EE 451 Homework 5

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Problem 1

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1. GIVEN THE SYSTEM $h(n) = \delta(n) + \delta(n-1)$ AND THE OUTPUT $y(n) = \delta(n) + 3\delta(n-1) + 2\delta(n-2)$, DETERMINE $x(n)$ USING FREQUENCY DOMAIN APPROACH AND THEN USE FOURIER TRANSFORM TO FIND $x(n)$.

SOLUTION:

$$h(n) = \delta(n) + \delta(n-1)$$

$$h(n) \xrightarrow{\mathcal{F}} H(\omega) = 1 + e^{j\omega}$$

$$y(n) = \delta(n) + 3\delta(n-1) + 2\delta(n-2)$$

$$y(n) \xrightarrow{\mathcal{F}} Y(\omega) = 1 + 3e^{-j\omega} + 2e^{-2j\omega}$$

NOTE: BOTH $H(\omega)$ AND $Y(\omega)$ ABOVE WERE DETERMINED WITH THE TEXTBOOK FOURIER TRANSFORM TABLE 4.6 AND FROM IN CLASS PROBLEM WORK THROUGH.

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{1 + 3e^{-j\omega} + 2e^{-2j\omega}}{1 + e^{-j\omega}}$$

FACTORING $Y(\omega)$:

$$1 + 3e^{-j\omega} + 2e^{-2j\omega} \qquad x = e^{-j\omega}$$

$$1 + 3x + 2x^2 = 0$$

$$(1 + x)(1 + 2x)$$

$$1 + 3e^{-j\omega} + 2e^{-2j\omega} = (1 + e^{-j\omega})(1 + 2e^{-j\omega})$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{\cancel{(1+e^{-j\omega})}(1 + 2e^{-j\omega})}{\cancel{(1 + e^{-j\omega})}}$$

$$\underline{X(\omega) = 1 + 2e^{-j\omega}}$$

$$x(n) = \sum_{n=-\infty}^{\infty} X(\omega) e^{j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} (\underbrace{1}_{\delta(n)} + 2e^{-j\omega}) e^{j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} 2\cancel{e^{-j\omega}}\,\cancel{e^{j\omega}}\, e^{n}$$

$$= \sum_{n=-\infty}^{\infty} 2e^{n} ?$$

$$\underline{x(n) = \delta(n) + 2\delta(n-1)}$$

Problem 2

2. AN IIR FILTER IS DESCRIBED BY THE FOLLOWING DIFFERENCE EQUATION.

$$y(n) = a y(n-1) + x(n) \quad , \quad y(-1) = 0$$

A. DETERMINE THE FREQUENCY RESPONSE $H(\omega)$ FOR $a = 0.9$.

SOLUTION:

$$y(n) - 0.9y(n-1) = x(n)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= Y(z) - 0.9(Y(z)z^{-1} + y(-1))$$

