#task4

#a)

restart;

with(orthopoly) :

$Pm :=$

proc(f, $r := -1..1$) **global** Cp;

$$Cp := n \rightarrow \frac{int(f(x) \cdot P(n, x), x = r)}{int(P^2(n, x), x = r)};$$

$(x, m) \rightarrow add(Cp(k) \cdot P(k, x), k = 0..m)$;

end proc:

$Tm :=$

proc(f, $r := -1..1$) **global** Ct;

$$Ct := n \rightarrow \frac{int\left(\frac{f(x) \cdot T(n, x)}{\sqrt{1 - x^2}}, x = -1..1\right)}{int\left(\frac{T^2(n, x)}{\sqrt{1 - x^2}}, x = -1..1\right)};$$

$(x, m) \rightarrow add(Ct(k) \cdot T(k, x), k = 0..m)$;

end proc:

$fourier :=$

proc ($f, x1, x2, t := x2 - x1$)

global $a0$; **global** an; **global** bn;

$$a0 := simplify\left(\frac{2}{t} \cdot int(f(x), x = x1..x2)\right);$$

$$an := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), x = x1..x2\right)\right) \text{ assuming } n :: posint;$$

$$bn := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), x = x1..x2\right)\right) \text{ assuming } n :: posint;$$

$$(x, k) \rightarrow \left(\frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), n = 1..k\right)\right) :$$

end proc:

$eps := 0.1$:

$r := -1..1$:

$f := x \rightarrow -\sin^3(2 \cdot x)$:

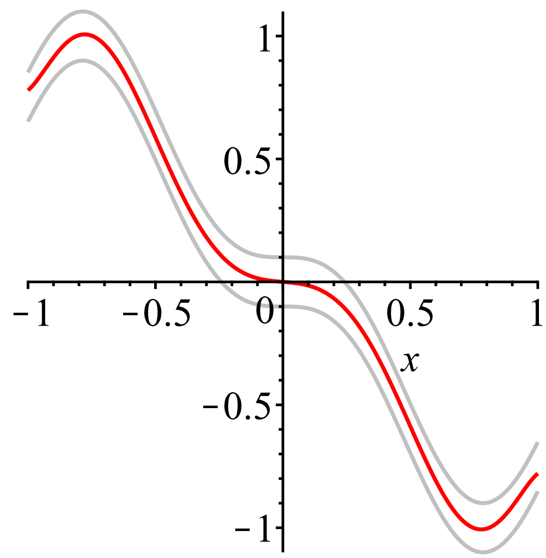
$fp := Pm(f)$:

$p := plot(fp(x, 7), x = r, color = red)$:

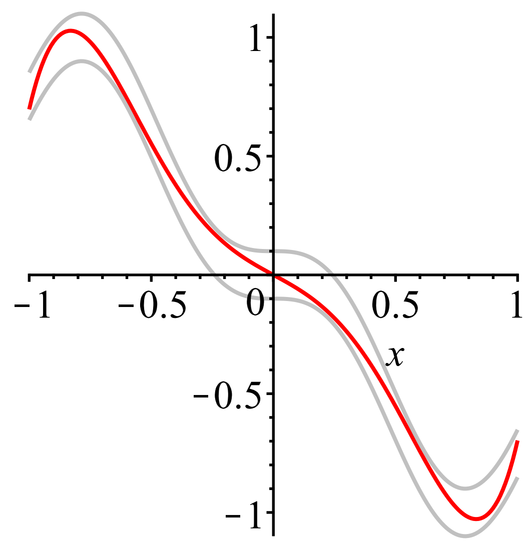
$p1 := plot(f(x) + eps, x = r, color = gray)$:

$p2 := plot(f(x) - eps, x = r, color = gray)$:

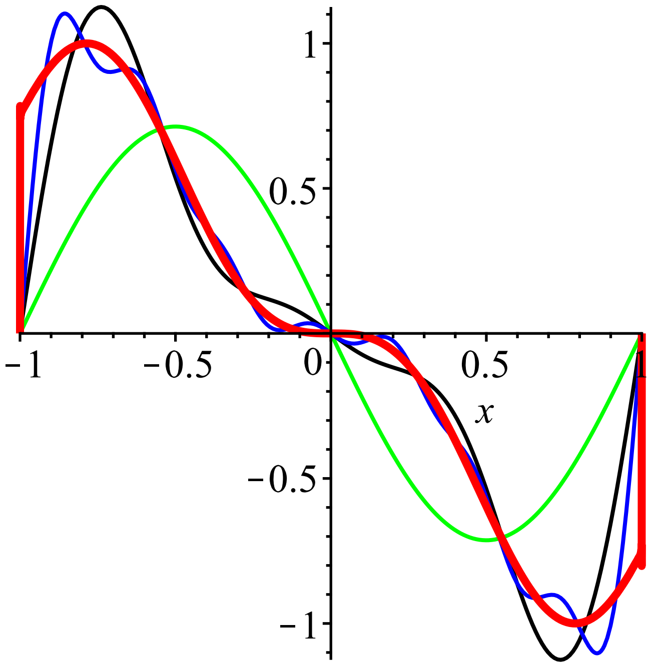
$plots[display]([p, p1, p2])$;



