In[5]:=
```
γ = 0; α = 0; f = 0; ω = .25;
```

In[6]:=
```
ndsol = NDSolve[{x''[t] + γ * x'[t] + x[t] - α * (x[t])^3 == f * Cos[ω * t],
    x'[0] == 0, x[0] == -1}, x, {t, 0, 5}]
```

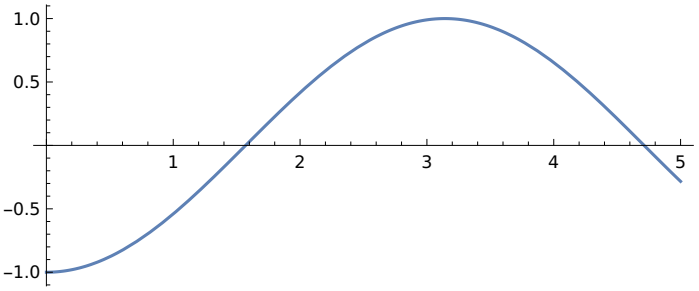
Out[6]= {{x → InterpolatingFunction[Domain: {{0., 5.}} Output: scalar]}}

In[7]:=
```
x[t] /. ndsol[[1]]
```

Out[7]= InterpolatingFunction[Domain: {{0., 5.}} Output: scalar][t]

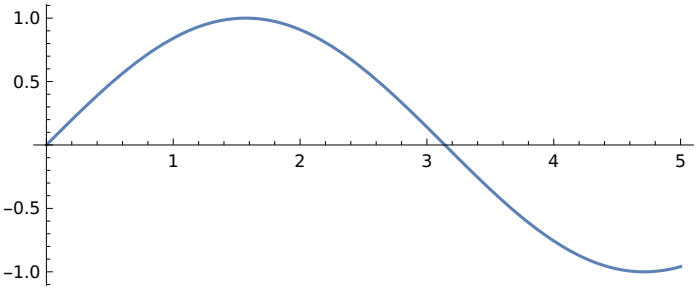
In[8]:=
```
Plot[Evaluate[{x[t]} /. ndsol[[1]], {t, 0, 5}, AspectRatio → Automatic]
```

Out[8]=



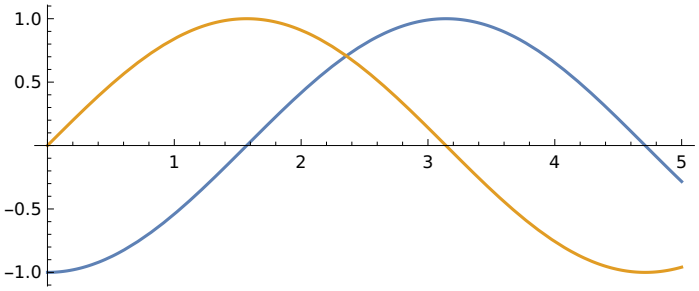
In[9]:=
```
Plot[Evaluate[{x'[t]} /. ndsol[[1]], {t, 0, 5}, AspectRatio → Automatic]
```

Out[9]=



In[10]:=
```
Plot[Evaluate[{x[t], x'[t]} /. ndsol[[1]], {t, 0, 5}, AspectRatio → Automatic]
```

Out[10]=



In[1]:=

```
γ = 0.1; α = -1; f = 10; ω = 1.5;
```

In[2]:=

```
ndsol = NDSolve[{x''[t] + γ * x'[t] + x[t] - α * (x[t])^3 == f * Cos[ω * t],
    x'[0] == 0, x[0] == 1}, x, {t, 0, 10}]
```

Out[2]= {{x → InterpolatingFunction[Domain: {{0., 10.}} Output: scalar]}}

In[3]:=

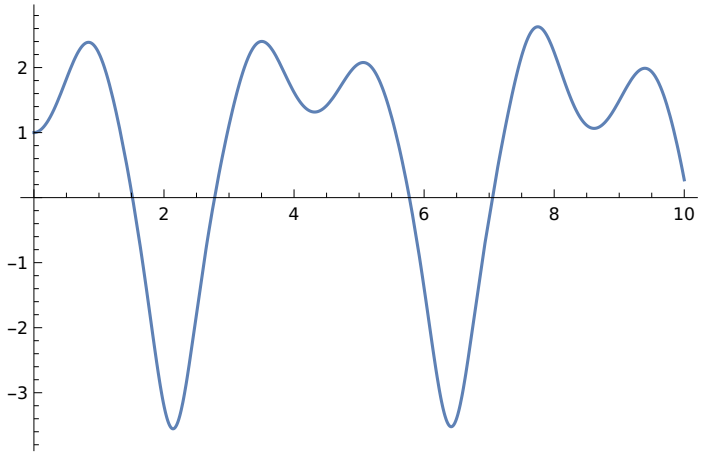
```
x[t] /. ndsol[[1]]
```

Out[3]= InterpolatingFunction[Domain: {{0., 10.}} Output: scalar][t]

