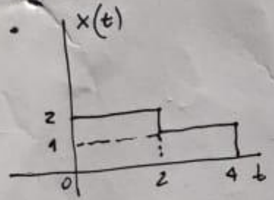
EJERCICIOS RESUELTOS

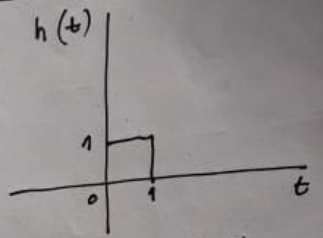
TEMA CONVOLUCIÓN

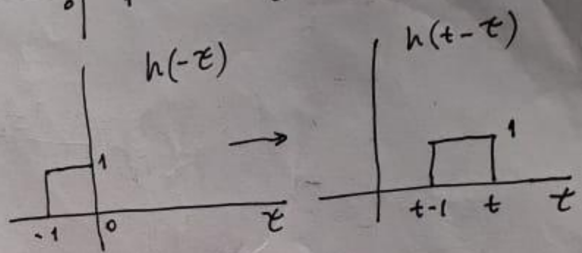
EJERCICIO 1

1

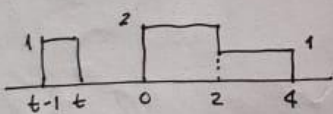
Ejercicio 3 - Convolución - Guía de TP.



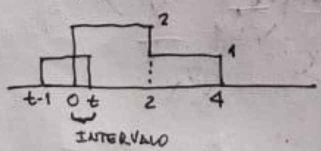




1er punto) $t<0 \Rightarrow y(t)=0$

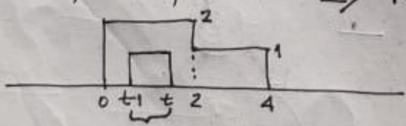


2do punto) $t>0$; $t-1<0 \Rightarrow 0<t<1$



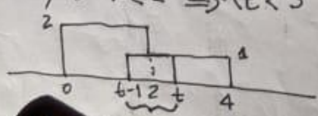
$$\int_0^t 2\,d\tau = 2|\tau|_0^t = \boxed{2t}$$

3er punto) $t>0$; $t<2$; $t-1>0 \Rightarrow 1<t<2$



$$\int_{t-1}^t 2\,d\tau = 2|t|_{t-1}^t$$
$$= 2(\cancel{t}-\cancel{t}+1)$$
$$= \boxed{2}$$

4to punto) $t>2$; $t-1<2 \Rightarrow 2<t<3$

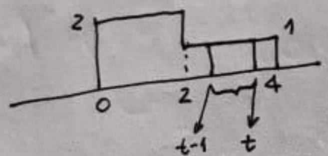


$$\int_{t-1}^2 2\,d\tau + \int_2^t d\tau = 2|\tau|_{t-1}^2 + |t|_2^t$$
$$= 2(2-t+1) + t-2$$
$$\boxed{= -t+4}$$

CIERRE EJERCICIO 1 – COMIENZO EJERCICIO 2

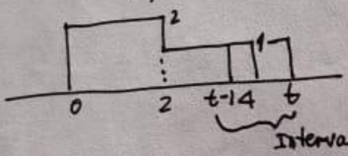
5to punto) $t<4$; $t-1>2 \Rightarrow 3<t<4$

$$\int_{t-1}^{t} d\tau = |\tau|_{t-1}^{t} = \cancel{t} - \cancel{t} + 1 \boxed{= 1}$$



6to punto) $t>4$; $t-1<4 \Rightarrow 4<t<5$

$$\int_{t-1}^{4} d\tau = |\tau|_{t-1}^{4} = (4-t+1) \boxed{= -t+5}$$



7mo intervalo) $t-1>4 \Rightarrow t>5 \Rightarrow \boxed{Y(t)=0}$

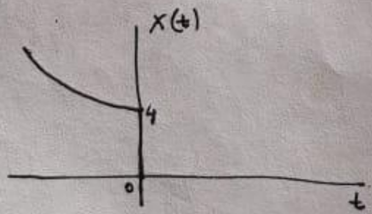
$$Y(t) = \begin{cases} 0 ; & t<0 \\ 2t ; & 0<t<1 \\ 2 ; & 1<t<2 \\ -t+4 ; & 2<t<3 \\ 1 ; & 3<t<4 \\ -t+5 ; & 4<t<5 \\ 0 ; & t>5 \end{cases}$$

