$
$$p4 := plot(ff(x, 5), x = r, color = black):$$
$$p5 := plot(ff(x, 7), x = r, color = green):$$
$$plots[display]([p1, p2, p3, p4, p5]);$$
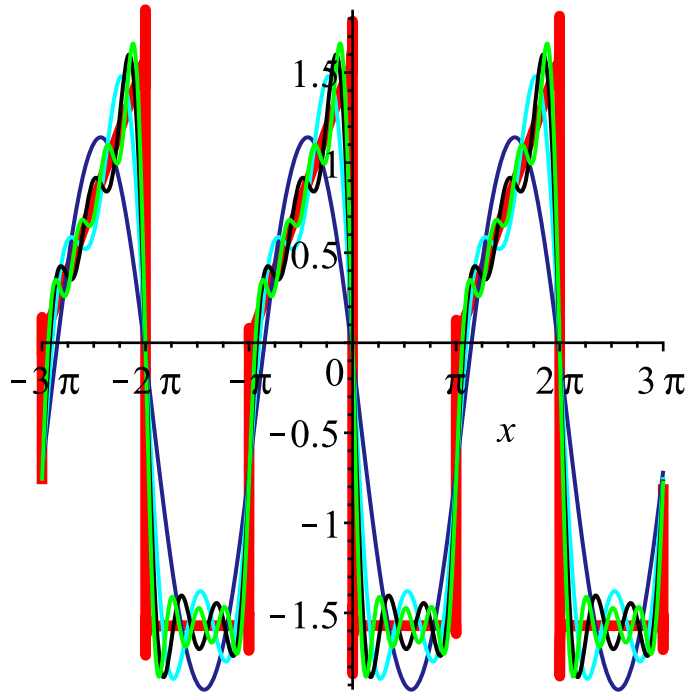
#task 2
 restart;
fourier :=
 proc $(f, x1, x2, t := x2 - x1)$
 global *a0*; **global** *an*; **global** *bn*;

 $a0 := simplify\left(\dfrac{2}{t} \cdot int(f(x), x = x1 ..x2) \right)$;

 $an := simplify\left(\dfrac{2}{t} \cdot int\left(f(x) \cdot \cos\left(\dfrac{2 \cdot \pi \cdot n \cdot x}{t} \right), x = x1 ..x2 \right) \right)$ assuming $n :: posint$;

 $bn := simplify\left(\dfrac{2}{t} \cdot int\left(f(x) \cdot \sin\left(\dfrac{2 \cdot \pi \cdot n \cdot x}{t} \right), x = x1 ..x2 \right) \right)$ assuming $n :: posint$;

 $(x, k) \rightarrow simplify\left(\dfrac{a0}{2} + sum\left(an \cdot \cos\left(\dfrac{2 \cdot \pi \cdot n \cdot x}{t} \right) + bn \cdot \sin\left(\dfrac{2 \cdot \pi \cdot n \cdot x}{t} \right), n = 1 ..k \right) \right) :$

 end proc:
 $a := 3 :$
 $b := 1 :$
 $c := -2 :$
 $x1 := 2 :$
 $x2 := 4 :$
 $f(x) := piecewise(0 < x < x1, a \cdot x + b, x1 \le x \le x2, c) :$
 $ff := fourier(f, 0, x2) :$
 a0;
 an;
 bn;