In[4]:=

```
Plot[Evaluate[{x[t]} /. ndsol[[1]]], {t, 0, 10}, AspectRatio → Automatic]
```

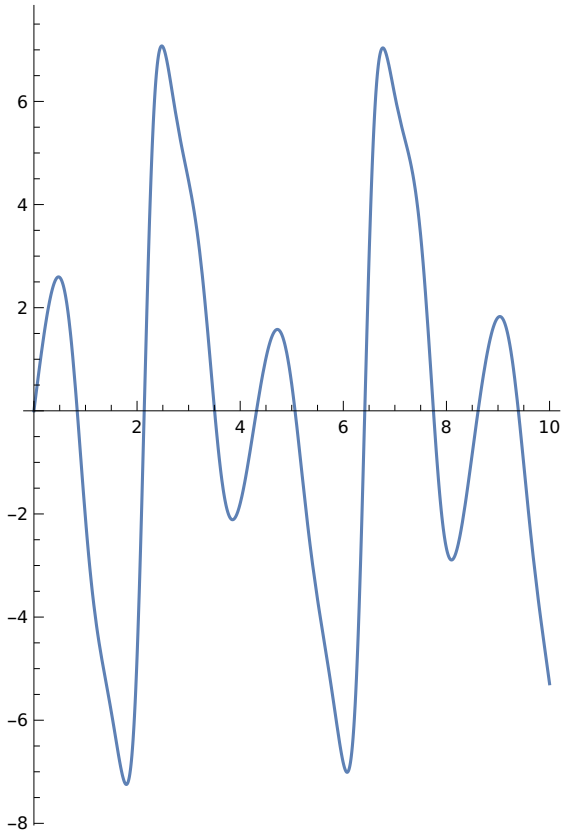
Out[4]=



In[6]:=

```
Plot[Evaluate[{x'[t]} /. ndsol[[1]], {t, 0, 10}, AspectRatio → Automatic]
```

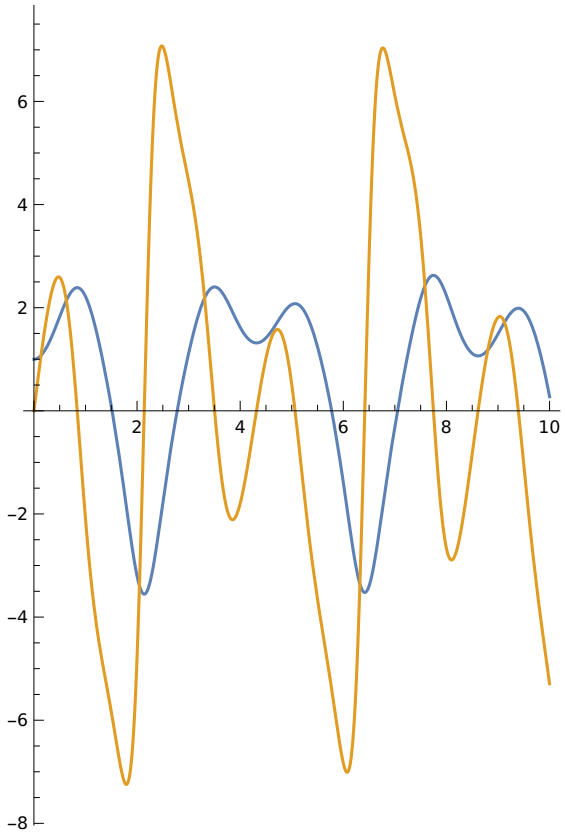
Out[6]=



In[8]:=

```
Plot[Evaluate[{x[t], x'[t]} /. ndsol[[1]], {t, 0, 10}, AspectRatio → Automatic]
```

Out[8]=



In[1]:=
```
eq = x''[t] + ω^2 * x[t] - ϵ * b * x'[t]^2 * x[t];
x[t_] = x0[t] + ϵ * x1[t]
eq
```

Out[2]= $x0[t] + \epsilon\, x1[t]$

Out[3]= $\omega^2 (x0[t] + \epsilon\, x1[t]) - b\, \epsilon\, (x0[t] + \epsilon\, x1[t])\, (x0'[t] + \epsilon\, x1'[t])^2 + x0''[t] + \epsilon\, x1''[t]$