Resolución Ej. 4 – Convolución –

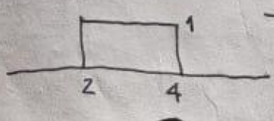
1

$x(t) = (e^{-2t}+3)\, u(-t)$

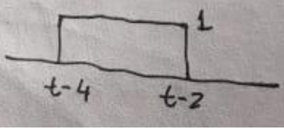
$$h(t) = \begin{cases} 1 ; & 2<t<4 \\ 0 ; & \forall \text{ otro } t \end{cases}$$



$h(\tau)$

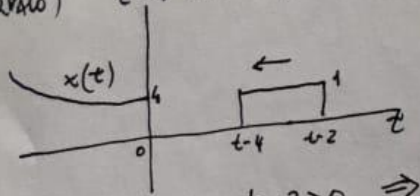


$h(t-\tau)$

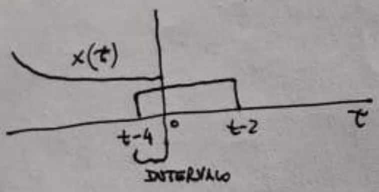


FINAL EJERCICIO 2

$1^{\underline{o}}$ INTERVALO) $t-4>0 \Rightarrow t>4$



$2^{\underline{do}}$ INTERVALO) $t-4<0\ ;\ t-2>0 \Rightarrow 2<t<4$



$$\int_{t-4}^{0}(e^{-2\tau}+3)\cdot 1 \cdot d\tau =$$

$$= \int_{t-4}^{0} e^{-2\tau}\,d\tau + \int_{t-4}^{0} 3\,d\tau =$$

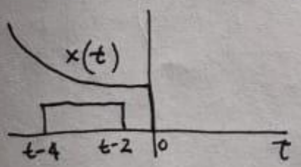
$$= -\frac{1}{2}\left| e^{-2\tau}\right|_{t-4}^{0} + 3\left|\tau\right|_{t-4}^{0} =$$

$$= -\frac{1}{2}\left(e^{0}-e^{-2(t-4)}\right) + 3(0-t+4)$$

$$= -\frac{1}{2} + \frac{1}{2}e^{-2t+8} - 3t + 12 =$$

$$\boxed{= \frac{1}{2}e^{-2t+8} - 3t + \frac{23}{2}}$$

3^{er} intervalo) $t-2<0 \Rightarrow t<2$



$$\int_{t-4}^{t-2}(e^{-2\tau}+3)\,d\tau = \int_{t-4}^{t-2} e^{-2\tau}\,d\tau + \int_{t-4}^{t-2} 3\,d\tau =$$

$$= -\frac{1}{2}\left|e^{-2\tau}\right|_{t-4}^{t-2} + 3\left|\tau\right|_{t-4}^{t-2}$$

$$= -\frac{1}{2}\left(e^{-2(t-2)} - e^{-2(t-4)}\right) + 3\left(\cancel{t}-2-\cancel{t}+4\right)$$

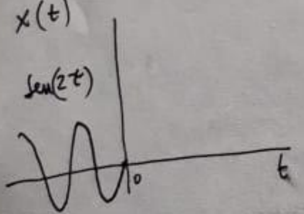
$$\boxed{= -\frac{1}{2}e^{-2t+4} + \frac{1}{2}e^{-2t+8} + 6}$$

