CONVOLUCIÓN DE SEÑALES **CONTINUAS**

Resolver un sistema LTI – SISTEMA LINEAL E INVARIANTE EN EL TIEMPO

y(t) = x(t) * h(t)

y(t) : SALIDA DEL SISTEMA – x(t) : ENTRADA DEL SISTEMA

h(t) : SEÑAL DE RESPUESTA AL IMPULSO DEL SISTEMA

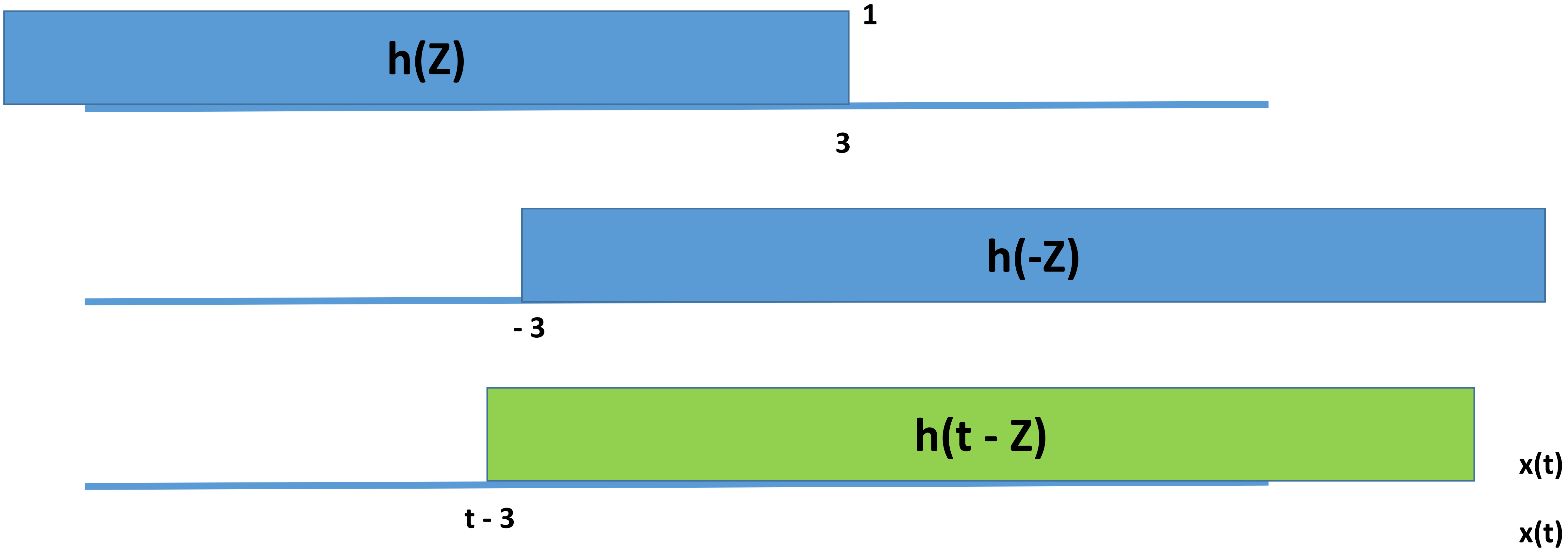
$$y(t) = \int_{-\infty}^{+\infty} x(\tau).h(t-\tau)d\tau = \int_{-\infty}^{+\infty} h(\tau).x(t-\tau)d\tau$$

y(t) = x(t) * h(t)

$$y(t) = \int_{-\infty}^{+\infty} x(\tau).h(t-\tau)d\tau = \int_{-\infty}^{+\infty} h(\tau).x(t-\tau)d\tau$$

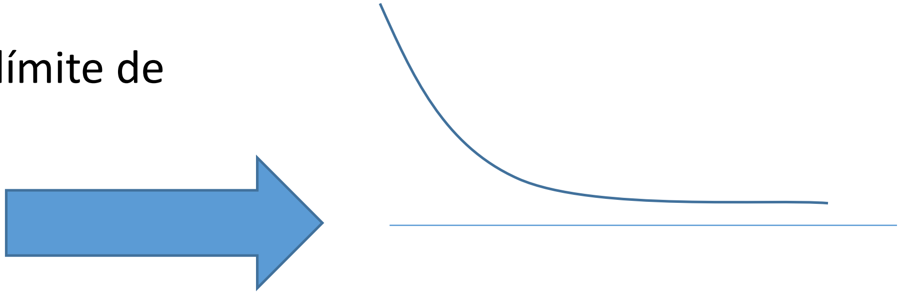
x(t) = e^{-3t} u(t)

