In[3]:=
```
sol = DSolve[y'[x] + y[x] * Tan[x] == Sin[2 * x], y[x], x]
```

Out[3]=
$$\{\{y[x] \to c_1 \text{Cos}[x] - 2 \text{Cos}[x]^2\}\}$$

In[4]:=
```
sol1 = DSolve[{z'[x] + z[x] Tan[x] == Sin[2 x], z[0] == 2}, z[x], x]
```

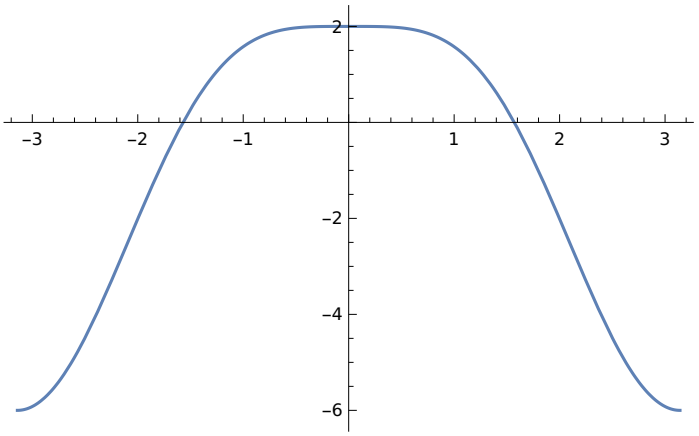
Out[4]=
$$\{\{z[x] \to -2 (-2 \text{Cos}[x] + \text{Cos}[x]^2)\}\}$$

In[31]:=
```
sol[[1]]
```

Out[31]=
$$\{\text{Tan}[x]\, y[x] + y'[x] == \text{Sin}[2\, x],\ \text{True}\}$$

In[6]:=
```
Plot[z[x] /. sol1[[1]], {x, -Pi, Pi}]
```

Out[6]=

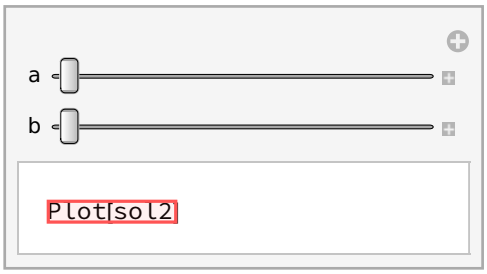



```
sol2 = DSolve[{a * y''[x] + b * y'[x] + c * y == 0, y[0] == 0, y'[0] == 1} y[x], x]
Manipulate[Plot[sol2], {a, 1, 4}, {b, 0, 2}]
```

DSolve: DSolve called with 2 arguments; 3 or more arguments are expected.

Out[8]=
$$\text{DSolve}[\{(c\, y + b\, y'[x] + a\, y''[x] == 0)\, y[x],\ (y[0] == 0)\, y[x],\ (y'[0] == 1)\, y[x]\},\ x]$$

Out[9]=



In[1]:=
```
sol3 = NDSolve[{x''[t] + γ * x'[t] + x[t] - α * x[t]^ 3 == f * Cos[ω t],
    γ[0] == 0.1, α[0] == -1, f[0] == 10, ω[0] == 1.5}, x, {t, 0, 10}]
```

NDSolve: The number of constraints (0) (initial conditions) is not equal to the total differential order of the system plus the number of discrete variables (2).

Out[1]=
$$\text{NDSolve}\left[\{x[t] - \alpha\, x[t]^3 + \gamma\, x'[t] + x''[t] == f\, \text{Cos}[t\, \omega],\right.$$
$$\left.\gamma[0] == 0.1,\ \alpha[0] == -1,\ f[0] == 10,\ \omega[0] == 1.5\},\ x,\ \{t,\ 0,\ 10\}\right]$$

In[57]:=
```
eq = x''[t] + ω^2 + ϵ b x[t]^3
```
Out[57]= $\omega^2 + b\,\epsilon\,(x0[t] + \epsilon\,x1[t])^3 + x0''[t] + \epsilon\,x1''[t]$

In[83]:=
```
xa[t_] = x0[t] + ϵ x1[t]
```
Out[83]= $x0[t] + \epsilon\,x1[t]$

In[84]:=
```
eq
```
Out[84]= $\omega^2 + b\,\epsilon\,(x0[t] + \epsilon\,x1[t])^3 + x0''[t] + \epsilon\,x1''[t]$

Out[32]= $\text{Information}\left[b\,\epsilon\,(x0[t] + \epsilon\,x1[t])^3 + \omega[t]^2 + x0''[t] + \epsilon\,x1''[t]\right]$

In[85]:=
```
Expand[eq]
```
Out[85]= $\omega^2 + b\,\epsilon\,x0[t]^3 + 3\,b\,\epsilon^2\,x0[t]^2\,x1[t] + 3\,b\,\epsilon^3\,x0[t]\,x1[t]^2 + b\,\epsilon^4\,x1[t]^3 + x0''[t] + \epsilon\,x1''[t]$