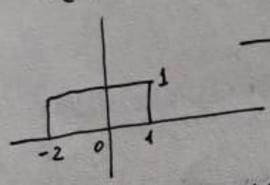
$$y(t) = \begin{cases} -\frac{1}{2}e^{-2t+4} + \frac{1}{2}e^{-2t+8} + 6 & ;\ t<2 \\ \frac{1}{2}e^{-2t+8} - 3t + \frac{23}{2} & ;\ 2<t<4 \\ 0 & ;\ t>4 \end{cases}$$

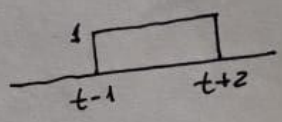
EJERCICIO 3

Resolución ejercicio 5 - CONVOLUCIÓN -

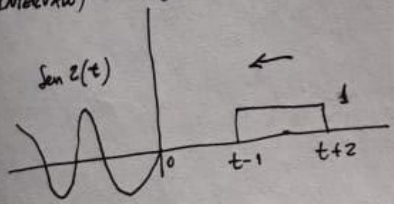
$x(t) = \text{sen}(2t)u(-t)$ $\qquad h(t) = u(t+2) - u(t-1)$



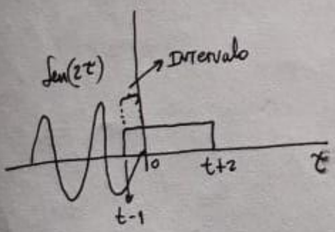




1er INTERVALO) $\quad t-1>0 \Rightarrow t>1 \Rightarrow y(t)=0$



2do INTERVALO) $\quad t-1<0\ ;\ t+2>0 \Rightarrow -2<t<1$



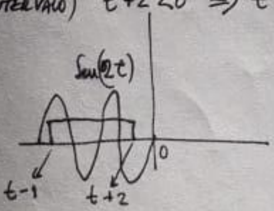
$$\int_{t-1}^{0} \text{sen}(2\tau)\cdot 1\, d\tau = \int_{t-1}^{0} \text{sen}(2\tau)\, d\tau =$$

$$= \frac{1}{2}\Big| -\cos(2\tau)\Big|_{t-1}^{0}$$

$$= \frac{1}{2}\left(-\cos(2.0) + \cos(2(t-1))\right)$$

$$\boxed{= -\frac{1}{2} + \frac{1}{2}\cos[2(t-1)]}$$

3er INTERVALO) $\quad t+2<0 \Rightarrow t<-2$



$$\int_{t-1}^{t+2} \text{sen}(2\tau)\, d\tau = \frac{1}{2}\Big| -\cos(2\tau)\Big|_{t-1}^{t+2}$$

$$= \frac{1}{2}\left(-\cos(2(t+2)) + \cos(2(t-1))\right)$$

$$\boxed{= -\frac{1}{2}\cos[2t+4] + \frac{1}{2}\cos[2t-2]}$$

EJERCICIO 4

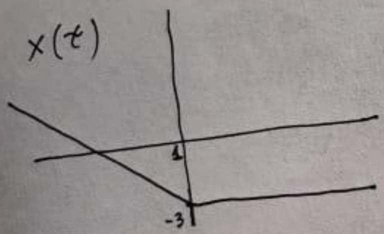
1

Resolución Ejercicio 6 - Convolución -

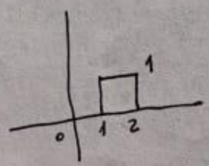
$$h(t) = u(t-1) - u(t-2)$$

$$x(t) = \begin{cases} -3t \; ; \; t < 1 \\ -3 \; ; \; t > 1 \end{cases}$$

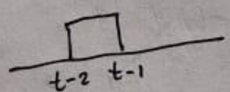
x(t)



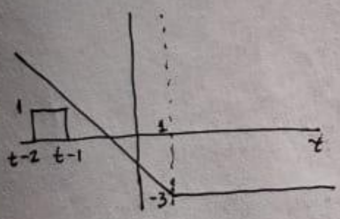
h(t)



h(t-τ)



1er intervalo) $t-1 < 1 \Rightarrow t < 2$



$$\int_{t-2}^{t-1} -3\tau \, d\tau = -\frac{3}{2} \left| \tau^2 \right|_{t-2}^{t-1}$$