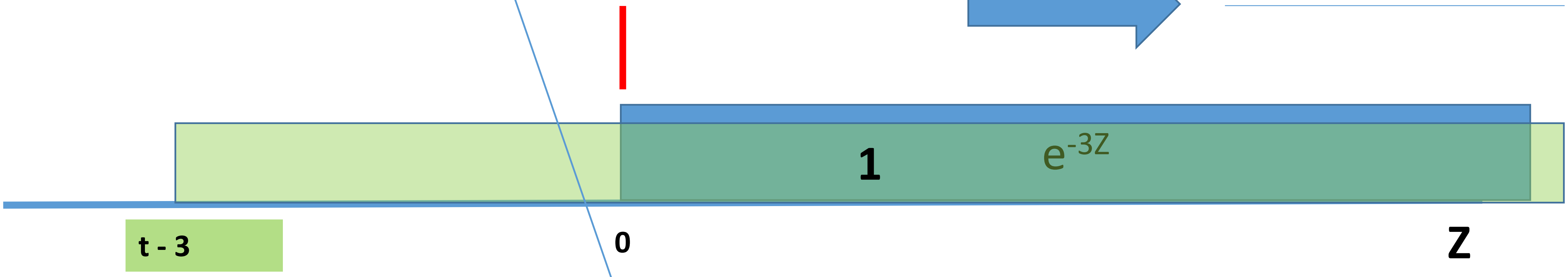
h(t) = u(-t+3) ---- h(Z)---h(t-Z) = u(-(t-Z)+3)=u(-t+Z+3)



$x(Z) = e^{-3Z}\,u(Z)$

Integral Impropia: Tiene solución cuando el límite de la señal en infinito tiene valor determinado.





t − 3 < 0
t < 3

$$y(t) = \int_{0}^{+\infty} e^{-3Z}\, 1 d\tau = \int_{0}^{+\infty} e^{-3Z}\, d\tau = -\frac{1}{3} e^{-3Z} \Big|_{0}^{+\infty} = -\frac{1}{3}(0 - 1) = \mathbf{\frac{1}{3}}$$

t − 3 > 0
t > 3

$$y(t) = \int_{t-3}^{+\infty} e^{-3Z}\, 1 d\tau = \int_{t-3}^{+\infty} e^{-3Z}\, d\tau = -\frac{1}{3} e^{-3Z} \Big|_{t-3}^{+\infty} = -\boldsymbol{\frac{1}{3}\left(0 - e^{-3(t-3)}\right)} = \boldsymbol{\frac{1}{3} e^{-3(t-3)}}$$