$$X(z) = z - 0.9z^{-1}$$

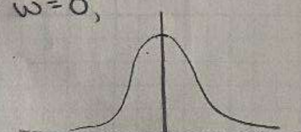
$$\frac{X(z)}{Y(z)} = H(z)$$

$$H(z) = \frac{1}{1 - 0.9z^{-1}} =$$

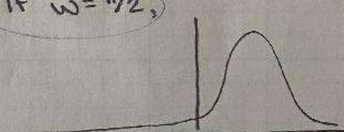
NOTE $z = e^{j\omega}$

$$H(\omega) = \frac{1}{1 - 0.9\omega}$$

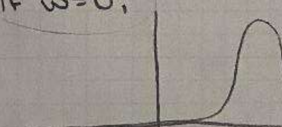
IF $\omega = 0$,



IF $\omega = \pi/2$,



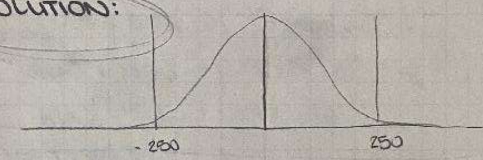
IF $\omega = 0$,

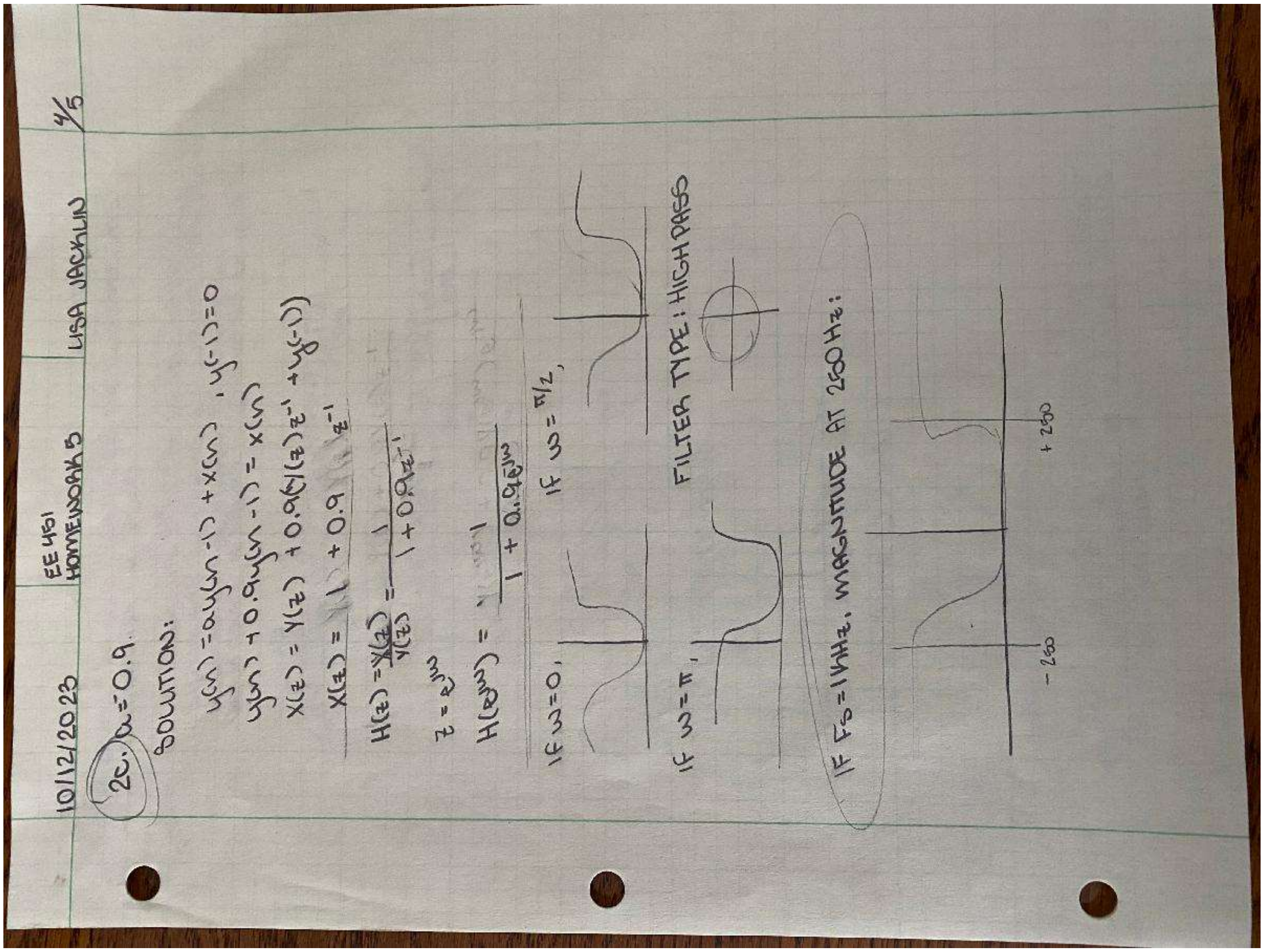


FILTER TYPE: LOW PASS

26. IF THIS FILTER IS USED IN PROCESSING OF ANALOG SIGNALS SAMPLED AT 1000 Hz, WHAT IS THE MAGNITUDE OF FREQUENCY RESPONSE AT 250 Hz?

