EE 451/651: Digital Signal Processing
Final Fall 2023

Duration: 120 minutes

Please submit all your Matlab work in one Livescript (.mlx) file. This Livescript must contain all the functions you used in your solutions (i.e., this Livescript should run if downloaded from Canvas without any other external file).

Indicate clearly which problem you are answering in your submitted files. For any plots, provide title indicating which problem the plot belongs to, and label properly x- and y-axis with names and units.

Any written parts must be captured and submitted to canvas, or handed to the instructor at the end of the exam.

Problem 1 (30 points)

An ideal lowpass, anti-aliasing, analog filter has the following desired characteristics:

$$H(z) = \begin{cases} 1, 0 \leq f \leq 2500\, Hz \\ 0, \qquad otherwise \end{cases}$$

The filter has to be replaced by an equivalent causal FIR digital filter with unit sample (impulse) response of 10 ms duration. The sampling frequency is 10 kHz. It is required that this filter response has linear phase and that the filter is designed using the method of windows (use rectangular window).

a) What is the length M of the FIR filter?
b) What is the cutoff frequency ω_c of the FIR filter?
c) Compute the values for the unit sample response, $h_{LP}(n)$ for the filter?
d) Using Matlab, plot the magnitude and phase response of $H(\omega)$
e) What is the attenuation (in dB) of the filter in the stopband?
f) What is the frequency resolution (in Hz) of the equivalent digital filter?

Problem 2 (40 points)

A first-order low pass Butterworth filter has to be replaced by an equivalent digital filter. The analog filter has unity gain at f = 0 Hz and the 3 dB cutoff frequency of f_c = 10 Hz. The sampling rate of the equivalent digital system is to be 200 Hz.

a) What is the expression $H(s)$ of the analog filter's transfer function?
b) Obtain an equivalent digital filter $H_{bi}(z)$ using the bilinear transformation method. Compute the filter coefficients a_{bi} and b_{bi}
c) Plot the magnitude and phase response of $H_{bi}(\omega)$
d) What is the -3 dB cutoff frequency ω_c of the digital filter?

e) Repeat part (b) for an equivalent digital filter $H_i(z)$ obtained using the impulse invariance method. Compute the filter coefficients a_i and b_i

f) Repeat part (c) for the filter $H_i(z)$

g) Repeat part (d) for the filter $H_i(z)$

h) Compare the magnitude of the frequency responses of $H_{bi}(\omega)$ and $H_i(\omega)$. Which filter has better magnitude response? Explain.

Problem 3 (30 points)

A digital signal $x(n)$, sampled with 10 kHz, need to be sent out as a signal with sampling rate of 15 kHz. Given that the original signal $x(n)$ occupies the spectrum $-\pi \leq \omega \leq \pi$, design a sampling frequency converter that can make this conversion without aliasing.

a) What are the lowest values for the interpolation value I and decimation value D of the converter?

b) Design a FIR digital filter $H_{fs}(\omega)$ that can be used to prevent aliasing from occurring in this conversion process. What is the cutoff frequency of this filter?

c) Using the frequency sampling method (optimum equi-ripple), compute the filter coefficients h_{fs} given the following specs:

- Ripple in the passband of 1 dB
- Attenuation in the stopband of 20 dB
- Transition bandwidth of 0.04π
- Cutoff frequency found in part (b)

d) Plot the magnitude and phase response of the filter $H_{fs}(\omega)$. Does the designed filter meet the specifications from part (c)?

e) At the end of this conversion process, what is the bandwidth occupied by the converted signal in the discrete time frequency domain (in ω)?