In[86]:=
```
Collect[Expand[eq], ϵ]
```
Out[86]= $\omega^2 + 3\,b\,\epsilon^2\,x0[t]^2\,x1[t] + 3\,b\,\epsilon^3\,x0[t]\,x1[t]^2 + b\,\epsilon^4\,x1[t]^3 + x0''[t] + \epsilon\,(b\,x0[t]^3 + x1''[t])$

In[35]:=
```
Coefficient[y^2 + 3 y + Sin[s] y, y, 2]
```
Out[35]= 1

In[62]:=
```
eq0 = Coefficient[Expand[eq], ϵ, 0]
```
Out[62]= $\omega^2 + x0''[t]$

In[63]:=
```
eq1 = Coefficient[Expand[eq], ϵ, 1]
```
Out[63]= $b\,x0[t]^3 + x1''[t]$

In[64]:=
```
s0 = DSolve[{eq0 == 0, x[0] == a, x0'[0] == 0}, x0, t]
```
Out[64]= $\left\{\left\{x0 \to \text{Function}\left[\{t\},\ \frac{1}{2}\left(2\,a - t^2\,\omega^2 - 2\,\epsilon\,x1[0]\right)\right]\right\}\right\}$

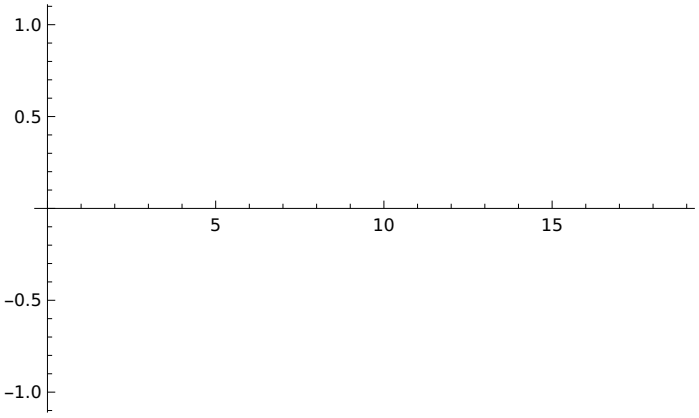
In[65]:=
```
x0 /. s0
```
Out[65]= $\left\{\text{Function}\left[\{t\},\ \frac{1}{2}\left(2\,a - t^2\,\omega^2 - 2\,\epsilon\,x1[0]\right)\right]\right\}$

In[66]:=
```
eq1 /. s0[[1]]
```
Out[66]= $\frac{1}{8}\, b\, (2\, a - t^2\, \omega^2 - 2\, \epsilon\, x1[0])^3 + x1''[t]$

In[87]:=
```
s1 = DSolve[{(eq1 /. s0[[1]]) == 0, x1[0] == 0, x1'[0] == 0}, x1, t]
```
Out[87]= $\left\{\left\{x1 \to \text{Function}\left[\{t\}, \frac{1}{2240}\right.\right.\right.$
$b\, (-1120\, a^3\, t^2 + 280\, a^2\, t^4\, \omega^2 - 56\, a\, t^6\, \omega^4 + 5\, t^8\, \omega^6 + 3360\, a^2\, t^2\, \epsilon\, x1[0] - 560\, a\, t^4\, \epsilon\, \omega^2\, x1[0] +$
$\left.\left.\left.\left. 56\, t^6\, \epsilon\, \omega^4\, x1[0] - 3360\, a\, t^2\, \epsilon^2\, x1[0]^2 + 280\, t^4\, \epsilon^2\, \omega^2\, x1[0]^2 + 1120\, t^2\, \epsilon^3\, x1[0]^3)\right]\right\}\right\}$

In[89]:=
```
xa[t]
```
Out[89]= $x0[t] + \epsilon\, x1[t]$

In[92]:=
```
x[t] /. s0[[1]] /. s1[[1]]
```
Out[92]= $\frac{1}{2}\, (2\, a - t^2\, \omega^2) + \frac{1}{2240}$
$b\, \epsilon\, (-1120\, a^3\, t^2 + 280\, a^2\, t^4\, \omega^2 - 56\, a\, t^6\, \omega^4 + 5\, t^8\, \omega^6 + 3360\, a^2\, t^2\, \epsilon\, x1[0] - 560\, a\, t^4\, \epsilon\, \omega^2\, x1[0] +$
$56\, t^6\, \epsilon\, \omega^4\, x1[0] - 3360\, a\, t^2\, \epsilon^2\, x1[0]^2 + 280\, t^4\, \epsilon^2\, \omega^2\, x1[0]^2 + 1120\, t^2\, \epsilon^3\, x1[0]^3)$

In[91]:=
```
Plot[Evaluate[xa[t] /. ϵ → {0, 0.1, 0.5, 0.9}], {t, 0, 6 π}]
```
Out[91]=


In[93]:=
```
nds = NDSolve[{X''[t] + ω^2 X[t] + 0.1 b X[t]^3 == 0, X[0] == a, X'[0] == 0}, X, {t, 0, 6 π}]
```
NDSolve: Initial condition a is not a number or a rectangular array of numbers.

