DEPARTMENT OF INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Bachelor's Thesis in Informatics: Games Engineering

Tsunami simulation in the ExaHyPE framework

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Tsunamisimulation innerhalb des ExaHyPE-Frameworks

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I confirm that this bachelor’s thesis in informatics: games engineering is my own work and I have documented all sources and material used.

Munich, 18.12.2017 Lisa Scheller

Acknowledgments

Abstract

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2.1.1 Definition

Partial differential equations (PDEs) are used to describe a multitude of problems. In contrast to ordinary differential equations (ODEs) PDEs contain partial derivatives, whereas ODEs only contain derivatives of one variable. There are a multitude of phenomenons that can be described by PDEs, ranging from fluid mechanics and traffic simulation over thermodynamic problems to quantum mechanics. In this bachelor's thesis we restrict ourselves to a certain subclass of PDEs, the so-called hyperbolic PDEs. The core equations of this thesis, the shallow water equations, which will be introduced in detail in the next section,as well as the standard advection equation, are hyperbolic PDEs.

Initially let

$$q_t + Aq_x = 0 \tag{2.1}$$

be a one-dimensional, homogenous, first-order system of PDEs. q is a vector from $\mathbb{R}^m$ that contains the unknowns of the equations and A a $m \times m$ matrix. If this equation shall describe a system of hyperbolic PDEs, the matrix A has to meet certain conditions. Firstly, the matrix has to be diagonizable [Lev04], thus it must be possible to describe the matrix in the following way:

$$A = S * D * S^{-1} \tag{2.2}$$

, where D is a matrix that is zero for all entries not on the diagonal, and S is a transformation matrix. Secondly, the eigenvalues of A have to be real. This leads to the following definition:

Definition 2.1.1. "A linear system of the form

$$q_t + Aq_x = 0$$

is called hyperbolic if the $m \times m$ matrix A is diagonizable with real eigenvalues."[Lev04]

Other types of PDEs are elliptic or parabolic PDEs.

2.1.2 Classification

There are several sub-groups of hyperbolic PDEs, in regard to the eigenvalues of A as well as in regard to the structure of the equations.

Eigenvalues

Since a symmetric matrix is always diagonizable with real eigenvalues, all symmetric matrices can be used to describe a hyperbolic problem [Lev04]. These equations are called symmetric hyperbolic.
If the matrix has distinct eigenvalues and is diagonizable, the system of PDEs is called strictly hyperbolic. [Lev04]
In contrast, a matrix that is not diagonizable but has real eigenvalues is called weakly hyperbolic.[Lev04]

Structure of equations

In the simplest case of a hyperbolic PDE A is not a matrix, but a constant scalar. Equation (2.1) simplifies to

$$q_t + aq_x = 0 \tag{2.3}$$

and a has to be real for the system to be hyperbolic.

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Bibliography

[Lev04] R. J. Leveque. *Finite Volume Methods for Hyperbolic Problems*. Cambridge Texts in Applied Mathematics. Cambridge University Press, 2004. ISBN: bla.