Homework-1

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1. **Find energy and power of the given function $x(t) = e^{-|t|}, t \in R$.**
2. **Find energy and power of the given function $x(t) = \cos(\frac{\pi}{2}t + \frac{\pi}{4}), t \in R$.**
3. **Find energy and power of the given function $x(t) = (1+j)e^{j\frac{\pi}{2}t},\ 0 < t < 10$.**
4. **Find energy of the given function $x(n) = cos\left(\frac{2\pi k_0 n}{N}\right), 0 \leq n \leq N-1$.**
5. **Find energy and power of the given function $x(t) = tan\left(\frac{\pi t}{2}\right), 0 < t < \frac{1}{2}$.**

Solutions (hand written)

1.

1. Find energy and power for $x(t) = e^{-|t|}$

Sol. $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |(e^{-|t|})|^2 dt$

$= \int_{-\infty}^{0} e^{-2|t|} dt + \int_{0}^{\infty} e^{-2|t|} dt$

$= \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt = \left.\frac{e^{2t}}{2}\right]_{-\infty}^{0} + \left.\frac{e^{-2t}}{(-2)}\right]_{0}^{\infty}$

$= \left[\frac{1}{2} - 0\right] + \left[0 - \frac{e^{0}}{(-2)}\right] = 1$ Joule

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{-|t|}|^2 dt$

$= \lim_{T\to\infty} \frac{1}{2T} \left[\int_{-T}^{0} e^{2t} dt + \int_{0}^{T} e^{-2t} dt\right]$

$= \lim_{T\to\infty} \frac{1}{2T} \left[\left.\frac{e^{2t}}{2}\right]_{-T}^{0} + \left.\frac{e^{-2t}}{(-2)}\right]_{0}^{T} \right]$

$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{1}{2} - \frac{e^{-2T}}{2} + \frac{e^{-2T}}{(-2)} + \frac{1}{2}\right]$

$= \lim_{T\to\infty} \frac{1}{2T} \left[1 - e^{-2T}\right]$

$= \frac{1}{\infty}[1-0] = 0$ Watt

$\therefore$ Energy = 1 Joule, Power = 0 Watt

$\therefore$ it is an energy signal

2.

2. $x(t) = \cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$, Find Energy & power

Sol. $E = \int_{-\infty}^{\infty} \left|\cos^2\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)\right| dt$

$\left[\because E = \int_{-\infty}^{\infty} |x(t)|^2 dt\right]$

$= \int_{-\infty}^{\infty} \frac{1 + \cos\left(\pi t + \frac{\pi}{2}\right)}{2} dt = \int_{-\infty}^{\infty} \frac{1 - \sin \pi t}{2} dt$

$= \frac{t}{2} + \frac{\cos(\pi t)}{2} \Big]_{-\infty}^{\infty}$

$= \underline{\underline{\infty}}$

$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos^2\left(\frac{\pi}{2}t + \frac{\pi}{4}\right) dt$

$= \lim_{T \to \infty} \frac{1}{2T} \left(\frac{t}{2} + \frac{\cos(\pi t)}{2}\right)\Big]_{-T}^{T}$

$= \lim_{T \to \infty} \frac{1}{2T} \left(\frac{T}{2} + \frac{\cos(\pi T)}{2} - \left(\frac{-T}{2} + \frac{\cos(\pi T)}{2}\right)\right)$

$= \lim_{T \to \infty} \frac{T}{2T} = \frac{1}{2}$ Watts

Therefore the given function is a power function since energy of it is infinity and power is finite.

3.

3. Find energy and power for $x(t) = (1+j)e^{\frac{j\pi t}{2}}$, $0 < t < 10$

sol. $E = \int_0^{10} |(1+j)e^{\frac{j\pi t}{2}}|^2 \, dt$

$= \int_0^{10} (\sqrt{1^2+1^2})^2 . |e^{j\pi t}| \, dt$

$= 2 \int_0^{10} |\cos \pi t + j \sin(\pi t)| \, dt$

$= 2 \times \sqrt{\cos^2 \pi t + \sin^2 \pi t} \int_0^{10} dt$

$= 2 \times 1 \times [10 - 0] = \underline{20 \text{ Joules}}$

3. $P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt = \frac{1}{10-0} \int_0^{10} |(1+j)e^{j\frac{\pi t}{2}}|^2 \, dt$

$= \frac{1}{10} (\sqrt{1^2+1^2})^2 (\sqrt{\cos^2 \frac{\pi}{2} t + \sin^2 \frac{\pi}{2} t})^2 \int_0^{10} dt$

$= \frac{1}{10} \times 2 \times 1 \times (10 - 0) = 2$ Watts

3. $P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_0^{10} (|x(t)|)^2 \, dt$

$= \lim_{T \to \infty} \frac{20}{2T}$ [∵ E is finite & equal to 20]

$= 0$

∴ P_∞ over infinite time interval is 0.

∴ The given function is an energy signal.

The power for the signal is finite when considered for a certain period of time, but when it is considered for infinite time interval, P = 0. Therefore this function can be considered an Energy signal.

4.

