4.17 CALCULAR LA TRANSFORMADA DE FOURIER DE CADA UNA DE LAS SIGUIENTES SEÑALES.

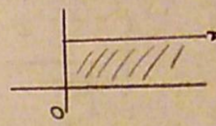
$$X(\omega) = \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j\omega\tau} \, d\tau$$

a) $[e^{-\alpha\tau} \cdot \cos \omega_0 \tau] \cdot u(\tau) \quad ; \quad \alpha > 0.$

1º paso, Acomodamos $x(\tau)$ en forma exponencial.

$$x(\tau) = \left[e^{-\alpha\tau} \cdot \frac{1}{2}\left(e^{j\omega_0\tau} + e^{-j\omega_0\tau}\right)\right] \cdot u(\tau)$$

$$\boxed{x(\tau) = \frac{1}{2}\left[e^{-\alpha\tau + j\omega_0\tau} + e^{-\alpha\tau - j\omega_0\tau}\right] \cdot u(\tau)}$$



2º PASO; Buscamos armar $X(\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} \underbrace{\frac{1}{2}\left[e^{-\tau(\alpha - j\omega_0)} + e^{-\tau(\alpha + j\omega_0)}\right] \cdot u(\tau)}_{x(\tau)} \cdot e^{-j\omega\tau} \, d\tau$$

por el escalón

$$= \frac{1}{2} \int_{0}^{\infty} \left[e^{-\tau(\alpha - j\omega_0 + j\omega)} + e^{-\tau(\alpha + j\omega_0 + j\omega)}\right] d\tau$$

$$= \frac{1}{2}\left[\left(\frac{-1}{\alpha - j\omega_0 + j\omega}\right) e^{-\tau(\alpha - j\omega_0 + j\omega)}\Big|_0^{\infty} + \left(-\frac{1}{\alpha + j\omega_0 + j\omega}\right) \cdot e^{-\tau(\alpha + j\omega_0 + j\omega)}\Big|_0^{\infty}\right]$$

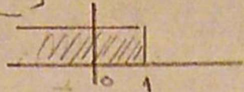
$$= -\frac{1}{2(\alpha - j\omega_0 + j\omega)}\left(\underbrace{e^{-\infty}}_{0} - \underbrace{e^{0}}_{1}\right) + \left(-\frac{1}{2(\alpha + j\omega_0 + j\omega)}\right)\left(\underbrace{e^{-\infty}}_{0} - \underbrace{e^{0}}_{1}\right)$$

$$\boxed{X(\omega) = \frac{1}{2(\alpha - j\omega_0 + j\omega)} + \frac{1}{2(\alpha + j\omega_0 + j\omega)}}$$

ⓑ $e^{2+\tau}\cdot u(-\tau+1)$

1) No hace falta acomodar en forma exp. así que reemplazo en fórmula

$$X(w)=\int_{-\infty}^{\infty}\Big[e^{2+\tau}\cdot \underbrace{u(-\tau+1)}_{\tau<1}\Big]\cdot e^{-jw\tau}\,d\tau$$



$$=\int_{-\infty}^{1} e^{2}\cdot e^{\tau(1-jw)}\,d\tau = \frac{e^{2}}{(1-jw)}\cdot e^{\tau(1-jw)}\Big|_{-\infty}^{1}$$

$$=\frac{e^{2}}{(1-jw)}\Big[e^{(1-jw)}-\underbrace{e^{-\infty}}_{0}\Big]\Rightarrow \boxed{X(w)=\frac{e^{3-jw}}{(1-jw)}}$$

ⓒ $e^{-3|\tau|}\cdot \operatorname{sen} 2\tau$

1º) trabajo con $x(t)$ para acomodar en forma exponencial

$$x(\tau)=e^{-3|\tau|}\cdot\frac{1}{2j}\left(e^{j2\tau}-e^{-j2\tau}\right)$$

2º) Armo $X(w)$ analizando intervalo de integración

$|\tau|$ significa que si → $\tau>0 \Rightarrow x(\tau)=e^{-3\tau}$ ∴ va de $(0 \text{ a } \infty)$

$\tau<0 \Rightarrow x(\tau)=e^{+3\tau}$ ∴ va de $(-\infty \text{ a } 0)$

Resolviendo

$$X(w)=\int_{-\infty}^{0} e^{3\tau}\cdot\frac{1}{2j}\left(e^{j2\tau}-e^{-j2\tau}\right)\cdot e^{-jw\tau}\,d\tau+\int_{0}^{\infty} e^{-3\tau}\,\frac{1}{2j}\left(e^{j2\tau}-e^{-j2\tau}\right)\cdot e^{-jw\tau}\,d\tau$$

