<u>**Homework 3**</u>

Course: DSAA, Monsoon 2017 @IIITS

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I. LTI Systems

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①(i) Considering the signal

$$x[n] = \alpha^n u[n]$$

$$\begin{aligned}
g[n] &= x[n] - \alpha x[n-1] \\
&= x[n] - \alpha[\alpha^{n-1} u[n-1]] \\
&= \alpha^n u[n] - \alpha^n u[n-1] \\
&= \alpha^n [u[n] - u[n-1]] = \alpha^n \delta[n]
\end{aligned}$$

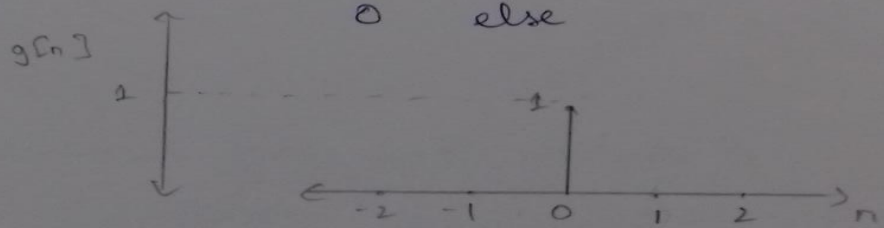
$$\begin{aligned}
g[0] &= \alpha^n [u[0] - u[-1]] \\
&= \alpha^n u[0] = \alpha^n
\end{aligned}$$

$$g[1] = \alpha^n [u[1] - u[0]] = 0$$

$\therefore \quad g[n] = (\frac{1}{2})^n \delta[n]$

exists only for $n = 0$

$$g[\hat{n}] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{else} \end{cases}$$



(ii)

$$\begin{aligned}
x[n] * h[n] &= (\tfrac{1}{2})^n \{u[n+2] - u[n-2]\} \\
&= (\tfrac{1}{2})^n u[n+2] - (\tfrac{1}{2})^n u[n-2]
\end{aligned}$$

$$= 4(\tfrac{1}{2})^{n+2} u[n+2] - \tfrac{1}{4}(\tfrac{1}{2})^{n-2} u[n-2] = 4x[n+2] - \tfrac{1}{4} x[n-2]$$

$$\therefore y[n] = x[n] * h[n] = 4x[n+2] - \tfrac{1}{4} x[n-2]$$

$$\delta[n] \rightarrow h[n]$$

$$h[n] = 4\delta[n+2] - \tfrac{1}{4}\delta[n-2]$$

$\therefore$ convolution signal $= 4\delta[n+2] - \frac{1}{4}\delta[n-2]$

Matlab Code:

MyConvolution function:

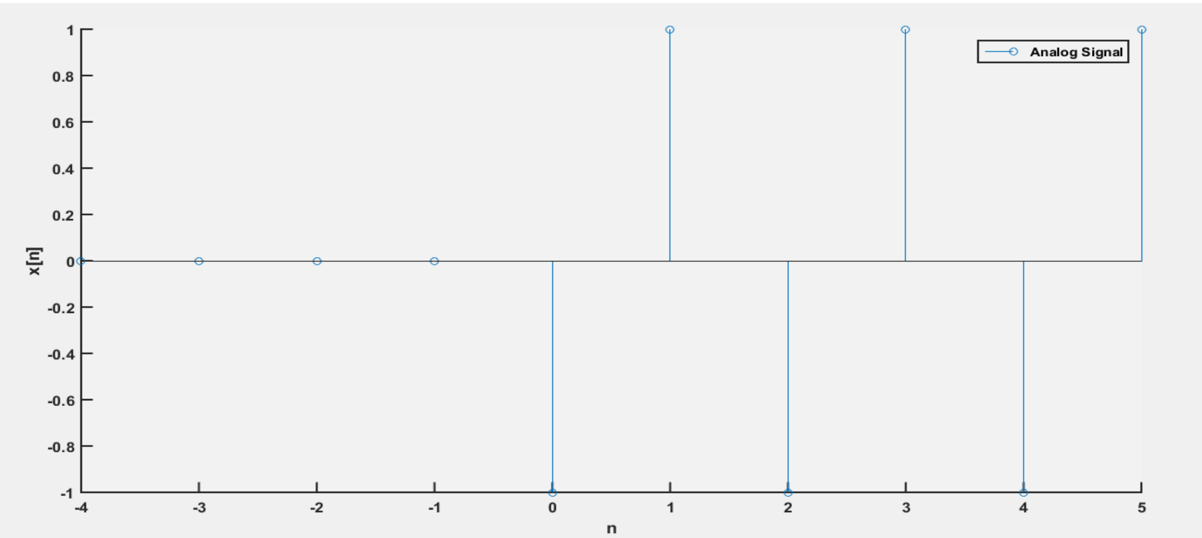
```
function [y] = myConvolution(x, h) m = length(x);
        n = length(h);
        L = m + n - 1;
        y = zeros(1, L);
        xe = zeros(1, L);
        he = zeros(1, L);
        xe(1: m) = x;
        he(1: n) = h;
        for i = 1: L;
                y(i) = 0;
                for j = 1: i;
                        y(i) = y(i) + he(j) * xe(i - (j - 1));
                end;
        end;
return;
```

