AJ INSTITUTE OF ENGINEERING & TECHNOLOGY

DEPT. OF TRAINING & PLACEMENT

Test-8

Permutation & Combination, Volume & surface area

Wednesday 26th June 2024

1. Find the value of $^{20}C_{17}$

   a. 1260
   b. 1140
   c. 2580
   d. 3200

Correct Option: (b)

Hint:

$$^{n}C_{r} = \frac{nPr!}{r!}$$

$$^{n}C_{r} = \frac{n!}{(r!)\,(n-r)!}$$

$$^{20}C_{17} = \frac{20!}{(17!)\,(20-17)!}$$

$$^{20}C_{17} = \frac{20 \times 19 \times 18 \times 17!}{(17!)\,(3)!}$$

$$^{20}C_{17} = \frac{20 \times 19 \times 18}{3 \times 2 \times 1}$$

$$^{20}C_{17} = 1140$$

2. How many words can be formed by using letters of the word ‘DELHI’?

   a. 50
   b. 72
   c. 85
   d. 120

Correct Option: (d)

Hint:

The word 'DELHI' contains 5 letters

Therefore, required number of words = ${}^5P_5 = 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$

120 words can be formed by using letters of the word 'DELHI'

3. Find the value of ${}^{50}P_2$

   a. 4500
   b. 3260
   c. 2450
   d. 1470

Correct Option: (c)

Hint:

$${}^nP_r = \frac{n!}{(n-r)!}$$

Here n = 50 and r = 2

$$\frac{50!}{(50-2)!} = \frac{50 \times 49 \times 48}{48!} = 50 \times 49 = 2450$$

Value of ${}^{50}P_2 = 2450$

4. Find the number of ways the letters of the word 'RUBBER can be arranged?

   a. 450
   b. 362
   c. 250
   d. 180

Correct Option: (d)

Hint:

The word 'RUBBER' contains 6 letters: 2R, 2B, 1 U, 1 E

Therefore,

$$\text{The required Number of ways: } \frac{N!}{(2R!) \times (2B!) \times (1U!) \times (1E!)}$$

$$= \frac{6!}{(2 \times 1) \times (2 \times 1) \times (1) \times \times (1)}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4}$$

$$= 6 \times 5 \times 3 \times 2$$

$$= 180$$

5. Find in how many different ways, the letters of the word 'LEADING' can be arranged in such way that the vowels always come together?

   a. 548
   b. 426
   c. 720
   d. 790

Correct Option: (c)

The letters in the word 'LEADING' are 7.

Number of vowels = EAI

Number of consonants: LDNG

Condition: Vowels always come together. Therefore, let's consider the vowels as one group 'EAI'

Hence, **vowels 'EAI' and consonants LDNG together form 5 letters.**

Therefore, these 5 letters can be arranged in 5! Ways = 5! = 5 × 4 × 3 × 2 = 120 ways

Letters in group of vowels 'EAI' can also be interchanged. So, these vowels can be arranged in 3! Ways = 6 ways.

All together, required number of ways = 120 x 6 = 720 ways

6. Out of 5 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

   a. 60
   b. 1200

c. 5230
d. 7200

Correct Option: (d)

Hint:

$${}^{n}C_{r} = \frac{n!}{(r!)\,(n-r)!}$$

3 consonants out of 5 can be selected in ${}^{5}C_{3}$ ways

2 vowels out of 4 can be selected in ${}^{4}C_{2}$ ways

In ${}^{5}C_{3}$ and ${}^{4}C_{2}$ ways the consonants and vowels can be selected.

Remember: ${}^{5}C_{3}$ and ${}^{4}C_{2}$, hence multiply the terms ${}^{5}C_{3} \times {}^{4}C_{2}$

$${}^{5}C_{3} \times {}^{4}C_{2} = \frac{5!}{3!\,(5-3)!} \times \frac{4!}{2!\,(4-2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 2} \times \frac{4 \times 3 \times 2 \times 1}{2 \times 2} = (5 \times 2) \times (3 \times 2)$$

60 groups can be made each having 3 consonants and 2 vowels

Each word contains 3 consonants and 2 vowels i.e 5 letters.