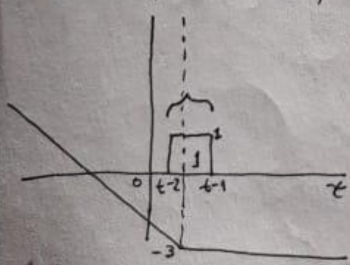
$$= -\frac{3}{2}\left((t-1)^2 - (t-2)^2\right)$$

$$= -\frac{3}{2}\left[\cancel{t^2} - 2t + 1 - \cancel{t^2} + 4t - 4\right]$$

$$= -\frac{3}{2}\left[2t - 3\right]$$

$$\boxed{= -3t + \frac{9}{2}}$$

2do intervalo) $t-1 > 1 \; ; \; t-2 < 1 \Rightarrow 2 < t < 3$



$$\int_{t-2}^{1} -3\tau \, d\tau + \int_{1}^{t-1} -3 \, d\tau =$$

$$= -\frac{3}{2} \left| \tau^2 \right|_{t-2}^{1} - 3 \left| \tau \right|_{1}^{t-1} =$$

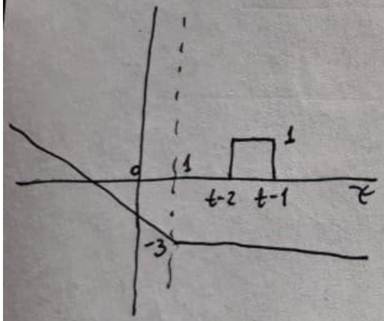
$$= -\frac{3}{2}\left(1^2 - (t-2)^2\right) - 3(t-1-1)$$

$$= -\frac{3}{2}\left(1 - t^2 + 4t - 4\right) - 3t + 6$$

$$= -\frac{3}{2}\left(-t^2+4t-3\right) - 3t + 6 = \frac{3}{2}t^2 - 6t + \frac{9}{2} - 3t + 6$$

$$\boxed{= \frac{3}{2}t^2 - 9t + \frac{21}{2}}$$

3º INTERVALO) $t-2>1$; $t>3$ $\Longrightarrow$



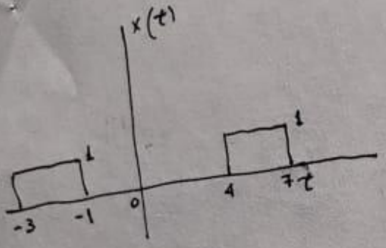
$$\int_{t-2}^{t-1} -3\,d\tau = -3\left(\cancel{t}-1-\cancel{t}+2\right)$$

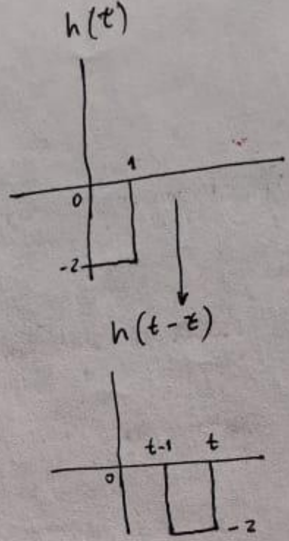
$$= -3\cdot 1 \boxed{= -3}$$

$$y(t) = \begin{cases} -3t + \frac{9}{2} & ; \ t<2 \\ \frac{3}{2}t^2 - 9t + \frac{21}{2} & ; \ 2<t<3 \\ -3 & ; \ t>3 \end{cases}$$

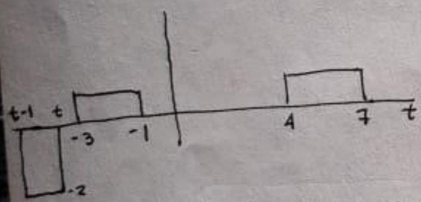
EJERCICIO 5

Resolución Ejercicio 7 - Convolución





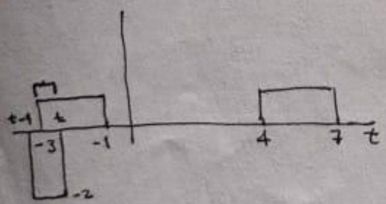
1er intervalo) $t < -3 \Rightarrow y(t) = 0$