4. Find Energy for $x(n) = \cos \frac{2\pi k_0 n}{N}$ $\quad 0 \le n \le N-1$.

Sol.

$$E = \sum_{n=0}^{N-1} \left| \cos\left(\frac{2\pi k_0 n}{N}\right)\right|^2$$

$$= \sum_{n=0}^{N-1} \cos^2\left(\frac{2\pi k_0 n}{N}\right)$$

$$= \sum_{n=0}^{N-1} \frac{1+\cos \frac{4\pi k_0 n}{N}}{2} = \frac{1}{2} \sum_{n=0}^{N-1} \left[1 + \cos \frac{4\pi k_0 n}{N}\right]$$

$$= \frac{N}{2} + \frac{1}{2} \sum_{n=0}^{N-1} \cos \frac{4\pi k_0 n}{N} = \frac{N}{2} + \frac{1}{2} \times \text{Real part} \left\{\sum_{n=0}^{N-1} e^{\frac{4\pi k_0 n j}{N}}\right\}$$

$$= \frac{N}{2} + \frac{1}{2} \times \text{Real part} \left\{ e^{\frac{4\pi k_0 (0) j}{N}} + e^{\frac{4\pi k_0 (1) j}{N}} + \dots e^{\frac{4\pi k_0 (N-1) j}{N}} \right\}$$

$$= \frac{N}{2} + \frac{1}{2} \times \text{Real part} \left\{ \frac{(1)\left(1 - e^{\frac{4\pi k_0 j}{N} \times N}\right)}{1 - e^{\frac{4\pi k_0 j}{N}}} \right\}$$

$$= \frac{N}{2} + \frac{1}{2} \times \text{Real part} \left\{ \frac{1-1}{1-e^{\frac{4\pi k_0 j}{N}}} \right\} = \frac{N}{2} + 0 = \frac{N}{2}$$

$$P = \frac{1}{N-1-0+1} \sum_{n=0}^{N-1} \left|\cos\left(\frac{2\pi k_0 n}{N}\right)\right|^2 \quad \left[\because P = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x(n)|^2\right]$$

$$= \frac{1}{N}\left[\frac{N}{2}\right] = \frac{1}{2} \text{ Watts}$$

4. $$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N-1} |x[n]|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1}\left(\frac{N}{2}\right) \quad \left[\because E \text{ is equal to } \frac{N}{2}\right]$$

$$= \lim_{N \to \infty} \frac{N}{4N+2} = \lim_{N \to \infty} \frac{1}{4 + \frac{2}{N}} = \frac{1}{4+0} = \frac{1}{4} \text{ Watts}$$

$\therefore$ P_∞ over infinite time interval is $\frac{1}{4}$.

Therefore, we observe that the power if finite for infinite time interval and energy is N/2 for any N, i.e. when N tends to infinity, Energy is also infinity. Thus, the given signal is a power signal.

5.

5. $x(t) = \tan\left(\frac{\pi}{2}t\right), \quad 0 < t < \frac{1}{2}$

$$E = \int_0^{\frac{1}{2}} \tan^2 \frac{\pi}{2}t \, dt$$

$$= \int_0^{\frac{1}{2}} \left[\sec^2 \frac{\pi}{2}t - 1\right] dt = \frac{2}{\pi}\tan\frac{\pi}{2}t - t \Big]_0^{\frac{1}{2}}$$

$$= \frac{2}{\pi}(1) - \frac{1}{2} + 0 - 0 = \frac{2}{\pi} - \frac{1}{2} = 0.1366$$

$$P = \frac{1}{\frac{1}{2} - 0} \int_0^{\frac{1}{2}} \tan^2 \frac{\pi}{2}t \, dt = 2 \times 0.1366 = 0.2732$$

5. $P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_0^{\frac{1}{2}} \tan^2\left(\frac{\pi}{2}t\right) dt, \quad 0 < t < \frac{1}{2}$

$$= \lim_{T \to \infty} \frac{1}{2T} \left(\frac{2}{\pi} - \frac{1}{2}\right) \qquad \left[\because E = \frac{2}{\pi} - \frac{1}{2} \text{ for given wave}\right]$$

$$= 0$$

$\therefore P_\infty$ for the given function is 0 over the ~~ime~~ infinite time interval

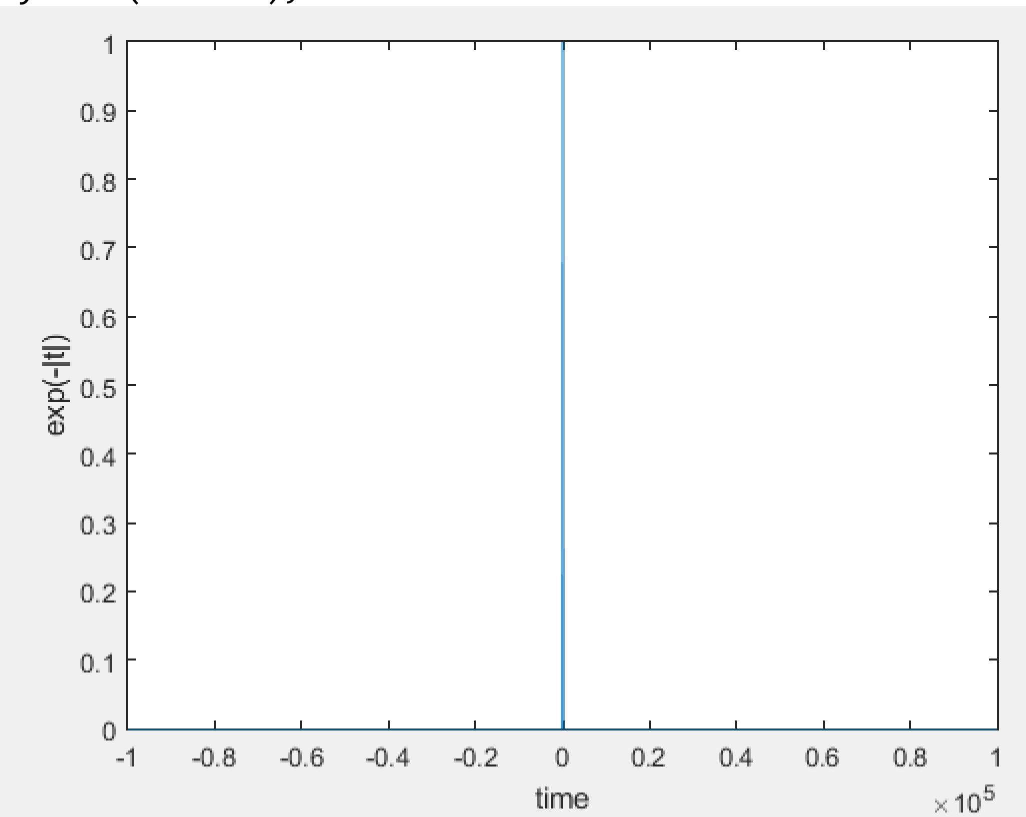
In this signal, we observe power is finite and non-zero for a period, but it is 0 when power is calculated for infinite time interval. Therefore the given function is an energy signal.