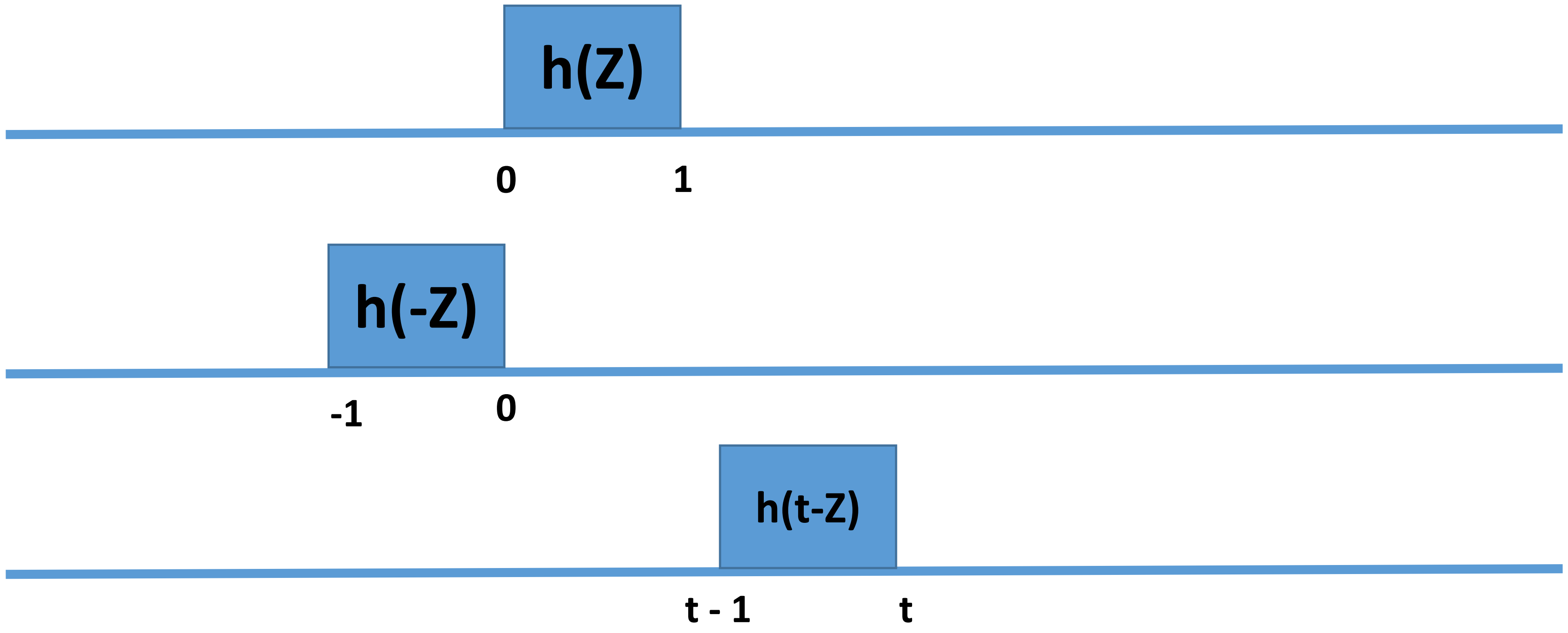
t

$y(t) = x(t) \; * \; h(t)$

$x(t) = 2[u(t) – u(t\text{-}2)] + [u(t – 2) – u(t – 4)]$

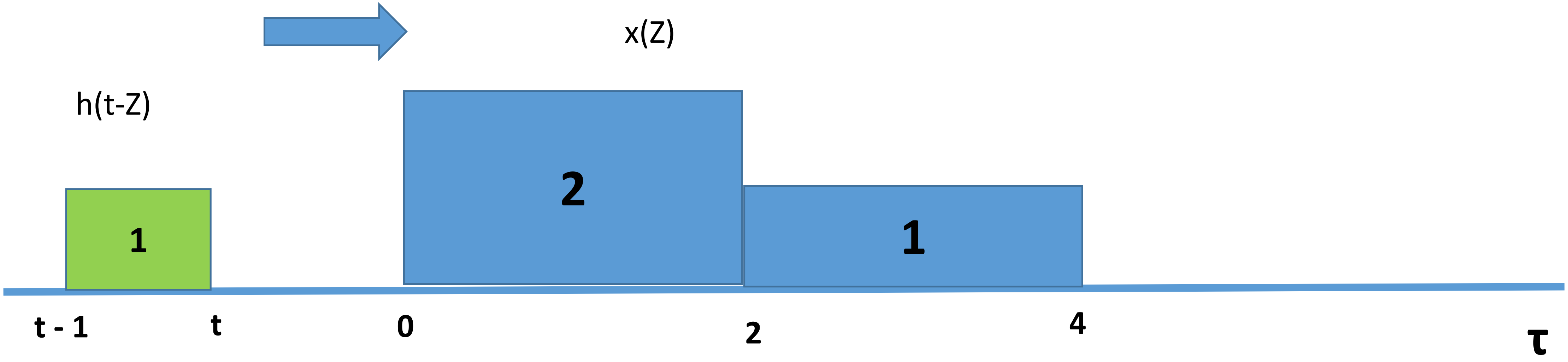
$h(t) = u(t) – u(t – 1)$ ------ $h(Z)$

Se desplaza "t" lugares



x(t)

