#task1

$$a := (n, x) \to \frac{n+3}{2n+1} \cdot \frac{1}{(4x^2 - 8x + 5)^n} :$$

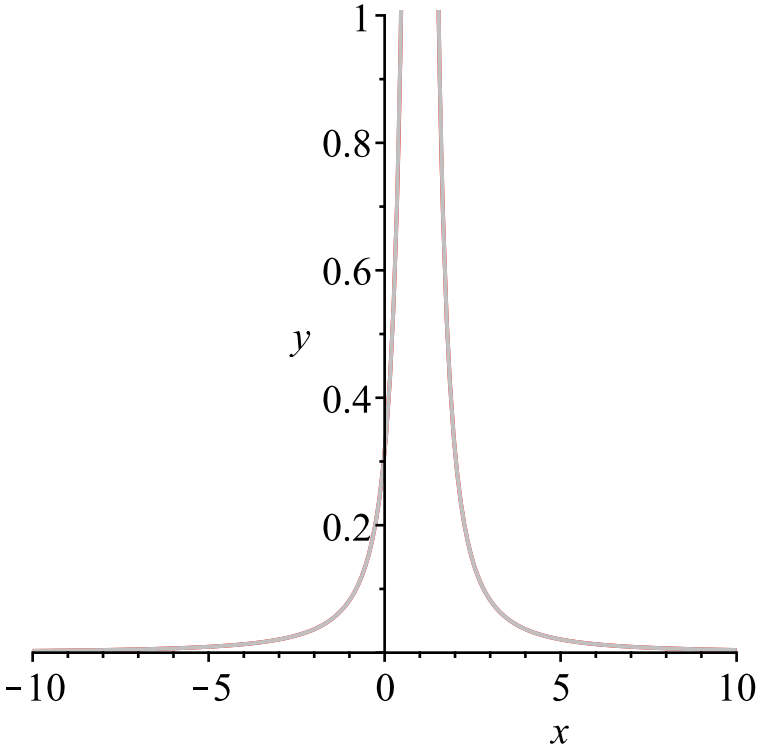
$$l := x \to limit\left(\frac{a(n+1, x)}{a(n, x)}, n = \text{infinity}\right) : l(x);$$

$$\frac{1}{4x^2 - 8x + 5} \quad (1)$$

$$solve(\{|l(x)| < 1\}, x);$$

$$\{x < 1\}, \{1 < x\} \quad (2)$$

$$rx := -10..10 :$$
$$ry := 0..1 :$$
$$p1 := plot(S(x), x = rx, y = ry) :$$
$$p2 := plot(Sw(x), x = rx, y = ry) :$$
$$plots[display](p1, p2, color = [red, gray]);$$



#task2

$$restart;$$

$$u := (x, n) \to \frac{(-1)^n \cdot x^n}{4n - 6} :$$

$$eps := 0.01 :$$

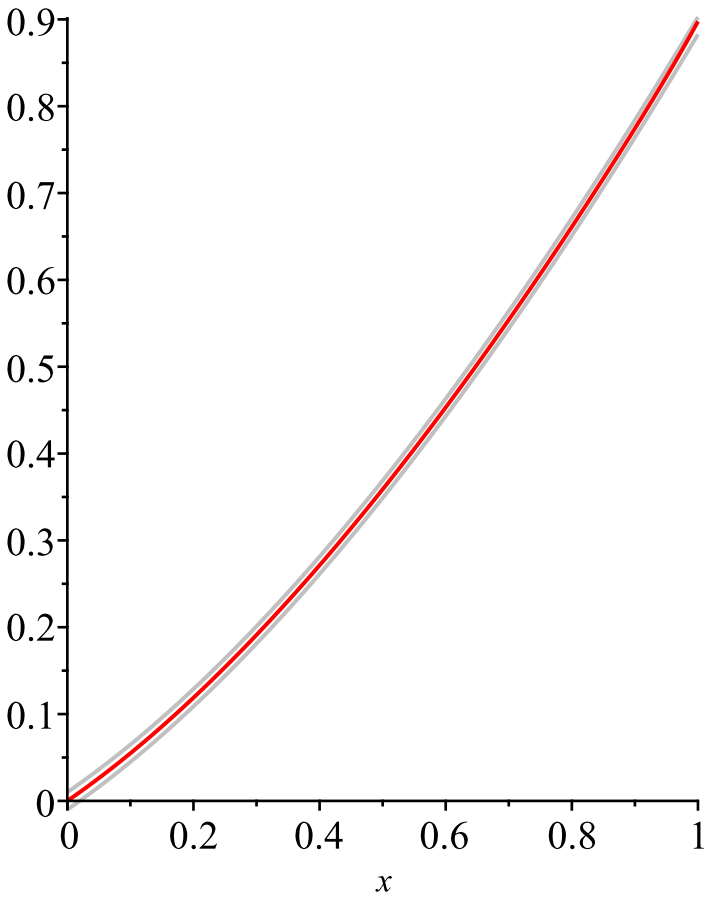
$$solve\left(\left\{\left|limit\left(\frac{u(x, n+1)}{u(x, n)}, n = \text{infinity}\right)\right| < 1\right\}\right);$$

$$\{-1 < x, x < 1\} \quad (3)$$

$$ne := solve\left(\frac{1}{4 \cdot ne - 2} = eps, ne\right);$$

$$ne := 25.50000000 \quad (4)$$

```
S :=
(
      proc(n1, n2 := infinity)
        if n2 = infinity then limit(sum(u(n1, n), n = 1 ..k), k = infinity)
        else sum(u(n1, n), n = 1 ..n2)
        end if
      end proc
 ) :
plot( [S(x) + eps, S(x, 26), S(x) - eps], x = 0 ..1, color = [gray, red, gray]);
```




```
#task3
restart;
eps := 0.001 :
f(x) := cos(x^2) :
taylor(f(x), x = 0, 17);
```

$$1 - \frac{1}{2} x^4 + \frac{1}{24} x^8 - \frac{1}{720} x^{12} + \frac{1}{40320} x^{16} + O(x^{20}) \qquad \textbf{(5)}$$

```
u(x, n) := (-1)^n·x^(4·n) / (2·n)! :
S :=
(
      proc(n1, n2 := infinity)
        if n2 = infinity then limit(sum(u(n1, n), n = 1 ..k), k = infinity)
        else sum(u(n1, n), n = 0 ..n2)
```

```
        end if
    end proc
):
fsolve(|u(1, n + 1)| = eps, n):
n_ε := evalf(%);
```

$$n_{\varepsilon} := 2.087088359 \tag{6}$$

```
evalf(|int(S(x, 3), x = 0..1) - int(f(x), x = 0..1)|);
```

$$1.4460\ 10^{-6} \tag{7}$$