2do intervalo) $t > -3$; $t-1 < -3 \Rightarrow -3 < t < -2$

$$\int_{-3}^{t} (-2)\cdot 1 \, d\tau = -2 \int_{-3}^{t} d\tau =$$

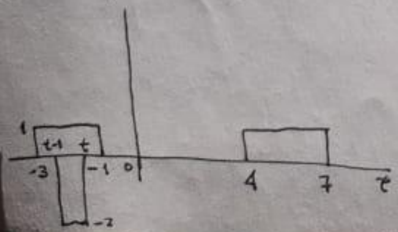
$$= -2 \tau \Big|_{-3}^{t} = -2\,(t-(-3)) \boxed{= -2t-6}$$



3er intervalo) $t < -1$; $t-1 > -3 \Rightarrow -2 < t < -1$

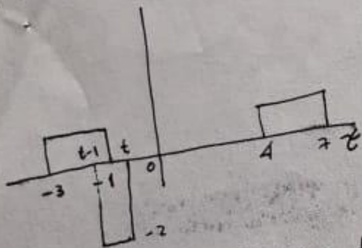
$$\int_{t-1}^{t} -2 \, d\tau = -2\, \tau \Big|_{t-1}^{t} = -2\,(\cancel{t} - \cancel{t} + 1)$$

$$\boxed{= -2}$$



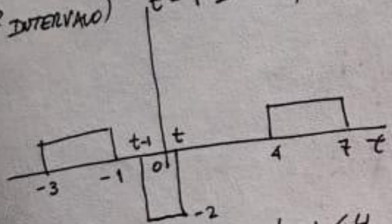
4to INTERVALO) $t>-1$; $t-1<-1 \Rightarrow -1<t<0$

$$\int_{t-1}^{-1} -2\,dt = -2\,t\Big|_{t-1}^{-1} = -2(\cancel{-1}-t\cancel{+1}) \quad \boxed{=2t}$$



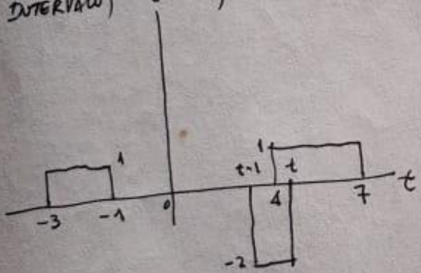
5to INTERVALO) $t-1>-1$; $t<4 \Rightarrow 0<t<4$

$y(t)=0$
(NO HAY CONVOLUCIÓN)



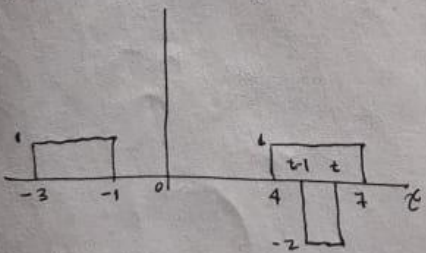
6to INTERVALO) $t>4$; $t-1<4 \Rightarrow 4<t<5$

$$\int_{4}^{t} -2\,dt = -2\,t\Big|_{4}^{t} = -2(t-4) \quad \boxed{=-2t+8}$$



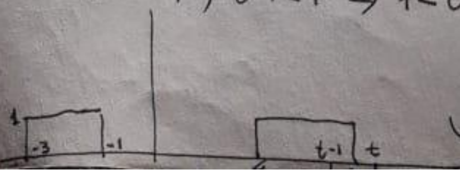
7mo INTERVALO) $t<7$; $t-1>4 \Rightarrow 5<t<7$

$$\int_{t-1}^{t} -2\,dt = -2\,t\Big|_{t-1}^{t} = -2(\cancel{t}-\cancel{t}+1) \quad \boxed{=-2}$$