$x(Z) = 2[u(Z) - u(Z-2)] + [u(Z - 2) - u(Z - 4)]$



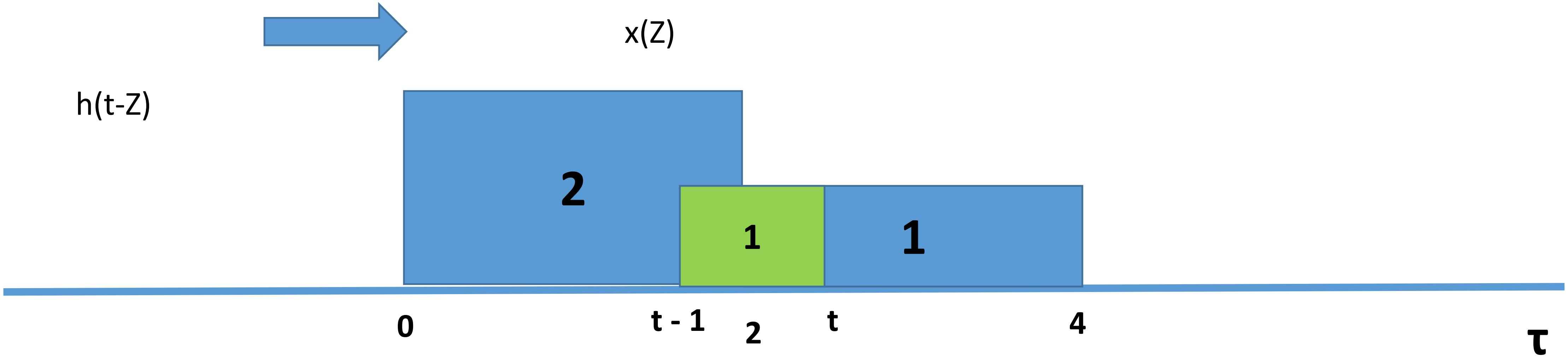
t < 0 ; y(t) = 0

t > 0
t - 1 < 0
0 < t < 1

$$y(t) = \int_0^t 2\,.\,1 d\tau = 2\int_0^t d\tau = 2Z\,\Big|_0^t = \boldsymbol{2t}$$

t

x(Z) = 2[u(Z) – u(Z-2)] + [u(Z – 2) – u(Z – 4)]



t < 2
t – 1 > 0
1 < t < 2

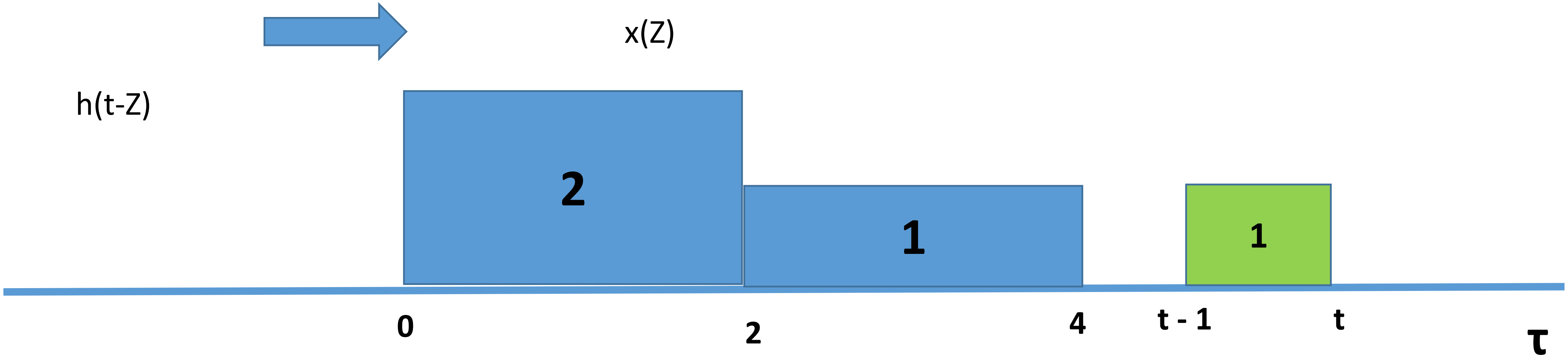
$$y(t) = \int_{t-1}^{t} 2\,.\,1 d\tau = 2\int_{t-1}^{t} d\tau = 2Z \begin{matrix} t \\ t-1 \end{matrix} = \mathbf{2}$$

t > 2
t - 1 < 2
2 < t < 3

$$y(t) = \int_{t-1}^{2} 2\,.\,1 d\tau + \int_{2}^{t} 1\,.\,1 d\tau = -\boldsymbol{t} + \mathbf{4}$$

t

x(Z) = 2[u(Z) – u(Z-2)] + [u(Z – 2) – u(Z – 4)]



t < 4
t – 1 > 2
3 < t < 4

$$y(t) = \int_{t-1}^{t} 1\,.\,1 d\tau = \int_{t-1}^{t} d\tau = Z\begin{matrix} t \\ t-1 \end{matrix} = \mathbf{1}$$

t > 4
t - 1 < 4
4 < t < 5

$$y(t) = \int_{t-1}^{4} 1\,.\,1 d\tau = \int_{t-1}^{4} d\tau = -\boldsymbol{t} + \mathbf{5}$$