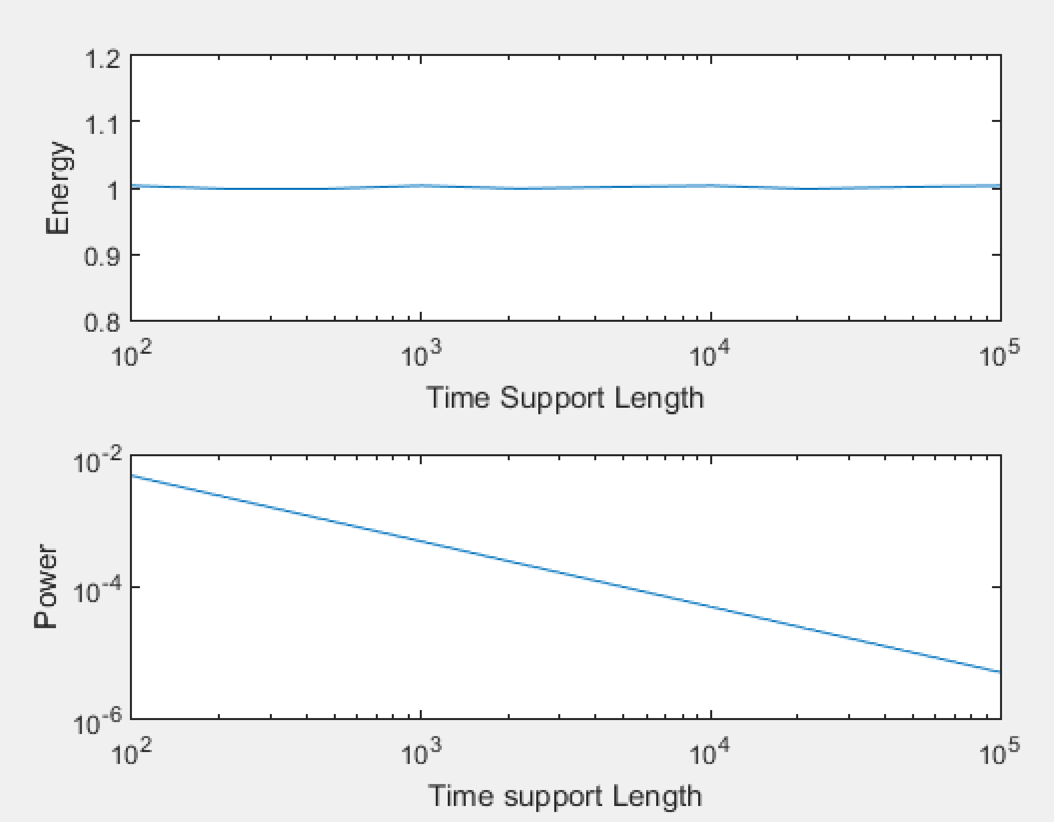
<u>MATLAB Codes:</u>

1.

```
clc;
clear all;
close all;
%%
Nt = logspace(2, 5, 10);
for nx = 1: length(Nt)
      t = -Nt(nx): 0.1: Nt(nx);
      xt = exp(-abs(t));
      plot(t, xt);
      xlabel('time');
```

```
    ylabel('exp(-|t|)');
    drawnow;
    pause(0.1);
    egx(nx) = trapz(t, xt.^2);
end;
powx = egx./ 2 * (Nt);
figure;
subplot(2, 1, 1);
semilogx(Nt, egx);
ylim([.8, 1.2]);
ylabel('Energy');
xlabel('Time Support Length');
subplot(2, 1, 2);
loglog(Nt, powx);
xlabel('Time support Length');
ylabel('Power');
```

