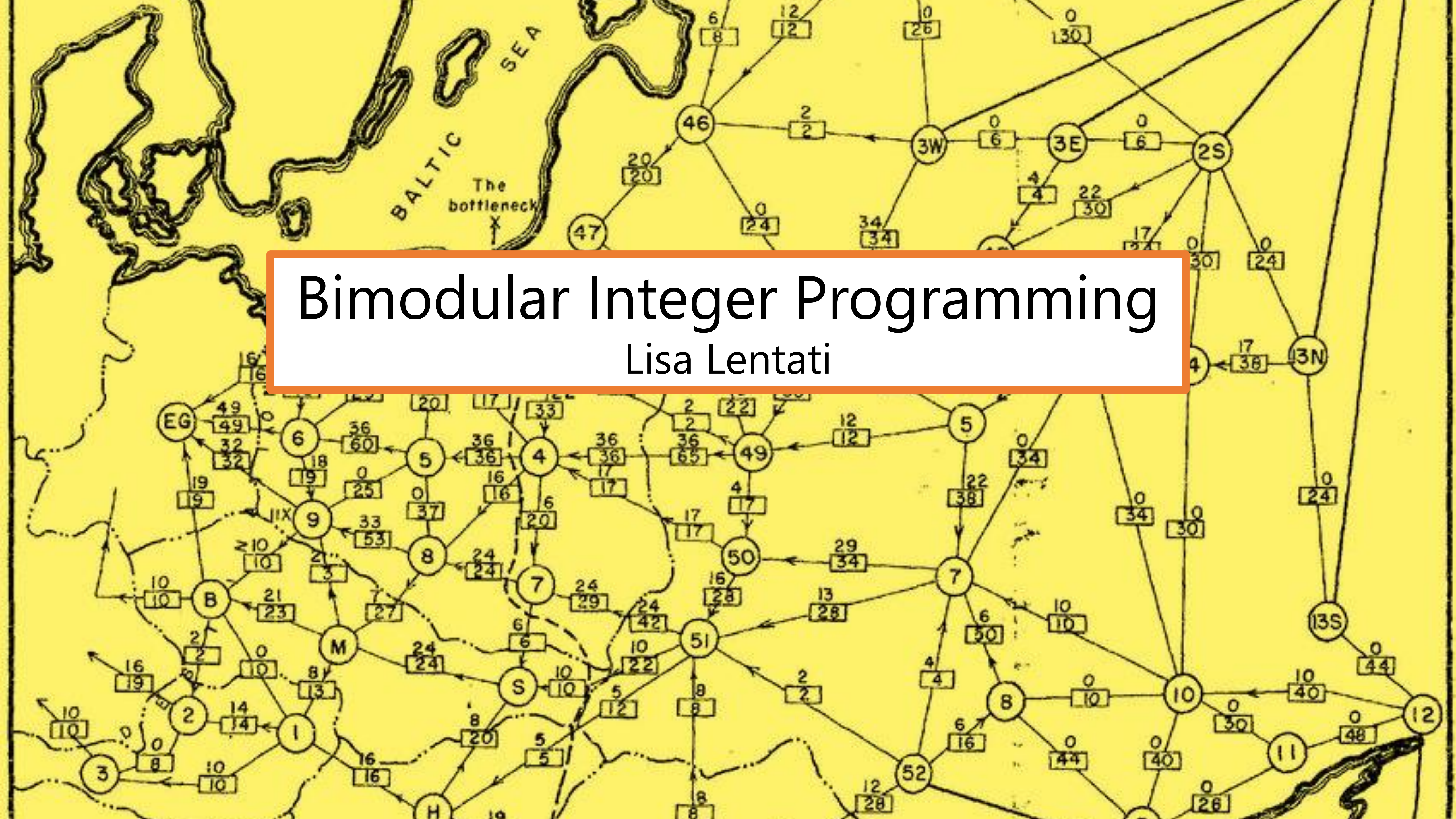


Bimodular Integer Programming

Lisa Lentati

Topic

- *Dissertation*: Solving bimodular integer programs in strongly polynomial time

  Supervisor: Dr. Giacomo Zambelli

- *Paper*: A strongly polynomial algorithm for bimodular integer linear programming (2017)