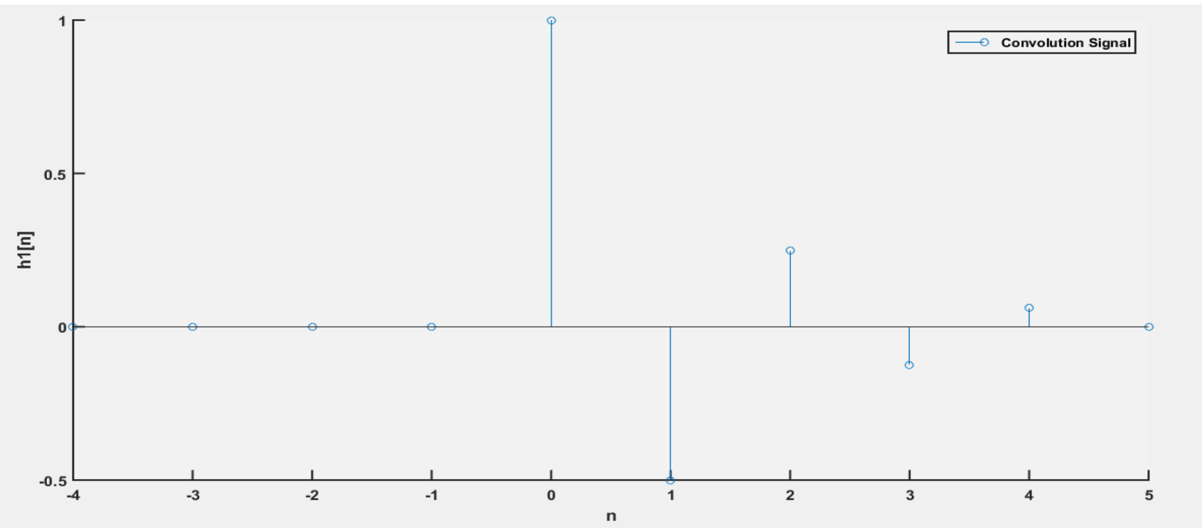
Original Code:

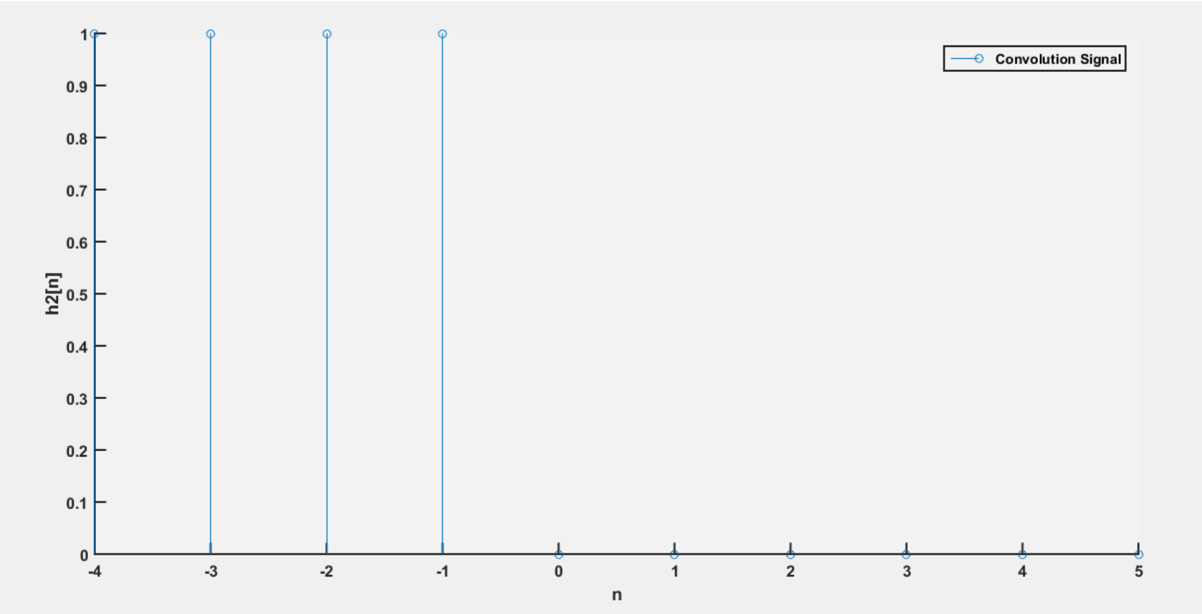
```
clc;
clear all;
close all;
%%
x1 = [0, 0, 0, 0, -1, 1, -1, 1, -1, 1];
h1 = [0, 0, 0, 0, (-1 / 2) ^ 0, (-1 / 2) ^ 1, (-1 / 2) ^ 2, (-1 / 2) ^ 3, (-1 / 2) ^ 4, 0];
h2 = [1, 1, 1, 1, 0, 0, 0, 0, 0, 0];
xaxis1 = -4: 1: 5;
stem(xaxis1, x1);
set(gca, 'Box', 'off', ..., 'FontSize', 12, ..., 'FontWeight', 'bold', ..., 'LineWidth', 1.5, ..., 'FontName', 'Helvetica', ..., 'Color', [0.95 0.95 0.95], ..., 'XGrid', 'off', ..., 'YGrid', 'off');
xlabel('n');
ylabel('x[n]');
legend('Analog Signal');
figure();
stem(xaxis1, h1);
set(gca, 'Box', 'off', ..., 'FontSize', 12, ..., 'FontWeight', 'bold', ..., 'LineWidth', 1.5, ..., 'FontName', 'Helvetica', ..., 'Color', [0.95 0.95 0.95], ..., 'XGrid', 'off', ..., 'YGrid', 'off');
xlabel('n');
ylabel('h1[n]');
legend('Convolution Signal');
figure();
stem(xaxis1, h2);
set(gca, 'Box', 'off', ..., 'FontSize', 12, ..., 'FontWeight', 'bold', ..., 'LineWidth', 1.5, ..., 'FontName', 'Helvetica', ..., 'Color', [0.95 0.95 0.95], ..., 'XGrid', 'off', ..., 'YGrid', 'off');
xlabel('n');
ylabel('h2[n]');
legend('Convolution Signal');
resultXaxis = -4: 1: 23;
w = myConvolution(x1, h1);
wAxis = -8: 1: 10;
figure();
stem(wAxis, w);
set(gca, 'Box', 'off', ..., 'FontSize', 12, ..., 'FontWeight', 'bold', ..., 'LineWidth', 1.5, ..., 'FontName', 'Helvetica', ..., 'Color', [0.95 0.95 0.95], ..., 'XGrid', 'off', ..., 'YGrid', 'off');
xlabel('n');
ylabel('w[n]');
```

```
legend('x[n]*h1[n]');
result1 = myConvolution(w, h2);
g = myConvolution(h1, h2);
gAxis = -4: 1: 14;
figure();
stem(gAxis, g);
set(gca, 'Box', 'off', ..., 'FontSize', 12, ..., 'FontWeight', 'bold', ..., 'LineWidth', 1.5, ..., 'FontName', 'Helvetica', ..., 'Color', [0.95 0.95 0.95], ..., 'XGrid', 'off', ..., 'YGrid', 'off');
xlabel('n');
ylabel('g[n]');
legend('h1[n]*h2[n]');
result2 = myConvolution(x1, g);
figure();
stem(resultXaxis, result1, '+');
hold on;
stem(resultXaxis, result2);
set(gca, 'Box', 'off', ..., 'FontSize', 12, ..., 'FontWeight', 'bold', ..., 'LineWidth', 1.5, ..., 'FontName', 'Helvetica', ..., 'Color', [0.95 0.95 0.95], ..., 'XGrid', 'off', ..., 'YGrid', 'off');
xlabel('n');
ylabel('y[n]');
legend('y1[n]', 'y2[n]');
```