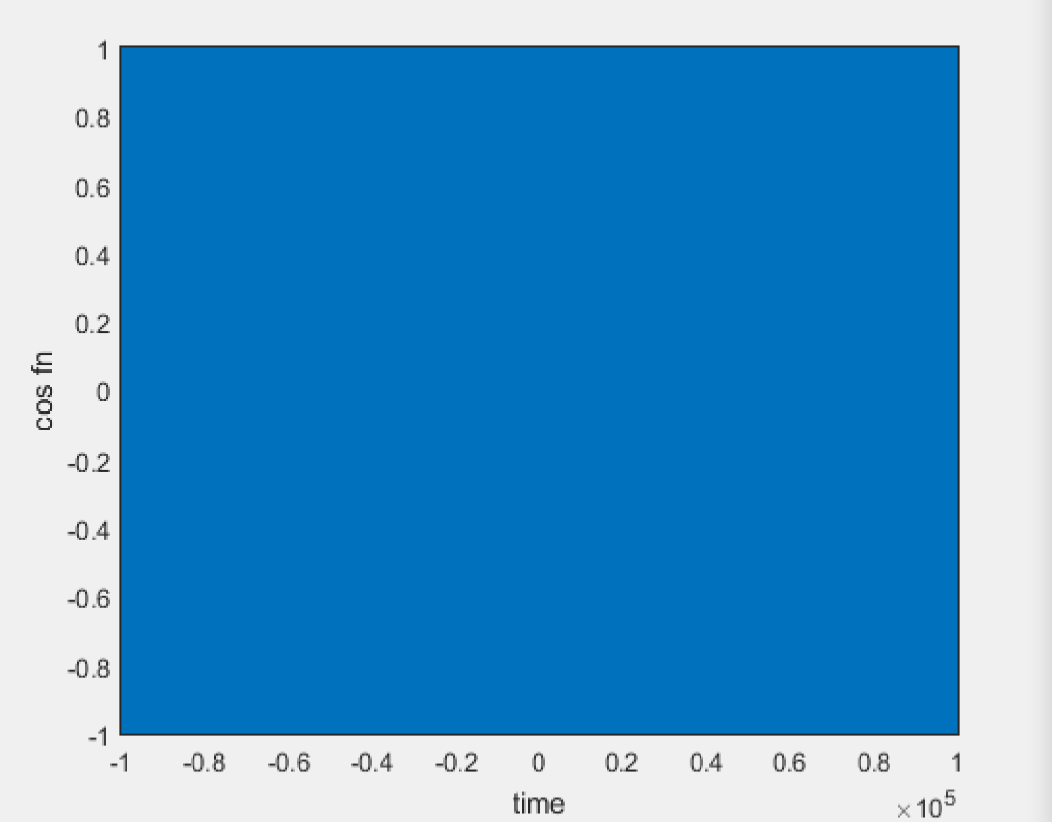


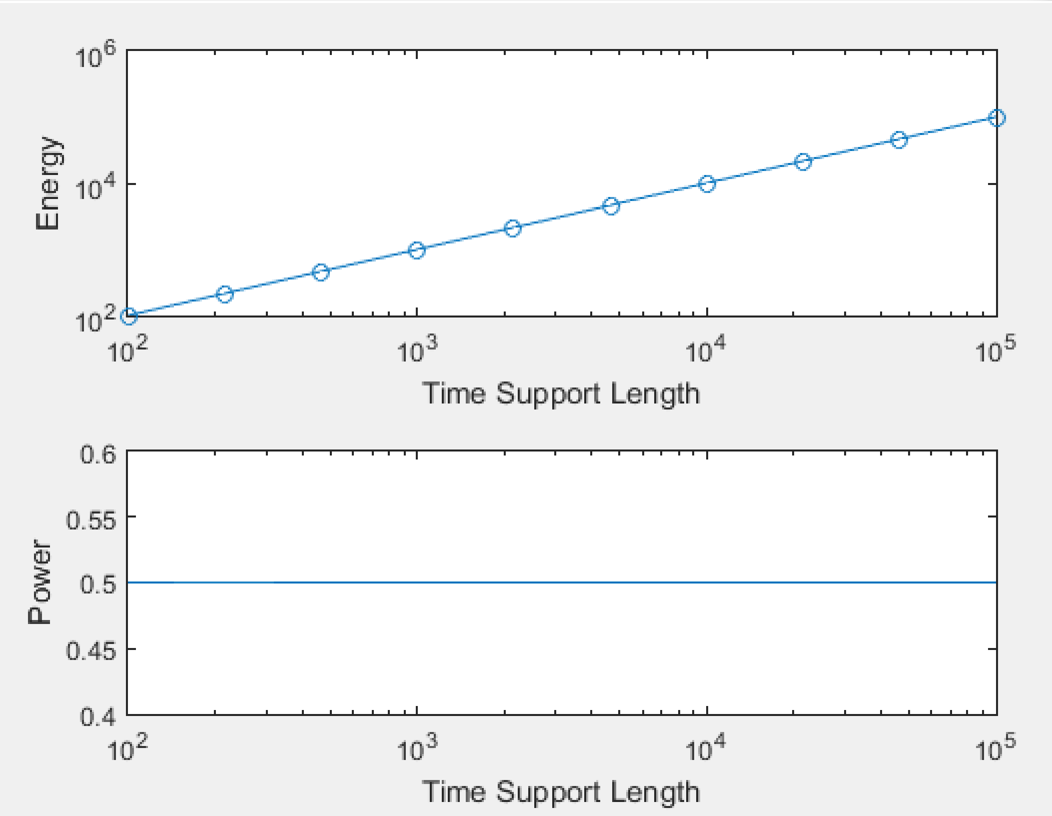


2.

```
clc
clear all
close all
%%
Nt = logspace(2, 5, 10);
for nx = 1: length(Nt)
      t = -Nt(nx): 0.1: Nt(nx);
      xt = cos(pi * t / 2 + pi / 4);
      plot(t, xt);
      xlabel('time');
      ylabel('cos fn');
      drawnow;
      pause(0.1);
      egx(nx) = trapz(t, xt.^2);
end;
powx = egx. / (2 * Nt);
figure;
subplot(2, 1, 1);
loglog(Nt, egx, 'o-');
xlabel('Time Support Length');
ylabel('Energy');
subplot(2, 1, 2);
semilogx(Nt, powx);
xlabel('Time Support Length');
ylabel('Power');
```

```
ylim([0.4, 0.6]);
```