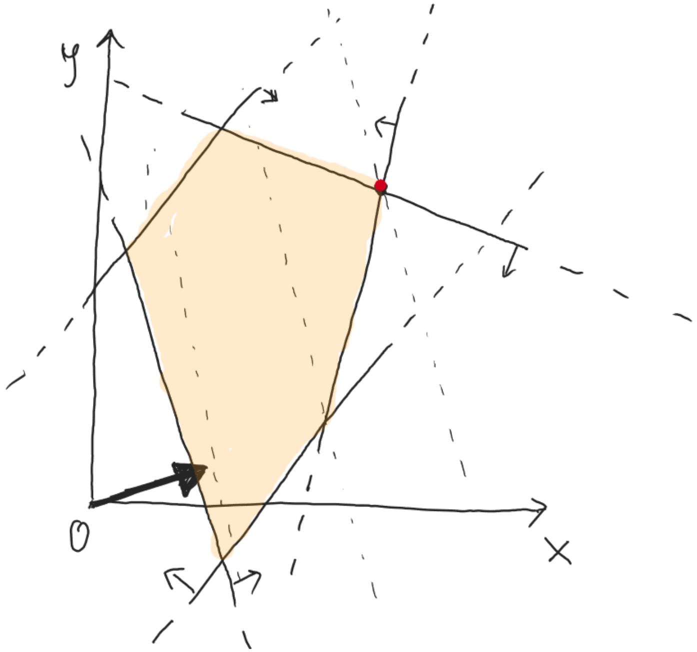
  Authors: S. Artmann, R. Weismantel, and R. Zenklusen

Structure

- Introduction to integer programming
- Bimodular integer programming
- Outline of an algorithm to solve bimodular integer programming

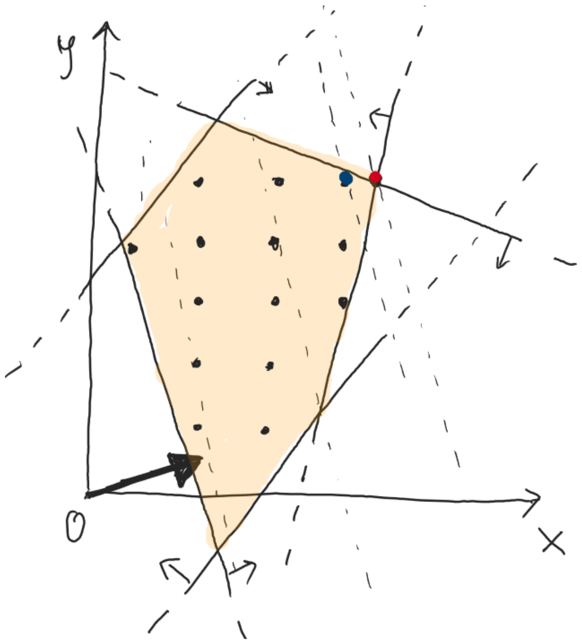
Linear programming

- $\max\{c^\top x : Ax \leq b, x \in \mathbb{R}^n\}$ where $c \in \mathbb{R}^n, b \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m\times n}$, rank($A$)$= n$



Integer programming

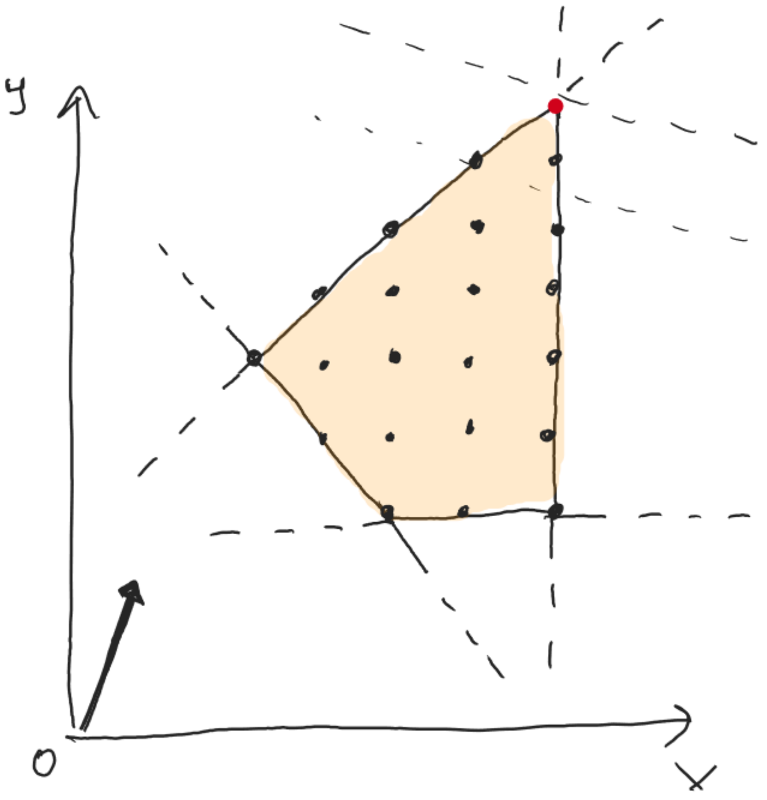
- $\max\{c^\top x : Ax \leq b, x \in \mathbb{Z}^n\}$ where $c \in \mathbb{Z}^n, b \in \mathbb{Z}^m$ and $A \in \mathbb{Z}^{m\times n}$, rank($A$)$= n$