In[4]:=
```
Expand[eq]
```

Out[4]= $\omega^2\, x0[t] + \epsilon\, \omega^2\, x1[t] - b\, \epsilon\, x0[t]\, x0'[t]^2 - b\, \epsilon^2\, x1[t]\, x0'[t]^2 - 2\, b\, \epsilon^2\, x0[t]\, x0'[t]\, x1'[t] -$
$2\, b\, \epsilon^3\, x1[t]\, x0'[t]\, x1'[t] - b\, \epsilon^3\, x0[t]\, x1'[t]^2 - b\, \epsilon^4\, x1[t]\, x1'[t]^2 + x0''[t] + \epsilon\, x1''[t]$

In[5]:=
```
Collect[eq, ϵ]
```

Out[5]= $\omega^2\, x0[t] - b\, \epsilon^4\, x1[t]\, x1'[t]^2 + \epsilon^2 \left(-b\, x1[t]\, x0'[t]^2 - 2\, b\, x0[t]\, x0'[t]\, x1'[t]\right) +$
$\epsilon^3 \left(-2\, b\, x1[t]\, x0'[t]\, x1'[t] - b\, x0[t]\, x1'[t]^2\right) + x0''[t] + \epsilon \left(\omega^2\, x1[t] - b\, x0[t]\, x0'[t]^2 + x1''[t]\right)$

In[6]:=
```
eq0 = Coefficient[Expand[eq], ϵ, 0]
```

Out[6]= $\omega^2\, x0[t] + x0''[t]$

In[7]:=
```
eq1 = Coefficient[Expand[eq], ϵ, 1]
```

Out[7]= $\omega^2\, x1[t] - b\, x0[t]\, x0'[t]^2 + x1''[t]$

In[8]:=
```
s0 = DSolve[{eq0 == 0, x0[0] == 0, x0'[0] == 1}, x0, t]
```

Out[8]= $\left\{\left\{x0 \to \text{Function}\left[\{t\}, \frac{\text{Sin}[t\, \omega]}{\omega}\right]\right\}\right\}$

In[9]:=
```
eq1 /. s0[[1]]
s1 = DSolve[{(eq1 /. s0[[1]]) == 0, x1[0] == 0, x1'[0] == 0}, x1, t]
```

Out[9]= $-\frac{b\, \text{Cos}[t\, \omega]^2\, \text{Sin}[t\, \omega]}{\omega} + \omega^2\, x1[t] + x1''[t]$

Out[10]= $\left\{\left\{x1 \to \text{Function}\left[\{t\}, -\frac{b \left(4\, t\, \omega\, \text{Cos}[t\, \omega] - 8\, \text{Sin}[t\, \omega] + 8\, \text{Cos}[t\, \omega]^4\, \text{Sin}[t\, \omega] - \text{Cos}[t\, \omega] \times \text{Sin}[4\, t\, \omega]\right)}{32\, \omega^3}\right]\right\}\right\}$

In[11]:=
```
x[t]
xa[t_] = x[t] /. s0[[1]] /. s1[[1]]
```

Out[11]= $x0[t] + \epsilon\, x1[t]$

Out[12]= $\frac{\text{Sin}[t\, \omega]}{\omega} - \frac{b\, \epsilon \left(4\, t\, \omega\, \text{Cos}[t\, \omega] - 8\, \text{Sin}[t\, \omega] + 8\, \text{Cos}[t\, \omega]^4\, \text{Sin}[t\, \omega] - \text{Cos}[t\, \omega] \times \text{Sin}[4\, t\, \omega]\right)}{32\, \omega^3}$

```
ω = b = 2;
nds1 = NDSolve[
  {X''[t] + ω^2 * X[t] - ϵ * b * X'[t]^2 * X[t] == 0, X[0] == 0, X'[0] == 1}, X, {t, 0, 6 * Pi}]
Plot[X[t] /. nds1[[1]], {t, 0, 6 * Pi}]
```

NDSolve: The function value {1., −0.00057904 + 0.00028952 ϵ} is not a list of numbers with dimensions {2} at {t, X[t], X′[t]} = {0.00014476, 0.00014476, 1.}.

Out[29]= {{X → InterpolatingFunction[Domain: {{0., 0.}} Output: scalar]}}

InterpolatingFunction: Input value {0.000385069} lies outside the range of data in the interpolating function. Extrapolation will be used.

Out[30]=