SOLUTION:





Problem 3

```
%Part a: moving_average_filter function creation!
%below is the code I setup.

% function [b, a, wl] = moving_average_filter(n)
% %--------------------------------------------------
% %based on notes from in class review of HW5, we know the following:
% b = [1/n 1/n 1/n];
% a = [1];
%
% %please note that I am using sudo code for much of this for my
% %understanding to be displayed.
% %thie issue here is going to be determining wl. To determine this value, we
% %need to have a nested function.
% function w = cutoff(wl)
%     %here is where abs(H(w)) - abs(H0)/ sqrt(2) comes in since we want to
%     %have a value determined to find -3dB based on the desciption of the
%     %problem.
%
%     %Since we have wl, we will have to calculate H(wl), this can be done by
%     %using filter (b,a,wl) since we are given these values and then we
%     %should be able to use fft to transform into the frequency domain.
%     y=filter(b,a,wl);
%     yf = fftshift(fft(y));
%
```

```
%     %using this, we should be able to use the main to subtract from H(0)
%     %and then divide by sqrt(2) which should give w.
%     yft = yf - yf(0)/sqrt(2);
%
% end
%
% wl = fzero(@cutoff, [zero, zero2]);
% %note that we need to find the first zero and the second zero here to
% %determine what we are trying to find.

%part b.
Fs3 = 1000;
Ts3 = 1/Fs3;
n = 2:50;
%although I do not have a program that will do this because it is only sudo
%code, I can take the values above to determine a sudo code that would
%theoretically display what I believe should be seen.
%first I need to use the N and fs values that I have been given and coded
%above.

%(a, b, wl) = moving_filter_average(n); % we need to have these given
%in order to plot values required.
%then we can do:
% stem(wl) %this would display what wl would be.

%c. now, if we were to plot for the same n value, we need to have the first
%and second zero values of the w1 for us to be albe to find the maximum
%between these which would be peak 2.

%wlz1 = fzero(wl = 0);

%lets asume that the sudo code above would display what we are expecting,
%all zeros for wl, then we can go with what in matlab wouldbe wlz1(1) and
%wlz1(2) noting that matlab counts from one. between these two values we
%would need to find the middle value.

%middle = (wlz1(1) + wlz1(2) )/2; %at this point in the reply, we should be
%able to determine the maximum height.
```