Out[93]= $\text{NDSolve}\left[\{\omega^2\, X[t] + 0.1\, b\, X[t]^3 + X''[t] == 0,\ X[0] == a,\ X'[0] == 0\},\ X,\ \{t,\ 0,\ 6\, \pi\}\right]$

In[94]:=
```
Plot[{X[t] /. nds[[1]], xa[t] /. ϵ → 0.1}, {t, 0, 6 π} , PlotStyle → {Green, Red}]
```

NDSolve: 0.000385069 cannot be used as a variable.

ReplaceAll: $\{\omega^2\, X[0.000385069] + 0.1\, b\, X[0.000385069]^3 + X''[0.000385069] == 0,\ X[0] == a,\ X'[0] == 0\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: $\{\omega^2\, X[0.000385069] + 0.1\, b\, X[0.000385069]^3 + X''[0.000385069] == 0.,\ X[0.] == a,\ X'[0.] == 0.\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolve: 0.38507 cannot be used as a variable.

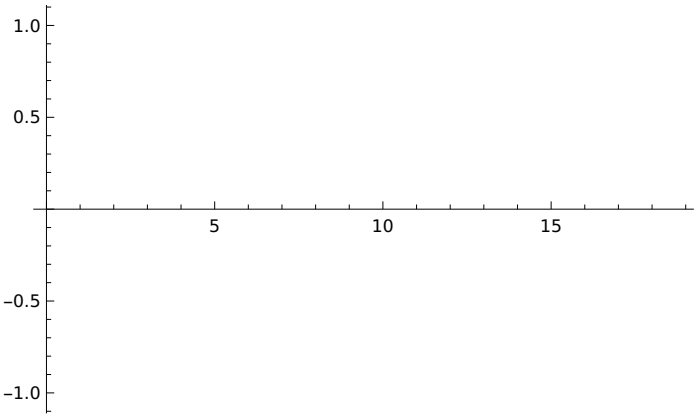
ReplaceAll: $\{\omega^2\, X[0.38507] + 0.1\, b\, X[0.38507]^3 + X''[0.38507] == 0,\ X[0] == a,\ X'[0] == 0\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

NDSolve: 0.769755 cannot be used as a variable.

General: Further output of NDSolve::dsvar will be suppressed during this calculation.

Out[94]=



In[77]:=
```
eq11 = x''[t] + ω^2 - ϵ b x'[t]^2 x[t]
```

Out[77]= $\omega^2 - b\,\epsilon\,(x0[t] + \epsilon\, x1[t])\,(x0'[t] + \epsilon\, x1'[t])^2 + x0''[t] + \epsilon\, x1''[t]$

In[78]:=
```
x[t_] = x0[t] + ϵ x1[t]
```

Out[78]= $x0[t] + \epsilon\, x1[t]$

In[79]:=
```
Expand[eq11]
```

Out[79]= $\omega^2 - b\,\epsilon\, x0[t]\, x0'[t]^2 - b\,\epsilon^2\, x1[t]\, x0'[t]^2 - 2\, b\,\epsilon^2\, x0[t]\, x0'[t]\, x1'[t] -$
$2\, b\,\epsilon^3\, x1[t]\, x0'[t]\, x1'[t] - b\,\epsilon^3\, x0[t]\, x1'[t]^2 - b\,\epsilon^4\, x1[t]\, x1'[t]^2 + x0''[t] + \epsilon\, x1''[t]$

In[80]:=
```
Collect[Expand[eq11], ϵ]
```

Out[80]= $\omega^2 - b\,\epsilon^4\, x1[t]\, x1'[t]^2 + \epsilon^2\,(-b\, x1[t]\, x0'[t]^2 - 2\, b\, x0[t]\, x0'[t]\, x1'[t]) +$
$\epsilon^3\,(-2\, b\, x1[t]\, x0'[t]\, x1'[t] - b\, x0[t]\, x1'[t]^2) + x0''[t] + \epsilon\,(-b\, x0[t]\, x0'[t]^2 + x1''[t])$

In[81]:=
```
eq10 = Coefficient[Expand[eq11], ϵ, 0]
```

Out[81]=
$$\omega^2 + x0''[t]$$

In[82]:=
```
s10 = DSolve[{eq0 == 0, x[0] == 0, x0'[0] == 0}, x0, t]
```

Out[82]=
$$\left\{\left\{x0 \to \text{Function}\left[\{t\}, \frac{1}{2}\left(-t^2\,\omega^2 - 2\,\epsilon\,x1[0]\right)\right]\right\}\right\}$$