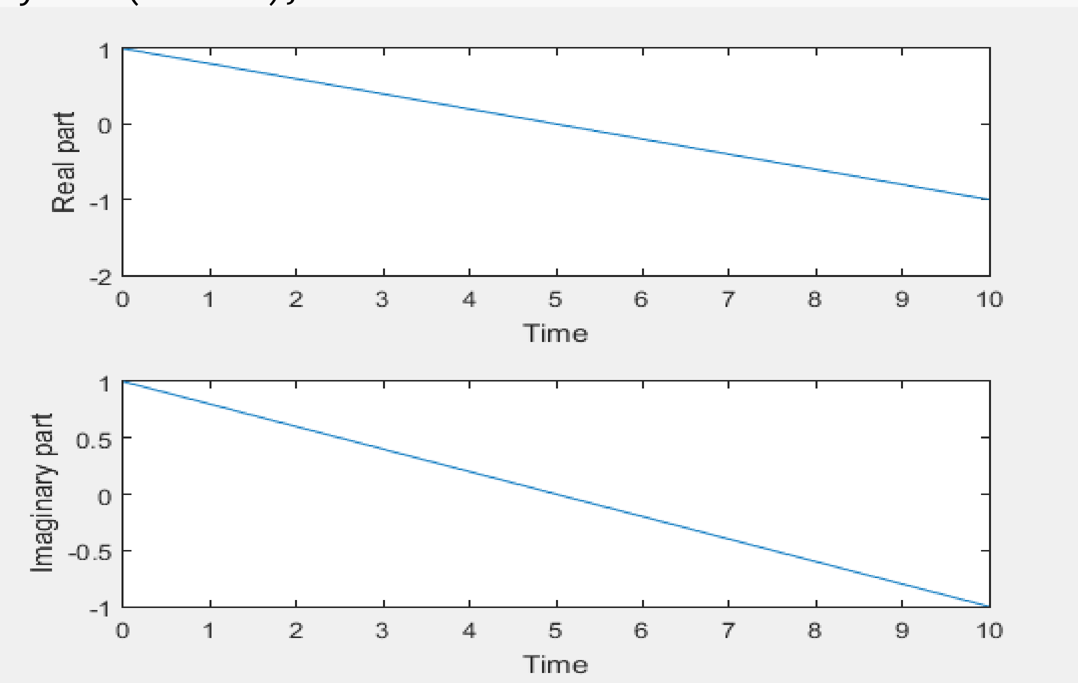


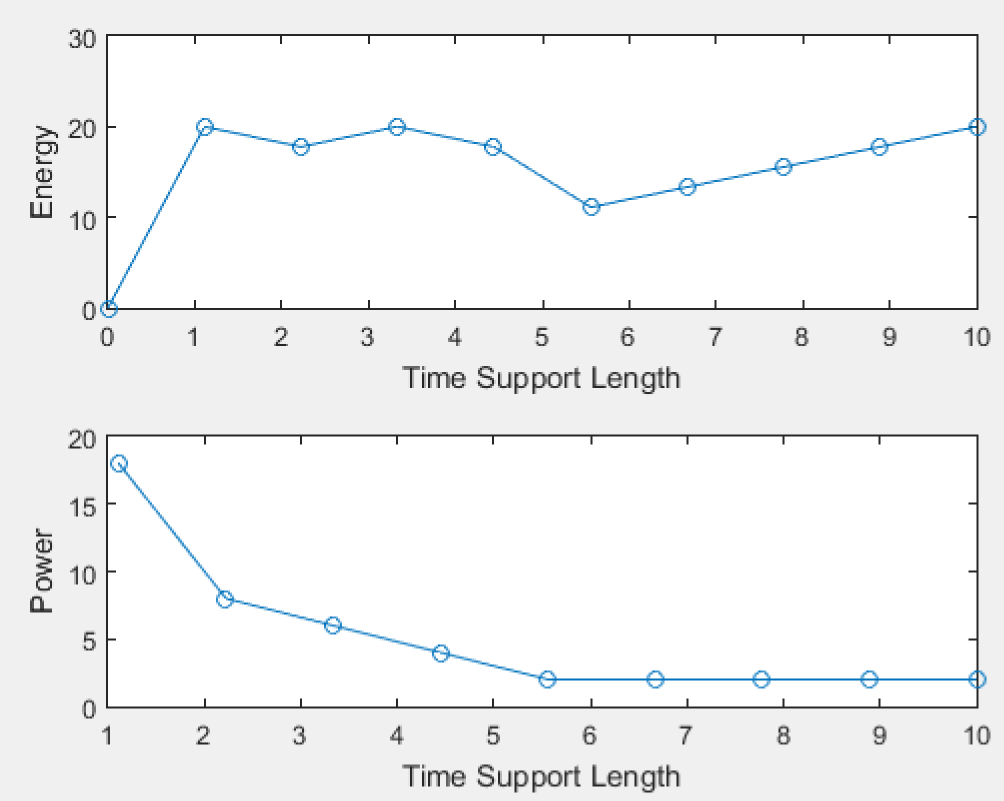


3.

```
clc
clear all
close all
```

```
%%
nt = 10;
inc = linspace(0, 10, 10);
for nx = 1: length(inc)
      t = 0: inc(nx): nt;
      xt = (1 + j) * exp(j * pi * t / 2);
      subplot(2, 1, 1);
      plot(t, real(xt));
      xlabel('Time');
      ylabel('Real part');
      subplot(2, 1, 2);
      plot(t, imag(xt));
      xlabel('Time');
      ylabel('Imaginary part');
      drawnow;
      pause(0.1);
      egx(nx) = trapz(t, abs(xt).^2);
end;
powx = egx./(inc);
figure;
subplot(2, 1, 1);
plot(inc, egx, '-o');
xlabel('Time Support Length');
ylabel('Energy');
subplot(2, 1, 2);
