Exercises 2 for the lecture
“Mathematische Modellierung in der Klimaforschung W16/17”

Due to Tuesday, 2016-11-22, Room A011 or via e-mail to
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Exercise 1 (Check the true solution)**.** Consider the transport equation

$$\begin{aligned} q_t + a q_x &= 0 \qquad \forall (x,t) \in \mathbb{R} \times \mathbb{R}_0^+ \\ q(x,0) &= q_0(x) \end{aligned} \tag{1}$$

for some $q_0 \in L^1_{\text{loc}}(\mathbb{R})$. Show that $u(x,t) = q_0(x - at)$ is a weak solution to equation (1). Thus it fulfills the equation

$$\int_{x_1}^{x_2} u(x,t_2) - u(x,t_1)\,\mathrm{d}x = \int_{t_1}^{t_2} a u(x_1,t) - a u(x_2,t)\,\mathrm{d}t \tag{2}$$

for all intervals $[x_1, x_2] \times [t_1, t_2] \subset \mathbb{R} \times \mathbb{R}_0^+$.

Exercise 2 (Stability)**.** Prove that the central difference method from the last exercise

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x} \cdot \frac{a(Q_{i+1}^n - Q_{i-1}^n)}{2} \tag{3}$$

is unconditionally unstable by using the von-Neumann stability analysis.

Exercise 3 (Accuracy)**.** Compute the Accuracy of method (3).

Exercise 4 (Burgers equation)**.** Consider our first non-linear equation, the burgers equation:

$$\begin{aligned} q_t + \left(\frac{1}{2} q^2\right)_x &= 0 \qquad \forall (x,t) \in \mathbb{R} \times \mathbb{R}_0^+ \\ q(x,0) &= q_0(x) \end{aligned} \tag{4}$$

Try solving the equation with a finite difference method with

(i) the Upwind approximation for the spatial derivative

(ii) the Central difference approximation for the spatial derivative

What do you observe?