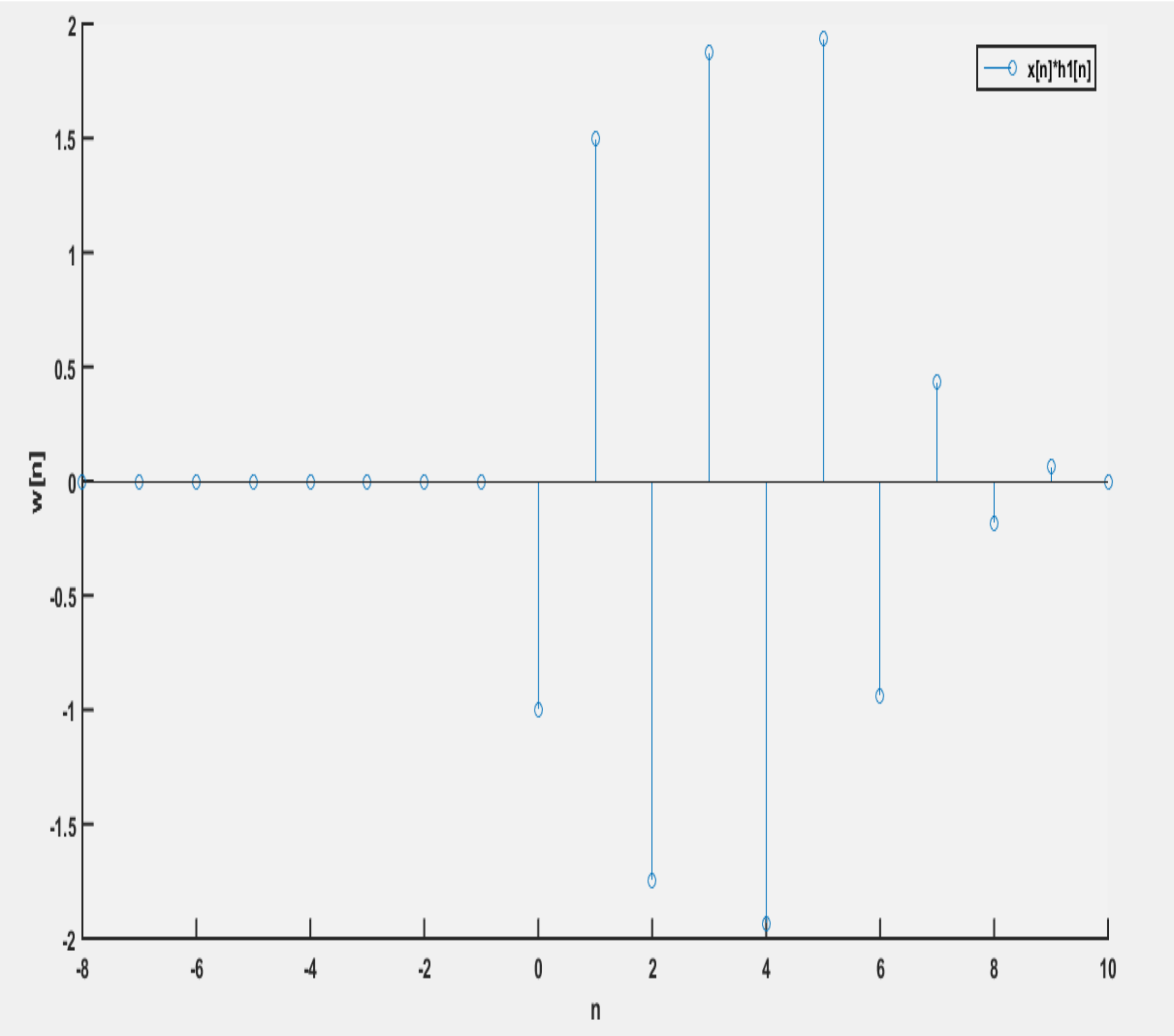
Results:



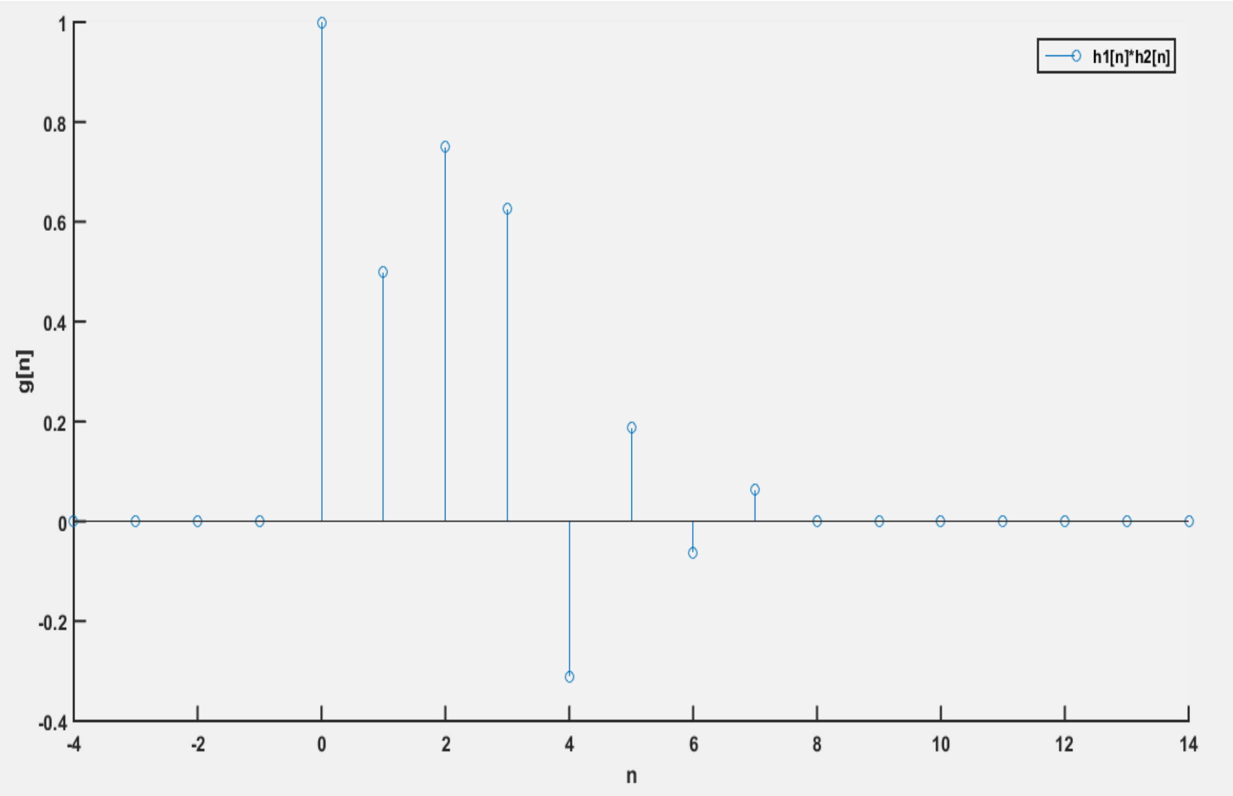




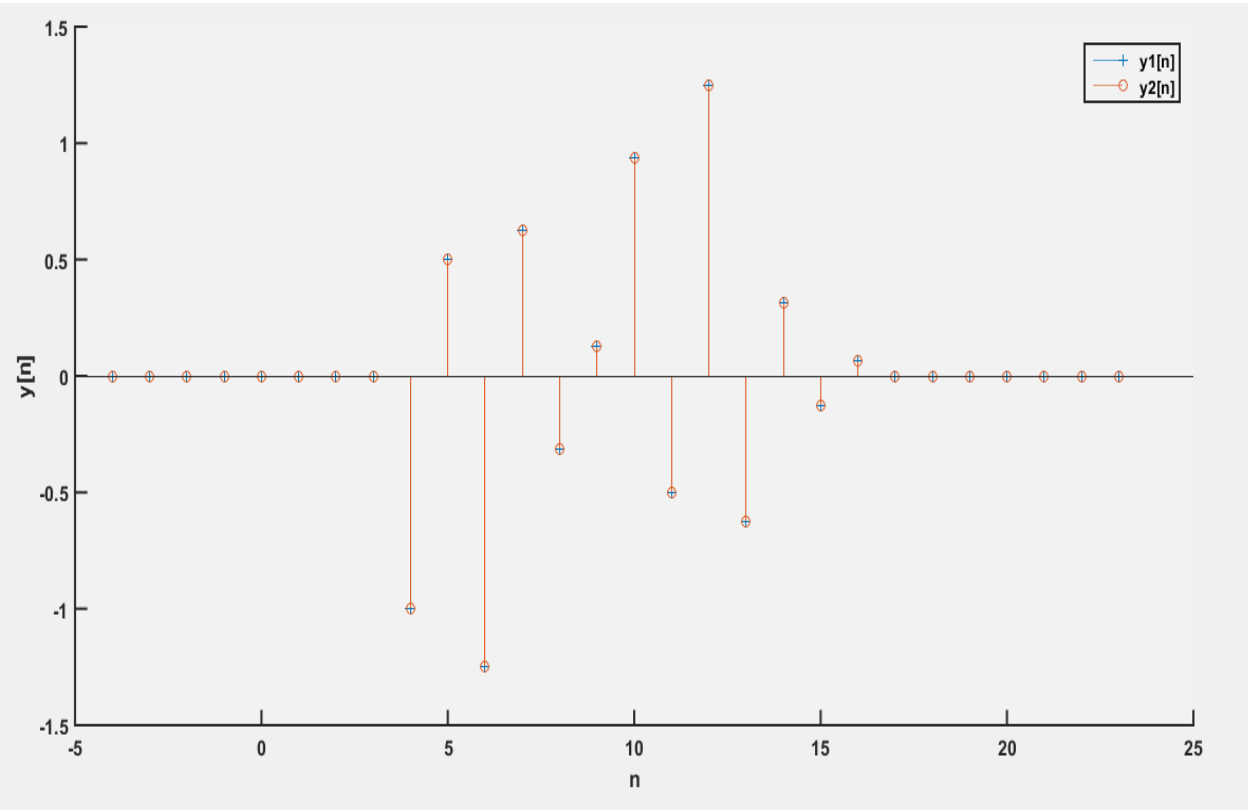
Convolution Signal
h2[n]
n



x[n]*h1[n]
w[n]
n



h1[n]*h2[n]
g[n]
n
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-4
-2
0
2
4
6
8
10
12
14



y1[n]
y2[n]
y[n]
n
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15
20
25

Theoretical part:

② Given

$$x[n] = (-1)^{n+1}\{u[n] - u[n-6]\}$$

$$= [\underset{\uparrow}{-1}, 1, -1, 1, -1, 1]$$

$$h_1[n] = [-\tfrac{1}{2}]^n \{u[n] - u[n-5]\}$$

$$= [\underset{\uparrow}{1}, -\tfrac{1}{2}, \tfrac{1}{4}, -\tfrac{1}{8}, \tfrac{1}{16}]$$