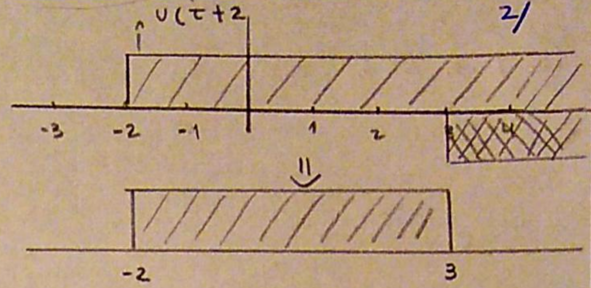
$$=\frac{1}{2j}\Big[\int_{-\infty}^{0} e^{\tau(3+j2-jw)}\,d\tau-\int_{-\infty}^{0} e^{\tau(3-2j-jw)}\,d\tau\Big]+\frac{1}{2j}\Big[\int_{0}^{\infty} e^{-\tau(3+2j+jw)}\,d\tau-\int_{0}^{\infty} e^{-\tau(3+2j+jw)}\,d\tau\Big]$$

$$=\frac{1}{2j}\Big[\frac{1}{3+2j-jw}\cdot\big(\underbrace{e^{0}}_{1}-\underbrace{e^{-\infty}}_{0}\big)-\frac{1}{3-2j-jw}\big(\underbrace{e^{0}}_{1}-\underbrace{e^{-\infty}}_{0}\big)\Big]+\frac{1}{2j}\Big[\frac{-1}{3-2j+jw}\big(\underbrace{e^{-\infty}}_{0}-\underbrace{e^{0}}_{1}\big)+\frac{1}{3+2j+jw}\big(\underbrace{e^{-\infty}}_{0}-\underbrace{e^{0}}_{1}\big)\Big]$$

$$=\frac{1}{6j+4\underbrace{j^2}_{-1}-2\underbrace{j^2}_{-1}w}-\frac{1}{6j-4\underbrace{j^2}_{-1}-2\underbrace{j^2}_{-1}w}+\frac{1}{6j-4\underbrace{j^2}_{-1}+2\underbrace{j^2}_{-1}w}-\frac{1}{6j+4\underbrace{j^2}_{-1}+2\underbrace{j^2}_{-1}w}\qquad \boxed{j^2=-1}$$

$$\boxed{X(w)=\frac{1}{6j-4+2w}-\frac{1}{6j+4+2w}+\frac{1}{6j+4-2w}-\frac{1}{6j-4-2w}}$$

(d) $e^{-3\tau}\left[\underbrace{u(\tau+2)-u(\tau-3)}_{[-2;3]}\right]$



a) Resolución por fórmula

$$X(w)=\int_{-2}^{3} e^{-3\tau}\cdot e^{-jw\tau}\,d\tau=\int_{-2}^{3} e^{-\tau(3+jw)}\,d\tau=\frac{-1}{3+jw}\,e^{-\tau(3+jw)}\Big|_{-2}^{3}$$

$$\boxed{X(w)=-\frac{1}{3+jw}\left(e^{-9-3jw}-e^{6+2jw}\right)}$$

b) también puede resolverse por tabla

- distribuyo

$$x(\tau)=e^{-3\tau}\cdot u(t+2)-e^{-3\tau}\cdot u(\tau-3)$$

- Igualo el desplazamiento en ambas partes

$$x(\tau)=e^{-3\tau}\cdot\frac{e^{-6}}{e^{-6}}\cdot u(\tau+2)-e^{-3\tau}\cdot\frac{e^{9}}{e^{9}}\cdot u(\tau-3)$$

- Reacomodo

$$x(\tau)=e^{6}\cdot e^{-3(\tau+2)}\cdot u(\tau+2)-e^{-9}\cdot e^{-3(\tau-3)}\cdot u(\tau-3)$$

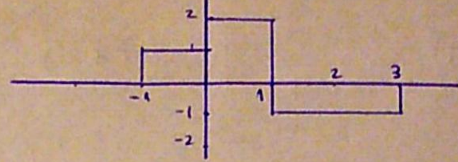
$\boxed{\tau_0=-2}$ $\boxed{a=3}$ $\boxed{\tau_0=3}$ $\boxed{a=-3}$

$\mathcal{F}\ e^{-a\tau}\cdot u(\tau)\Rightarrow\frac{1}{a+jw}$ + Prop. Despl. $x(t-\tau_0)\to e^{-jw\tau_0}\cdot X(w)$