plot(inc, powx, '-o');
xlabel('Time Support Length');
ylabel('Power');
```

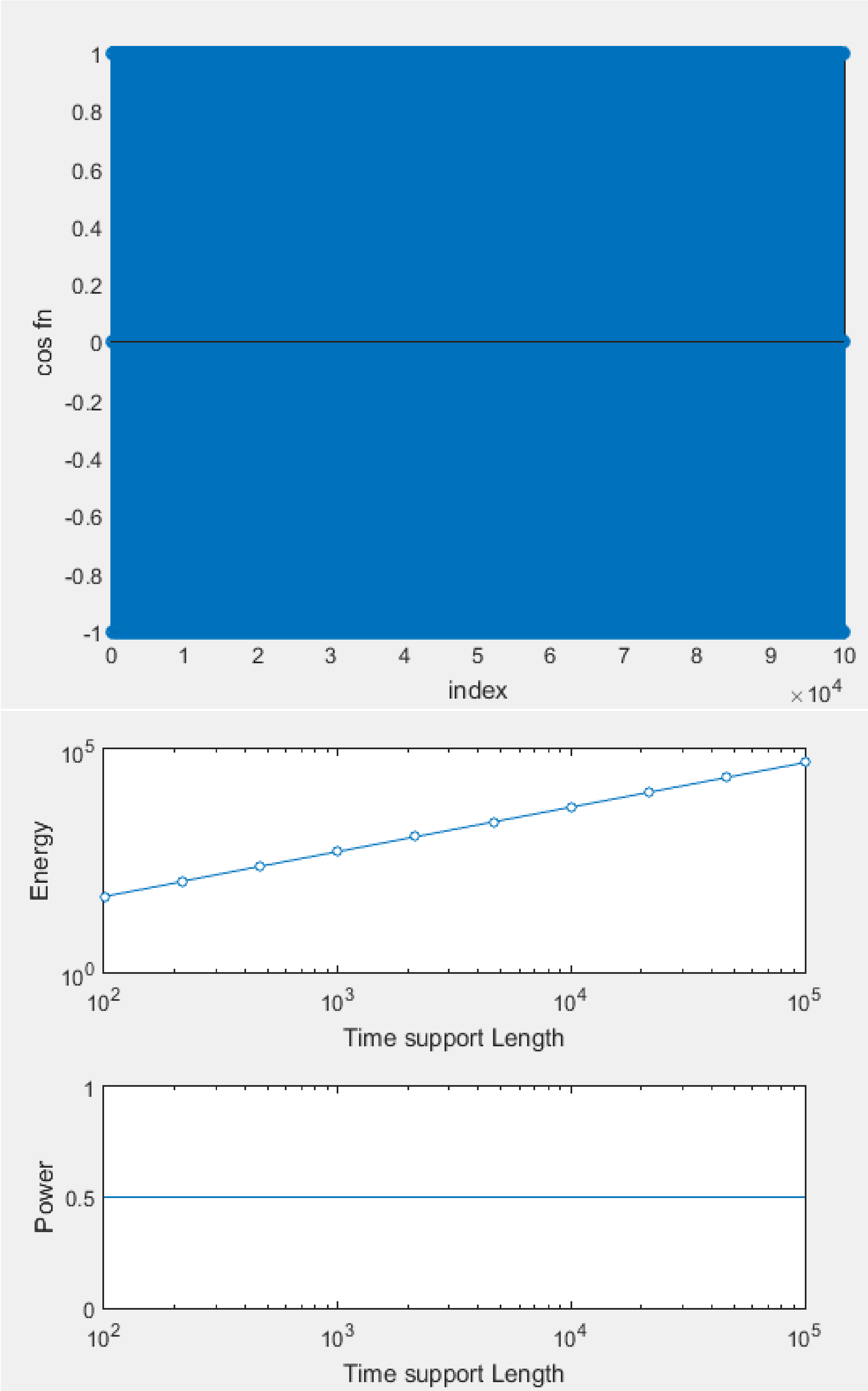




4.

```
clc
clear all
close all
%%
Nt = logspace(2, 5, 10);
for nx = 1: length(Nt)
      n = 0: Nt(nx) - 1;
      k_0 = floor(Nt(nx) / 4);
      xt = cos(2 * pi * k_0 * n / Nt(nx));
      stem(n, xt);
      xlabel('index');
      ylabel('cos fn');
      drawnow;
      pause(0.1);
      egx(nx) = sum(abs(xt). ^ 2);
end;
powx = egx./(Nt);
figure;
subplot(2, 1, 1);
loglog(Nt, egx, 'o-', 'MarkerSize', 4, 'MarkerFaceColor', 'w');
ylabel('Energy');
xlabel('Time support Length');
subplot(2, 1, 2);
semilogx(Nt, powx);
xlabel('Time support Length');
ylabel('Power');
```