Problem 4

```
load ekg.mat %to upload the data file from HW5
who %to determine what variables were loaded from the file
```

```
Your variables are:

Fs          Ts          ekg         ekg60filter ekge60      f0
Fs3         Ts3         ekg60Fft    ekg60n      ekgmin60    fft_signal
N           a           ekg60Final  ekg60npad   ekgpad      n
Q           b           ekg60filt   ekgFft      elim60      t
```

```
Fs = 1000;
Ts = 1/Fs; %sampling time
```

```
N = 800; %sample size for each

t = 0:Ts:(N-1)*Ts; %time frame

subplot(2,1,1);
stem(t, ekg);
title("Problem 4a: ekg vs time");

subplot(2,1,2);
stem(t, ekg60n);
title("Problem 4a: ekg60n vs time");
%now, changing both ekg and ekg60n to digital frequency of 0<w<2pi and
%padding the signal until N = 1024. note that here we are suposed to
%utilize fft as instructed.

%first for ekg
ekgpad = [ekg, zeros(1,1024 - N)]; %could have used dft function here
ekgFft = fft(ekgpad);
subplot(2,1,1);
plot(abs(ekgFft));
title("Problem4b: ekg frequency response");

%now for ekg60n
ekg60npad = [ekg60n, zeros(1, 1024 - N)];
ekg60Fft = fft(ekg60npad);
subplot(2,1,2);
plot(abs(ekg60Fft));
title("Problem4b: ekg60n Frequency Response");
```

```
%now we are going to try and remove the peak at 60Hz, and minimize the
```

```
%random noise of the signal ekg60n.
n = 0:1:N-1;
elim60 = cos(2*pi*30*n*Ts);
ekgmin60 = ekg60n ;

% Apply Fast Fourier Transform (FFT)
subplot(2,1,2);
fft_signal = fft(ekgmin60);
ekge60 = fft(fft_signal, 1024);
plot(abs(fft_signal));
title("Problem 4c: ekg60n Removal of 60Hz");
```

```
%now, using filter(b,a,x) I can go ahead and filter the data of the
%function ekg60n to see if it matches or is close to ekg which is the ideal
%signal.
% Design a notch filter to remove the 60Hz component
f0 = 60; % Frequency to notch out (60Hz)
Q = 10;  % Quality factor (adjust as needed)
[b, a] = iirnotch(f0/(Fs/2), f0/(Fs/2)/Q);
%note that iirnotch will let me use the frequency sample to determine where
%60hz is and will give me the coefficients for this!

ekg60filt = filter(b, a, ekg60n); %don't forget to transform again!
ekg60filter = dft(ekg60filt, 1024);
```

```
Transformation matrix for DFT
  Columns 1 through 5

   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i
   1.0000 + 0.0000i   1.0000 - 0.0061i   0.9999 - 0.0123i   0.9998 - 0.0184i   0.9997 - 0.0245i
```

```
   1.0000 + 0.0000i   0.9999 - 0.0123i   0.9997 - 0.0245i   0.9993 - 0.0368i   0.9988 - 0.0491i
   1.0000 + 0.0000i   0.9998 - 0.0184i   0.9993 - 0.0368i   0.9985 - 0.0552i   0.9973 - 0.0736i
   1.0000 + 0.0000i   0.9997 - 0.0245i   0.9988 - 0.0491i   0.9973 - 0.0736i   0.9952 - 0.0980i
   1.0000 + 0.0000i   0.9995 - 0.0307i   0.9981 - 0.0613i   0.9958 - 0.0919i   0.9925 - 0.1224i
   1.0000 + 0.0000i   0.9993 - 0.0368i   0.9973 - 0.0736i   0.9939 - 0.1102i   0.9892 - 0.1467i
   1.0000 + 0.0000i   0.9991 - 0.0429i   0.9963 - 0.0858i   0.9917 - 0.1285i   0.9853 - 0.1710i
   1.0000 + 0.0000i   0.9988 - 0.0491i   0.9952 - 0.0980i   0.9892 - 0.1467i   0.9808 - 0.1951i
   1.0000 + 0.0000i   0.9985 - 0.0552i   0.9939 - 0.1102i   0.9863 - 0.1649i   0.9757 - 0.2191i
   1.0000 + 0.0000i   0.9981 - 0.0613i   0.9925 - 0.1224i   0.9831 - 0.1830i   0.9700 - 0.2430i
```