$ft := Tm(f):$
$t := plot(ft(x, 6), x = r, color = red):$
$p1 := plot(f(x) + eps, x = r, color = gray):$
$p2 := plot(f(x) - eps, x = r, color = gray):$
$plots[display]([t, p1, p2]);$



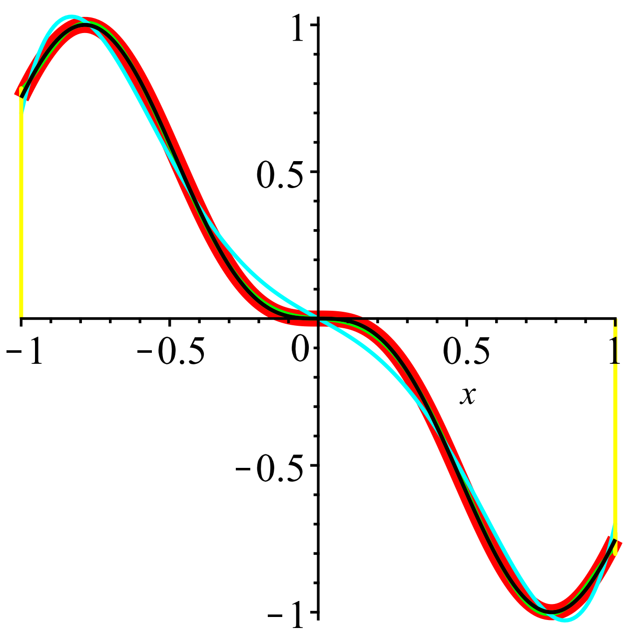
$ff := fourier(f, -1, 1):$
$r := -1 .. 1:$
$ff\_plot1 := plot(ff(x, 1), x = r, color = green):$
$ff\_plot2 := plot(ff(x, 3), x = r, color = black):$
$ff\_plot3 := plot(ff(x, 7), x = r, color = blue):$
$ff\_plot4 := plot(ff(x, 10000), x = r, color = red, thickness = 3):$
$plots[display]([ff\_plot1, ff\_plot2, ff\_plot3, ff\_plot4]);$



taylor($f(x), x=0, 25$);
fst := $x \rightarrow$ *convert*(*taylor*($f(x), x=0, 25$), *polynom*) :

$$(-8)\,x^3 + 16\,x^5 - \frac{208}{15}\,x^7 + \frac{1312}{189}\,x^9 - \frac{10736}{4725}\,x^{11} + \frac{2336}{4455}\,x^{13} - \frac{19131872}{212837625}\,x^{15} + \frac{506432}{42567525}\,x^{17} - \frac{448144}{357847875}\,x^{19} + \frac{25358432}{236238154425}\,x^{21} - \frac{5457575584}{714620417135625}\,x^{23} + \mathrm{O}\left(x^{25}\right) \quad \textbf{(1)}$$

plot([$f(x)$, *fp*(x, 7), *ft*(x, 6), *ff*(x, 10000), *fst*(x)], $x=r$, *color* = [*red*, *green*, *cyan*, *yellow*, *black*], *thickness* = [6, 1, 1, 1, 1]);



#b)
restart;

$with(orthopoly):$
$Pm :=$
proc$(f, r := -1..1)$ **global** Cp;

$$Cp := n \rightarrow \frac{int(f(x) \cdot P(n, x), x = r)}{int\left(P^2(n, x), x = r\right)};$$

$(x, m) \rightarrow add(Cp(k) \cdot P(k, x), k = 0..m);$
end proc:

$Tm :=$
proc$(f, r := -1..1)$ **global** Ct;

$$Ct := n \rightarrow \frac{int\left(\frac{f(x) \cdot T(n, x)}{\sqrt{1 - x^2}}, x = -1..1\right)}{int\left(\frac{T^2(n, x)}{\sqrt{1 - x^2}}, x = -1..1\right)};$$

$(x, m) \rightarrow add(Ct(k) \cdot T(k, x), k = 0..m);$
end proc:

$fourier :=$
proc $(f, x1, x2, t := x2 - x1)$
global $a0$; **global** an; **global** bn;