- utilizo $\mathcal{F}_6$ + Prop Despl

$$X(w)=e^{6}\left[\frac{1}{3+jw}\right]\cdot e^{-j(-2)w}-e^{-9}\cdot\left[\frac{1}{3+jw}\right]\cdot e^{-j(3)w}=\boxed{-\frac{1}{(3+jw)}\left[-e^{6+2jw}+e^{-9-3jw}\right]}$$

ⓔ $x(t)$ como la figura



$$x(\tau) = \begin{cases} 1 & -1 < \tau < 0 \\ 2 & 0 < \tau < 1 \\ -1 & 1 < \tau < 3 \end{cases}$$

$$X(w) = \int_{-1}^{0} (1) \cdot e^{-jw\tau} d\tau + \int_{0}^{1} (2) \cdot e^{-jw\tau} d\tau + \int_{1}^{3} (-1) \cdot e^{-jw\tau} d\tau$$

$$= \frac{-1}{jw} \left[e^{-jw\tau} \Big|_{-1}^{0} + 2\, e^{-jw\tau} \Big|_{0}^{1} + (-1) \cdot e^{-jw\tau} \Big|_{1}^{3} \right]$$

$$= -\frac{1}{jw} \left[(\underbrace{e^{0}}_{1} - e^{jw}) + 2\,(e^{-jw} - \underbrace{e^{0}}_{1}) - 1\,(e^{-3jw} - e^{-jw}) \right]$$

$$= \frac{1}{jw} \left[-1 + e^{jw} - 2e^{-jw} + 2 + e^{-3jw} - e^{-jw} \right]$$

$$\boxed{X(w) = \frac{1}{jw} \left[1 + e^{jw} - 3e^{-jw} + e^{-3jw} \right]}$$

Fyg novan.

ⓗ $x(t) = \sum_{k=0}^{\infty} \alpha^{k} \cdot \delta(t - kt)$ $\quad |\alpha| < 1 \Rightarrow \boxed{1 < \alpha < 1}$

$$X(w) = \sum_{k=0}^{\infty} \alpha^{k} \int_{-\infty}^{\infty} \underbrace{\delta(\tau - kT)}_{\text{sólo tiene valor en } kT} \cdot e^{-jw\tau} d\tau \Rightarrow \boxed{X(w) = \sum_{k=0}^{\infty} \alpha^{k} \cdot e^{-jw(kT)}} \text{ con } |\alpha| < 1$$

También se puede resolver por tabla F_{12}

ⓘ $x(t) = [\tau \cdot e^{-2\tau} \cdot \text{sen } 4\tau] \, u(\tau)$ $\quad x_1(\tau)$, x_2

- Para entrar a la tabla, reagrupo $x(t)$ en 2 $x_1(\tau)$ y $x_2(\tau)$ y transformo

$$x_1(\tau) = \tau \cdot e^{-2\tau} \cdot u(\tau) \Rightarrow X_1(w) = \frac{1}{(2 + jw)^2} \qquad a = 2$$

F_{13} $\tau \cdot e^{-a\tau} \cdot u(\tau) \Rightarrow \frac{1}{(a + jw)^2}$

$$x_2(\tau) = \text{sen } 4\tau \Rightarrow X_2(w) = \frac{\pi}{j} \left[\delta(w-4) - \delta(w+4) \right] \qquad w_0 = 4$$

F_4 $\text{sen } w_0 \tau = \frac{\pi}{j} (\delta(w - w_0) - \delta(w + w_0))$

- Por propiedad $x_1(\tau) \cdot x_2(\tau) \Rightarrow \frac{1}{2\pi} X(w) * Y(w)$. Para la convolución uso la prop. del impulso como elemento neutro.

$\Rightarrow$ cálculo auxiliar

$$\frac{1}{(2 + jw)^2} * \delta(w - 4) = \frac{1}{(2 + j(w-4))^2}$$

$$\frac{1}{(2 + jw)^2} * \delta(w + 4) = \frac{1}{(2 + j(w+4))^2}$$

$$X(w) = \frac{1}{2\cancel{\pi}} \cdot \frac{\cancel{\pi}}{j} \left(\frac{1}{(2 + j(w-4))^2} - \frac{1}{(2 + j(w+4))^2} \right) \Rightarrow \boxed{X(w) = \frac{1}{j} \left[\frac{1}{(2 + j(w-4))^2} - \frac{1}{(2 + j(w+4))^2} \right]}$$

(j) $\operatorname{sen} t + \cos\left(2\pi t + \frac{\pi}{4}\right)$

$$\boxed{X(w) = \frac{\pi}{j}\left[\delta(w-1) - \delta(w+1)\right] + e^{j\frac{\pi}{4}}\pi\delta(w-2\pi) + e^{-j\frac{\pi}{4}}\pi\,\delta(w+2\pi)}$$