Hint:

If N different objects are to be arranged, then they can be arranged in N! ways.

5 letters can be arranged in 5! Ways.

5! = 5 x 4 x 3 x 2 x 1 = 120

Required number of words = (120 x 60) = 7200

7. In what ways can a group of 6 boys and 2 girls be made out of the total of 7 boys and 3 girls?

A. 50
B. 120
C. 21
D. 20

Answer: C

Answer with the explanation:

We know that ${}^{n}C_{r} = {}^{n}C_{(n-r)}$

The combination of 6 boys out of 7 and 2 girls out of 3 can be represented as ${}^{7}C_{6} + {}^{3}C_{2}$

Therefore, the required number of ways = ${}^{7}C_{6} * {}^{3}C_{2} = {}^{7}C(7-6) * {}^{3}C_{(3-2)} = \frac{7}{1} * \frac{3}{1} = 21$

Hence, in 21 ways the group of 6 boys and 2 girls can be made.

8. Out of a group of 7 boys and 6 girls, five boys are selected to form a team so that at least 3 boys are there on the team. In how many ways can it be done?

A. 645
B. 734
C. 756
D. 612

Answer: C

Answer with the explanation:

We may have 5 men only, 4 men and 1 woman, and 3 men and 2 women in the committee.

So, the combination will be

as we know that

$${}^{n}C_{r} = \frac{n\,(n-1)\,(n-2)\,\ldots \text{upto r factors}}{r!}$$

So, $({}^{7}C_{3} * {}^{6}C_{2}) + ({}^{7}C_{4} * {}^{6}C_{1}) + ({}^{7}C_{5})$

Or, $\left[\frac{7*6*5}{3*2*1} * \frac{6*5}{2*1}\right] + \left[\frac{7*6*5*4}{4*3*2*1} * \frac{6}{1}\right] + \left[\frac{7*6*5*4*3}{5*4*3*2*1}\right]$

Or, 525 +210+21 = 756

So, there are 756 ways to form a committee.

9. A box contains 2 red balls, 3 black balls, and 4 white balls. Find the number of ways by which 3 balls can be drawn from the box in which at least 1 black ball should be present.

A. 64
B. 48
C. 32
D. 96

Answer: A

Answer with the explanation:

The possible combination could be (1 black ball and 2 non-black balls), (2 black balls and 1 non- black ball), and (only 3 black balls).

Therefore the required number of combinations = $({}^{3}C_{1} * {}^{6}C_{2}) + ({}^{3}C_{2} * {}^{6}C_{1}) + ({}^{3}C_{3})$

r, $\left[\frac{3}{1} * \frac{6*5}{2*1}\right] + \left[\frac{3*2}{2*1} * \frac{6}{1}\right] + \left[\frac{3*2*1}{3*2*1}\right]$ = 45+18+1 = 64

10. Out of 7 constants and 4 vowels, how many words of 3 constants and 2 vowels can be formed?

A. 21020
B. 25200
C. 10500
D. 21400

Answer: B

Answer with the explanation:

We can combine 3 consonants and 2 vowels out of 7 consonants and 4 vowels in a way

${}^{7}C_{3} * {}^{4}C_{2} = \left[\frac{7*6*5}{3*2*1} * \frac{4*3}{2*1}\right]$ = 210

That means 210 groups having 3 consonants and 2 vowels.

Each group consists of 5 words that mean the possible arrangement of the letters is 5!

Or, 5! = 5*4*3*2*1 = 120 ways.

Therefore, the required number of words = 210*120 = 25200.

11. How much 4-digit number can be formed from the digits 2, 3, 4, 5, 6, and 7 which are divisible by 5 in such a way that digits should not repeat.

A. 25
B. 30
C. 35
D. 60

Answer: D

Answer with the explanation:

A number is divisible by 5 if the number ends with 0 or 5, but we don't have 0 in the given digits that means 5 should come at the unit place.