$$h_2[n] = u[n+4] - u[n]$$

$$= [1, 1, 1, 1, \underset{\uparrow}{0}, 0, 0, 0, 0, 0]$$

$$w[n] = x[n] * h_1[n]$$

$$w[0] = x[0]h_1[0] = -1$$

$$w[1] = x[0]h_1[1] + x[1]h_1[0] = \tfrac{1}{2} + 1 = 1.5$$

$$w[2] = x[0]h_1[2] + x[1]h_1[1] + x[2]h_1[0]$$

$$= \tfrac{-1}{4} + (\tfrac{-1}{2}) - 1 = \underline{\underline{-1.75}}$$

$$w[3] = x[0]h_1[3] + x[1]h_1[2] + x[2]h_1[1] + x[3]h_1[0]$$

$$= \tfrac{1}{8} + \tfrac{1}{4} + \tfrac{1}{2} + 1 = 1.875$$

$$w[4] = x[0]h_1[4] + x[1]h_1[3] + x[2]h_1[2] + x[3]h_1[1]$$

$$+ x[4]h_1[0] = -\tfrac{1}{16} - \tfrac{1}{8} - \tfrac{1}{4} - \tfrac{1}{2} - 1 = -1.9375$$

$$w[5] = x[0]h_1[5] + x[1]h_1[4] + x[2]h_1[3] + x[3]h_1[2]$$

$$+ x[4]h_1[1] + x[5]h_1[0]$$

$$= 0 + \tfrac{1}{16} + \tfrac{1}{8} + \tfrac{1}{4} + \tfrac{1}{2} + 1 = 1.9375$$

$$w[6] = \cancel{x[1]h_1[4]}\; x[2]h_1[4] + x[3]h_1[3] + x[4]h_1[2]$$

$$+ x[5]h_1[1] = -\tfrac{1}{16} - \tfrac{1}{8} - \tfrac{1}{4} - \tfrac{1}{2} = -0.9375$$

$$w[7] = x[3]h_1[4] + x[4]h_1[3] + x[5]h_1[2]$$

$$= (1)(\tfrac{1}{16}) + (-1)(-\tfrac{1}{8}) + \tfrac{1}{4} = 0.4375$$

$w[8] = x[4]h_1[4] + x[5]h_1[3] = \frac{-1}{16} - \frac{1}{8} = -0.1875$

$w[9] = x[5]h_1[4] = 0.0625$

$$w = [\underset{\uparrow}{-1}, 1.5, -1.75, 1.875, -1.9375, 1.9375, -0.9375, 0.4375, -0.1875, 0.0625]$$

$$h_2 = [1, 1, 1, 1, \underset{\uparrow}{0}, 0, 0, 0, 0, 0]$$

$w[n] * h_2[n]$

$y[-4] = w[0]h_2[-4]$ $\cancel{+w[1]h_2[}$ $= -1$

$y[-3] = w[0]h_2[-3] + w[1]h_2[-4] = 0.5$

$y[-2] = w[0]h_2[-2] + w[1]h_2[-3] + w[2]h_2[-4] = -1.25$

$y[-1] = w[0]h_2[-1] + w[1]h_2[-2] + w[2]h_2[-3] + w[3]h_2[-4]$

$= -1 + 1.5 - 1.75 + 1.875 \cancel{-1.9375} = 0.625$

$y[0] = w[0]h_2[0] + w[1] + w[2] + w[3] + w[4]$

$= -0.3125$

$y[1] = w[2] + w[3] + w[4] + w[5] = 0.125$

$y[2] = w[3] + w[4] + w[5] + w[6] = 0.9375$

$y[3] = w[4] + w[5] + w[6] + w[7] = -0.5$

$y[4] = w[5] + w[6] + w[7] + w[8] = 1.25$

$y[5] = w[6] + w[7] + w[8] + w[9] = -0.625$

$y[6] = w[7] + w[8] + w[9] = 0.3125$

$y[7] = w[8] + w[9] = -0.125$

$y[8] = w[9] = 0.0625$

$$y = [-1, 0.5, -1.25, 0.625, \underset{\uparrow}{-0.3125}, 0.125, 0.9375, -0.5, 1.25, -0.625, 0.3125, -0.125, 0.0625]$$

Second method:-

$x = [\underset{\uparrow}{-1}, 1, -1, 1, -1, 1]$

$h_1 = [\underset{\uparrow}{1}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}]$