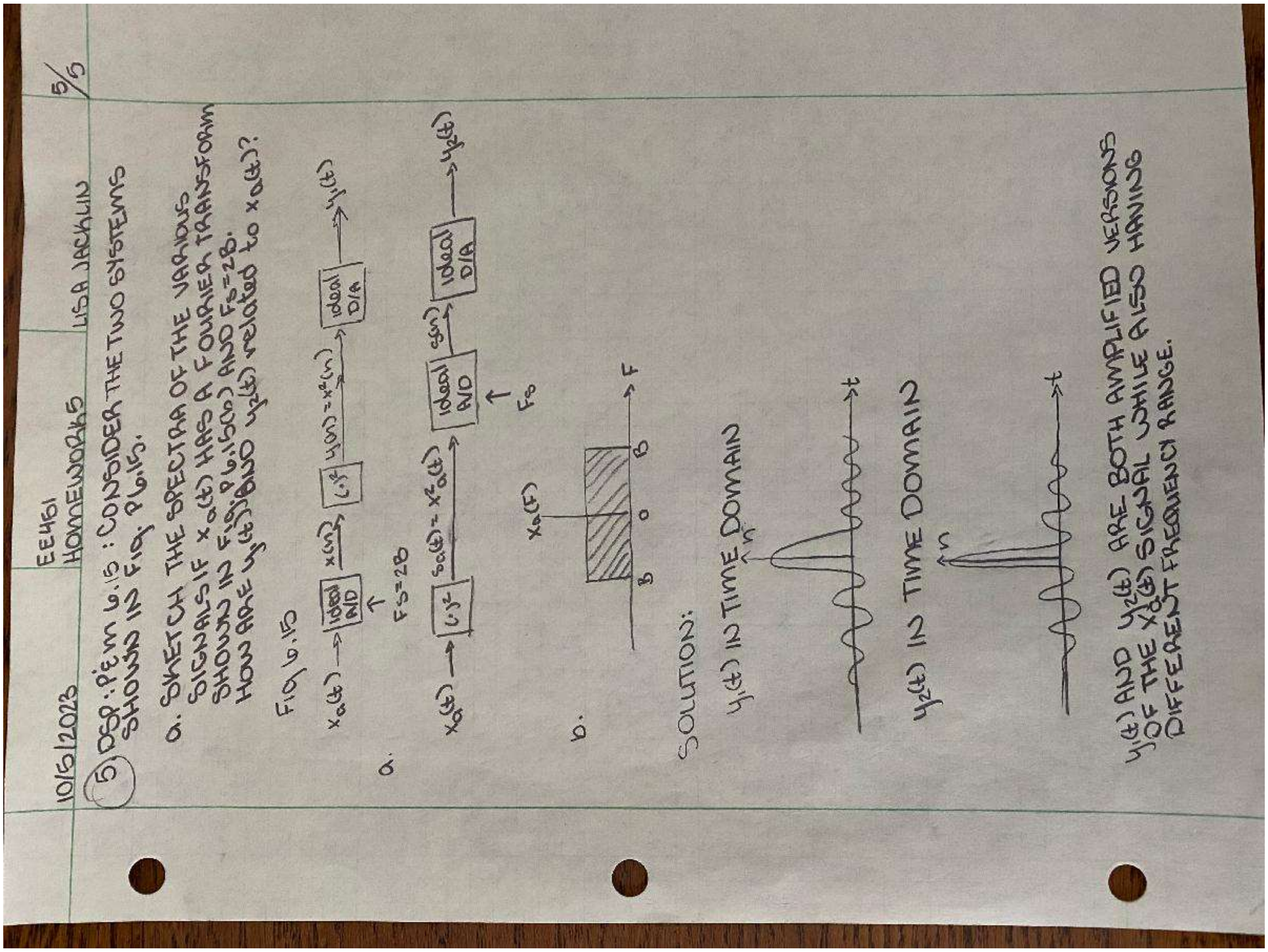
```
Xk = 1024×1 complex

10^2 ×
   3.2014 + 0.0000i
  -1.1395 - 2.0792i
  -0.9471 + 0.6608i
   0.4561 + 1.4156i
   1.6369 - 1.7286i
  -2.5212 - 0.3686i
   0.3696 + 2.1618i
   1.9588 - 0.2368i
  -0.5113 - 2.0227i
  -1.4393 + 1.3409i
```

```
subplot(2,1,2);
plot(abs(ekg60filter));
title("Problem 4d: filter out 60hz with Filter command");
```

```
%comment on results:
%as can be seen by comparing the two different ways of going about clearing
%the 60hz signal, although there is still noise, using the dft function
%created works far better to eliminate spikes, but perhaps not as well at
```

```
%eliminating general noise throughout the signal.
```

Problem 5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5 DSP: P'EM 6.15: CONSIDER THE TWO SYSTEMS SHOWN IN FIG. P6.15.

a. SKETCH THE SPECTRA OF THE VARIOUS SIGNALS IF $x_a(t)$ HAS A FOURIER TRANSFORM SHOWN IN FIG. P6.15(b) AND $F_s = 2B$. HOW ARE $y_1(t)$ AND $y_2(t)$ related to $x_a(t)$?

FIG 6.15

a. $x_a(t) \rightarrow$ ideal A/D $\xrightarrow{x(n)} (\cdot)^2 \xrightarrow{y(n)=x^2(n)}$ ideal D/A $\rightarrow y_1(t)$, $F_s = 2B$

$x_a(t) \rightarrow (\cdot)^2 \xrightarrow{s_a(t)=x_a^2(t)}$ ideal A/D $\xrightarrow{s(n)}$ ideal D/A $\rightarrow y_2(t)$, F_s

b. $X_a(F)$: $-B$, 0, B, F

SOLUTION:

$y_1(t)$ IN TIME DOMAIN

$y_2(t)$ IN TIME DOMAIN

$y_1(t)$ AND $y_2(t)$ ARE BOTH AMPLIFIED VERSIONS OF THE $x_a(t)$ SIGNAL WHILE ALSO HAVING DIFFERENT FREQUENCY RANGE.