Homework – 02

Name: Dash Subhadeep

Roll no. 201601021

Solutions

Homework 2

Name: Subhadeep Dash
Roll no 201601021

Solutions

1. Given $x(t)$ is a real valued signal and its sampling rate is $\Omega_s = 8000\pi$ rad/s

Since the sampling rate is Ω_s, $X(\Omega)$ will vanish for $|\Omega| \geq \frac{\Omega_s}{2}$.
[From Sampling Theorem]

$\therefore$ $X(\Omega) = 0$ or will vanish for $\Omega > 4000\pi$ rad/s

2. We can obtain the signal from the Fourier transform through the following formula:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega$$

and the formula for Fourier Transform of a signal is

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

Given Nyquist rate is Ω_1.

(i) $y_1(t) = y(t) + y(t-1)$

Applying Fourier Transform on both sides,

$$Y_1(\Omega) = \int_{-\infty}^{\infty} y(t) e^{-j\Omega t} dt + \int_{-\infty}^{\infty} y(t-1) e^{-j\Omega t} dt$$

$$= \int_{-\infty}^{\infty} y(t) e^{-j\Omega t} dt + e^{j\Omega} \int_{-\infty}^{\infty} y(t-1) e^{-j\Omega(t-1)} dt$$

Let $t' = t-1$

Differentiating on both sides

$dt' = dt$

$$\therefore Y_1(\Omega) = \int_{-\infty}^{\infty} y(t)e^{-j\Omega t}dt + \left[\int_{-\infty}^{\infty} y(t')e^{-j\Omega t'}dt'\right]e^{j\Omega}$$

$$= Y(\Omega) + Y(\Omega)e^{j\Omega}$$

$$= e^{j\Omega}(Y(\Omega)) + Y(\Omega)$$

$$= Y(\Omega)[1+e^{j\Omega}]$$

$\therefore$ $Y(\Omega)$ is 0 for $|\Omega| > \frac{\Omega_1}{2}$.

$\therefore$ Nyquist rate for $y_1(t)$ is also Ω_1.

(ii) $y_2(t) = \frac{d}{dt}x(t)$

We know

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)e^{j\Omega t}d\Omega$$

where $x(t)$ is inverse fourier transform of $X(\Omega)$

Differentiating both sides

$$\frac{d}{dt}x(t) = \frac{1}{2\pi}\frac{d}{dt}\int_{-\infty}^{\infty} X(\Omega)e^{j\Omega t}d\Omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}[j\Omega X(\Omega)]e^{j\Omega t}d\Omega$$

$\therefore$ $\frac{d}{dt}x(t)$ is inverse Fourier Transform for $j\Omega X(\Omega)$ i.e. $Y_2(\Omega) = j\Omega X(\Omega)$

$\because$ Nyquist rate for $x(t)$ is Ω_1,

$$Y_2(\Omega) = 0 \quad \forall\ |\Omega| > \frac{\Omega_1}{2}$$

$$\left[\because X(\Omega) = 0 \ \forall\ |\Omega| > \frac{\Omega_1}{2}\right]$$

$\therefore$ Nyquist rate for $y_2(t)$ is also Ω_1.

(iii) $y_3(t) = y(t)\exp(j\Omega_0 t)$

Applying Fourier Transform on both sides

$$Y_3(\Omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\Omega t} \cdot e^{(j\Omega_0 t)}\, dt$$

$$= \int_{-\infty}^{\infty} y(t)\, e^{-j(\Omega-\Omega_0)t}\, dt$$

$$= Y(\Omega - \Omega_0)$$

Nyquist rate for $y(t)$ is Ω_1.

$\therefore$ $Y(\Omega-\Omega_0) = 0$ for $|\Omega - \Omega_0| > \frac{\Omega_1}{2}$.

$\therefore$ Nyquist rate is $|\Omega - \Omega_0| \times 2$ for $y_3(t)$.

③ Let $x_1(t) = \cos(2\pi(200)t) = \cos(2\pi(f_1)t) = \cos(\Omega_1 t)$

$x_2(t) = \cos(2\pi(400)t) = \cos(2\pi(f_2)t) = \cos(\Omega_2 t)$

$$z(t) = \cos(\Omega_1 t)\cos(\Omega_2 t)$$

$$z(t) = \frac{1}{2}\left[\cos((\Omega_1+\Omega_2)t) + \cos((\Omega_1-\Omega_2)t)\right]$$

We use fft function in Matlab and then plot the values against fvec.

We observe peaks in the graph.

The spectral regions give the maximum frequency.