Now, one of the remaining 5 digits (2, 3, 4, 6, and 7) can come at the tens place.
Similarly, we can fill the hundreds place by one of the remaining 4 digits.
Therefore, the thousands place can be filled by one of the remaining 3 digits.
Hence, the required number of the numbers = 1*5*4*3 = 60.

12. In what ways the letters of the word "CRICKET" can be arranged to form the different new words so that the vowels always come together?

A. 120
B. 240
C. 360
D. 480

Answer: A

Answer with the explanation:

The word CRICKET has 7 different letters, but ATQ, the vowels should always come together.
Now, let the vowels IE as a single entity.
Therefore, the number of letters is CRCKT = 5 in which C is repeated twice, and IE = 1
Since the total number of letters = 5+1 = 6
So the arrangement (permutation) would be = $\frac{5!}{2!} = \frac{5*4*3*2!}{2!}$ = 5*4*3= 60 ways.

Note: Here 2! is taken in the denominator, because the letter C is repeated twice.

Now, the vowels IE can be arranged in 2 different ways, i.e., ${}^{2}P_{2}$ = 2! = 2*1 = 2 ways.

So, the new words that can be formed = 60*2 = 120.

13. In what ways the letters of the word **ACTORS** can arrange so that the vowels occupy only the even positions?

A. 212

B. 144

C. 576

D. 400

Answer: B

Answer with the explanation:

The word ACTORS consist of 6 letters in which there are 2 vowels and 4 consonants

ATQ, the vowels A, O can be placed at any of the position out of 2, 4, and 6.

That means the number of ways to arrange the vowels = ${}^{3}P_{2} = \frac{3!}{(3-2)!} = \frac{3!}{1!}$ = 3*2 = 6 ways

Similarly, the number of ways to arrange the consonants = ${}^{4}P_{4} = \frac{4!}{(4-4)!}$ = 4*3*2*1 = 24 ways

Note: we know that 0! = 1

Therefore the required numbers of ways = 6*24 = 144

14. How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

a. 40

b. 400

c. 5040

d. 2520

Answer: Option ©

Explanation:

'LOGARITHMS' contains 10 different letters.

Required number of words = Number of arrangements of 10 letters, taking 4 at a time.

$= {}^{10}P_{4}$

$= (10 \times 9 \times 8 \times 7)$

= 5040.

15. In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together?

a.10080

b.4989600

c.120960

d. None of these

Answer: Option ©

Explanation:

In the word 'MATHEMATICS', we treat the vowels AEAI as one letter.

Thus, we have MTHMTCS (AEAI).

Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different.

∴ Number of ways of arranging these letters = $\frac{8!}{(2!)(2!)}$ = 10080.

Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.

Number of ways of arranging these letters = $\frac{4!}{2!}$ = 12.

∴ Required number of words = (10080 × 12) = 120960.

16. The surface area of a cube is 600 cm2. The length of its diagonal is

A. 10/√3 cm
B. 10 / √2 cm
C. 10 √3 cm
D. 10 √2 cm

Answer: C

Explanation:

Surface area of cube = $6\ a^2$

$600 = 6\ a^2$

$a^2 = 100$

$a = \sqrt{100} = 10$ cm

Diagonal of cube = $\sqrt{3}\ a = \sqrt{3} \times 10 = 10\sqrt{3}$ cm

17) The length of the longest rod that can be placed in a room 30 m long, 24 m broad and 18 m high, is

A. 30 m

B. $15\sqrt{2}$ m

C. $30\sqrt{2}$ m

D. 60 m

Answer: C

Explanation:

Length of room = 30 m

Breadth of room = 24 m

Height of room = 18 m

Length of longest rod = diagonal of room = $\sqrt{(l^2 + b^2 + h^2)}$ units

$= \sqrt{30^2 + 24^2 + 18^2} = \sqrt{900 + 576 + 324} = \sqrt{1800}$

$= \sqrt{2 \times 3 \times 3 \times 10 \times 10} = 30\sqrt{2}$ m

18) The dimensions of a hall are 40 m, 25 m and 20 m. If each person requires 200 cubic meters, find the number of persons who can be accommodated in the hall.