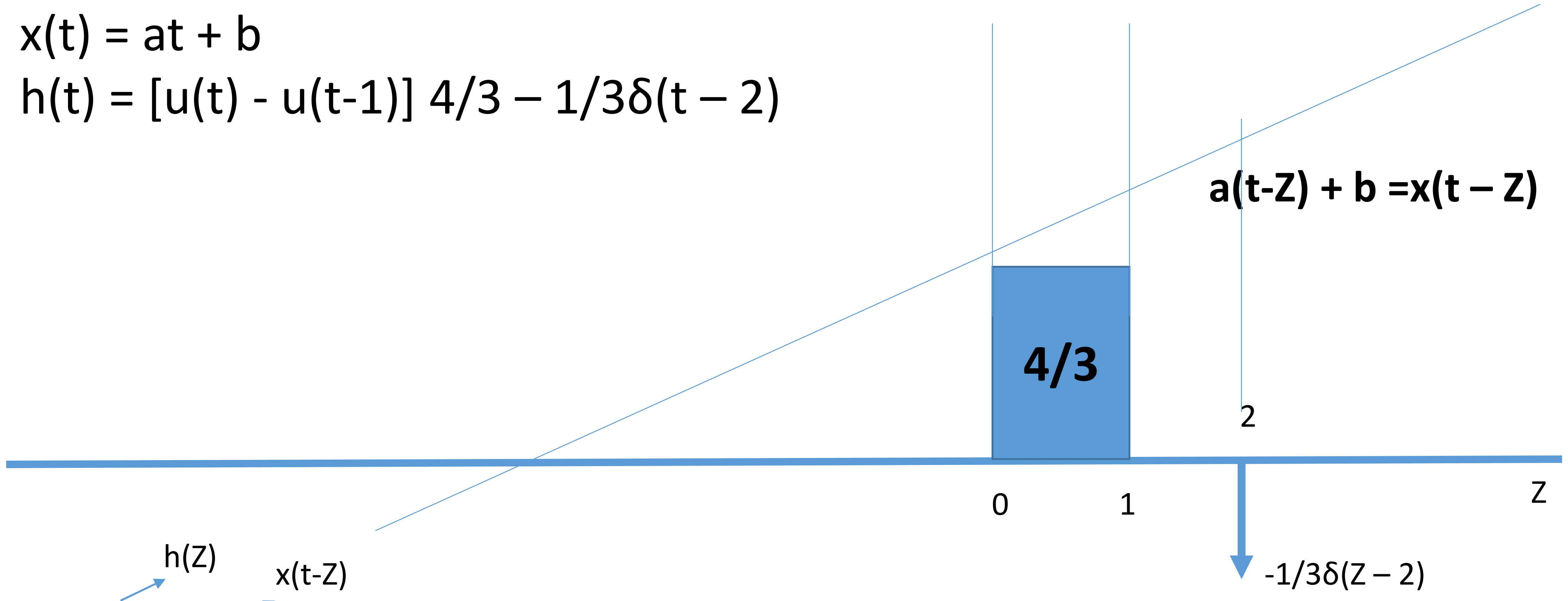
t – 1 > 4
t > 5 **; y(t) = 0**

t

$y(t) = x(t) * h(t)$

$x(t) = at + b$

$h(t) = [u(t) - u(t-1)]\, 4/3 - 1/3\delta(t-2)$



h(Z) x(t-Z)

$$y(t) = \int_0^1 \frac{4}{3}(a(t \ -Z)+b)\,d\tau + \left(-\frac{\mathbf{1}}{\mathbf{3}}\right)(\mathbf{a}(\mathbf{t}-\mathbf{2})+\mathbf{b}) = \int_0^1 \frac{4}{3}\left(a(t \ -Z)\right)d\tau + \int_0^1 \frac{4}{3}b\,d\tau + \left(-\frac{\mathbf{1}}{\mathbf{3}}\right)(\mathbf{a}(\mathbf{t}-\mathbf{2})+\mathbf{b}$$