$h_2 = [1, 1, 1, 1, \underset{\uparrow}{0}, 0, 0, 0, 0, 0]$

$g[-4] = h_1[0]\, h_2[-4] = 1$

$g[-3] = h_1[0]\, h_2[-3] + h_1[1] h_2[-4] = \frac{1}{2} = 0.5$

$g[-2] = h_1[0]\, h_2[-2] + h_1[1]\, h_2[-3] + h_1[2] h_2[-4]$

$= 0.75$

$g[-1] = h_1[0] + h_1[1] + h_1[2] + h_1[3] = 0.625$

$g[0] = h_1[1] + h_1[2] + h_1[3] + h_1[4] = -0.3125$

$g[1] = h_1[2] + h_1[3] + h_1[4] = 0.1875$

$g[2] = h_1[3] + h_1[4] = -0.0625$

$g[3] = h_1[4] = 0.0625$

$g = [1, 0.5, 0.75, 0.625, \underset{\uparrow}{-0.3125}, 0.1875, -0.0625,$
$0.0625]$

$y = x[n] * g[n]$

$y[-4] = x[0] g[-4] = -1$

$y[-3] = x[1] g[-4] + x[0] g[-3] = 0.5$

$y[-2] = x[0] g[-2] + x[1] g[-3] + x[2] g[-4] = -1.25$

$y[-1] = x[0] g[-1] + x[1] g[-2] + x[2] g[-3] + x[3] g[-4]$

$= 1.75 - 1.125 = 0.625$

$y[0] = x[0] g[0] + x[1] g[-1] + x[2] g[-2] + x[3] g[-3]$
$+ x[4] g[-4]$

$= -0.3125$

$y[1] = x[0]g[1] + x[1]g[0] + x[2]g[-1] + x[3]g[-2]$
$+ x[4]g[-3] + x[5]g[-4]$
$= -g[1] + g[0] - g[-1] + g[-2] - g[-3] + g[-4] = 0.125$

$y[2] = x[0]g[2] + x[2]g[0] + x[1]g[1] + x[3]g[-1]$
$+ x[4]g[-2] + x[5]g[-3]$
$= -g[2] + g[1] - g[0] + g[-1] - g[-2] + g[-3] = 0.9375$

$y[3] = -g[3] + g[2] - g[1] + g[0] - g[-1] + g[-2]$

$= -0.5$

$y[4] = \cancel{-g[4]} + g[3] - g[2] + g[1] - g[0] + g[-1] = 1.25$

$y[5] = \cancel{g[4]} - g[3] + g[2] - g[1] + g[0] = -0.625$

$y[6] = g[3] - g[2] + g[1] \cancel{- g[0]}$

$= 0.3125$

$y[7] = -g[3] + g[2]$

$= -0.125$

$y[8] = g[3] = 0.0625$

$\therefore y = [-1, 0.5, -1.25, 0.625, -0.3125,$
$0.125, 0.9375, -0.5, 1.25,$
$-0.625, 0.3125, -0.125]$

$\cancel{y[9]}$

$\therefore$ y obtained from both methods is same.

Discussion:

1. **We see that in the first question, the graph g[n] is $\alpha^n * \delta[n]$, i.e. the graph's value is α^n only when $n = 0$. Therefore, we get a value = 1 for the graph only when n = 0, else it is 0.**
2. **In second part of the first problem, we have converted the output in terms of input signal. We have substituted it with $\delta[n]$ signal to obtain the convolution signal.**
3. **In second problem, we must prove the associativity property of convolution.**
4. **So, we first convolute the first two signals $x[n]$ and $h_1[n]$, by which we obtain $w[n]$ and further convoluting $w[n]$ with $h_2[n]$ gives $y_1[n]$.**
5. **Later to prove associativity, we convolute $h_1[n]$ and $h_2[n]$ first and convolute the resultant signal $g[n]$ with $x[n]$ to obtain $y_2[n]$.**
6. **We observe the resultant graphs in the results part here.**
7. **We conclude that $y_1[n] = y_2[n]$ from the graph and the results calculated theoretically also depict the same.**
8. **Therefore, convolution satisfies associativity property.**

********Thanks for Reading********