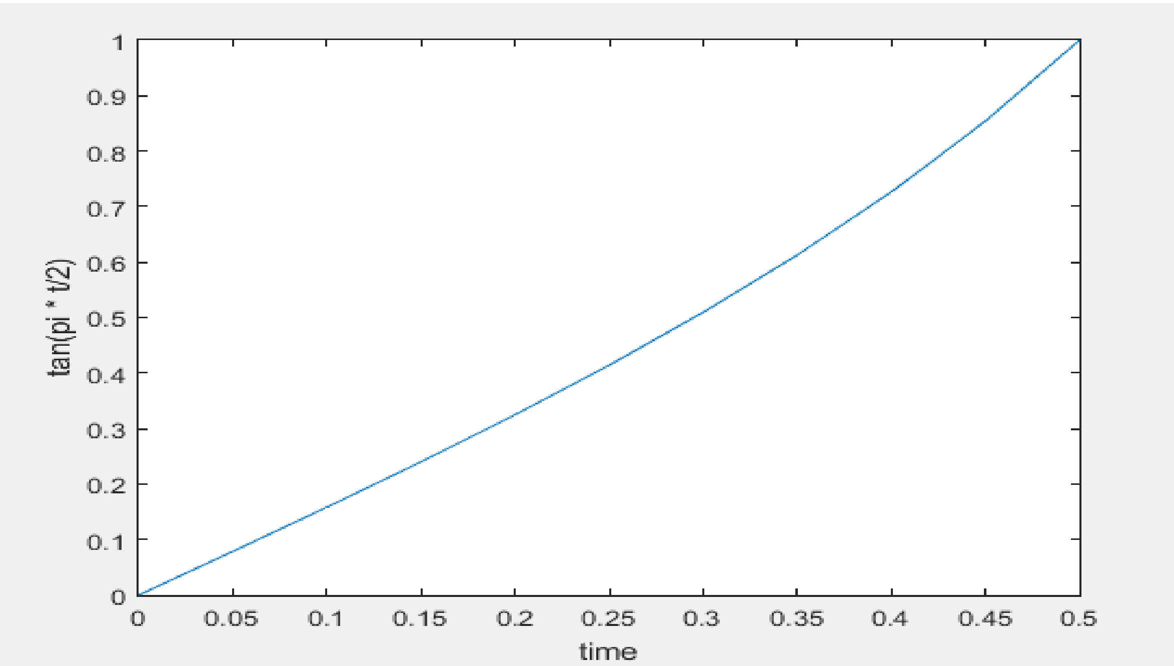
```
ylim([0, 1]);
```

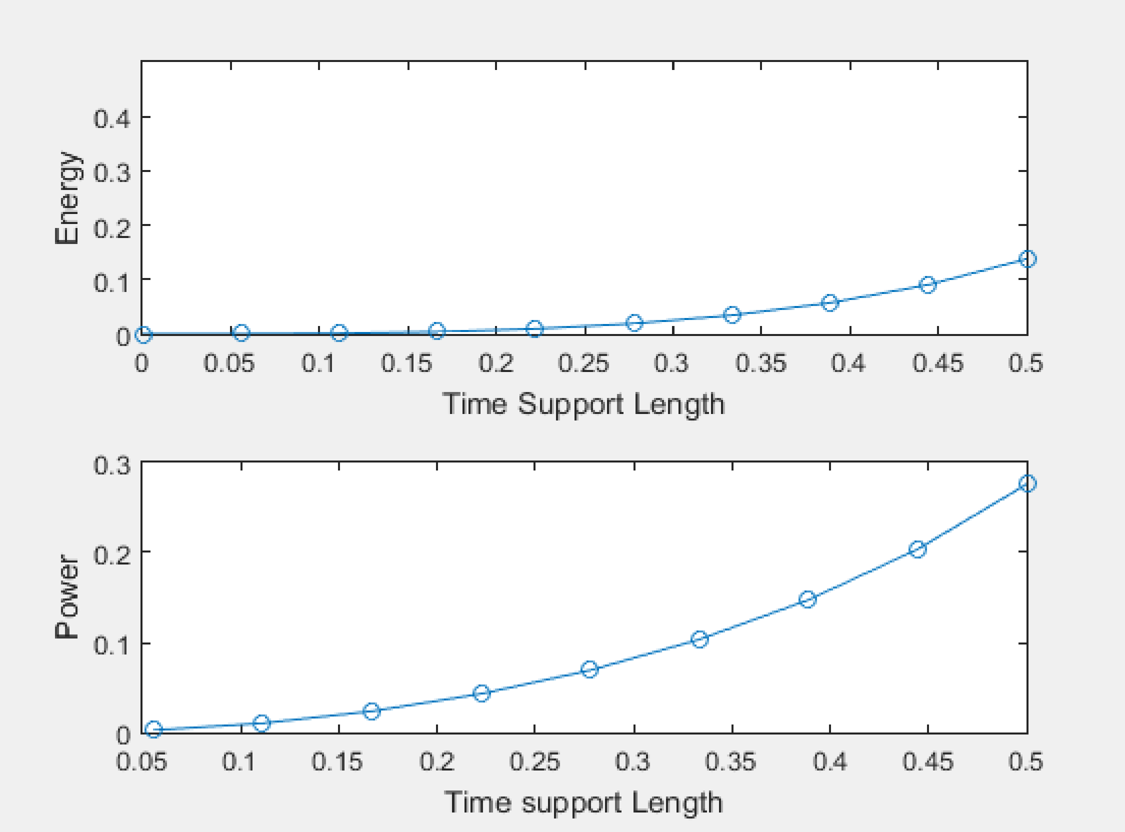


5.

```
clc
clear all
```

```
close all
%%
Nt = linspace(0, 0.5, 10);
for nx = 1: length(Nt)
      t = 0: Nt(nx)/10: Nt(nx);
      xt = tan(pi * t / 2);
      plot(t, xt);
      xlabel('time');
      ylabel('tan(pi * t/2)');
      drawnow;
      pause(0.1);
      egx(nx) = trapz(t, xt. ^ 2);
end;
powx = egx./(Nt);
figure;
subplot(2, 1, 1);
plot(Nt, egx);
ylim([0, 0.5]);
ylabel('Energy');
xlabel('Time Support Length');
subplot(2, 1, 2);
plot(Nt, powx);
xlabel('Time support Length');
ylabel('Power');
```





<u>Discussions</u>

Common Discussion on these 5 problems:

1. **For an analog signal $x(t)$, Total Energy over the time interval $t_1 \le t \le t_2$ is**
$$E = \int_{t1}^{t2} |x(t)|^2 dt$$
2. **For an analog signal $x(t)$, Average Power over the time interval $t_1 \le t \le t_2$ is**
$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 dt$$
3. **For an analog signal $x(t)$, Power over infinite time interval is**
$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$
4. **For a discrete signal $x[n]$, Total Energy over the time interval $n_1 \le n \le n_2$ is**
$$E = \sum_{n=n_1}^{n_2} |x[n]|^2$$
5. **For a discrete signal $x[n]$, Average Power over the time interval $n_1 \le n \le n_2$ is**
$$P = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$
6. **For a discrete signal $x[n]$, Average Power over infinite time interval is**
$$P = \lim_{N \to \infty} \frac{1}{2N + 1} \sum_{n=-N}^{N} |x(t)|^2$$

Reference:

Signals and Systems, Page no. 6, 7

Basic Matlab Commands:

1. **logspace(a, b, n): Generates a logarithmically spaced vector of n points between 10^a and 10^b.**
2. **linspace(a, b, n): Generates a linearly spaced vector of n points between a and b.**
3. **semilogx/semilogy(x, y): Plots data as logarithmic scales for the x-axis/ y-axis.**
4. **loglog(x, y): Plots data as logarithmic scales for each axis.**

Discussion on first problem:

1. **The graph depicts that the energy value for the given function is approximately 1 and the power is seen to be a decreasing function which eventually becomes around 0 when the time is a large value.**
2. **Therefore, the given function is an Energy function as the value is finite and approximately 1 always and Power tends to become zero when T is large enough.**

Discussion on second problem:

1. **The graph conveys that the energy is a linearly increasing function which tends to be ∞, when the time t tends to ∞.**