(k) $\underbrace{\frac{\operatorname{sen}\pi t}{\pi t}}_{x_1} \cdot \underbrace{\frac{\operatorname{sen} 2\pi(t-1)}{\pi(t-1)}}_{y_1}$

Tenemos multiplicación de 2 señales

x prop: la multiplicación en el tiempo $\Rightarrow$ convolución en la frecuencia

P Dual. $x(t)\cdot y(t) \Rightarrow \frac{1}{2\pi} X(w) * Y(w)$

1- transformamos $x_1(t)$ e $y_1(t)$

$x_1(t) = \frac{\operatorname{sen}\pi t}{\pi t}$ Fq $\frac{\operatorname{sen} Wt}{\pi t} \Rightarrow X(w) = \begin{cases} 1 & |w| < W \\ 0 & |w| > W \end{cases}$ $\boxed{X_1(w) = \begin{cases} 1 & |w| < \pi \\ 0 & |w| > \pi \end{cases}}$

$\boxed{W = \pi}$

$W = 2\pi$

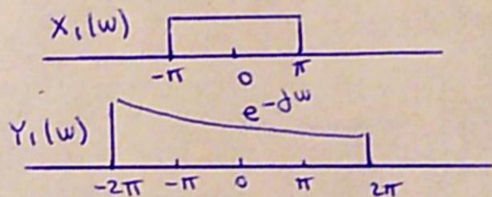
$y_1(t) = \frac{\operatorname{sen} 2\pi(t-1)}{\pi(t-1)}$ Fq + Prop. Desplaz $x(t-t_0) \to e^{-jwt_0}\cdot X(w)$

$t_0 = 1$.

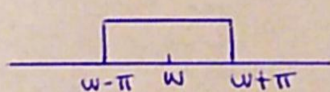
$y_1(t_1) = \frac{\operatorname{sen} 2\pi t_1}{\pi t_1} \Rightarrow Y(w_1) = \begin{cases} 1 & |w| < 2\pi \\ 0 & |w| > 2\pi \end{cases}$ + despl. $\to$ $\boxed{Y_1(w) = \begin{cases} 1\cdot e^{-jw(1)} & |w| < 2\pi \\ 0 & |w| > 2\pi \end{cases}}$

2- Aplico convolución

$X(w) = \frac{1}{2\pi} X_1(w) * Y_1(w)$

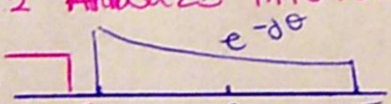


2.1 Invierto y desplazo $X_1(w)$

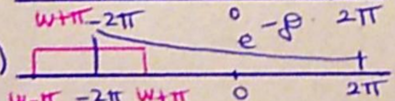


$$X(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y_1(\theta)\cdot X_1(w-\theta)\,d\theta$$

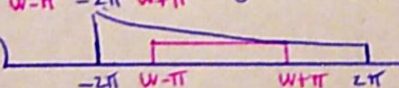
2.2 Analizo intervalos

1º) 

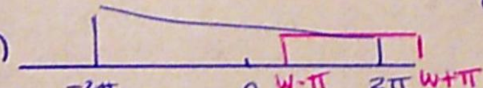
$w+\pi < -2\pi$ $\quad$ $w < -3\pi$ $\quad$ $X(w) = 0$.

2º) 

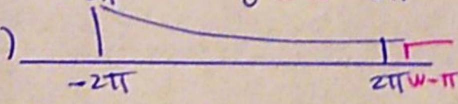
$-2\pi < w+\pi < 0$ $\quad$ $-3\pi < w < -\pi$ $\quad$ $X(w) = \frac{1}{2\pi}\int_{-2\pi}^{w+\pi} e^{-j\theta}\,d\theta \ldots$

3º) 

$0 < w+\pi < 2\pi$ $\quad$ $-\pi < w < \pi$ $\quad$ $X(w) = \frac{1}{2\pi}\int_{w-\pi}^{w+\pi} e^{-j\theta}\,d\theta \ldots$

4º) 

$2\pi < w+\pi < 4\pi$ $\quad$ $\pi < w < 3\pi$ $\quad$ $X(w) = \frac{1}{2\pi}\int_{w-\pi}^{2\pi} e^{-j\theta}\,d\theta \ldots$

5º) 

$4\pi < w+\pi$ $\quad$ $w > 3\pi$ $\quad$ $X(w) = 0$