$$y(t) = \frac{4}{3}\int_0^1 at\,d\tau - \frac{4}{3}a\int_0^1 Z\,d\tau + \frac{4}{3}b\int_0^1 d\tau + \left(-\frac{\mathbf{1}}{\mathbf{3}}\right)\mathbf{a}(\mathbf{t}-\mathbf{2}) - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{b} =$$

$$y(t) = \frac{4}{3}\int_0^1 at\,d\tau - \frac{4}{3}\int_0^1 Z\,d\tau + \frac{4}{3}b\int_0^1 d\tau + \left(-\frac{\mathbf{1}}{\mathbf{3}}\right)\mathbf{a(t-2)} - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{b} =$$

$$y(t) = \frac{4}{3}at\int_0^1 d\tau - \frac{4}{3}a\int_0^1 Z\,d\tau + \frac{4}{3}b\int_0^1 d\tau + \left(-\frac{\mathbf{1}}{\mathbf{3}}\right)\mathbf{at} + \frac{\mathbf{2}}{\mathbf{3}}\mathbf{a} - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{b} =$$

$$y(t) = \frac{4}{3}at\,(1-0) - \frac{4}{3}a\left(\frac{1^2}{2} - \frac{0^2}{2}\right) + \frac{4}{3}b(1-0) - \frac{1}{3}\mathrm{at} + \frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{a} - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{b} =$$

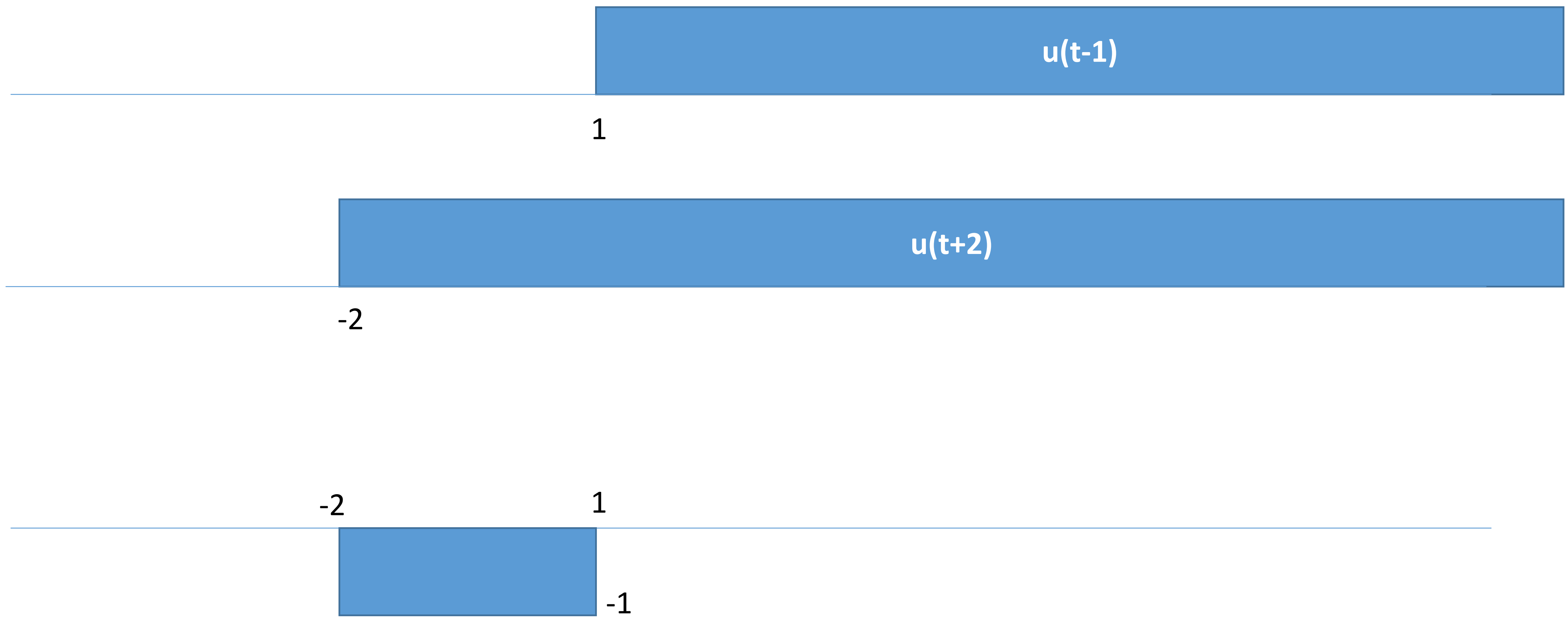
$$y(t) = \frac{4}{3}at\; - \frac{4}{3}a\left(\frac{1}{2}\right) + \frac{4}{3}b - \frac{1}{2}\mathrm{at} + \frac{2}{3}a - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{b} =$$

