


In[5]:=


$\gamma = 0; \alpha = 0; f = 0; \omega = .25;$

In[6]:=

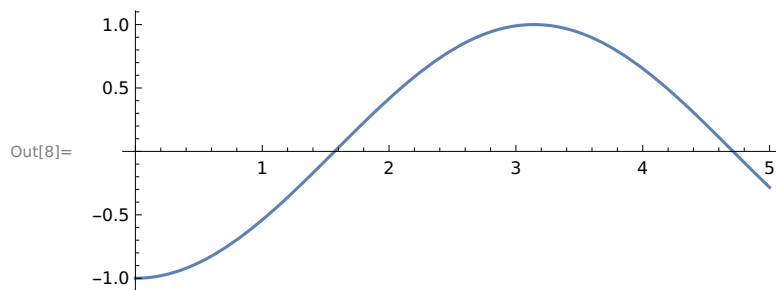
`ndsol = NDSolve[{x'[t] + γ * x'[t] + x[t] - α * (x[t])^3 == f * Cos[ω * t],
x'[0] == 0, x[0] == -1}, x, {t, 0, 5}]`

Out[6]= `{{x → InterpolatingFunction[ Domain: {{0., 5.}} Output: scalar]}}`

In[7]:= `x[t] /. ndsol[[1]]`

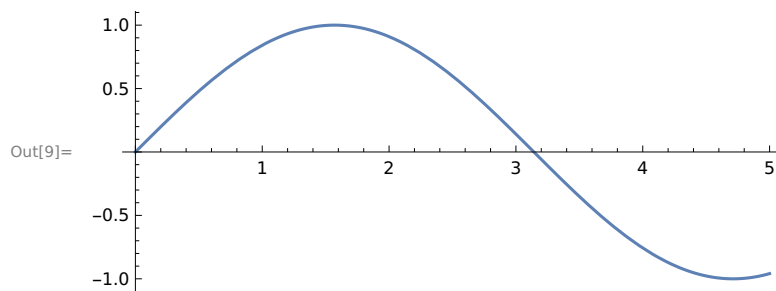
Out[7]= `InterpolatingFunction[ Domain: {{0., 5.}} Output: scalar]][t]`

In[8]:= `Plot[Evaluate[{x[t]} /. ndsol[[1]], {t, 0, 5}, AspectRatio → Automatic]`



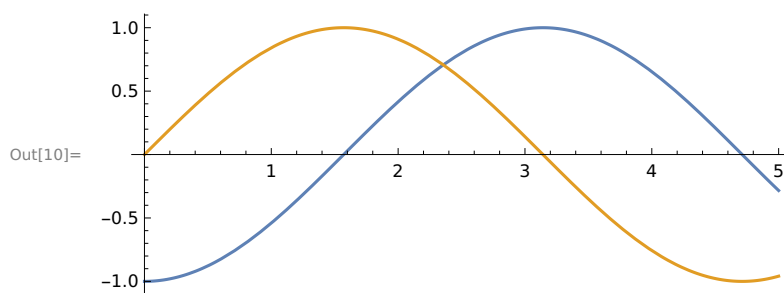
In[9]:=

`Plot[Evaluate[{x'[t]} /. ndsol[[1]], {t, 0, 5}, AspectRatio → Automatic]`



In[10]:=

`Plot[Evaluate[{x[t], x'[t]} /. ndsol[[1]], {t, 0, 5}, AspectRatio → Automatic]`



In[1]:=

```
 $\gamma = 0.1; \alpha = -1; f = 10; \omega = 1.5;$ 
```

In[2]:=

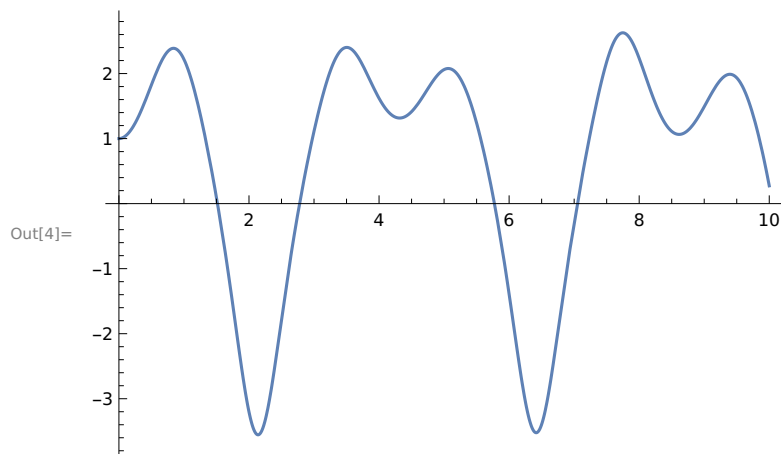
```
ndsol = NDSolve[{x'[t] +  $\gamma$  * x'[t] + x[t] -  $\alpha$  * (x[t])^3 == f * Cos[ $\omega$  * t],  
x'[0] == 0, x[0] == 1}, x, {t, 0, 10}]
```