A. 150

B. 140

C. 120

D. 100

Answer: D

Explanation:

Length of the hall = 40 m

Breadth of hall= 25 m

Height of hall = 20 m

Volume of the hall = l x b x h

$= 40 \times 25 \times 20 = 20{,}000\ m^3$

Space occupied by each person = $200m^3$

Number of person that can accommodate in the hall = $\frac{\text{volume of the hall}}{\text{space occupied by one person}} = \frac{20000}{200} = 100$ persons

19) A water tank is 30 m long, 20 m wide and 12 m deep. It is made of iron sheet which is 3 m wide. The tank is open at the top. If the cost of iron sheet is Rs. 10 per meter, what is the total cost of iron sheet required to build the tank?

A. Rs. 6000
B. Rs. 8000
C. Rs. 9000
D. Rs. 10,000

Answer: A

Explanation:

Length of water tank = 30 m
Width of water tank = 20 m
Depth of water tank = 12 m

Area of water tank = 2(lb+bh +hl) - lb

$= 2 (30 \times 20 + 20 \times 12 + 12 \times 30) - 30 \times 20$
$= 2 (600 + 240 + 360) - 600$
$= 2 (1200) - 600$
$= 2400 - 600 = 1800\ m^2$

Area of iron sheet (L*B) = area of tank = $1800\ m^2$

$L * B = 1800$
$L \times 3 = 1800$
$L = 1800/3 = 600\ m$

Given that, the cost of iron sheet is Rs. 10 per meter.
So, the total cost of iron sheet to build the tank = 10 x 600 = Rs. 6000

20) In a shower 10 cm of rain falls. What is the volume of water that falls on 1.5 hectares of ground?

A. $1500\ m^3$
B. $1400\ m^3$
C. $1200\ m^3$
D. $1000\ m^3$

Answer: A

Explanation:

1 hectare = 10,000 m^2

1.5 hectare = 1.5 x 10 x 1000 = 15,000 m^2

Depth = 10 cm of rainfall = $\frac{10}{100}$

∴ Volume of water = Area * Depth

15,000 * $\frac{10}{100}$ = 1500 m3

21) The dimensions of a piece of iron in the shape of a cuboid are 270 cm x 100 cm x 64 cm. If it is melted and recast into a cube, find the surface of the cube.

A. 14,400 cm^2

B. 44,200 cm^2

C. 57,600 cm^2

D. 86,400 cm^2

Answer: D

Explanation:

Volume of iron piece = l x b x h

= 270 x 100 x 64 = 1,728,000 cm^3

Volume of cuboid = volume of cube

$1{,}728{,}000 = a^3$

$a = \sqrt[3]{1{,}728{,}000} = \sqrt[3]{2 \times 2 \times 2 \times 6 \times 6 \times 6 \times 10 \times 10 \times 10} = 2 \times 6 \times 10 = 120$

Surface area of cube = 6 a^2

= 6 x 1202= 86,400 cm^2

22) Surface area of a sphere is 5544 cm2. Its volume will be

A. 30808 cm^3

B. 38808 cm^3

C. 380808 cm^3

D. 380800 cm^3

Answer: B

Explanation:

Surface area of sphere = $4\pi r^2 = 5544\ cm^2$

$\pi r^2 = \frac{5544}{4} = 1386$

$\frac{22}{7} r^2 = 1386$

$r^2 = 1386 \times \frac{7}{22} = 63 \times 7 = 441$

$r = \sqrt{441} = \sqrt{21 \times 21} = 21\ cm$

Volume of a sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 4 \times 22 \times 21 \times 21 = 38808\ cm^3$

$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

Volume of a sphere $= 4 \times 22 \times 21 \times 21 = 38808\ cm^3$

23) If two spheres have their radii in the ratio 2: 3, the ratio of their volumes is

A. 8 : 27

B. 4 : 9

C. 6 : 12

D. 1 : 15

Answer: A

Explanation:

Let the radius of both the spheres be 'r' and 'R', respectively.

r: R = 2: 3

Volume of sphere = $\frac{4}{3}\pi r^3$

Ratio between the volume of spheres = $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{2^3}{3^3} = \frac{8}{27}$