$$y(t) = \frac{4}{3}at\; - \frac{2}{3}a + \frac{4}{3}b - \frac{1}{3}\mathrm{at} + \frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{a} - \frac{1}{3}\mathrm{b} = \frac{\mathbf{3}}{\mathbf{3}}\boldsymbol{at} + \frac{\mathbf{3}}{\mathbf{3}}\boldsymbol{b} = \boldsymbol{at} + \boldsymbol{b}$$

$$y(t) = x(t) \; * \; h(t)$$

$$x(t) = e^{-3t}\,[u(t-1) - u(t+2)]$$

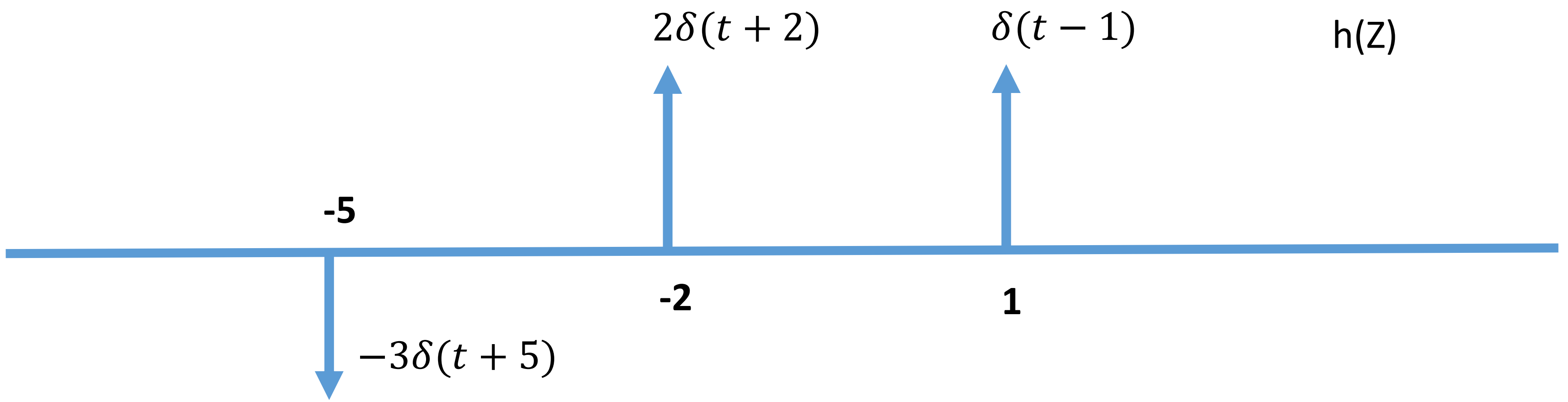
$$h(t) = \delta(t-1) + 2\delta(t+2) - 3\delta(t+5)$$