Out[2]= $\left\{ \left\{ x \rightarrow \text{InterpolatingFunction} \left[\left\{ \left\{ \begin{array}{c} \text{Domain: } \{0., 10.\} \\ \text{Output: scalar} \end{array} \right\} \right\} \right] \right\} \right\}$

In[3]:= $x[t] /. \text{ndsol}[[1]]$

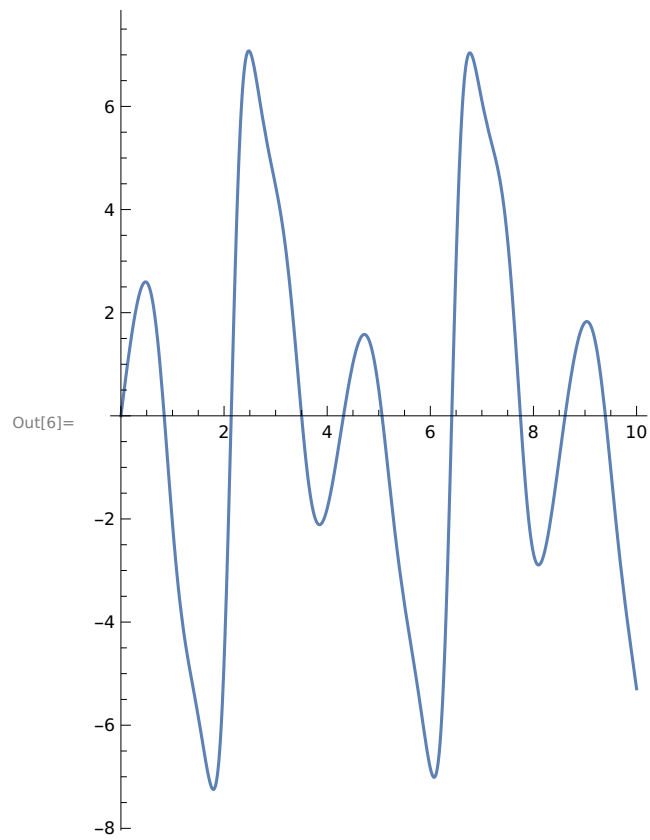
Out[3]= $\text{InterpolatingFunction} \left[\left\{ \left\{ \begin{array}{c} \text{Domain: } \{0., 10.\} \\ \text{Output: scalar} \end{array} \right\} \right\} \right][t]$

In[4]:= $\text{Plot}[\text{Evaluate}[\{x[t]\} /. \text{ndsol}[[1]], \{t, 0, 10\}, \text{AspectRatio} \rightarrow \text{Automatic}]$



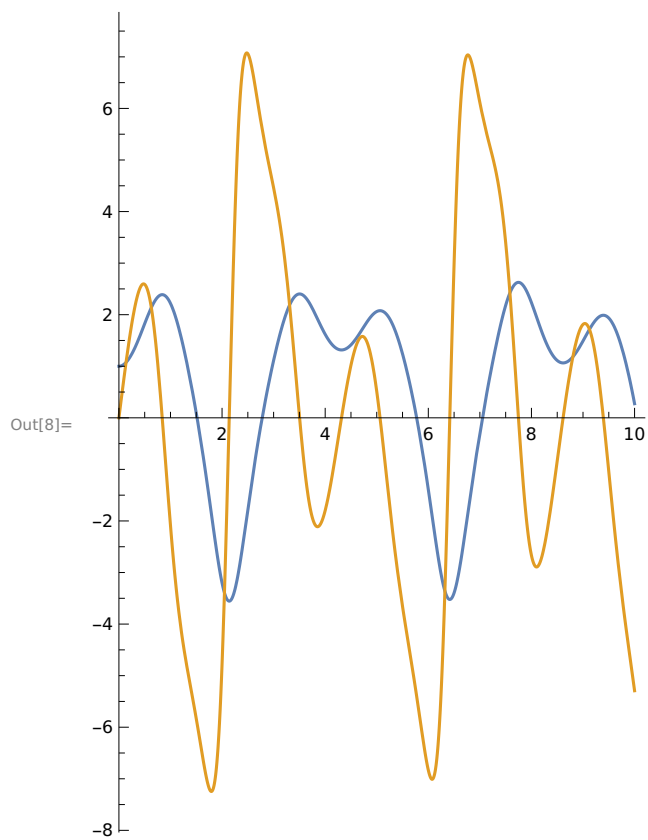
In[6]:=

```
Plot[Evaluate[{x'[t]} /. ndsol[[1]], {t, 0, 10}, AspectRatio -> Automatic]
```