8vo INTERVALO) $t>7$; $t-1<7 \Rightarrow 7<t<8$

$$\int_{t-1}^{7} -2\,dt = -2\,t\Big|_{t-1}^{7} = -2(7-t+1) \quad \boxed{2t-16}$$



$9^{\underline{no}}$ INTERVALO) $t-1>7 \Rightarrow t>8$

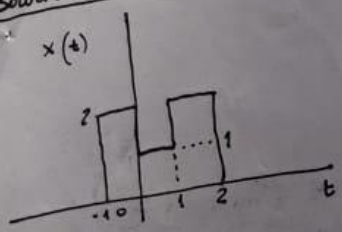
$$\Downarrow$$

$$y(t) = 0$$

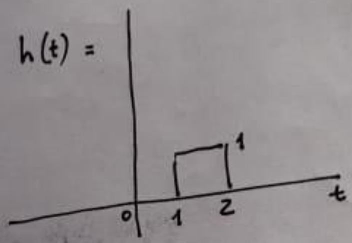
$$y(t) = \begin{cases} 0 \ ; & t<-3 \\ -2t-6 \ ; & -3<t<-2 \\ -2 \ ; & -2<t<-1 \\ 2t \ ; & -1<t<0 \\ 0 \ ; & 0<t<4 \\ -2t+8 \ ; & 4<t<5 \\ -2 \ ; & 5<t<7 \\ 2t-16 \ ; & 7<t<8 \\ 0 \ ; & t>8 \end{cases}$$

EJERCICIO 6

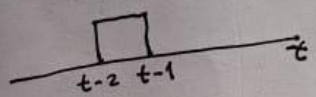
RESOLUCIÓN EJERCICIO 8 - CONVOLUCIÓN -



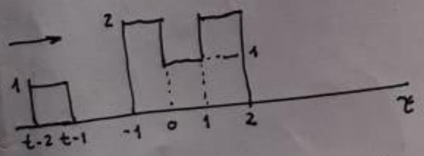
h(t) =



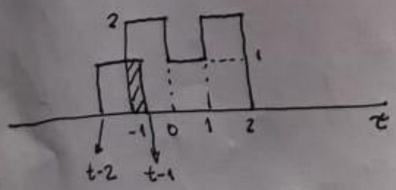
$h(t-\tau)$



1er INTERVALO) $t-1<-1 \Rightarrow t<0 \Rightarrow y(t)=0$

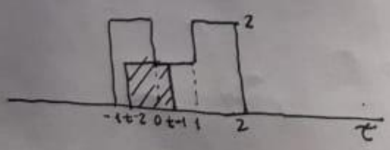


2do INTERVALO) $t-1>-1 \; ; \; t-2<-1 \Rightarrow 0<t<1$



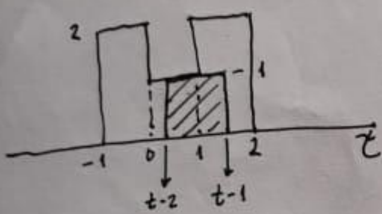
$$\int_{-1}^{t-1} 2\,d\tau = 2\,|\tau|_{-1}^{t-1} = 2\left(t-1-(-1)\right)$$
$$\boxed{=2t}$$

3er INTERVALO) $t-1>0 \; ; \; t-2<0 \Rightarrow 1<t<2$



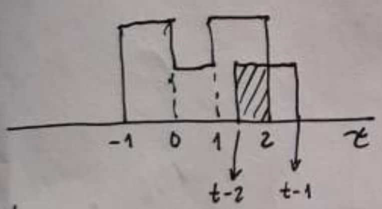
$$\int_{t-2}^{0} 2\,d\tau + \int_{0}^{t-1} d\tau = 2\,|\tau|_{t-2}^{0} + |\tau|_{0}^{t-1}$$
$$= 2(0-t+2) + (t-1)$$
$$= -2t+4+t-1$$
$$\boxed{= -t+3}$$