$$2$$

$$\dfrac{6 \, (-1)^n - 6}{\pi^2 \, n^2}$$

$$\frac{3 + 9 \, (-1)^{1+n}}{\pi \, n}$$

$r := -x2 .. x2 :$
$p1 := plot(\mathit{ff}(x, 250), x = r, color = red, thickness = 3) :$
$p2 := plot(\mathit{ff}(x, 1), x = r, color = navy) :$
$p3 := plot(\mathit{ff}(x, 3), x = r, color = cyan) :$
$p4 := plot(\mathit{ff}(x, 5), x = r, color = black) :$
$p5 := plot(\mathit{ff}(x, 7), x = r, color = green) :$
$plots[display]([p1, p2, p3, p4, p5]);$
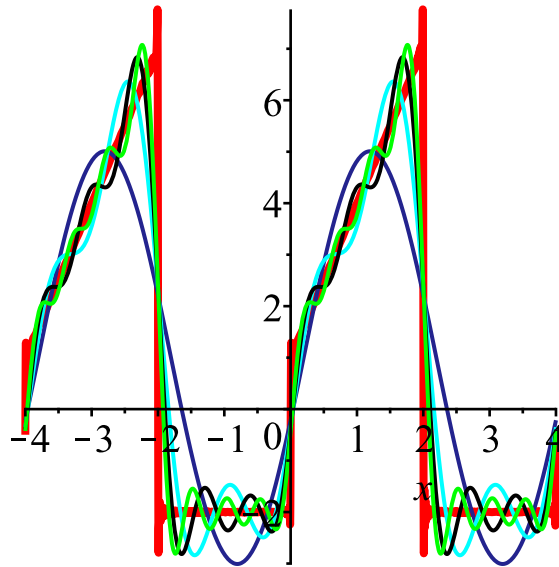
$plot([f(x), \mathit{ff}(x, 250)], x = r, thickness = [3, 1], color = [red, green]);$
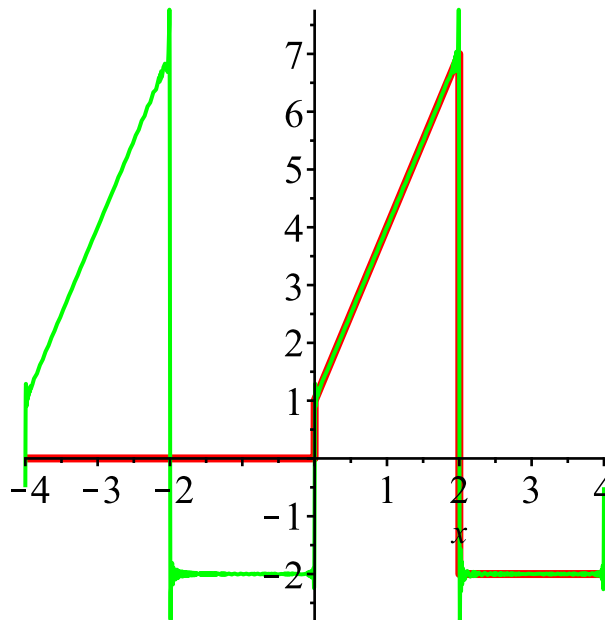
#task3
restart;
fourier :=
proc $(f, x1, x2, t := x2 - x1)$
global $a0$; **global** an; **global** bn;

$$a0 := simplify\left(\frac{2}{t} \cdot int(f(x), x = x1 .. x2)\right);$$

$$an := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), x = x1 .. x2\right)\right) \text{ assuming } n :: posint;$$

$$bn := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), x = x1 .. x2\right)\right) \text{ assuming } n :: posint;$$

$$(x, k) \rightarrow \left(\frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), n = 1 .. k\right)\right) :$$

end proc:

fourier_cos :=
proc$(f, x1, x2)$
global *a0*; **global** *an*;

$$a0 := simplify\left(\frac{2}{x2} \cdot int(f(x), x = x1 .. x2)\right);$$

$$an := simplify\left(\frac{2}{x2} \cdot int\left(f(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{x2}\right), x = x1 .. x2\right)\right) \text{ assuming } n :: posint;$$

$$(x, k) \rightarrow \left(\frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{x2}\right), n = 1 .. k\right)\right) :$$

end proc:

fourier_sin :=
proc$(f, x1, x2)$
global *bn*;

$$bn := simplify\left(\frac{2}{x2} \cdot int\left(f(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{x2}\right), x = x1 .. x2\right)\right) \text{ assuming } n :: posint;$$

$$(x, k) \rightarrow \left(sum\left(bn \cdot \sin\left(\frac{\pi \cdot n \cdot x}{x2}\right), n = 1 .. k\right)\right) :$$

end proc:
$x1 := 0 :$
$x2 := 6 :$

$$f(x) := piecewise\left(0 < x < 4, \frac{(x-2)^2}{2}, 4 \leq x \leq 6, -x + 6\right) :$$