In[8]:=

```
Plot[Evaluate[{x[t], x'[t]} /. ndsol[1]], {t, 0, 10}, AspectRatio → Automatic]
```



In[1]:= **eq = x'[t] + ω^2 * x[t] - ϵ * b * x'[t]^2 * x[t];**

x[t_] = x0[t] + ϵ * x1[t]

eq

Out[2]= $x_0[t] + \epsilon x_1[t]$

Out[3]= $\omega^2 (x_0[t] + \epsilon x_1[t]) - b \epsilon (x_0[t] + \epsilon x_1[t]) (x_0'[t] + \epsilon x_1'[t])^2 + x_0''[t] + \epsilon x_1''[t]$

In[4]:=

Expand[eq]

Out[4]= $\omega^2 x_0[t] + \epsilon \omega^2 x_1[t] - b \epsilon x_0[t] x_0'[t]^2 - b \epsilon^2 x_1[t] x_0'[t]^2 - 2 b \epsilon^2 x_0[t] x_0'[t] x_1'[t] - 2 b \epsilon^3 x_1[t] x_0'[t] x_1'[t] - b \epsilon^3 x_0[t] x_1'[t]^2 - b \epsilon^4 x_1[t] x_1'[t]^2 + x_0''[t] + \epsilon x_1''[t]$

In[5]:=

Collect[eq, ϵ]

Out[5]= $\omega^2 x_0[t] - b \epsilon^4 x_1[t] x_1'[t]^2 + \epsilon^2 (-b x_1[t] x_0'[t]^2 - 2 b x_0[t] x_0'[t] x_1'[t]) + \epsilon^3 (-2 b x_1[t] x_0'[t] x_1'[t] - b x_0[t] x_1'[t]^2) + x_0''[t] + \epsilon (\omega^2 x_1[t] - b x_0[t] x_0'[t]^2 + x_1''[t])$

In[6]:= **eq0 = Coefficient[Expand[eq], ϵ , 0]**

Out[6]= $\omega^2 x_0[t] + x_0''[t]$

In[7]:= **eq1 = Coefficient[Expand[eq], ϵ , 1]**

Out[7]= $\omega^2 x_1[t] - b x_0[t] x_0'[t]^2 + x_1''[t]$

In[8]:= **s0 = DSolve[{eq0 == 0, x0[0] == 0, x0'[0] == 1}, x0, t]**

Out[8]= $\left\{ \left\{ x_0 \rightarrow \text{Function}[\{t\}, \frac{\sin[t \omega]}{\omega}] \right\} \right\}$

In[9]:=

eq1 /. s0[[1]]

s1 = DSolve[{(eq1 /. s0[[1]]) == 0, x1[0] == 0, x1'[0] == 0}, x1, t]

Out[9]= $-\frac{b \cos[t \omega]^2 \sin[t \omega]}{\omega} + \omega^2 x_1[t] + x_1''[t]$

Out[10]= $\left\{ \left\{ x_1 \rightarrow \text{Function}[\{t\}, -\frac{b (4 t \omega \cos[t \omega] - 8 \sin[t \omega] + 8 \cos[t \omega]^4 \sin[t \omega] - \cos[t \omega] \times \sin[4 t \omega])}{32 \omega^3}] \right\} \right\}$

In[11]:= **x[t]**

xa[t_] = x[t] /. s0[[1]] /. s1[[1]]

Out[11]= $x_0[t] + \epsilon x_1[t]$

Out[12]= $\frac{\sin[t \omega]}{\omega} - \frac{b \epsilon (4 t \omega \cos[t \omega] - 8 \sin[t \omega] + 8 \cos[t \omega]^4 \sin[t \omega] - \cos[t \omega] \times \sin[4 t \omega])}{32 \omega^3}$

```

ω = b = 2;
nds1 = NDSolve[
  {X'[t] + ω^2 * X[t] - ε * b * X'[t]^2 * X[t] == 0, X[0] == 0, X'[0] == 1}, X, {t, 0, 6 * Pi}]
Plot[X[t] /. nds1[[1]], {t, 0, 6 * Pi}]

```

NDSolve: The function value {1., -0.00057904 + 0.00028952 ϵ } is not a list of numbers with dimensions {2} at {t, X[t], X'[t]} = {0.00014476, 0.00014476, 1.}.

Out[29]= $\left\{ \left\{ X \rightarrow \text{InterpolatingFunction}\left[\begin{array}{c} \text{Domain: } \{0., 0.\} \\ \text{Output: scalar} \end{array} \right] \right\} \right\}$

InterpolatingFunction: Input value {0.000385069} lies outside the range of data in the interpolating function. Extrapolation will be used.