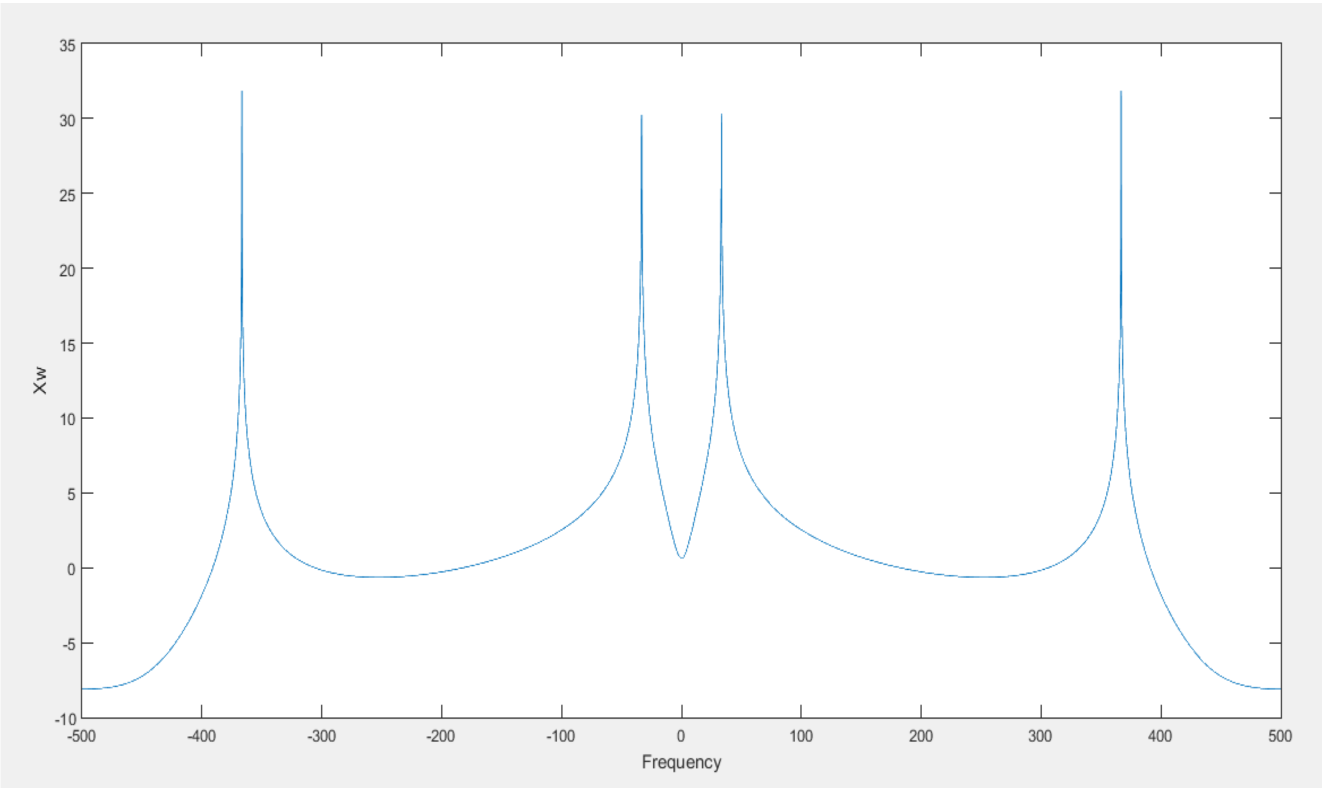
We observe maximum frequency is $\Omega_1 + \Omega_2$.

$\therefore$ Nyquist rate $= 2 * |\Omega_1 + \Omega_2|$.

Matlab Code

```
clc
clear all
close all
%%
Fs = 1000;
t = 0: 1 / Fs: 2 * pi;
w0 = 2 * pi * Fs / 5;
w1 = 2 * pi * Fs / 6;
%fft is for fourier transform and xt is the function with product of functions cos((w0)*t) and cos((w1)*t), where w0 and w1 are the maximum frequencies of their respective functions.
xt = (cos((w0) * t).*cos((w1) * t));
L = length(xt);
Y = fft(xt, L);
fvec = Fs / 2 * linspace(-1, 1, L);
Xw = fftshift(Y);
plot(fvec, 10 * log10(abs(Xw)));
xlabel('Frequency');
ylabel('Xw');
```

Matlab plot



Therefore, the Nyquist rate from the plotted graph is $2 * F_{max} = 2 * |F_1 + F_2| = 2 * 366.67 = 733.33$.

Discussion

Discussion on problem 1:

1. **It is a direct application of Sampling theorem where the sampling frequency should be at least twice the maximum frequency to avoid aliasing.**
2. **Therefore, the maximum frequency that can be sampled without losing values is sampling frequency/2.**

Discussion on problem 2:

1. **The problems are solved using Fourier transform and Inverse Fourier Transform formulae.**
   Fourier transform:
$$X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$
   Inverse Fourier transform:
$$x(t) = \frac{1}{2 * \pi}\int_{-\infty}^{\infty} X(\Omega)e^{j\Omega t}d\Omega$$
2. **For the third subproblem Nyquist rate for $y_3(t) = |\Omega - \Omega_0| * 2$, where $|\Omega - \Omega_0| > \frac{\Omega_1}{2}$. Therefore, $|\Omega| > \frac{2\Omega_0 + \Omega_1}{2}$, which can be stated as new Nyquist rate for $y_3(t)$.**

Discussion on Matlab code and procedure of problem 3:

1. **We have taken $\Omega_1 = 2 * \pi * \frac{F_s}{5}$ and $\Omega_2 = 2 * \pi * \frac{F_s}{6}$.**
2. **We have applied fft directly in the Matlab code to obtain the Fourier transform of the signal.**
3. **fftshift is required to plot the graph with 0 at the centre so that Fourier transform can be obtained at both positive and negative sides.**
4. **From the written solutions part, it's clear that maximum frequency is $\Omega_1 + \Omega_2$ and the same can be seen in the above image.**
5. **The maximum frequency Ω_{max} is sum of $\Omega_1 = 2 * \pi * \frac{F_s}{5}$ and $\Omega_2 = 2 * \pi * \frac{F_s}{6}$.**
6. **From the plotted graph, $F_{max} = 366.67 = 200 + 166.67 = \frac{1000}{5} + \frac{1000}{6} = F_1 + F_2 = \frac{F_s}{5} + \frac{F_s}{6}$.**
7. **Therefore, the four peaks in the graph are $F_1 + F_2, F_1 - F_2, F_2 - F_1, -(F_1 + F_2)$.**
8. **The Nyquist Rate is twice the maximum frequency i. e. $2 * |\Omega_1 + \Omega_2|$.**

******Thanks******