$$a0 := simplify\left(\frac{2}{t} \cdot int(f(x), x = x1..x2)\right);$$

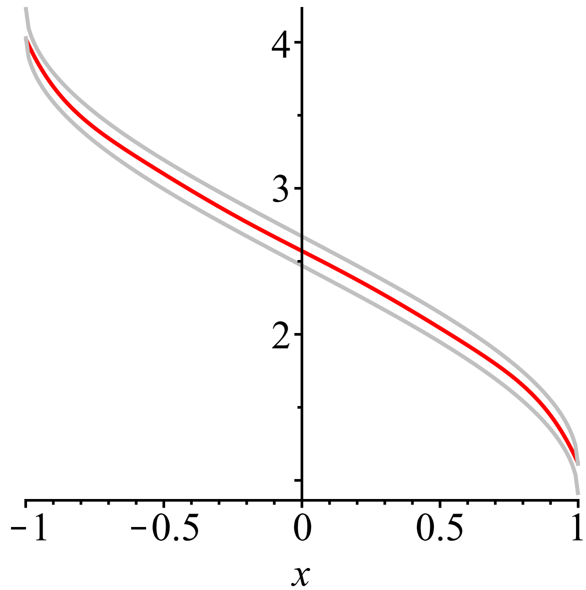
$$an := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), x = x1..x2\right)\right) \text{ assuming } n :: posint;$$

$$bn := simplify\left(\frac{2}{t} \cdot int\left(f(x) \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), x = x1..x2\right)\right) \text{ assuming } n :: posint;$$

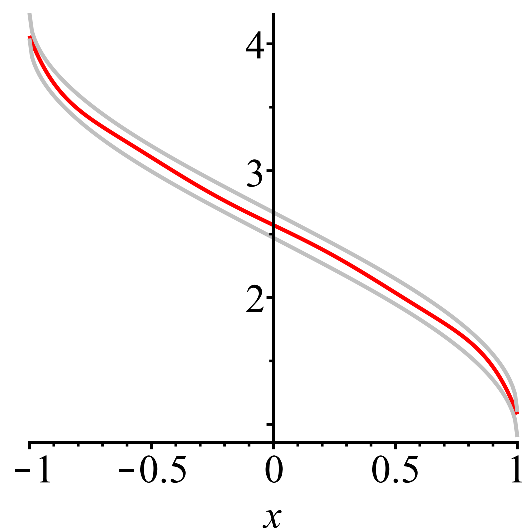
$$(x, k) \rightarrow \left(\frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{t}\right), n = 1..k\right)\right):$$

end proc:

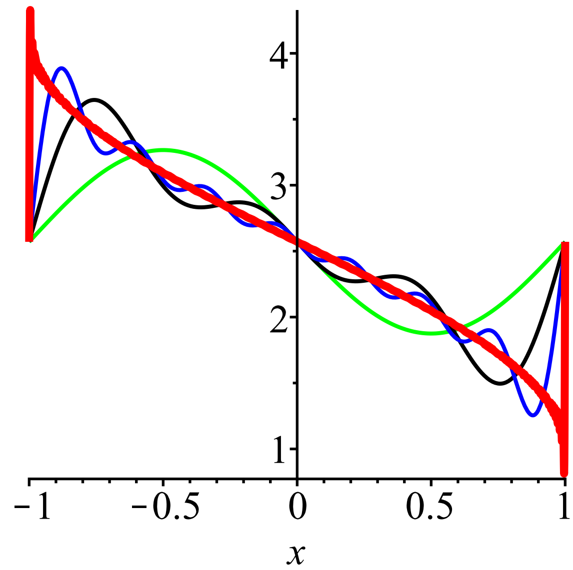
$eps := 0.1:$
$r := -1..1:$
$f := x \rightarrow \arccos(x) + 1:$
$fp := Pm(f):$
$p := plot(fp(x, 8), x = r, color = red):$
$p1 := plot(f(x) + eps, x = r, color = gray):$
$p2 := plot(f(x) - eps, x = r, color = gray):$
$plots[display]([p, p1, p2]);$




```
ft := Tm(f) :
t := plot(ft(x, 7), x = r, color = red) :
p1 := plot(f(x) + eps, x = r, color = gray) :
p2 := plot(f(x) - eps, x = r, color = gray) :
plots[display]([t, p1, p2]);
```




```
ff := fourier(f, -1, 1) :
ff_plot1 := plot(ff(x, 1), x = r, color = green) :
ff_plot2 := plot(ff(x, 3), x = r, color = black) :
ff_plot3 := plot(ff(x, 7), x = r, color = blue) :
ff_plot4 := plot(ff(x, 250), x = r, color = red, thickness = 3) :
plots[display]([ff_plot1, ff_plot2, ff_plot3, ff_plot4]);
```



$taylor(f(x), x=0, 25);$
$fst := x \rightarrow convert(taylor(f(x), x=0, 25), polynom):$

$$\frac{\pi}{2}+1-x-\frac{1}{6}x^3-\frac{3}{40}x^5-\frac{5}{112}x^7-\frac{35}{1152}x^9-\frac{63}{2816}x^{11}-\frac{231}{13312}x^{13}-\frac{143}{10240}x^{15}-\frac{6435}{557056}x^{17}-\frac{12155}{1245184}x^{19}-\frac{46189}{5505024}x^{21}-\frac{88179}{12058624}x^{23}+\mathrm{O}(x^{25}) \quad \textbf{(2)}$$

$plot([f(x), fp(x, 7), ft(x, 6), ff(x, 250), fst(x)], x=r, color=[red, green, cyan, yellow, black],$
$thickness=[6, 1, 1, 1, 1]);$