2. **The power is clearly constant as shown in the graph and has a finite value of $\frac{1}{2}$.**
3. **Therefore, the given function is a Power function as the value of Power is finite for any t and energy tends to ∞ for $t = \infty$.**

Discussion on third problem:

1. **Here, the time interval is confined from 0 to 10, that is, the energy and power of the signal is to be calculated during this period and further, adding to it, the given function is a complex one, i.e. absolute function is to be used while calculating power and energy.**
2. **Graphs for both real and imaginary part are plotted.**
3. **The energy graph is seen to be a zigzag pattern which attains the value of 20 at $t = 10$, which is our required energy. Calculations can be seen in the hand-written part.**
4. **The power graph shows constant value of 2 at $t = 10$.**
5. **Therefore, the given function is an energy function as it has a finite energy and power calculated over infinite interval of time is equal to 0.**

Discussion on fourth problem:

1. **In this case, the signal is discrete and time interval is given to be from 0 to some $N - 1$ where $N \in R$.**
2. **In the calculation part, the value of energy is known to be $\frac{N}{2}$, for every $N \in R$ and the same can be seen in graph too.**
3. **We observe a power of $\frac{1}{2}$ units, when calculated in the given time interval for n, where as if we calculate the power for infinite time interval, we get a constant value of $\frac{1}{4}$ units, which means that the power of the function is constant.**
4. **We observe that when N tends to infinity, the energy value also tends to infinity.**
5. **Therefore, the given function is a power signal.**

Discussion on fifth problem:

1. **Again here, the signal is analog unlike the previous one and is over the time interval $0 < t < \frac{1}{2}$.**
2. **The energy and power are not seen to have any regular pattern, yet we attain a value of $\frac{2}{\pi} - \frac{1}{2} = 0.1366$ for Energy at $t = \frac{1}{2}$.**
3. **The power is 0.2732 at $t = \frac{1}{2}$, hence proving our theoretical values of power and energy.**
4. **When the power is calculated over infinite time interval, we get power = 0, since energy is a finite value and when t tends to become infinity, it takes power to become 0 as seen per calculations.**
5. **We can conclude that the given signal is an Energy signal.**

**********Thank you**********