Totally unimodular integer programs

- <u>Totally unimodular</u> (TU) matrix: all sub-determinants have values 0, ± 1

- <u>Unimodular</u> matrix: the $n \times n$ sub-determinants have values 0, ± 1



Can we extend the result further?

- Yes!
- $n \times n$ sub-determinants have values 0, ±1, ±2 .
- A bimodular integer program (BIP) is an optimization problem of the form,

$$\max\{c^{\top}x : Ax \le b, x \in \mathbb{Z}^n\}$$

  where $c \in \mathbb{Z}^n, b \in \mathbb{Z}^m$ and $A \in \mathbb{Z}^{m\times n}$, rank($A$)= n and the $n \times n$ sub-determinants of A are at most 2 in absolute value.

Outline of the algorithm

1. Transform BIP to a TU problem with an additional parity constraint

$$\max\{c^\top x \colon Ax \le 0, x \in \mathbb{Z}^n_{\ge 0}, \textstyle\sum_{i \in S} x_i \ \text{odd}\} \qquad \text{(CPTU)}$$

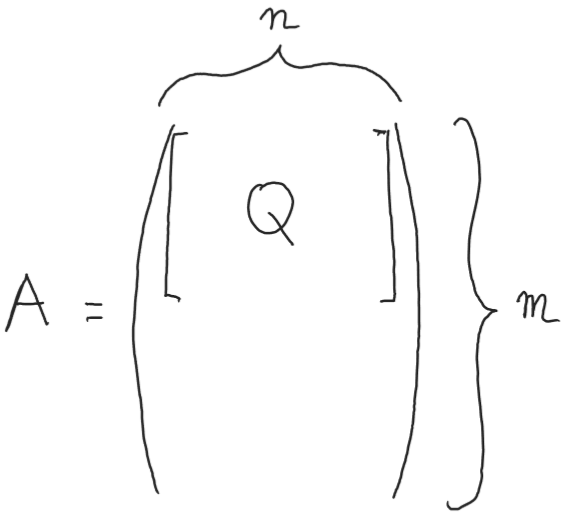
2. Iteratively decompose CPTU into smaller CPTU problems
   - The number of subproblems doesn't grow exponentially

3. Solving the base problems

Thank you!

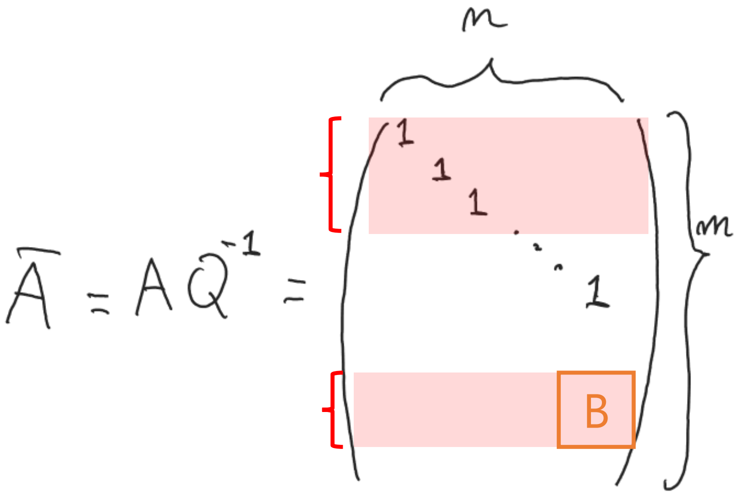
- Slides and dissertation: https://github.com/LisaLentati/bip

Appendix: from unimodular to TU



$\mathrm{rank}(A) = n$

$\det(Q) = \pm 1$



- $\max\{c^\top x : Ax \leq b,\ x \in \mathbb{Z}^n\}$, with A unimodular
- $x = \ Q^{-1}\mathrm{y}$
- $\max\{c^\top Q^{-1}\mathrm{y} : \bar{A}\mathrm{y} \leq bQ^{-1},\ \mathrm{y} \in \mathbb{Z}^n\}$, with $\bar{A}$ totally unimodular