$ff := fourier(f, x1, x2) :$
$a0;$
$an;$
$bn;$

$$\frac{14}{9}$$

$$\frac{9\,\pi\,n \cos\left(\dfrac{4\,\pi\,n}{3}\right) + 3\,\pi\,n - 9\,\sin\left(\dfrac{4\,\pi\,n}{3}\right)}{\pi^3\,n^3}$$

$$\frac{2\,\pi^2\,n^2 + 9\,\pi\,n \sin\left(\dfrac{4\,\pi\,n}{3}\right) + 9\,\cos\left(\dfrac{4\,\pi\,n}{3}\right) - 9}{\pi^3\,n^3}$$

(3)

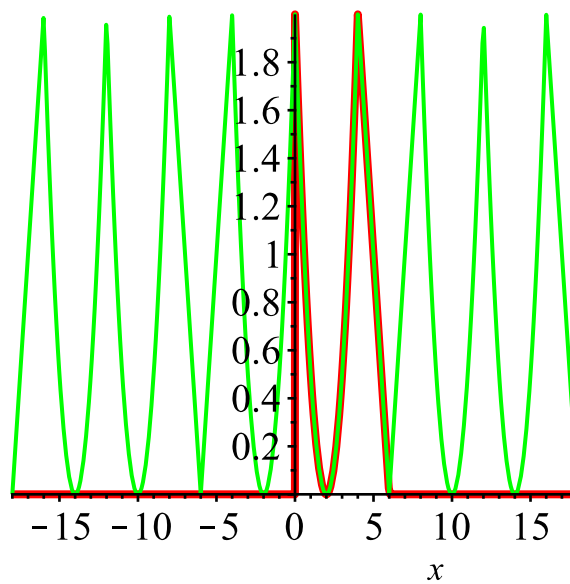$plot([f(x), ff(x, 10000)], x = -3 \cdot x2 .. x2 \cdot 3, thickness = [3, 1], color = [red, green]);$
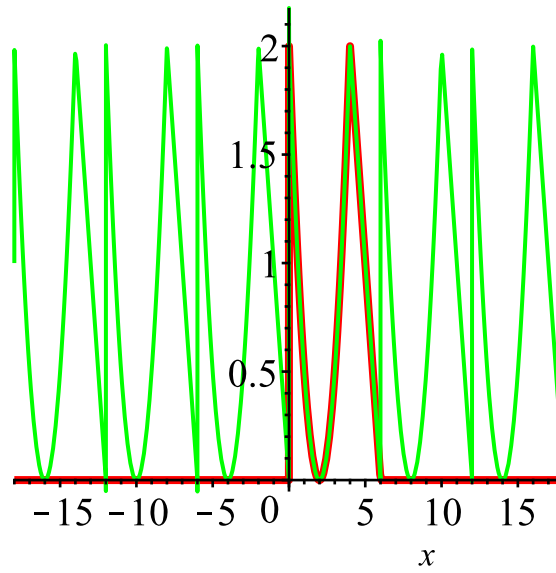
$fcos := fourier\_cos(f, x1, x2) :$
$a0;$
$an;$

$$\frac{14}{9}$$

$$\frac{12 \pi (-1)^{n+1} n + 36 \pi n \cos\left(\frac{2 \pi n}{3}\right) + 24 \pi n - 72 \sin\left(\frac{2 \pi n}{3}\right)}{\pi^3 n^3}$$

(4)

$plot([f(x), fcos(x, 10000)], x = -3 \cdot x2 .. x2 \cdot 3, thickness = [3, 1], color = [red, green]);$

$fsin := fourier\_sin(f, x1, x2) :$
$bn;$

$$\frac{4\pi^2 n^2 + 36\pi n \sin\left(\dfrac{2\pi n}{3}\right) + 72\cos\left(\dfrac{2\pi n}{3}\right) - 72}{\pi^3 n^3} \tag{5}$$

$plot([\,f(x), fsin(x, 10000)\,], x = -3\cdot x2\mathinner{..}x2\cdot 3, thickness = [3, 1], color = [red, green]);$