=8:27

24) The radius of hemisphere is 3 cm. The ratio of its volume to the total surface area is

A. 1: 3

B. 2: 1

C. 1: 1

D. 2: 3

Answer: D

Explanation:

Radius of hemisphere = 3 cm

$$\frac{\text{Volume of hemisphere}}{\text{Total surafce area of hemisphere}} = \frac{\frac{2}{3}\pi r^3}{3\pi r^2} = \frac{\frac{2}{3}r}{3} = \frac{2}{9} \times 3 = \frac{2}{3} = 2:3$$

Ratio = 2: 3

25) The length, breadth, and height of a brick are 10 cm, 4 cm, and 3 cm, respectively. Find the surface area of the brick?

A. 154 cm^2

B. 156 cm^2

C. 160 cm^2

D. 164 cm^2

Answer: D

Explanation:

Surface area of a Cuboid = 2(lb+ bh+ hl) cm^2

So,

Surface area of a brick = 2(10*4+4*3+3*10) cm^2

= 2(82) cm^2= 164 cm^2

26) A 6 m long and 4 m wide cistern contains water up to a breadth of 1 m 25 cm. Find the total surface area of the surface immersed in water?

A. 42 m^2

B. 49 m^2

C. 52 m^2

D. 64 m^2

Answer: B

Explanation:

Area of the wet surface =

2[lb+ bh+ hl] - lb = 2 [bh+ lh] + lb

= 2[(4*1.25+6*1.25)] +6*4 = 49 m^2

Solution 2:

The area of the wet surface= Area of the base + area of the two walls 2 * (4 *1.25) + area of the other two walls 2 * (6 * 1.25) cm

= 6*4 + 2 (4*1.25) + 2 (6*1.25)

= 24+ 10 + 15 = 49 m^2

27) A boat of length 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm when a man gets into it. What is the mass of the man?

A. 50 Kg
B. 60 Kg
C. 70 Kg
D. 80 Kg

Answer: B

Explanation:

In this type of question, first, we will calculate the volume of water displaces then we will multiply it with the density of water.

1 cm = 0.06 m

Volume of water displaced = 3*2*0.01 = 0.06 m cube

Mass of Man = Volume of water displaced * Density of water (around 1000 kg/m^3)

= 0.06 * 1000 = 60 kg

28) If the curved surface area of a sphere is same as the curved surface area of a hemisphere, find the radius of the hemisphere.

A. Same as that of the sphere.
B. $\sqrt{2}$ times that of the sphere.
C. $\sqrt{3}$ times that of the sphere.
D. 2 times that of the sphere.

Answer: B

Explanation:

Curved surface area of a sphere = Curved surface area of a hemisphere

$$4 \pi R^2 = 2 \pi r^2$$

$$2 R^2 = r^2$$

Radius of hemisphere i.e. 'r' = $\sqrt{2 R^2} = \sqrt{2}\ R$

29) How many bricks, each measuring 25cm*11.25cm*6cm, will be needed to build a wall 8m*6m*22.5m?

A. 610000
B. 620000
C. 630000
D. 640000

Answer: D

Explanation:

To solve this type of question, simply divide the volume of the wall with the volume of brick to get the required number of bricks.

Number of bricks:

$$= \frac{\text{Volume of wall}}{\text{volume of 1 brick}} = \frac{800*600*2250}{25*11.25*6} = 640000$$

30) The cost of the paint is Rs. 36.50 per kg. If 1 kg of paint covers 16 square feet, how much will it cost to paint outside of a cube having 8 feet each side

A. 850
B. 860
C. 876
D. 886

Answer: C

Explanation:

We will first calculate the Surface area of the cube, and then we will calculate the quantity of paint required.

Surface area = $6a^2$

$=6*8^2=384$ sq feet

16 square feet is covered by 1 kg of paint.

384 square feet will be covered by = $\frac{384}{16}$ = 24kg of paint

As per the question, the cost of the paint is Rs. 36.50 per kg.

So, the total cost of painting =36.50*24

=Rs.876