$4^{\underline{to}}$ INTERVALO) $t-1>1 \; ; \; t-2<1 \Rightarrow 2<t<3$



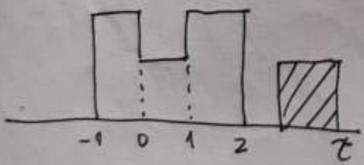
$$\int_{t-2}^{1} dt + \int_{1}^{t-1} 2\,dt = \left| t \right|_{t-2}^{1} + 2\left| t \right|_{1}^{t-1}$$
$$= 1-t+2+2(t-1-1)$$
$$= 3-t+2t-4$$
$$\boxed{= t-1}$$

$5^{\underline{to}}$ INTERVALO) $t-1>2 \; ; \; t-2<2 \Rightarrow 3<t<4$



$$\int_{t-2}^{2} 2\,dt = 2\left| t \right|_{t-2}^{2} = (2-t+2)\,2$$
$$\boxed{= -2t+8}$$

$6^{\underline{to}}$ INTERVALO) $t-2>2 \Rightarrow t>4 \Rightarrow y(t)=0$



$$y(t) = \begin{cases} 0 \; ; & t<0 \\ 2t \; ; & 0<t<1 \\ -t+3 \; ; & 1<t<2 \\ t-1 \; ; & 2<t<3 \\ -2t+8 \; ; & 3<t<4 \\ 0 \; ; & t>4 \end{cases}$$

EJERCICIOS 7 y 8 (EJEMPLOS DE RESOLUCIONES ANALÍTICAS)

RESOLUCIÓN EJERCICIO 9 – CONVOLUCIÓN –

$x(t) = -e^{-2t} u(t) =$

$h(t) = 3u(t)$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} -e^{-2\tau} \underbrace{u(\tau)}_{\tau>0} \cdot 3\, \underbrace{u(t-\tau)}_{\substack{t-\tau>0 \\ -\tau>-t \\ \tau<t}}\, d\tau$$

$$= \underbrace{\int_0^t -3e^{-2\tau}\, d\tau}_{\text{VALE CUANDO } t>0} = \frac{-3}{-2} \left| e^{-2\tau} \right|_0^t = \frac{3}{2}\left(e^{-2t} - 1\right)$$

$$\boxed{= \frac{3}{2} e^{-2t} - \frac{3}{2}}$$

Solución:

$$y(t) = \begin{cases} 0 \; ; \; t<0 \\ \frac{3}{2} e^{-2t} - \frac{3}{2} \; ; \; t>0 \end{cases}$$

RESOLUCIÓN EJERCICIO 10 – CONVOLUCIÓN –

$x(t) = \delta(t+3) + \delta(t+5) \qquad h(t) = \operatorname{Sen}(2t)\, u(t)$

$$y(t) = \int_{-\infty}^{+\infty} \operatorname{Sen}(2\tau)\, u(\tau) \cdot \left[\delta(t-\tau+3) + \delta(t-\tau+5)\right] d\tau$$

$$y(t) = \int_{-\infty}^{+\infty} \operatorname{Sen}(2\tau)\, \underbrace{u(\tau)}_{\tau>0}\, \delta(t-\underbrace{\tau}_{\tau=t+3}+3)\, d\tau + \int_{-\infty}^{+\infty} \operatorname{Sen}(2\tau)\, \underbrace{u(\tau)}_{\tau>0}\, \delta(t-\underbrace{\tau}_{\tau=t+5}+5)\, d\tau$$

$$y(t) = \underbrace{\operatorname{Sen}[2(t+3)]}_{\text{sólo cuando } t+3>0} + \underbrace{\operatorname{Sen}[2(t+5)]}_{\text{cuando } t+5>0}$$

$$y(t) = \begin{cases} 0 \; ; \; t<-5 \\ \operatorname{Sen}[2(t+5)] \; ; \; -5<t<-3 \\ \operatorname{Sen}(2(t+3)) + \operatorname{Sen}(2(t+5)) \; ; \; t>-3 \end{cases}$$