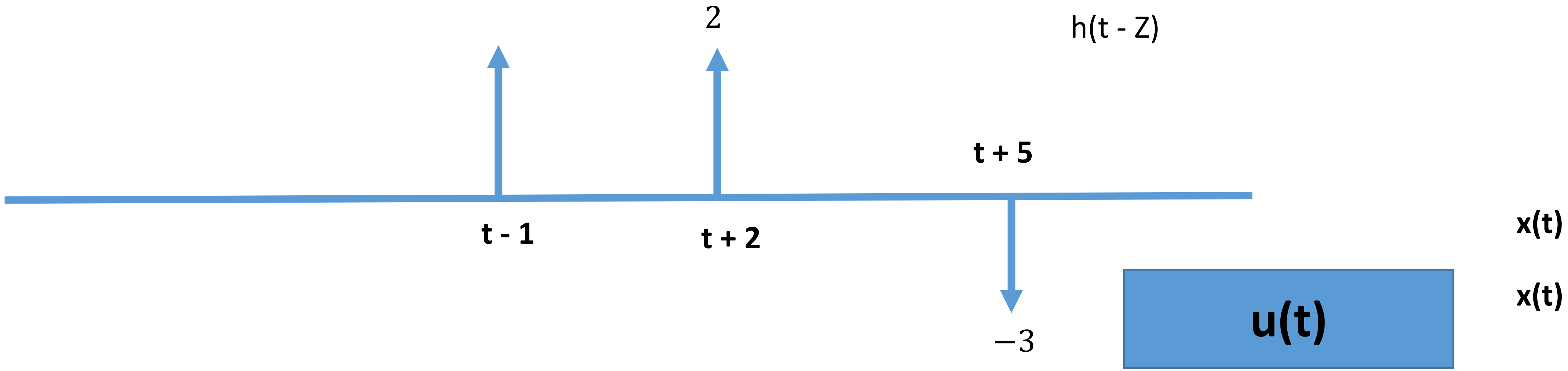


$y(t) = x(t) \; * \; h(t)$

$x(t) = e^{-3t}\,[u(t-1) - u(t+2)]$

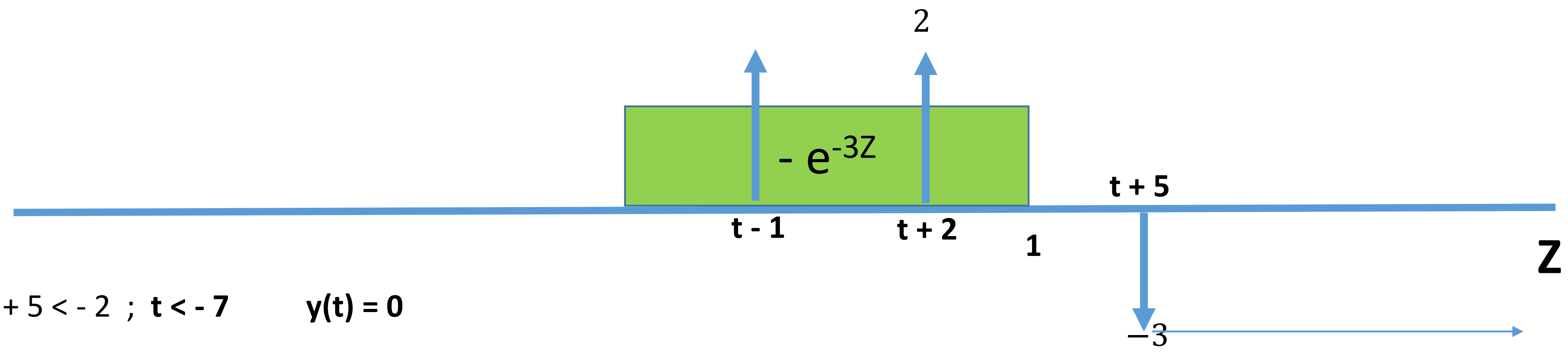
$h(t) = \delta(t-1) + 2\delta(t+2) - 3\delta(t+5)$ -- h(Z)





$x(Z) = e^{-3Z}\,[u(Z-1) - u(Z+2)]$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau).h(t-\tau)d\tau = \int_{-\infty}^{+\infty} h(\tau).x(t-\tau)d\tau$$



1) t + 5 < - 2 ; **t < - 7** **y(t) = 0**

2) t + 5 > - 2

t + 5 < 1 **- 7 < t < - 4** $y(t) = (-3)\,(-e^{-3(t+5)})$

3) t + 2 > -2 ; **- 4 < t < -1** $y(t) = -\,2\,e^{-3\,(t+2)}$

t + 2 < 1

4) t – 1 > -2 ; **-1 < t < 2** $y(t) = -\,e^{-3(t-1)}$

t – 1 < 1

t

5) t – 1 > 1 ; **t > 2** **y(t) = 0**

1) $t + 5 < -2$; $\mathbf{t < -7}$ ---------------------→ $y(t) = 0$

2) $t + 5 > -2$

$t + 5 < 1$ $\mathbf{-7 < t < -4}$ -------→ $y(t) = 3e^{-3(t+5)}$

3) $t + 2 > -2$; $\mathbf{-4 < t < -1}$ -----→ $y(t) = -2e^{-3(t+2)}$

$t + 2 < 1$

4) $t - 1 > -2$; $-\mathbf{1 < t < 2}$ **------→** $y(t) = -e^{-3(t-1)}$

$t - 1 < 1$

5) $t - 1 > 1$; $t > 2$ ----------------→ $\mathbf{y(t) = 0}$